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Maxwellian Circuits-Based Analysis of Loaded Wire Antennas and Scatterers

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Abstract—Based on the recently proposed Maxwellian circuit (MC) theory, a new method to analyze wire antennas and scatterers loaded with linear lumped elements is demonstrated in this letter. To effectively incorporate the load boundary condition into the numerical solution, the MC model is solved herein using finite element method (FEM).

Index Terms—Finite element method (FEM), loaded wire antennas/scatterers, Maxwellian circuits (MC).

I. INTRODUCTION

LOADED wire antennas and scatterers are used in wide range of applications including modulated scattering techniques devised for electromagnetic fields measurements [1], [2], material characterization [1], antenna pattern synthesis, radar cross-section (RCS) control and optimization [4], etc. The design and optimization of loaded wire antennas and scatterers involve analyzing the near- and far-field parameters over a wide range of frequencies and/or excitations and/or loading conditions. An integral equation formulation with the method-of-moment (MoM) solution is typically applied to such problems [5].

The recently proposed theory of Maxwellian circuit (MC) [6] has provided solutions identical to integral equation solutions describing radiation and scattering from thin wire structures. Although the MC theory has been used subsequently to analyze linear wire antennas and scatterers [7], [8], its application for analyzing loaded wire structures has yet to be demonstrated.

In this letter, we investigate the use of the MC theory for the problem of loaded wire antennas and scatterers. Unlike the solution method presented in [7] and [8] which is based on Euler forward formula and requires solving for the voltages and currents simultaneously (in essence doubling the number of unknowns), the solution developed here is based on the finite element method (FEM). When linear elements are used for the FEM solution, the number of unknowns is reduced to half of those required for the Euler forward formula. Furthermore, with the proposed model, the load can be placed virtually anywhere along the wire and more than one load can be modeled simultaneously. The utility of the FEM to solve MC model differential equations and handling the load condition is demonstrated by considering various radiation and scattering scenarios.

Fig. 1. (a) Wire problem geometry. (b) Schematic of scattering from a loaded wire.

II. PRELIMINARY THEORY

For a thin wire of radius $a$ and length $L$ aligned along the $z$ axis as shown in Fig. 1(a), Pocklington’s integral equation (IE) is written with the reduced kernel as [9]

$$\int_{0}^{L} I(z') K(z, z') dz' = \frac{-j k}{\eta} E_z(z)$$

where

$$K(z, z') = \frac{e^{-j k R}}{4 \pi R^5} (I + j k R (2 R^2 - 3 a^2) + (k a R)^2$$

and $E_z(z)$ is the incident electric field component on the wire. For radiation problems with the impressed voltage at the feed terminals modeled as delta gap source, i.e., $V^i(z) = V_0 \delta(z - z_f)$, where $z_f$ is the feed point, $E_z(z)$ is replaced with $V_0 \delta(z - z_f)/l$ with $l$ being the length of the gap. Pocklington’s IE is typically solved for the current over the wire using the MoM [5].

In [6] and [7], it has been shown that the solution of the Pocklington’s IE is identical to the solution obtained from the following ordinary differential equation describing an equivalent transmission line (TL) model:

$$\frac{dV(z)}{dz} = -Z(z)I(z) + \alpha(z)V(z) + E_z^0(z)$$

$$\frac{dI(z)}{dz} = -Y(z)V(z) + \beta(z)I(z)$$

$$I(0) = I(L) = 0$$

with the per-unit parameters $Z(z) = R(z) + j \omega L(z)$ and $Y(z) = G(z) + j \omega C(z)$. For scatterers and antennas made of perfect electric conductors, the resistance $R$ can be interpreted as the per-unit radiation resistance, and the per-unit conductance $G$ is zero. In this model, radiation and scattering are accounted for by introducing the dependent current and voltage
sources, through the parameters \( \alpha(z) \) and \( \beta(z) \), to the conventional TL equations. The MC parameters \( Z(z), \alpha(z), Y(z), \) and \( \beta(z) \) are determined from two independent solutions of the homogeneous form of the Pocklington’s IE obtained by enforcing two boundary conditions as outlined in [6] and [7]. The salient feature of the MC is that the model parameters are said to be dependent only upon the geometry of the problem, i.e., they are independent of the excitation and terminal boundary conditions.

III. FEM FORMULATION

Equation (3) can be written as

\[
V = \frac{-1}{Y} \frac{dI}{dz} + CI
\]

(5)

where \( C = \beta/Y \). Using (5) into (2), the TL equation can be written as

\[
-\frac{d}{dz} \left[ \frac{1}{Y} \frac{dI}{dz} \right] + \frac{1}{Y} \left[ \alpha + \beta \right] \frac{dI}{dz} + [Z - \alpha C + C']I - E_z^2 = 0
\]

(6)

where \( C' = dC/dz \).

It is important to notice that the above equation reduces to the conventional TL equation when the parameters \( q \) and \( \beta \) are set to zero. The above equation is linear and admits the following quadratic form, i.e., weak-form

\[
\int_0^L \left( \frac{1}{Y} \frac{d\Phi}{dz} \frac{dI}{dz} + \frac{1}{Y} \left[ \alpha + \beta \right] \Phi \frac{dI}{dz} + [Z - \alpha C + C'] \Phi I - \Phi E_z^2 \right) \times dz = \left[ \Phi \frac{1}{Y} \frac{dI}{dz} \right]_0^L = 0
\]

(7)

where \( \Phi(z) \) is the weighting function. The weak-form in (7) is obtained via multiplying (6) by \( \Phi(z) \) and integrating over the length of the wire whereby the second derivative term is integrated by parts. Using (5), the secondary variable appearing in the last term of (7) can be expressed as

\[
\Phi \frac{1}{Y} \frac{dI}{dz} = -\Phi V + \Phi C I.
\]

(8)

The weak-form in (7) can be solved for the current using FEM. For instance, when \( N \) linear elements each of length \( l = L/N \) are used, the weak-form over the \( n \)th element can be written as

\[
Z^n T^n = g^n + b^n
\]

(9)

where the \( n \)th element’s impedance matrix, \( Z^n \), can be found assuming the corresponding TL parameters \( Y^n, \alpha^n, \beta^n, Z^n, C^n \), and \( C' \) are constant over the \( n \)th element support as,

\[
Z^n = \frac{1}{Y^n} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{1}{2Y^n} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}
+ \frac{I}{\omega C^n + C'} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}
- \begin{bmatrix} -C^n & 0 \\ 0 & C^{n+1} \end{bmatrix}.
\]

(10)

The \( n \)th element’s excitation and boundary vectors are found, respectively, as

\[
g^n = \frac{IE_z^n}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad b^n = \begin{bmatrix} V_{1n} \\ -V_{2n} \end{bmatrix}
\]

(11)

where \( E_z^n \) is the field excitation over the \( n \)th element, \( V_{1n} \) and \( -V_{2n} \) are the element’s nodal voltages.

After enforcing continuity of the current and voltage, and subsequently obtaining the final system of element equations, i.e., the impedance matrix \( Z \), the excitation vector \( g \), and the boundary vector \( b \), the current solution is given by

\[
I = Z^{-1}(g + b).
\]

(12)

The voltage continuity requires that \( V_{1n+1} - V_{2n} = 0 \) when no external source is introduced over the \( n \)th element.

The above solution was validated by comparison to the MoM solution for different scattering and radiation scenarios. For example, Fig. 2 shows the magnitude and phase of the current on a dipole antenna of length \( L = \lambda \) and radius 0.001\( \lambda \) fed at \( z = \lambda/4 \).

The general analysis of straight wire antennas and scatterers loaded with lumped linear elements can be found in [5]. Since the subsequent Maxwellian circuit solution depends on the general theory presented in [5], we shall briefly discuss this theory. The scattering and radiation from loaded wire structures can be analyzed by assuming linear network theory assuming the loads are linear or can be linearized. Consider a wire loaded with arbitrary impedance \( Z_L \) (or admittance \( Y_L = 1/Z_L \)). The two terminals of this load identifies a port, say port 1, with well defined current \( I_1 \) and voltage \( V_1 \). In the scattering problem, the wire is illuminated by an electric field produced by a source, for instance, an infinitesimal dipole as shown in Fig. 1(b). In this case, the terminals of the infinitesimal dipole identifies a second port, port 2, with current \( I_2 \) and voltage \( V_2 \). In the radiation problem, the second port is the feed port on the same wire.
The port currents and voltages are related by the short circuit equations as,

\[
\begin{bmatrix}
Y_{11} & Y_{12} \\
Y_{21} & Y_{22}
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = \begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

(13)

where \(Y_{11}\) and \(Y_{22}\) are the self and mutual admittances. In addition, the load imposes the following condition on the voltage across port 1:

\[
V_1 = -\frac{I_1}{Y_L}.
\]

(14)

When the illuminating source is far from the scatterer, \(Y_{11}\) reduces to the input admittance as the scatterer is fed at the load location. Notice that \(Y_{12} = I_c/V_2\), where \(I_c\) is the load terminal current when the load is short circuited. Solving the above two equations for \(V_1\) yields

\[
V_1 = V_c = -\frac{I_c}{Y_{11} + Y_L}.
\]

(15)

The addition of the load has no bearing on the MC model parameter extraction. Furthermore, the FEM development is not changed except for the additional boundary condition on the voltage. The load boundary condition can be handled effectively as follows. Assuming the load is inserted between two elements, say the \(n_L\)th and \((n_L + 1)\)th elements, the voltage drop across the load is written as

\[
-V_2^{n_L} + V_1^{n_L+1} = V_c.
\]

(16)

The load boundary condition can be incorporated into the FEM solution of (12) by inserting the equivalent voltage \(V_c\) in the appropriate location (row) in the boundary vector \(\mathbf{b}\). Multiple linear loads can be treated by following the same procedure after obtaining their equivalent voltages.

V. NUMERICAL RESULTS

In the following implementation, pulse basis functions with point matching are used for the MoM and linear elements are used for the FEM (\(N = 100\)). In line with [5], a loaded dipole antenna of length \(L = \lambda\) and length-diameter ratio \(L/2a = 74.2\) is considered here. The dipole is fed at \(z = L/4\) and loaded at \(z = L/2\) with the impedance, \(Z_L = \infty\). Fig. 3 shows the current distributions obtained using both techniques. These results compare well to their counterparts in [5, Fig. 9(b)].

VI. CONCLUSION

Based on the MC theory, a new method to analyze wire antennas and scatterers loaded with linear lumped elements has been demonstrated. The MC model is solved using FEM whereby the load boundary condition is incorporated as an equivalent voltage source. It has been shown that the developed method yields accurate results for both radiation and scattering scenarios.

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