A three-phase soft switching rectifier based on a ZCS zeta converter

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A Three-Phase Soft Switching Rectifier Based on a ZCS Zeta Converter

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Abstract—A new three-phase high power factor rectifier based on Zeta converter is presented. A resonant inductor and capacitor are added to the circuit to create zero-current switching condition for the switch. The circuit is operated in the PWM mode by putting a parallel switch and diode in series with the resonant capacitor. Different operating modes of the new topology are presented in detail. The simulation results verify the analysis.

Index Terms— Soft switching PWM; Three-phase PFC; Zero-current switching

I. INTRODUCTION

Three-phase rectifiers with power factor correction are widely used due to their high power factor, high efficiency, reduced harmonic content in their output current, low distortion in the input voltage, and low losses in the transmission lines [1]-[5]. In order to further reduce the switching losses, quasi-resonant zero current switching technique has been applied to three-phase rectifiers [1]. However, the quasi-resonant zero current technique results in high voltage and current stresses on the switches. Furthermore, switching frequency has to be variable for line and load regulation purposes. As a result of this, the quasi-resonant technique is not a good choice for the practicing engineers since the transformer and the inductor need to be designed based on the lowest switching frequency. Furthermore, optimal design of input filter is not attainable due to drastic switching frequency variations.

Due to the drawbacks of the quasi-resonant circuit, a three-phase soft switching PWM technique using constant-frequency control is introduced in this paper. It has the advantages of both the PWM and quasi-resonant techniques. In this circuit, the resonant process can be interrupted and the resonant time can be controlled. In the blocking state of the resonance, the circuit is operated in PWM mode. After the blocking state, the resonant process can be continued. So the circuit can control its output voltage by both regulating its switching frequency and duty ratio. This approach has been applied to a Zeta converter as described in the following sections.

II. PRINCIPLE OF OPERATION

The proposed circuit of the Zeta converter using three-phase soft switching PWM technique is shown in Fig. 1. It consists of an uncontrollable rectification circuit and a Zeta dc-dc converter. The zero current switching of the main switch Q1 is realized by the resonance between inductor $L_i$ and capacitor $C_r$.

To simplify the analysis, the following assumptions are made.

- The semiconductors are considered ideal.
- The three inductances are the same. The inductance is $L$.
- The switching frequency of the converter is far greater than the input line voltage.
- Capacitor $C_f$ and $C_i$ are large enough to make their voltages constant and equal to $V_0$ [3].
- The line voltage is considered constant during a switching period.

In Fig. 1, there are three diodes conducting at the same time because of the effect of input inductors. In every cycle, there are twelve $\pi/6$ periods. They can be analyzed in the same way. So the analysis of one period can reflect the operation of the whole cycle. The period of $u_a \geq u_b \geq 0 \geq u_c$ is analyzed in the following. This

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period can be divided into seven intervals. The equivalent circuits of these intervals are shown in Fig. 2.

(a) interval 1

(b) interval 2

(c) interval 3

(d) interval 4

(e) interval 5

(f) interval 6

(g) interval 7

Fig. 2. equivalent circuits of all the intervals
(1) Interval 1: $t_0 - t_1$

Before $Q_1$ is turned on, current flows through diode $D_B$. Initial values are $i_{tf1} = i_{tf1}(t_0)$ and $i_{tf2} = i_{tf2}(t_0)$. Then, switch $Q_1$ is turned on under zero current condition because of the effect of inductor $L_r$. After $Q_1$ is turned on, current $i_{c1}$ flowing through capacitor $C_r$ increases gradually. When $i_{c1} = i_{tf2}$, diode $D_B$ is cut off and this interval is finished.

In this interval:

\[
\begin{align*}
&\left\{ \begin{array}{l}
\frac{d}{dt}u_e + \frac{2V_d}{3} = L_r \frac{di_e}{dt} \\
V_d = L_r \frac{di_{f1}}{dt} - V_0 \\
\frac{L_{f1}}{dt} = -V_0 \\
i_c = -i_{fr}
\end{array} \right. \\
\end{align*}
\]

From (1), the current $i_{c1}$ and $i_{tf2}$ can be obtained as follows:

\[
i_{c1} = i_{fr} - i_{tf1} = \left[ \frac{2}{3L + 2L_r} V_0 - \frac{3}{3L + 2L_r} u_e + \frac{V_0}{L_{f1}} \right] t - i_{tf1}(t_0)
\]

\[
i_{tf2} = -\frac{V_0}{L_{f2}} t + i_{tf2}(t_0)
\]

Time $t_1$ is defined as the time instant when $i_{c1} = i_{tf2}$. Then $t_1$ can be expressed as:

\[
t_1 = \frac{i_{tf1}(t_0) + i_{tf2}(t_0)}{A},
\]

where,

\[
A = \left[ \frac{2}{3L + 2L_r} V_0 - \frac{3}{3L + 2L_r} u_e + \frac{V_0}{L_{f1}} + \frac{V_0}{L_{f2}} \right]
\]

(2) Interval 2: $t_1 - t_2$

When interval 1 is finished, diode $D_B$ is cut off, current begins to flow through $D_A$ and capacitor $C_r$ begins to be charged. Inductors $L_r$ and $L_{f1}$ and capacitors $C_1$ and $C_r$ begin to resonate. When the voltage of $C_r$ rises to its maximum, interval 2 is finished. Initial value is $u_{cr} = 0$. In interval 2:

\[
-V_d + L_r \frac{di_{fr}}{dt} + L_{f1} \frac{di_{f1}}{dt} = 0
\]

And

\[
\left\{ \begin{array}{l}
\frac{d}{dt}u_c + \frac{2V_d}{3} = L_r \frac{di_e}{dt} = -L_r \frac{di_{fr}}{dt} \\
i_{fr} = i_{tf1} + i_{cr} + i_{tf2} \\
i_{cr} = C_r \frac{du_{cr}}{dt}
\end{array} \right. \\
\]

From (2) and (3), the following can be obtained.

\[
\left\{ \begin{array}{l}
V_d = \frac{3}{2} \left[ -L_r \frac{di_{fr}}{dt} - u_e \right] \\
i_{fr} = i_{tf1} + i_{tf2} + C_r \frac{du_{cr}}{dt}
\end{array} \right. \\
\]

In this interval:

\[
\left\{ \begin{array}{l}
\frac{di_{f1}}{dt} = u_{cr} - V_0 \\
\frac{di_{f2}}{dt} = u_{cr} - V_0
\end{array} \right.
\]

Substituting (4) and (5) into (2), equation (6) can be obtained.

\[
\frac{d^2 u_{cr}}{dt^2} + B^2 u_{cr} = C
\]

where,

\[
B^2 = \frac{2}{(3L + 2L_r)C_r} + \frac{1}{L_{f1}C_r} + \frac{1}{L_{f2}C_r}
\]

\[
C = \left[ \frac{1}{L_{f1}C_r} + \frac{1}{L_{f2}C_r} + \frac{3}{3L + 2L_r} \right] V_0 - \frac{3V_C}{3L + 2L_r}
\]

Since $u_{cr} = 0$ when $t = t_1$ and currents $i_{fr}$, $i_{tf1}$, $i_{tf2}$ cannot be changed suddenly, $i_{cr}$ is equal to zero at the time when $t = t_1$. From (6), the following can be obtained.

\[
u_{cr} = -\frac{C}{B^2} \cos B(t - t_1) + \frac{C}{B^2}
\]

From (7), when $t = t_1 + \frac{\pi}{B}$, $u_{cr}$ reaches its maximum $\frac{2C}{B^2}$ and the interval is finished. This time instant is defined as $t_2$ and $t_2 = t_1 + \frac{\pi}{B}$. At this time instant, the following can be obtained.
where \( D = \left[ \frac{2}{3}(L_f + \frac{L_f L_{f2}}{L_f + L_{f2}}) + L \right]^{-1} \).

From (10), when interval 3 is finished, the following can be obtained.

\[
\begin{cases}
    i_{lf1} = -\frac{D L_{f2}}{L_{f1} + L_{f2}} u_c (t - t_2) + i_{lf1}(t_2) \\
    i_{lf2} = -\frac{D L_{f1}}{L_{f1} + L_{f2}} u_c (t - t_2) + i_{lf2}(t_2)
\end{cases}
\]

Equations in (11) show the initial values of interval 4.

(4) Interval 4: \( t_3 - t_4 \)

When \( t = t_3 \), switch \( Q_2 \) is turned on and \( C_r \) begins to discharge. Inductors \( L_r \) and \( L_{f1} \) and capacitors \( C_1 \) and \( C_r \) continue to resonate. At the time of \( t_4 \), the current \( i_b \) reaches the zero. Interval 4 is finished. Where the initial values are

\[ u_{cr} = \frac{2C}{B^2}, \quad i_{cr} = 0 \]

In this interval, the circuit has the same structure as interval 2. Using the same method of interval 2, equation (12) can be obtained.

\[
\begin{align*}
    u_{cr} &= \frac{C}{B^2} \cos B(t - t_3) + \frac{C}{B^2} \\
    \frac{di_b}{dt} &= -\frac{1}{2L} u_c + \frac{u_b}{2L} + \frac{u_c}{2L} + \frac{u_b}{2L} \\
    \frac{di_{lf1}}{dt} &= \frac{1}{L_f1} (u_{cr} - V_0) \\
    \frac{di_{lf2}}{dt} &= \frac{1}{L_f2} (u_{cr} - V_0)
\end{align*}
\]

Substituting (12) into (15), the expressions for \( i_{lf1} \) and \( i_{lf2} \) can be obtained. Then substituting these expressions into (16) equation (17) can be obtained.

\[
i_{lf} = i_{lf1} + i_{lf2} + C_r \frac{d^2 u_{cr}}{dt^2}
\]
\[
\frac{di_b}{dt} = \frac{1}{2} \left( \frac{C}{L_f r_1 B^2} + \frac{C}{L_f r_2 B^2} - C_r C \right) \cos(B(t - t_3)) \frac{1}{2} \left( \frac{C}{L_f B^2} + \frac{V_0}{L_f r_1} - \frac{V_0}{L_f r_2} \right) + \frac{u_c + u_b}{2L_r L} \]
\]

From (17), \( i_b \) can be expressed as the following:

\[
i_b = \frac{C}{2B} \left( \frac{1}{L_f r_1 B^2} + \frac{1}{L_f r_2 B^2} - C_r C \right) \sin(B(t - t_3)) + \frac{1}{2} \left( \frac{C}{L_f B^2} + \frac{V_0}{L_f r_1} - \frac{V_0}{L_f r_2} \right) + \frac{u_c + u_b}{2L_r L} \]
\]

From (18), when \( t = t_4 \), \( i_b \) is equal to zero and interval 4 is finished. The final values are \( u_{cr} = u_{cr}(t_4) \) and \( i_{cr} = i_{cr}(t_4) \).

(5) Interval 5: \( t_4 - t_5 \)

Current \( i_a \) is equal to \( -i_c \) when \( t = t_4 \). Inductors \( L_r \) and \( L_{f_1} \) and capacitors \( C_1 \) and \( C_r \) continue to resonate. Here, \( i_a = i_{lr} = -i_c \). When \( t = t_5 \), \( i_{lr} \) reaches zero and interval 5 is finished.

In this interval:

\[
\begin{align*}
i_{lr} & = i_{lf_1} + i_{lf_2} + C_r \frac{du_{cr}}{dt} \\
\frac{di_{lf_1}}{dt} & = u_{cr} - V_0 \\
\frac{di_{lf_2}}{dt} & = u_{cr} - V_0
\end{align*}
\]

From (19),

\[
\frac{di_{lr}}{dt} = L_r C_r \frac{du_{cr}}{dt} + \left( \frac{1}{L_{f_1}} + \frac{1}{L_{f_2}} \right) u_{cr} - \left( \frac{1}{L_{f_1}} + \frac{1}{L_{f_2}} \right) V_0
\]

In this interval:

\[
\frac{dV_d}{dt} = u_a - u_c - 2L \frac{di_{lr}}{dt}
\]

(21)

From (20) and (21) equation (22) can be obtained.

\[
-V_d + \frac{di_{lr}}{dt} + L_{f_1} \frac{di_{lf_1}}{dt} + L_{f_2} \frac{di_{lf_2}}{dt} = 0
\]

(22)

Substituting (19), (20), (21) into (22) yields

\[
\frac{d^2 u_{cr}}{dt^2} + g^2 u_{cr} = h
\]

where,

\[
\frac{1}{L_{f_1}} + \frac{1}{L_{f_2}} = \frac{u_a - u_c}{L_r} + \frac{1}{2L_r L} = g^2
\]

(23)

Solving (23) and substituting the initial condition \( u_{cr} = u_{cr}(t_4) \), \( i_{cr} = i_{cr}(t_4) \) when \( t = t_4 \) yields

\[
u_{cr} = \left[ u_{cr}(t_4) - \frac{h}{g^2} \right] \cos(g(t - t_4)) + \frac{i_{cr}(t_4)}{gC_r} \sin(g(t - t_4)) + \frac{h}{g^2}
\]

(24)

Substituting (24) and the initial condition \( i_{lr} = i_{lr}(t_4) \) when \( t = t_4 \) into (20) yields

\[
i_{lr} = m \sin(g(t - t_4)) + u_{cr}(t_4) + k(t - t_4) + \frac{i_{cr}(t_4)}{n}
\]

(25)

where,

\[
m = \left[ u_{cr}(t_4) - \frac{h}{g^2} \right] \left( \frac{1}{L_{f_1} g} + \frac{1}{L_{f_2} g} - C_r g \right)
\]

(26)

Time instant \( t_5 \) when \( i_{lr} = 0 \) can be obtained from (25). Currents \( i_a \) and \( i_c \) both reach zero when \( t = t_5 \) and switch \( Q_1 \) can be turned off under zero current condition. At this moment, \( u_{cr} = u_{cr}(t_5) \), \( i_{cr} = i_{cr}(t_5) \).

(6) Inverter 6: \( t_5 - t_6 \)

Voltage \( u_{cr} \) is not equal to zero when \( t = t_5 \). In the interval 6, \( C_r \) keeps on discharging. \( u_{cr} \) reaches zero when \( t = t_6 \) and interval 6 is finished. Switch \( Q_2 \) can be turned off under zero voltage condition at this moment. Then, current begins to flow through diode \( D_B \).

In the interval 6,

\[
i_{cr} = C_r \frac{du_{cr}}{dt}
\]

(26)

\[
\frac{di_{lf_1}}{dt} = i_{cr} + \frac{di_{lf_2}}{dt} \frac{L_{f_2}}{L_{f_1} + L_{f_2}} u_{cr} - V_0
\]

(27)

Substituting (26) into (27) yields
\[ \frac{d^2 u_{cr}}{dt^2} + \frac{L_{f1} + L_{f2}}{C_r L_{f1} L_{f2}} u_{cr} - \frac{L_{f1} + L_{f2}}{C_r L_{f1} L_{f2}} V_e = 0 \]  \tag{28}

Defining \( \frac{L_{f1} + L_{f2}}{C_r L_{f1} L_{f2}} = p^2 \), solving (28) and substituting \( u_{cr} = u_{cr}(t_s) \) and \( i_{cr} = i_{cr}(t_s) \) when \( t = t_s \), yields

\[ u_{cr} = \sqrt{E^2 + F^2} \sin \left( P(t - t_s) + \varphi \right) + V_o \]  \tag{29}

where, \( E = u_{cr}(t_s) - V_o \), \( F = \frac{i_{cr}(t_s)}{C_r p} \), \( \tan \varphi = \frac{E}{F} \).

When \( t = t_o \), \( u_{cr} \) is equal to zero and interval 6 is finished. Where,

\[ t_o = \frac{1}{P} \left[ \arcsin \left( \frac{-V_o}{\sqrt{E^2 + F^2}} \right) - \varphi \right] + t_4 \]  \tag{30}

(7) **Interval 7 (PWM mode):** \( t_{o} - t_{7} \)

In this interval, voltage \( u_{cr} \) keeps equal to zero and current flows through \( D_3 \). The length of this interval \( T_7 = t_7 - t_o \) is decided by the PWM control strategy.

## VIII. SELECTION OF CIRCUIT PARAMETERS

As mentioned above, intervals 3 and 7 are both operated under PWM mode. So the output voltage can be changed by regulating the duty ratio of the switch. Regulating the duty ratio rather than the frequency of the switch is the main characteristic of the proposed circuit.

In interval 6, in order to make sure that \( u_{cr} \) can be equal to zero, the circuit parameters should make \( \frac{V_o}{\sqrt{E^2 + F^2}} < 1 \) valid.

## IX. SIMULATION RESULTS

A MATLAB model for three-phase zero-current switching Zeta converter has been developed. The three-phase soft switching PWM technique is used in this model. The parameters of this model are:

\[ u_{\phi} = 311 \sin \left( 100 \pi t + \frac{2i}{3} \pi \right), \phi = a, b, c, i = 0, 1, 2. \]

The switching period of \( Q_1 \) is \( T_s = 50 \mu s \). Duty ratio is 0.6.

\[ L = 100 \mu H, L_r = 3 \mu H, L_{f1} = 1 mH, L_{f2} = 1 mH, C_1 = 0.01 F, C_r = 1 \mu F, C_f = 0.01 F, R_{id} = 10 \Omega \]

Fig. 3 depicts the voltage and current waveforms of phase a and there is nearly no phase shift between these waveforms, as expected by the analysis.

## Conclusion

The three-phase soft switching PWM technique based on a zero-current switching Zeta converter is introduced in this paper. This technique controls the output by regulating the duty ratio rather than the switching frequency. By using this topology, high power factor and optimized design are realizable. The principle of operation and the circuit parameters selection are also provided. The simulation results verify the theoretical analysis. The three-phase soft switching PWM technique is a very practical and promising technique.