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Abstract— **Frequency-domain equalization (FDE) is an effective technique for high data rate wireless communication systems suffering from very long intersymbol interference. Most of existing FDE algorithms are limited to quasi-static or slow time-varying fading channels, where least mean squares (LMS) or recursive least squares (RLS) adaptive algorithms were utilized for channel estimation. In this paper, we employ interpolation method to develop channel estimation algorithm in the frequency domain. We show that the new channel estimation algorithm can significantly outperform LMS and RLS algorithms. Numerical examples demonstrate that the new algorithm can track time-varying fading channels with Doppler up to $300 - 400$ Hz. This means, for 1.9 GHz carrier frequency band, the new algorithm can provide good bit error rate performance even if the mobile is moving at a high speed of $170 - 228$ kilo-meters per hour, while the fading channel impulse response is 60 taps long.**

I. Introduction

Single carrier frequency-domain equalization (SC-FDE) has been shown to be an attractive equalization scheme for broadband wireless channels which has very long impulse response memory. Compared to orthogonal frequency division multiplex (OFDM), a single carrier system with FDE has similar performance and signal processing complexity but lower peak-to-average power ratio and less sensitivity to carrier frequency errors, and this arises from the use of single carrier modulation. Moreover, compared to time-domain equalization, SC-FDE has less computational complexity and better convergence properties [1] to achieve the same or better performance in severe frequency-selective fading channels.

Recent years, SC-FDE has received increasing attention in the literature [2]-[17]. Among the existing techniques, SC-FDE is often designed according to one of the following three channel assumptions: 1) the fading channel coefficients are assumed to be perfectly known at the receiver [6], [8], [10], [11], [13], [15], [16], then frequency-domain linear equalizers or decision feedback equalizers are analyzed and/or designed accordingly; 2) the fading channel are assumed to be constant for a frame consisting of one training block and many data blocks, the fading channel is estimated via the training block and utilized for equalization for the entire frame [2], [3], [4], without adaptive receiver processing; 3) the fading channel is assumed to be static for at least one block but varying within a frame, which consists of a few training blocks (at the beginning of the frame) and many data blocks, then adaptive FDE is developed by employing least mean squares (LMS) or recursive least squares (RLS) adaptive processing in the frequency domain [5], [14], [17]. The equalizers developed based on the first two assumptions have demonstrated significant performance gain of frequency-domain equalization over time-domain equalization, however, they may not be applicable to practical systems over time-varying channels with satisfactory performance. The adaptive equalizers derived from the third assumption has achieved substantial advancement in dealing with slow time-varying frequency-selective channels compared with these non-adaptive SC-FDEs. However, as indicated in the examples of [5], [14], [17], the adaptive SC-FDEs employing LMS or RLS algorithms can degrade significantly for moderate and fast moving mobiles.

In this paper, we employ interpolation method to propose a new algorithm for frequency-domain channel estimation for severe time-varying and frequency-selective fading channels. Our new algorithms are developed by employing a frame structure which consists of one training block and many data blocks. The training block is utilized to estimate fading channel transfer function of the block. The fading channel transfer functions of the data blocks are estimated by interpolating the channel transfer functions of the training blocks at the current frame and the next frame. Noise variance is also estimated at the training blocks. Channel equalization is performed in the frequency domain by employing the estimated channel transfer functions and noise variance.

II. System Models and Preliminaries

Consider an n_R-branch diversity system with one transmit antenna and n_R receive antennas as shown in Fig. 1. At the transmitter, the baseband data sequence $\{x(k)\}$ is periodically added cyclic prefix (CP) and modulated onto a single carrier frequency for transmission across the time-varying and frequency-selective fading diversity channel. At the receiver, the CP is removed at each branch, the fast Fourier transform (FFT) is utilized to convert the time-domain signal to frequency-domain signal, frequency-domain channel estimation, equalization and diversity combining are employed to mitigate inter-symbol interference. An inverse FFT (IFFT) is equipped to convert frequency-domain signal to time-domain signal for demodulating and detecting the baseband data sequence $\{\hat{x}(k)\}$.

To facilitate frequency-domain channel estimation and equalization for broadband wireless systems over time-varying and frequency-selective fading channels, we propose that the baseband signal sequence is partitioned in frames having a

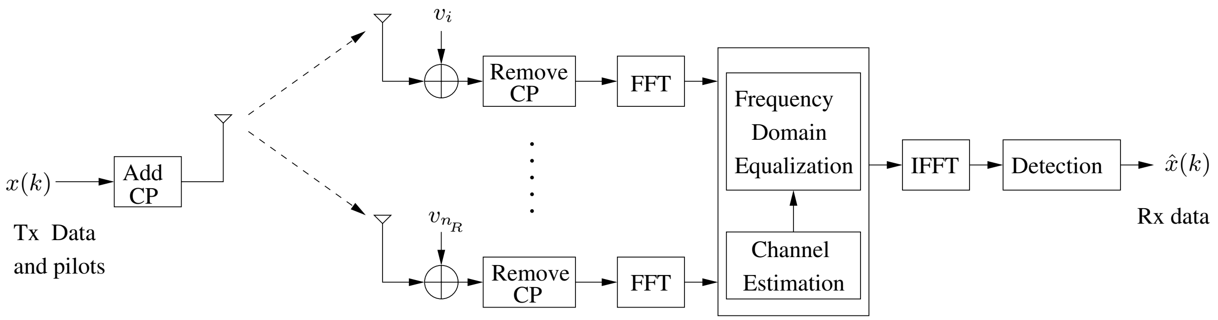

Fig. 1. The simplified block diagram of a single carrier SIMO wireless system with frequency-domain channel estimation and equalization.

time duration T_f as shown in Fig. 2. Each frame contains N_f signal blocks, where the first block is a training block designed for channel estimation and noise variance estimation. Each block contains N_c symbols of CP and N symbols of data (or training) sequence. The block time duration $T_b = T_c + T_d = (N_c + N)T_s$, where T_s is the symbol period.

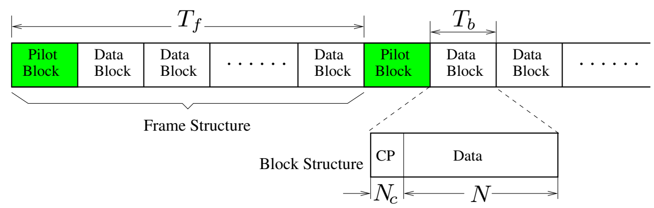

Fig. 2. The frame structure.

A. Time-Domain System Model

Let $x_p(k)$ be the kth transmitted symbol at pth block of a frame, $y_{p,i}(k)$ be the kth received symbol at pth block of the frame at the ith receive branch. Then the received pth block signal of the frame at the ith branch is given by

$$y_{p,i}(k) = \sum_{l=1}^{L} h_{p,i}(l,k)x_p(k+1-l)+v_{p,i}(k),$$
$$k = -N_c+1, -N_c+2, \cdots, N; \quad p = 1, 2, \cdots, N_f \quad (1)$$

where $v_{p,i}(k)$ is additive white Gaussian noise with average power σ^2 at the ith receive branch, L is the channel length, $h_{p,i}(l, k)$ is the baseband equivalent channel response of the composite time-varying frequency-selective fading channel of the ith branch. The composite channel is the cascade of the transmit pulse shaping filter, physical fading channel, and receive matched filter. It is known that the N_c ($N_c \geq L-1$) symbols of CP is chosen to satisfy

$$x_p(k) = x_p(k+N), \qquad k = -N_c+1, \cdots, -1, 0. \qquad (2)$$

After removing the CP, the pth block received data symbols at the ith branch can be expressed by (3) on top of next page, or in a compact matrix-vector form as

$$\mathbf{y}_{p,i} = \mathbf{T}_{p,i}\mathbf{x}_p + \mathbf{v}_{p,i}. \qquad (4)$$

In principle, if the receiver has perfect knowledge of the time-domain channel matrices $\{\mathbf{T}_{p,i}\}_{i=1}^{n_R}$, then the pth block transmitted data $\mathbf{x}_p$ can be estimated and detected via the minimum mean square error (MMSE) criterion. The standard solution is

$$\hat{\mathbf{x}}_p = \left[\sum_{i=1}^{n_R} \mathbf{T}_{p,i}^h \mathbf{T}_{p,i} + \sigma^2 \mathbf{I}_N\right]^{-1} \left[\sum_{i=1}^{n_R} \mathbf{T}_{p,i}^h \mathbf{y}_{p,i}\right]. \qquad (5)$$

However, this time-domain MMSE equalizer requires the inversion of an $N \times N$ Hermitian matrix that needs $\mathcal{Q}(N^2)$ operations, which is infeasible for larger N. On the other hand, if N is chosen to be small, the data efficiency $\frac{N}{N+L-1}$ will be low for large channel length L, which is harmful for high data rate broadband wireless communication systems.

B. Frequency-Domain System Model

Let $\mathbf{F}$ be the normalized FFT matrix of size $N \times N$, *i.e.*, its (m, n)th element is given by $\frac{1}{\sqrt{N}} \exp\left(\frac{-j2\pi(m-1)(n-1)}{N}\right)$, taking the FFT of the received signal and transmitted signal and keeping in mind that $\mathbf{F}^h\mathbf{F} = \mathbf{I}_N$, one can obtain the frequency-domain representation as follows

$$\begin{aligned} \mathbf{Y}_{p,i} \triangleq \mathbf{F}\mathbf{y}_{p,i} &= \mathbf{F}\mathbf{T}_{p,i}\mathbf{F}^h\mathbf{F}\mathbf{x}_p + \mathbf{F}\mathbf{v}_{p,i} \\ &= \mathbf{H}_{p,i}\mathbf{X}_p + \mathbf{V}_{p,i} \end{aligned} \qquad (6)$$

where $\mathbf{H}_{p,i} = \mathbf{F}\mathbf{T}_{p,i}\mathbf{F}^h$ is the frequency-domain channel matrix for the pth block at the ith branch. The frequency-domain MMSE equalization is given by

$$\hat{\mathbf{X}}_p = \left[\sum_{i=1}^{n_R} \mathbf{H}_{p,i}^h \mathbf{H}_{p,i} + \sigma^2 \mathbf{I}_N\right]^{-1} \left[\sum_{i=1}^{n_R} \mathbf{H}_{p,i}^h \mathbf{Y}_{p,i}\right]. \qquad (7)$$

For general time-varying and frequency-selective fading channels, the time-domain channel matrix $\mathbf{T}_{pi}$ is not circulant, and frequency-domain channel matrix $\mathbf{H}_{p,i}$ is not diagonal. Therefore, the frequency-domain MMSE equalization (7) has no advantage over its time-domain counterpart in terms of computational complexity, and the frequency tones of the received signal $\mathbf{Y}_{p,i}$ are not orthogonal. However, if the fading channel coefficients remain constant within a block, then $\mathbf{T}_{pi}$ is circulant and $\mathbf{H}_{p,i}$ is diagonal. Consequently, the frequency tones $\{Y_{p,i}(m)\}_{m=1}^{N}$ of the received signal are orthogonal, and

$$\begin{bmatrix} y_{p,i}(1) \\ y_{p,i}(2) \\ \vdots \\ y_{p,i}(L) \\ \vdots \\ \vdots \\ y_{p,i}(N) \end{bmatrix} = \begin{bmatrix} h_{p,i}(1,1) & 0 & \cdots & 0 & h_{p,i}(L,1) & \cdots & h_{p,i}(2,1) \\ h_{p,i}(2,2) & h_{p,i}(1,2) & 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & h_{p,i}(L,L-1) \\ h_{p,i}(L,L) & \ddots & \ddots & h_{p,i}(1,L) & 0 & \ddots & 0 \\ 0 & h_{p,i}(L,L+1) & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_{p,i}(L,N) & \cdots & \cdots & h_{p,i}(1,N) \end{bmatrix} \begin{bmatrix} x_p(1) \\ x_p(2) \\ \vdots \\ x_p(L) \\ \vdots \\ \vdots \\ x_p(N) \end{bmatrix} + \begin{bmatrix} v_{p,i}(1) \\ v_{p,i}(2) \\ \vdots \\ v_{p,i}(L) \\ \vdots \\ \vdots \\ v_{p,i}(N) \end{bmatrix} \quad (3)$$

the frequency-domain input-output relationship and equalization are simplified as

$$Y_{p,i}(m) = H_{p,i}(m)X_p(m)+V_{p,i}(m), \quad m = 1,2,\cdots,N \quad (8)$$

$$\hat{X}_p(m) = \left[\sum_{i=1}^{n_R}|H_{p,i}(m)|^2+\sigma^2\right]^{-1}\left[\sum_{i=1}^{n_R}H_{p,i}^h(m)Y_{p,i}(m)\right] \quad (9)$$

where $Y_{p,i}(m)$, $H_{p,i}(m)$, $X_p(m)$ and $V_{p,i}(m)$ are the discrete Fourier transform of the corresponding time-domain signals, given by

$$Y_{p,i}(m) = \frac{1}{\sqrt{N}}\sum_{k=1}^{N} y_{p,i}(k)\exp\left(\frac{-j2\pi(k-1)(m-1)}{N}\right) \quad (10)$$

$$H_{p,i}(m) = \sum_{l=1}^{L} h_{p,i}(l,\frac{N}{2})\exp\left(\frac{-j2\pi(l-1)(m-1)}{N}\right) \quad (11)$$

$$X_p(m) = \frac{1}{\sqrt{N}}\sum_{k=1}^{N} x_p(k)\exp\left(\frac{-j2\pi(k-1)(m-1)}{N}\right) \quad (12)$$

$$V_{p,i}(m) = \frac{1}{\sqrt{N}}\sum_{k=1}^{N} v_{p,i}(k)\exp\left(\frac{-j2\pi(k-1)(m-1)}{N}\right). \quad (13)$$

When the block time duration T_b is smaller than the channel coherence time, the fading channel coefficients remains approximately constant for the entire block, and we can use (8) to approximately describe the time-varying wireless fading channel in the frequency domain. In this paper, new algorithms for frequency-domain channel estimation and channel equalization will be developed based on (8) for time-varying frequency-selective fading channels, we will show via numerical examples that this approximation works well for realistic broadband wireless communication systems with both low mobility and high mobility users.

III. Channel Estimation and Noise Variance Estimation

In this section, we present an interpolation algorithm for channel estimation in the frequency domain, and a technique for noise variance estimation.

A. Frequency-Domain Channel Estimation

For the training block, $p = 1$, both the transmitted signal $X_1(m)$ and received signal $Y_{1,i}(m)$ are known. The frequency-domain channel transfer function $H_{1,i}(m)$ at the training block can be estimated by least squares (LS) criterion as follows:

$$\tilde{H}_{1,i}(m) = \frac{Y_{1,i}(m)}{X_1(m)} = H_{1,i}(m) + \frac{V_{1,i}(m)}{X_1(m)}. \quad (14)$$

The estimate $\tilde{H}_{1,i}(m)$ can be improved by a frequency-domain filter to reduce noise. Although various frequency-domain filters can be employed, a common technique is to transform $\tilde{H}_{1,i}(m)$ into the time domain with an IFFT, and use an L-size window mask to remove the noise beyond the channel length, then transform the time-domain channel coefficients back to the frequency domain with an FFT. This procedure was originally proposed in OFDM systems [19]. The noise-reduced channel estimation of the training block can be represented by

$$\hat{H}_{1,i}(m) = H_{1,i}(m) + \frac{\hat{V}_{1,i}(m)}{X_1(m)}, \quad m = 1,2,\cdots,N \quad (15)$$

where $\hat{V}_{1,i}(m)$ is equal to $\sum_{k=1}^{L}\frac{v_{1,i}(k)}{\sqrt{N}}\exp\left(\frac{-j2\pi(k-1)(m-1)}{N}\right)$.

Provided $v_{1,i}(k)$ is AWGN with average power σ^2, one can easily conclude that $V_{1,i}(m)$ and $\hat{V}_{1,i}(m)$ are zero-mean Gaussian with average power being σ^2 and $\frac{\sigma^2 L}{N}$, respectively. Therefore, the noise average power is reduced by a factor $\frac{N}{L}$ via the FFT-based frequency-domain filter.

According to eqn. (15), a desired property of the training sequence is to have constant $|X_1(m)|^2$ for all m, so that noise amplification on certain frequency tones can be avoided. Although many sequences can achieve this property, a good solution is to adopt Chu sequences [20] as the training sequence, because Chu sequences have constant magnitude in both frequency domain and time domain, which avoids the peak-to-average power ratio problem at the transmitter. In this paper, we choose Chu sequences as the training sequence to ensure $|X_1(m)|^2 = 1, \forall m$.

Similar to $\hat{H}_{1,i}(m)$ which denotes the noise-reduced estimated channel transfer function at the ith receive branch for the training block of the current frame, let $\hat{H}_{N_f+1,i}(m) = H_{N_f+1,i}(m)+\frac{\hat{V}_{N_f+1,i}(m)}{X_1(m)}$ be the noise-reduced estimated transfer functions of the training block of the next frame. Define the column vector $\hat{\mathbf{H}}_i(m) = \left[\begin{array}{cc}\hat{H}_{1,i}(m) & \hat{H}_{N_f+1,i}(m)\end{array}\right]^t$, and let $\mathbf{C}_{p,i}(m)$ be an interpolation row vector corresponding to the pth block in the current frame for the ith receive branch.

Then the transfer function of the pth block of the current frame is estimated by

$$\hat{H}_{p,i}(m) = \mathbf{C}_{p,i}(m)\hat{\mathbf{H}}_i(m), \quad p = 1, 2, \cdots, N_f \quad (16)$$

and the estimation error for the ith branch is given by

$$E_{p,i}(m) = H_{p,i}(m) - \hat{H}_{p,i}(m) = H_{p,i}(m) - \mathbf{C}_{p,i}(m)\hat{\mathbf{H}}_i(m). \quad (17)$$

The optimal solution for $\mathbf{C}_{p,i}(m)$ to minimize the mean square estimation error is given by

$$\mathbf{C}_{p,i}(m) = \mathcal{E}\left\{H_{p,i}(m)\hat{\mathbf{H}}_i^h(m)\right\}\left[\mathcal{E}\left\{\hat{\mathbf{H}}_i(m)\hat{\mathbf{H}}_i^h(m)\right\}\right]^{-1} \quad (18)$$

$$= \left[\ \mathcal{E}\left\{H_{p,i}(m)H_{1,i}^*(m)\right\} \quad \mathcal{E}\left\{H_{p,i}(m)H_{N_f+1,i}^*(m)\right\}\ \right] \times \begin{bmatrix} \mathcal{E}\left\{|H_{1,i}(m)|^2\right\} + \frac{\sigma^2 L}{N} & \mathcal{E}\left\{H_{1,i}(m)H_{N_f+1,i}^*(m)\right\} \\ \mathcal{E}\left\{H_{N_f+1,i}(m)H_{1,i}^*(m)\right\} & \mathcal{E}\left\{\left|H_{N_f+1,i}(m)\right|^2\right\} + \frac{\sigma^2 L}{N} \end{bmatrix}^{-1} \quad (19)$$

We are now in a position to present the frequency-domain channel estimation and its mean square errors of data blocks over Rayleigh fading channels.

Proposition 1: For frequency-selective Rayleigh fading, the interpolation row vector $\mathbf{C}_{p,i}(m)$ and the minimum mean square error $\varepsilon_{p,i}(m) = \mathcal{E}\left\{\left|H_{p,i}(m) - \hat{H}_{p,i}(m)\right|^2\right\}$ can be simplified to be

$$\mathbf{C}_p = \begin{bmatrix} J_0\left[\omega_d(p-1)T_b\right] \\ J_0\left[\omega_d\left(N_f+1-p\right)T_b\right] \end{bmatrix}^t \begin{bmatrix} 1 + \frac{\sigma^2 L}{N} & J_0\left(\omega_d N_f T_b\right) \\ J_0\left(\omega_d N_f T_b\right) & 1 + \frac{\sigma^2 L}{N} \end{bmatrix}^{-1} \quad (20)$$

$$\varepsilon_p = 1 - \begin{bmatrix} J_0\left[\omega_d(p-1)T_b\right] \\ J_0\left[\omega_d\left(N_f+1-p\right)T_b\right] \end{bmatrix}^t \begin{bmatrix} 1 + \frac{\sigma^2 L}{N} & J_0\left(\omega_d N_f T_b\right) \\ J_0\left(\omega_d N_f T_b\right) & 1 + \frac{\sigma^2 L}{N} \end{bmatrix}^{-1} \times \begin{bmatrix} J_0\left[\omega_d p T_b\right] \\ J_0\left[\omega_d\left(N_f-p\right)T_b\right] \end{bmatrix} \quad (21)$$

where $J_0(\cdot)$ is the zero-order Bessel function of the first kind, and $\omega_d = 2\pi f_d$ is the maximum angular Doppler frequency.

Proof: The proof is omitted for brevity.

It is noted that for Rayleigh fading channels, once N_f and T_b are chosen, $\mathbf{C}_p$ depends on the maximum Doppler frequency f_d and the noise average power σ^2. The estimation of f_d can be done by the algorithm presented in [22] and the estimation of σ^2 is given in the next subsection.

It is also noted that interpolation-based channel estimation methods have been previously studied for OFDM systems [23] and for frequency flat fading channels [24]. The algorithm presented in this paper is to demonstrate that the interpolation-based channel estimation can deal with much higher Doppler than the LMS and RLS algorithms for single-carrier broadband wireless systems.

B. Noise Variance Estimation

Let $\tilde{h}_{1,i}(l)$ and $\tilde{h}_{N_f+1,i}(l)$ be the N-point IFFT of $\tilde{H}_{1,i}(m)$ and $\tilde{H}_{N_f+1,i}(m)$, respectively, where $l = 1, 2, \cdots, N$. As we know that the channel length is L and $|X_1(m)|^2 = 1$, hence $\tilde{h}_{1,i}(l)$ and $\tilde{h}_{N_f+1,i}(l)$ for $l = L+1, L+2, \cdots, N$ are noise with the same variance σ^2. Therefore, σ^2 can be estimated by

$$\hat{\sigma}^2 = \frac{1}{2(N-L)} \sum_{l=L+1}^{N} \left[\left|\tilde{h}_{1,i}(l)\right|^2 + \left|\tilde{h}_{N_f+1,i}(l)\right|^2\right]. \quad (22)$$

It should be pointed that estimating σ^2 needs only small fractional computations as shown in (22), because $\tilde{h}_{1,i}(l)$ and $\tilde{h}_{N_f+1,i}(l)$ are obtained when we estimate $\hat{H}_{1,i}(m)$ and $\hat{H}_{N_f+1,i}(m)$ by the frequency-domain filter using IFFT and FFT.

IV. Frequency-Domain Channel Equalization

According to eqn. (8), the pth block received signals at the n_R branches are given in frequency domain as follows:

$$\begin{bmatrix} Y_{p,1}(m) \\ Y_{p,2}(m) \\ \vdots \\ Y_{p,n_R}(m) \end{bmatrix} = \begin{bmatrix} H_{p,1}(m) \\ H_{p,2}(m) \\ \vdots \\ H_{p,n_R}(m) \end{bmatrix} X_p(m) + \begin{bmatrix} V_{p,1}(m) \\ V_{p,2}(m) \\ \vdots \\ V_{p,n_R}(m) \end{bmatrix}. \quad (23)$$

This equation can be written in a compact form as follows:

$$\mathbf{Y}_p(m) = \mathbf{H}_p(m)X_p(m) + \mathbf{V}_p(m). \quad (24)$$

We are now in a position to state the following result.

Proposition 2: The output of the frequency-domain MMSE equalizer is given by

$$\hat{X}_p(m) = \left[\hat{\mathbf{H}}_p^h(m)\hat{\mathbf{H}}_p(m) + \varepsilon_p + \sigma^2\right]^{-1} \hat{\mathbf{H}}_p^h(m)\mathbf{Y}_p(m) \quad (25)$$

where $\hat{\mathbf{H}}_p(m) = \left[\ \hat{H}_{p,1}(m) \quad \hat{H}_{p,2}(m) \quad \cdots \quad \hat{H}_{p,n_R}(m)\ \right]^t$ is the noise-reduced estimated transfer function vector of the pth block.

Proof: From (17) we have $\mathbf{H}_p(m) = \hat{\mathbf{H}}_p(m) + \mathbf{E}_p(m)$ with $\mathbf{E}_p(m) = \left[\ E_{p,1}(m) \quad E_{p,2}(m) \quad \cdots \quad E_{p,n_R}(m)\ \right]^t$ being the estimation error vector. Replacing $\mathbf{H}_p(m)$ by $\hat{\mathbf{H}}_p(m)+\mathbf{E}_p(m)$, (24) yields

$$\mathbf{Y}_p(m) = \hat{\mathbf{H}}_p(m)X_p(m) + \mathbf{E}_p(m)X_p(m) + \mathbf{V}_p(m). \quad (26)$$

Let $\mathbf{W}_p(m)$ be the frequency-domain equalizer row vector, the output of the equalizer is given by $\hat{X}_p(m) = \mathbf{W}_p(m)\mathbf{Y}_p(m)$. The equalization error vector is given by

$$E_{X_p}(m) = X_p(m) - \hat{X}_p(m) = X_p(m) - \mathbf{W}_p(m)\mathbf{Y}_p(m). \quad (27)$$

Adopting the MMSE criterion, we find the equalizer row vector given by

$$\begin{aligned} \mathbf{W}_p(m) &= \mathcal{E}\left\{X_p(m)\mathbf{Y}_p^h(m)\right\}\left[\mathcal{E}\left\{\mathbf{Y}_p(m)\mathbf{Y}_p^h(m)\right\}\right]^{-1} \\ &= \hat{\mathbf{H}}^h(m)\left[\hat{\mathbf{H}}(m)\hat{\mathbf{H}}^h(m) + \mathcal{E}\left\{\mathbf{E}_p(m)\mathbf{E}_p^h(m)\right\}\right. \\ &\quad \left. + \mathcal{E}\left\{\mathbf{V}_p(m)\mathbf{V}_p^h(m)\right\}\right]^{-1} \\ &= \hat{\mathbf{H}}^h(m)\left[\hat{\mathbf{H}}(m)\hat{\mathbf{H}}^h(m) + \varepsilon_p\mathbf{I}_{n_R} + \sigma^2\mathbf{I}_{n_R}\right]^{-1} \\ &= \left[\hat{\mathbf{H}}^h(m)\hat{\mathbf{H}}(m) + \varepsilon_p + \sigma^2\right]^{-1}\hat{\mathbf{H}}^h(m) \end{aligned} \quad (28)$$

where the last equality is obtained by using the matrix inversion lemma [25]. This completes the proof.

Finally, applying IFFT on the frequency domain equalized data sequence $\hat{X}_p(m)$, $m = 1, 2, \cdots, N$, we obtain the pth block estimated data sequence $\hat{x}_p(k)$, $k = 1, 2, \cdots, N$ in the time domain.

V. Simulation Results

The performance evaluation of the proposed algorithms has been carried out by extensive computer simulations with various system parameters and fading channels. For convenient comparison, we present numerical examples based on two previously reported wireless systems.

System A: we adopt the 60-tap frequency-selective Rayleigh fading channel, where the average power of the first 20 taps ramps up linearly and the last 40 taps ramps down linearly, as described in [5], and the fading channel is normalized to have total average power as one. We choose FFT size $N = 256$, symbol interval $T_s = 0.25\mu s$ and QPSK modulation, which are the same as these of [5]. We further choose frame length $N_f = 10$ to have the same data efficiency as that of [5] for the LMS and RLS adaptations, which employed 10 training blocks at the beginning of every frame and each frame consisted of 100 blocks.

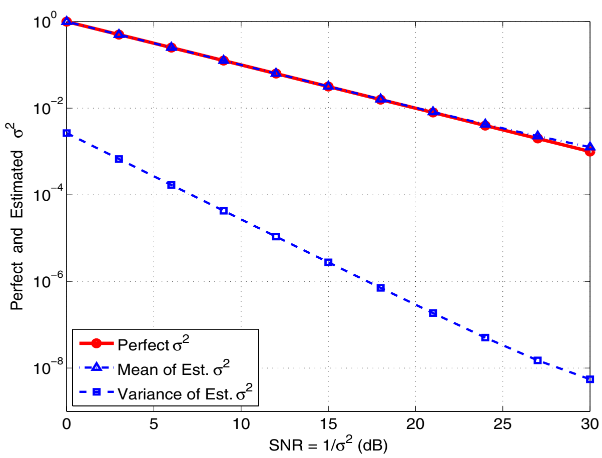

Fig. 3. Performance of the noise variance estimation algorithm.

The performance of the proposed noise variance estimation technique is shown in Fig. 3. As can be seen, the estimation technique provides unbiased estimation for the noise variance $\hat{\sigma}^2$, and the variance of the estimated $\hat{\sigma}^2$ is small, which indicates that the estimation algorithm is accurate.

Fig. 4 shows the BER performance of a two-antenna receiver and a four-antenna receiver equipped with our proposed algorithms when the Doppler is 200 Hz, which is equivalent to a mobile speed of 114 km/h at carrier frequency of 1.9 GHz. As can be seen, both diversity receivers with our algorithms have only about 1 dB degradation from the ideal receiver with perfect channel fading information. Moreover, for the four-antenna receiver, our algorithm with 200 Hz Doppler has the same performance as the LMS and RLS algorithms of [5] with quasi-static channel, and for the two-antenna receiver, our algorithm with $f_d = 200$ Hz is slightly better than the RLS algorithm but slightly worse than the LMS algorithm of [5] when they are operated with quasi-static channel. The LMS and RLS algorithms will degrade 3-6 dB at BER = 10^{-4} when the Doppler is 200 Hz, as pointed out by the author of [5].

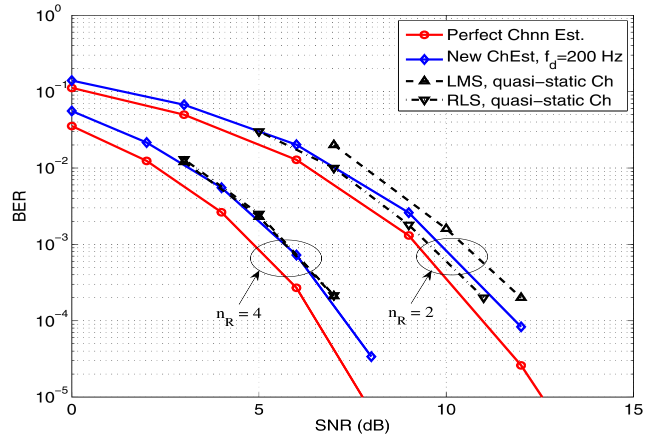

Fig. 4. BER versus SNR of diversity receivers with our proposed algorithms at $f_d = 200$ Hz and those of [5] for quasi-static channel.

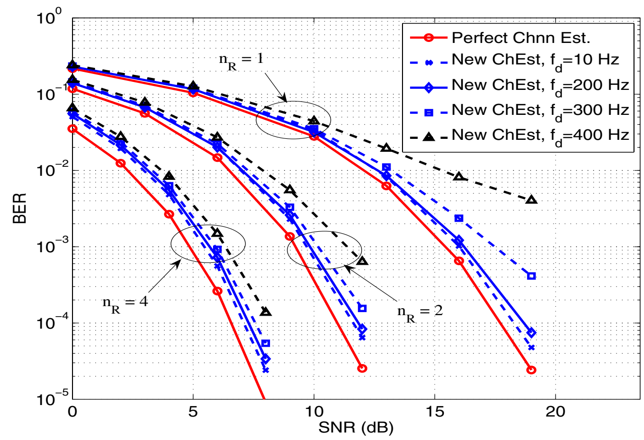

Fig. 5. BER versus SNR of diversity receivers with various Doppler spreads.

Fig. 5 depicts the BER performance of single-branch receiver, two-branch diversity receiver and four-branch diversity receiver over various Doppler spreads up to 400 Hz. From this figure, it is observed that the BER degradation due to larger Doppler tends to be smaller when the diversity order increases.

Clearly, our proposed algorithms can effectively cope with severe fading channels which has very long impulse response and large Doppler shift.

System B: We adopt the same 11-tap frequency-selective Rayleigh fading channel whose l-th tap has average power given by $1.2257 \exp(-0.8l)$, as described in [17]. We choose frame length $N_b = 10$, FFT size $N = 128$, CP length $N_c = 10$, symbol interval $T_s = 0.5\mu s$, receive antenna number $M = 1$ and QPSK modulation. Therefore, the data efficiency is $\frac{N}{N+N_c} \times \frac{N_b-1}{N_b} = 83.5\%$, which is slightly higher than the data efficiency of 82.8% in [17].

Figure 6 shows the BER results of the single-branch receiver employing our proposed frequency-domain channel equalization incorporated our proposed noise variance estimation and channel estimation algorithms with various Doppler frequencies $f_d = 20, 50, 100, 200$ and 300 Hz. For comparison purpose, the results of MMSE equalizers based on perfect channel knowledge and RLS adaptive algorithm [17] with normalized Doppler $f_d T_s = 1 \times 10^{-5}$, *i.e.*, $f_d = 20$ Hz are also included. As can be seen from the BER results, for Doppler frequency up to 50 Hz, our proposed algorithms is less than 1 dB away from the ideal case with perfect channel knowledge at BER of 10^{-5}. For Doppler up to 300 Hz, our algorithms still provide better results than that of the RLS algorithm in [17] with Doppler $f_d = 20$ Hz. This indicates that our algorithm can handle 15 times higher Doppler than the RLS algorithm in [17], and still provides better BER performance and maintains slightly higher data efficiency. The cost we pay for the proposed algorithm is using 128-point FFT and IFFT while the RLS algorithm in [17] employs 64-point FFT and IFFT. However, our channel estimation algorithm has lower computational complexity than the channel tracking algorithm with RLS adaptation.

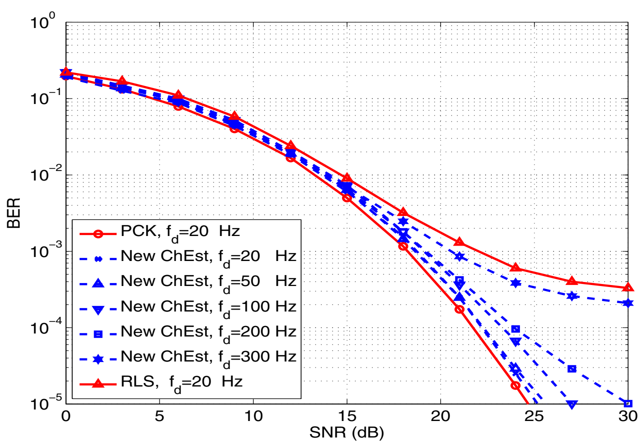

Fig. 6. BER versus SNR of a single-antenna receiver.

VI. Conclusion

In this paper, we have presented algorithms for fading channel estimation, noise variance estimation and fading channel equalization in the frequency domain for diversity channels. It has been demonstrated via examples that the proposed algorithms perform very well for broadband wireless communication systems which encounter very long impulse response and very fast time-varying fading channels. Numerical results have shown that our algorithm has 3-6 dB gain over the LMS and/or RLS algorithms in [5] at 200Hz Doppler, and our algorithm can handle 15 times higher Doppler than the RLS algorithm in [17].

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