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# A Radar Sensing Algorithm by Gabor Theory

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## Abstract

*In this paper, an alternative Target Density Function(TDF) is proposed for narrowband radar model. This is achieved by estimating a new target density function by Gabor theory. It is shown how Gabor transform can be used to obtaining wideband target density function by transmitting a waveform which is a kernel for this transform. The windowing characteristics of this theory is plausible to reaching an accurate result. The presented wideband target density function is developed in a various manner different from the conventional methods.*

**Keywords:** narrowband radar, target density function, Gabor theory, SAR-ISAR.

## 1. Introduction

Target density function(TDF) is the reflectivity of spatially and continuously distributed targets and it is an important characteristic of radar imaging. TDF is known by different names such as ambiguity function, density function, object(target), object reflectivity function, doubly-spread reflectivity function, and reflection coefficient [1–6].

There are two well known approaches on TDF. First one considers point scatterers reflected off the target scatterer centers. Integration of all point scatterers is able to obtain the whole object. This radar imaging technique is based on inverse Fourier transform(IFT) and used mostly in inverse synthetic aperture radar(SAR) studies [7–12].

Second method on TDF is a dense target environment approach credited to Fowle and Naparst considering the ambiguity functions with two variables as range and velocity [13, 14]. As an advanced way, Naparst technique measures the the closeness of the targets to each other in a dense target environment rather than typical radar imaging.

This paper establishes a new target density function whereby Gabor theory is applied to narrowband radars. The proposed target density function has similar characteristics with Naparst-Fowle approach. However, it is developed by utilizing of Gabor theory different from the ambiguity functions used in their model. Beside, while the similar density function in SAR-ISAR method is obtained by Fourier transform, here it is produced by more plausible Gabor transforms based on the time-frequency analyze.

## 2. Preliminaries Of Density Functions

In this section, the background of the target density functions consists of the following techniques;

- SAR-ISAR reflectivity functions
- Naparst's target density functions

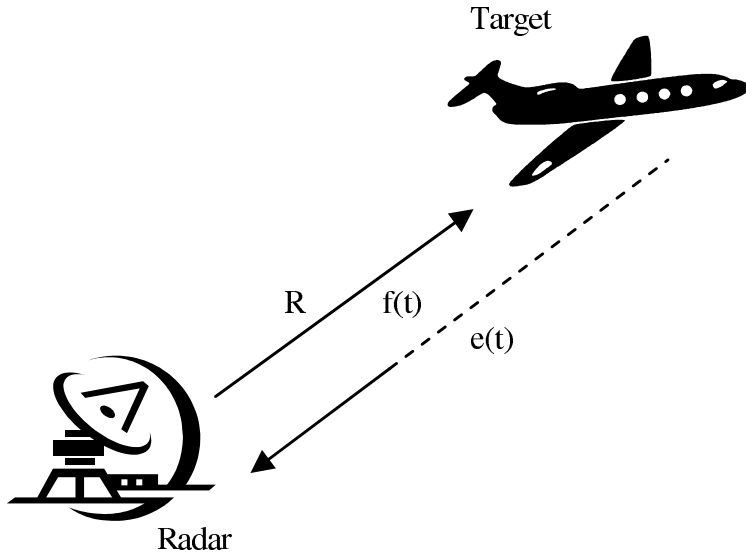
### 2.1. SAR - ISAR Reflectivity Functions

Synthetic aperture radar(SAR) is a well known radar imaging technique used for earth surface imaging. Coherent SAR imaging is an alternative approach to remote sensing that provides contribution to the imaging over visible/infrared sensing technology [7, 9, 10, 15–18].

Let consider Figure 1 for SAR receiving mode. If the target is composed of continuum point targets(scatterers), by the superposition principle, the echo(reflected signal)  $e(t)$ , from such a target at  $x, y, z$  points is:

$$e(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x, y, z) f\left(t - \frac{2R(x, y, z)}{c}\right) dx dy dz \quad (1)$$

where  $f$  is the transmitted signal function,  $\rho$  is reflectivity function,  $R$  is the range, and  $c$  is the speed of light. As stated in Equation 1, the returned signal  $e(t)$  is a de-



**Figure 1. Radar-target Detection.**

layed and time-scaled version of the transmitted signal,  $f(t)$ .

As a target density function (TDF) term, *reflection coefficient* is used in ISAR (Inverse Synthetic Aperture Radar) image formation. With respect to superposition theorem, the reflection coefficient is integration of all point scatterers. Summation of the point scatterers represents the whole object [7–10, 12, 15–18] in reflectivity coefficient function form,  $\rho(x, y, z)$  given in Equation 1. SAR systems are designed by moving a real aperture or antenna through a series of positions along the flight track. This corresponds to multi-aperture SAR imaging [19].

As for ISAR systems, imaging is based on similar principles to SAR imaging. However, they have different configuration. In SAR imaging, the radar is flying in space and the object is stationary, while in ISAR imaging, the object is moving and the radar is stationary. Target motion is the essence of the difference between SAR and ISAR [7, 9, 10, 20, 21].

ISAR is considered as an inverse Fourier transform (IFT) of a 3-D object on a 2-D [7, 9, 10, 22]. For simplification, if Equation (1) is expressed in two dimensions, after demodulation and some pre-filtering processes, the measured ISAR signal becomes:

$$e(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x, y) e^{-j2\pi f_0 \frac{2R_p(t)}{c}} dx dy \quad (2)$$

for  $2R_p(t)/c \leq t \leq T_{PRI} + 2R_p(t)/c$ . Here,  $T_{PRI}$  is pulse interval repetition,  $c$  is the speed of light,  $f_0$  is carrier frequency, and  $R_p(t)$  is the range from the radar to the

point-scatterer, given as:

$$R_p(t) = R(t) + x \cos[\theta(t) - \alpha] - y \sin[\theta(t) - \alpha] \quad (3)$$

where  $\alpha$  is the azimuth angle and  $\theta(t)$  is the rotation angle. If Inverse Fourier Transform of Equation 2 is taken, the image function  $\rho(x, y)$  is obtained as the following.

$$\rho(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(f_x, f_y) e^{j2\pi(xf_x - yf_y) \frac{2R_p(t)}{c}} df_x df_y \quad (4)$$

where

$$f_x = \frac{2f_0}{c} \cos\theta(t), \text{ and } f_y = \frac{2f_0}{c} \sin\theta(t) \quad (5)$$

## 2.2. Target Density Functions

First *Density* term related to the target density function is called by Fowle et al [13]. Fowle is focused on the problem of the detection and resolution in two dimensions of a large number of targets in a fixed part of the target space and by inspiring of ambiguity functions. Then, *Dense target environment* term is used by Naparst's paper [14] considering Fowle work. This approach is based on ambiguity and cross-ambiguity functions [23–25] as well. In his work, the dense-target environment is defined as the closeness of a lot of targets at a distance, which their velocities are so close to each other. Density of targets at distance  $x$  and velocity  $y$  is given as  $D(x, y)$ . In this case, the echo or the reflected signal from targets is

$$e(t) = \int_0^{\infty} \int_{-\infty}^{\infty} D(x, y) \sqrt{y} s(y(t-x)) dx dy \quad (6)$$

In this approach, it is assumed that all targets are illuminated equally. As seen, the target density function is composed of the range and velocity variables similar to the ambiguity functions. Reconstruction of the target density function in Naparst algorithm is finalized as follows (see Ref [14] for the details);

$$D(x, y) = \sum_{n,m=0}^{\infty} \langle e_n, s_m \rangle A_{nm}(x, y) \quad (7)$$

where  $s_m$  are signals sent out and  $e_n$  are their echoes. The cross-ambiguity function of the signals sent out ( $s_1, s_2, \dots$ ) is

$$A_{nm}(x, y) = \int_{-\infty}^{\infty} s_n(y(t-x)) \bar{s}_m(t) dt \quad (8)$$

## 3. Narrowband Approach to New Target Density Functions

In this section, an alternative target density functions (TDF) for active sensor is studied by Gabor theory. The new target density function is generated by

considering a narrowband approach. The proposed approach and its comparison with the other techniques are analyzed respectively as the following.

### 3.1. Gabor Theory to Narrowband Target Density Functions

Here, it is shown how Gabor transform can be used to narrowband radars by transmitting a waveform which is a kernel for these transforms. Let consider the general the narrowband model [26]

$$e(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D(x,y)e^{-jyt} \psi(t-x) dx dy \quad (9)$$

Let's define

$$\psi(t) = \psi_{\perp}^*(t) \quad (10)$$

and

$$-y = \omega, \quad dy = -d\omega \quad (11)$$

and uniquely define  $D_{\perp}$  to be the function such that

$$D_{\perp}(x, \omega) = D(x,y) \quad (12)$$

Then Equation 9 can be rewritten as

$$e(t) = - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_{\perp}(x, \omega) e^{j\omega t} \psi_{\perp}^*(t-x) d\omega dx \quad (13)$$

If both side of Equation 13 is multiplied by  $\frac{1}{2\pi\|\psi_{\perp}\|^2}$ , it yields

$$-\frac{1}{2\pi\|\psi_{\perp}\|^2} e(t) = \frac{1}{2\pi\|\psi_{\perp}\|^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_{\perp}(x, \omega) \times e^{j\omega t} \psi_{\perp}^*(t-x) d\omega dx \quad (14)$$

if the left side is taken as

$$r(t) = -\frac{1}{2\pi\|\psi_{\perp}\|^2} e(t) \quad (15)$$

it is rewritten as follows.

$$r(t) = \frac{1}{2\pi\|\psi_{\perp}\|^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_{\perp}(x, \omega) e^{j\omega t} \psi_{\perp}^*(t-x) d\omega dx \quad (16)$$

This is quite similar or slightly modified form of inverse Gabor transform [27] which is known Short Term Fourier Transform. The regular Gabor transform of Equation 16 yields  $D_{\perp}(x, \omega)$  as

$$D_{\perp}(x, \omega) = \int_{-\infty}^{\infty} e^{-j\omega t} \psi_{\perp}(t-x) r(t) dt \quad (17)$$

which is desired result. Thus, it is satisfied as a desired target density function which is capable of measuring the the closeness of the targets to each other in a dense target environment by Gabor transform.

### 3.2. Comparison

The developed technique here is inspired partly by analogy to Fowle-Naparst and SAR-ISAR approaches.

- *Comparing to Fowle-Naparst:* As an advanced work of Fowle's study, Naparst has developed a target density function for a high dense target environment with multiple targets, whose velocities are close to each other. In general this TDF acts like a separator for the targets at the distance with a given velocity. The significant difference is relevant to the used models. While Fowle-Naparst technique takes advantage of the wideband model and cross-ambiguity functions, the proposed alternative approach here is studied by utilizing of the narrowband model, and it is developed by Gabor theory.
- *Comparing to ISAR:* Main difference between the target density functions in this study and ISAR arises from the utilized techniques. Although ISAR and the narrowband have the target density functions(TDF), while the ISAR TDF presents an imaging kernel, the similar TDF in this study is in form of a separator or comparator at the distance with a given velocity. Beside that, ISAR is based on the restricted Fourier-inverse Fourier transforms, in this study, the alternative target density function is produced by the transform in more flexible (non stationarity) form by Gabor transform. This can lead better convergency.

## 4. Discussion and Conclusion

Gabor theory was applied to active sensors such as radars by estimating an alternative target density function. It was shown how Gabor theory can be used as an approach to narrowband model by active sensors by transmitting a waveform which is a kernel for this transform. The functional and methodic characteristics here differ the developed target density function from Naparst-Fowle and SAR-ISAR techniques. Main contributions of this study are summarized below.

- *A novel target density function :* New target density function was presented for a novel narrowband model.
- *Windowing Function:* The developed target density function was achieved by windowing functions which is capable of the time-frequency analyzes such as Gabor transform.
- *Gabor theory to narrowband radars:* The alternative target density function on narrowband radars

was developed by making use of Gabor theory. This was generated by transmitting a new waveform which is a kernel for Gabor transform such as variable window function.

Future work will concentrate on the use of the extrapolation of this theoretical study in order to obtain improvements in implementation of the developed target density function.

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