Decentralized State Feedback and Near Optimal Adaptive Neural Network Control of Interconnected Nonlinear Discrete-Time Systems

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Decentralized State Feedback and Near Optimal Adaptive Neural Network Control of Interconnected Nonlinear Discrete-time Systems

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Abstract— **In this paper, first a novel decentralized state feedback stabilization controller is introduced for a class of nonlinear interconnected discrete-time systems in affine form with unknown subsystem dynamics, control gain matrix, and interconnection dynamics by employing neural networks (NNs). Subsequently, the optimal control problem of decentralized nonlinear discrete-time system is considered with unknown internal subsystem and interconnection dynamics while assuming that the control gain matrix is known. For the near optimal controller development, the direct neural dynamic programming technique is utilized to solve the Hamilton-Jacobi-Bellman (HJB) equation forward-in-time. The decentralized optimal controller design for each subsystem utilizes the critic-actor structure by using NNs. All NN parameters are tuned online. By using Lyapunov techniques it is shown that all subsystems signals are uniformly ultimately bounded (UUB) for stabilization of such systems.**

I. INTRODUCTION

In the recent years, there has been a great interest in the decentralized control of interconnected nonlinear systems using neural networks (NNs) [1-8]. In large-scale systems such as power systems, the feedback delays degrade the controller performance thus necessitating more decentralized control techniques. The decentralized state feedback control effort has focused mainly on nonlinear continuous-time systems [1-5] and limited effort in discrete-time case [6-8]. Although for many applications, continuous-time controller design can be considered, in practice discrete-time control approaches are preferred for computer implementations [9] since continuous-time controller designs render unsatisfactory performance when implemented using low sampled hardware [10]. Therefore, decentralized controller development in discrete-time has been explicitly considered.

In [6], the discrete-time NN controller design for a class of interconnected nonlinear systems is considered where the interconnected terms are considered to be over bounded by a constant. Moreover, the control gain matrix is taken to be unity (i.e. $g(x)=1$). In [7][8], a stabilizing robust controller is proposed by assuming the dynamics are known beforehand. Therefore, the NNs are not utilized in the controller design.

On the other hand, the objective of an optimal controller is to minimize a cost function [11] while ensuring stability. In general, the optimal control of linear systems can be obtained by solving the Riccati equation [11]. However, the optimal control of nonlinear discrete time systems often requires solving the nonlinear Hamilton-Jacobi-Bellman (HJB) equation. Although extensive theoretical work has been done on nonlinear optimal for discrete-time systems [12][13], obtaining a closed-form solution for the HJB equations is still extremely hard.

Therefore, approximate solutions referred to as approximate dynamic programming (ADP) have been proposed to solve the HJB equation forward-in-time for discrete-time nonlinear optimal regulation [12][13]. These solutions are based on policy-value iterations for discrete-time nonlinear systems to solve the nonlinear HJB equation offline. Neural networks (NN) are utilized to approximate the unknown nonlinear functions. The drawback with off-line solutions is the need for large data sets for NN training and a long training procedure.

In this work, first a novel decentralized NN state feedback controller is developed for a class of interconnected nonlinear discrete-time system in Brunovsky canonical form where the restrictions in [6] are relaxed while approximating the unknown subsystem internal dynamics and input coefficient matrix via NNs. The interconnected terms are also considered unknown as opposed to [7][8]. A single NN is used to approximate the control gain matrix $g(x)$ as well as the subsystem internal dynamics $f(x)$ for each subsystem. By using the subsystem state vector, the overall closed-loop stability of the nonlinear system for stabilization is presented.

II. INTERCONNECTED NONLINEAR SYSTEM- FEEDBACK CONTROLLER DESIGN

Consider the class of N interconnected subsystems defined in Brunovsky Canonical form as

$$x_{i1}(k+1)=x_{i2}(k);\ldots;x_{in-1}(k+1)=x_{in}(k)\,;x_{in}(k+1)=f_i(x_i(k))+ g_i(x_i(k))u_i+\Delta_i(x)\,;y_i(k)=x_{i1}(k) \quad (1)$$

where index i represents the subsystem number, N is the number of subsystems, n is the order of the subsystem, $f_i(x_i(k))$, represents subsystem nonlinear internal dynamics, $g_i(x_i(k))$ is the input gain matrix, $\Delta_i(x)$ denotes interconnected terms of the subsystem 'i' with $x=[x_1^T,\ldots,x_N^T]^T$, $x_i=[x_{i1},\ldots,x_{in}]^T$ for $1\le i\le N$.

Define regulation error as

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$$z_{ip}(k) = x_{ip}(k) - x_{ipd} \quad (2)$$

for $1 \le i \le N$ and $1 \le p \le n$, where $x_{ipd} = 0$ is the desired set point for stabilization for the state $x_{ip}(k)$ for $1 \le p \le n$. Next, define the filtered regulation error as

$$r_i(k) = [\lambda_i \ 1]^T z_i(k) \quad (3)$$

where $z_i(k) = [z_{i1}(k) \ z_{i2}(k) \ \ldots z_{in}(k)]^T$ and $\lambda_i = [\lambda_{i1} \ \lambda_{i2} \ \ldots \lambda_{i,n-1}]$. The coefficients λ_{i1} through λ_{ip} are selected such that the poles of the characteristic equation $\zeta(q) = \lambda_{i1} + \lambda_{i2}q + \cdots + \lambda_{i,n-1}q^{n-2} + q^{n-1}$ are inside the unit disc. Before we proceed, the following mild assumptions and definition are needed.

Assumption 1: Let the interconnection terms (weak in nature) in (1) be bounded above in a compact set Ω such that

$$|\Delta_i(x)| \le \sum_{j=1}^{N} \delta_{ij}(r_j) \le \delta_{i0}(x) + \sum_{j=1}^{N} \gamma_{ij}|r_j| \quad (4)$$

where δ_{0i} and γ_{ij} are known small positive constants for $1 \le i \le N$ and $1 \le j \le n$ in contrast with [6].

Assumption 2: The input gain of each subsystem in (1) is bounded away from zero and is bounded in the compact set Ω. Without loss of generality, we assume that it satisfies

$$0 < g_{i\min} \le g_i(x_i(k)) \le g_{i\max} \quad (5)$$

in a compact set Ω where $g_{i\min}$ and $g_{i\max}$ are positive real constants..

A. State Feedback Controller Design

In this part we develop a NN stabilizing controller which employs the filtered regulation error and NN function approximation capability and a novel NN weight estimate tuning scheme. The stability criterion is then elaborated to show the stability of the filtered regulation error as well as NN weight estimates.

Starting with (3), the filtered error dynamics can be written by using (1) and (2) as

$r_i(k+1) = [\lambda_i \ 1]^T z_i(k+1) = f_i(x_i(k)) - x_{ind} + [0 \ \lambda_i]^T z_i + g_i(x_i(k))u_i + \Delta_i(x)$.

The ideal stabilizing control input can be defined as $u_i = u_i^* = u_{id} + K_i r_i(k)$ where $u_{id} = -g_i(x_i(k))^{-1}\left(f_i(x_i(k)) - x_{ind} + [0 \ \lambda_i]^T z_i\right)$. This results in asymptotically stable dynamics $r_i(k+1) = K_i r_i(k)$ with $K_i < 1$ being a positive design constant. However, in practical applications u_{id} is not available since the internal dynamics $f_i(x_i(k))$ and control gain matrix $g_i(x_i(k))$ are unknown.

Thus, we employ NN function approximation property to approximate u_{id} as

$$u_{id} = -g_i(x_i(k))^{-1}\left(f_i(x_i(k)) - x_{ind} + [0 \ \lambda_i]^T z_i\right) = W_i^T \rho_i(x_i, x_{ind}) - \varepsilon_i(x_i, x_{ind})) \quad (6)$$

where W_i is the target NN weight matrix, $\rho_i(.)$ is the activation function, and $\varepsilon_i(.)$ is the approximation error which satisfies $\|\varepsilon_i(.)\| \le \varepsilon_{i\max}$. In practice, the target weights W_i and approximation error ε_i are not available either and only an estimation of the NN weights is available. Thus, u_{id} is approximated as $\hat{u}_{id}$ by a NN to obtain the control input u_i as

$$u_i = \hat{u}_{id} + K_i r_i(k) = \hat{W}_i^T \rho_i(x_i, x_{ind}) + K_i r_i(k) \quad (7)$$

where $\hat{W}_i^T$ is the NN weight estimation matrix. Define the weight estimation error as $\tilde{W}_i = \hat{W}_i - W_i$.

Consequently, by using (7) and adding and subtracting u_{id} in (6), the filtered error (3) dynamics becomes

$$r_i(k+1) = [\lambda_i \ 1]^T z_i(k+1) = g_i(x_i(k))\left(\tilde{W}_i^T \rho_i + \varepsilon_i + K_i r_i\right) + \Delta_i(x) \quad (8)$$

Define the NN weight update law as

$$\hat{W}_i^T(k+1) = c_i \hat{W}_i(k) - c_i^{-1} \alpha_i \rho_i r_i(k+1) \quad (9)$$

where $c_i < 1$ is a positive design constant. By subtracting the ideal weights from (9), we have

$$\tilde{W}_i^T(k+1) = c_i \tilde{W}_i(k) - c_i^{-1} \alpha_i \rho_i r_i(k+1) - (I - c_i) W_i \quad (10)$$

The presence of the parameter $c_i < 1$ in the above update law provides stability of the weights where (9) becomes a stable system with input $r_i(k+1)$, and thus, prevents the NN weight estimates from remaining large after the filtered regulation error becomes small, as opposed to conventional update laws [6][14] where $c_i = 1$.

B. Stability analysis

In this part we introduce the following theorem to show that the nonlinear discrete-time interconnected system (1) along with controller (7) and the NN weight update law (9) are stable while the filtered regulation errors $r_i(k)$ and weight estimation errors $\tilde{W}_i(k)$ of the individual subsystems are bounded in the presence of unknown internal dynamics $f_i(x_i(k))$ and control gain matrix $g_i(x_i(k))$, and unknown interconnection terms $\Delta_i(x)$ for $1 \le i \le N$.

Once the filtered regulation error $r_i(k)$ is proven bounded, it is treated as a bounded input for the linear time-invariant system (3) as $\lambda_{i1} z_{i1}(k) + \cdots + \lambda_{i,n-1} z_{i1}(k+n-2) + z_{i1}(k+n-1) = r_i(k)$ which yields bounded results for the output $z_{i1}(k)$ for all $1 \le i \le N$.

Theorem 1 ***(Decentralized State Feedback NN Controller):*** *Consider the nonlinear discrete-time interconnected system given by (1). Consider that the Assumptions 1 and 2 hold and $x_{ipd} = 0$ (for all $1 \le i \le N$ and $1 \le p \le n$) and initial conditions for system (1) are bounded in the compact set Ω. Let the unknown nonlinearities in each subsystem be approximated by a NN whose weight update is provided by (9). Then there exist a set of control gains K_i and filtered error coefficients λ_i, associated with the given control inputs (7) such that the filtered error $r_i(k)$ as well as the NN weight estimation error $\tilde{W}_i$ are UUB for all $1 \le i \le N$.*

Proof. Define the overall Lyapunov function candidate $L = L_W + L_r$, where $L_r(k) = \sum_{i=1}^{N}\left(r_i(k)\Big/\sqrt{g_i(x(k-1))}\right)^2$ and

$L_W(k)=\sum_{i=1}^{N} 1/\alpha_i \tilde{W}_i^T(k)\tilde{W}_i(k)$. Then, the first difference of the Lyapunov function due to the first term becomes

$$\Delta L_r=\sum_{i=1}^{N}\left(r_i(k+1)\Big/\sqrt{g_i(x(k))}\right)^2-\sum_{i=1}^{N}\left(r_i(k)\Big/\sqrt{g_i(x(k-1))}\right)^2 \quad (11)$$

Substituting the filtered error (8) into (11) and expanding the terms, we obtain

$$\Delta L_r=\sum_{i=1}^{N}\Delta L_{ri}=\sum_{i=1}^{N}\left(\sqrt{g_{i(k)}}(\tilde{W}_i^T\rho_i(x_i)+\varepsilon_i+K_ir_i)+\Delta_i(x)\right)^2-\sum_{i=1}^{N}\left(\frac{r_i(k)}{\sqrt{g_{i(k-1)}}}\right)^2 \quad (12)$$

Next, by using (10), the first difference due to the second term in the overall Lyapunov function candidate is obtained

$$\Delta L_W=1/\alpha_i\left(c_i\tilde{W}_i(k)-c_i^{-1}\alpha_i\rho_ir_i(k+1)-(I-c_i)W_i\right)^2-1/\alpha_i\tilde{W}_i^T(k)\tilde{W}_i(k) \quad (13)$$

Expanding the first difference of the overall Lyapunov function candidate $\Delta L=\Delta L_r+\Delta L_W$ by using (11) and (12) and expanding the terms, applying the Cauchy-Schwarz inequality $(a_1+a_2+\cdots+a_n)^2 \le n(a_1^2+a_2^2+\cdots+a_n^2)$, $\rho_i^T\tilde{W}_i\tilde{W}_i^T\rho_i=\tilde{W}_i^T\rho_i\rho_i^T\tilde{W}_i$, Assumption 1, and $\sum_{i=1}^{N}\beta_i\left|\Delta_i(x)\right|^2 \le \sum_{i=1}^{N}\beta_i((N+1)\delta_{i0}^2+(N+1)\sum_{j=1}^{N}\gamma_{ij}^2r_j^2= \sum_{i=1}^{N}\beta_i(N+1)\delta_{i0}^2+\sum_{i=1}^{N}\sum_{j=1}^{N}\beta_j(N+1)\gamma_{ji}^2r_i^2$ where $\beta_i=\beta_i(x_i)$ is an arbitrary real-valued function of x_i, ΔL becomes

$$\Delta L\le\sum_{i=1}^{N}(C'_{ir}r_i^2+C'_{iw}\left\|\tilde{W}_i(k)\right\|^2+C'_{i\varepsilon}) \quad (14)$$

where

$$C'_{i\varepsilon}=\sum_{i=1}^{N}\begin{pmatrix}\left(1+g_{i\max}+g_i(k)^2+4(1-c_i)c_i^{-1}g_{i\max}^2+4\alpha_ic_i^{-2}\left\|\rho_i\right\|^2g_i(k)^2\right)\varepsilon_{i\max}^2\\ +\frac{1}{\alpha_i}(1-c_i)^2\left\|W_i\right\|^2+\frac{c_i(1-c_i)}{\alpha_i}\left\|W_i\right\|^2+(1-c_i)c_i^{-1}\left\|W_i\right\|^2\left\|\rho_i\right\|_{\max}^2\\ +2(I-c_i)^2\left\|W_i\right\|^2\left\|\rho_i\right\|_{\max}^2+(2+\frac{1}{g_{i\min}}+4(1-c_i)c_i^{-1}+4\alpha_ic_i^{-2}\left\|\rho_i\right\|_{\max}^2)(N+1)\delta_{0i}^2\end{pmatrix},$$

$$C'_{ir}=\frac{1}{g_{i\max}}-\left(1+g_{i\max}+g_{i\max}^2+4(1-c_i)c_i^{-1}g_{i\max}^2+4\alpha_ic_i^{-2}\left\|\rho_i\right\|_{\max}^2g_{i\max}^2\right)K_i^2-\sum_{j=1}^{N}\left[2+1/g_{j\min}+4(1-c_j)c_j^{-1}+4\alpha_jc_j^{-2}\left\|\rho_j\right\|_{\max}^2\right](N+1)\gamma_{ji}^2$$

, $C'_{iw}=\frac{1-c_i}{\alpha_i}+C'_{i\rho w}$, and $C'_{i\rho w}=\begin{pmatrix}g_{i\min}-4(1-c_i)c_i^{-1}g_{i\max}^2\\ -4\alpha_ic_i^{-2}\left\|\rho_i\right\|_{\max}^2g_{i\max}^2\end{pmatrix}\left\|\rho_i\right\|_{\max}^2$.

Therefore, $\Delta L\le 0$ in (14) provided the filtered error and weight estimation errors hold for all $1\le i\le N$ as

$$\left|r_i\right|>\sqrt{C'_{i\varepsilon}/C'_{ir}}\text{, or}\left\|\tilde{W}_i^T(k)\right\|>\sqrt{C'_{i\varepsilon}/C'_{iw}}. \quad (15)$$

This guaranties the boundedness of the weight estimation error $\tilde{W}_i(k)$ and filtered error $r_i(k)$ which in turn shows that the errors $z_i(k)$ are UUB for all $1\le i\le N$ as explained. ■

The bounds can be reduced through $C'_{i\varepsilon}$ which can be achieved if the constant c_i is selected to be close to one, whereas K_i and α_i are selected to be small positive constants for $1\le i\le N$, which, in turn, causes the stability bounds (15) to decrease. Also, the interconnection bound parameters γ_{ji} and δ_{0i} (for $1\le i\le N$) need to be small which can be achieved when the effect of the interconnection dynamics are small or *weak* in nature. On the other hand, the NN function approximation error $\varepsilon_{i\max}$ in $C'_{i\varepsilon}$ can be made small by increasing the number of NN hidden-layer neurons [14] for $1\le i\le N$.

III. Decentralized Optimal Control

In this section our goal is to find optimal control inputs $u_i(x_i(k))$ or also denoted here as $u_i(k)$ for $1\le i\le N$ in order to stabilize the interconnected system (1) while minimizing the infinite horizon cost function

$$J(x(k))=Q(x(k))+u^TRu+J(x(k+1)) \quad (16)$$

with $Q(x)=\sum_{i=1}^{N}Q_i(x_i)$ and $R=diag(R_1,\ldots,R_N)$ where $Q(x)\in\Re^+$ is a positive definite function of the overall interconnected system states, $R\in\Re^{N\times N}$ is positive definite matrix, and $u(k)=u(x(k))=[u_1(x_1(k)),\ldots,u_N(x_N(k))]^T$ where $u_i(x_i(k))$ is only a function of the *i*th subsystem states (for $1\le i\le N$).

Define $g(x(k))=diag(\bar{g}_1(x_1(k)),\ldots,\bar{g}_N(x_N(k)))$, $\bar{g}_i(x_i(k))_{n\times 1}=[0,\ldots,g_i(x_i(k))]^T$, $f(x(k))=[\bar{f}_1^T(x(k)),\ldots,\bar{f}_N^T(x(k))]^T$, $\bar{f}_i(x(k))_{n\times 1}=[x_{i2}(k),\ldots,x_{i,n-1}(k),f_i(x_i(k))+\Delta_i(x(k))]^T$ for $1\le i\le N$. Also, define the subsystem cost function (17) as

$$J_i(x(k))=Q_i(x_i(k))+u_i(x(k))^TR_iu_i(x(k))+J_i(x(k+1))\le\infty \quad (17)$$

By stationarity condition [11], the optimal policy that minimizes the cost function $J_i(.)$ can be obtained by

$$u_i^*(x(k))=-\frac{1}{2}R_i^{-1}\bar{g}_i(x_i(k))^T\,\partial J_i^*(x(k+1))\big/\partial x_i(k+1) \quad (18)$$

Substituting the subsystem optimal control policy (18) into equation (17) results in the nonlinear partial difference HJB equation where the subsystem optimal policy and cost function are obtained as functions of the overall interconnected system state vector $x(k)$ due to the presence of the interconnection term, whereas the control policy $u_i(.)$ has to be a function of subsystem state vector, $x_i(k)$, only to be synthesized.

Define the notations "$J_i^*(.)$" and "$u_i^*(.)$" obtained by (17) and (18) to represent the optimal value of the cost function "$J_i(.)$" and control policy "$u_i(.)$", respectively. Consequently, the optimal control of the interconnected system can be viewed as the subsystem optimal problems corresponding to the cost functions (17) for $1\le i\le N$. As mentioned earlier, due to unavailability of the overall system state vector, a nearly optimal subsystem policy can be defined which leads to the nearly optimal policy of the overall interconnected system. However, finding a solution for the subsystem optimal policy that minimizes (17) in the presence of unknown interconnection terms is still generally hard.

Consequently, in this paper, we use an approximation of the subsystem cost function and optimal policy which are obtained through subsystem states only. Here, in order to obtain the nearly optimal control policy which is a function of subsystem states, $u_i'^*(.)$ is used as an approximation of $u_i(.)$ by using only subsystem states. Moreover, " $J_i'^*(.)$ " is an approximation of $J_i^*(.)$ when only subsystem states and $u_i'^*(.)$ are employed. Now, assume that

$$J_i^*(x(k)) = J_i'^*(x_i(k)) + \overline{\vartheta}_{1i}(x(k));\ u_i^*(x(k)) = u_i'^*(x_i(k)) + \vartheta_{2i}(x(k)) \quad (19)$$

where the terms $\overline{\vartheta}_{1i}(x(k))$ and $\vartheta_{2i}(x(k))$ reflect the effects of the interconnection terms to be discussed later. Subsequently, the nearly optimal subsystem policy $u_i'^*(x_i(k))$ and the corresponding subsystem cost function $J_i'^*(x_i(k))$ for $1 \le i \le N$ are obtained. From (17) and (19), the subsystem cost function $J_i'^*(x_i(k))$ satisfies

$$J_i'^*(x_i(k)) = Q_i(x_i(k)) + u_i'^*(x_i(k))^T R_i u_i'^*(x_i(k)) + J_i'^*(x_i(k+1)) + \aleph(x(k)) \quad (20)$$

where $\aleph(x(k)) = J_i^*(x(k)) - \overline{\vartheta}_{1i}(x(k) - Q_i(x_i(k)) - \big(u_i^*(x_i(k)) - \vartheta_{2i}(x(k))\big)^T R_i \big(u_i^*(x_i(k)) - \vartheta_{2i}(x(k))\big) - \big(J_i^*(x_i(k+1)) - \overline{\vartheta}_{1i}(x(k+1))\big)$

Note that $x(k+1) = f(x(k)) + g(x(k))u(x(k)) = \Im(x(k))$, and thus, $x(k) = \Im^{-1}(x(k+1))$. Hence, the term $\aleph(x(k)) = J_i^*(x(k)) - \overline{\vartheta}_{1i}(x(k) - Q_i(x_i(k) - u_i'^*(x_i(k))^T R_i u_i'^*(x_i(k))) - J_i'^*(x_i(k+1))$ can be written as $\aleph(x(k)) = \vartheta_{1i}(x(k+1))$.

Assumption 3: Let the terms $\vartheta_{1i}(x(k)$ in (20) be bounded above as a function of the interconnection terms in the compact set Ω such that $|\vartheta_{1i}(x)| \le \vartheta_{1i0}(x) + \sum_{j=1}^{N} \kappa_{ij} |\Delta_j(x)|$, where $|\vartheta_{1i0}(x)| \le \vartheta_{1i0M}$, with ϑ_{1i0M} and κ_{ij} are positive constants for $1 \le i \le N$.

By using the stationarity condition [11] $\partial J_i^*(x_i(k))/\partial u_i(x(k)) = 0$, equation (18) yields

$$u_i^*(x(k)) = u_i'^*(x_i(k)) + \vartheta_{2i}(x(k)) = -0.5 R_i^{-1} \overline{g}_i(x_i(k))^T \frac{\partial J_i^*(x(k+1))}{\partial x_i(k+1)} = -0.5 R_i^{-1} \overline{g}_i(x_i(k))^T \partial J_i'^*(x_i(k+1))/\partial x_i(k+1) + \vartheta_{3i}(x(k)) \quad (21)$$

where

$$\vartheta_{3i}(x(k)) = -0.5 R_i^{-1} \overline{g}_i(x_i(k))^T \partial \overline{\vartheta}_{1i}(x(k+1))/\partial x_i(k+1)$$

Since the HJB equation (20) has no known closed-form solution, we use neural networks (NN), with subsystem states as their inputs, to approximate the cost function $J_i'^*(x_i(k))$ as well as the nearly optimal policy $u_i'^*(x_i(k))$ in a forward-in-time manner. In addition, the effect of the interconnection terms is overcome by augmenting a feedforward term similar to the tracking problem [15] while the nearly optimal control design for each subsystem is performed simultaneously.

Define $J_i'^*(x_i(k)) = W_{ci}^T \phi_i(x_i(k)) + \varepsilon_{ci}$ (22)

and $u_i'^*(x_i(k)) = u_{io}(x_i(k)) + u_{iF}(x_i(k)) = W_{ai}^T \psi_i(x_i(k)) - \varepsilon_{ai} + F_i(x_i(k))$ (23)

respectively, where $u_{io}(x_i(k)) = W_{ai}^T \psi_i(x_i(k)) - \varepsilon_{ai}$ is the optimal policy, $u_{Fi}(x_i(k)) = F_i(x_i(k))$ is the feedforward term, W_{ci} and W_{ai} are the target subsystem critic and action NN weights which are assumed to be bounded satisfying $\|W_{ci}\| \le W_{ciM}$, $\|W_{ai}\| \le W_{aiM}$, with ε_{ci} and ε_{ai} denote approximation errors satisfying $|\varepsilon_{ci}| \le \varepsilon_{ciM}$, $|\varepsilon_{ai}| \le \varepsilon_{aiM}$, with $\phi_i(.)$ and $\psi_i(.)$ being the NN activation function vectors for the critic and action NNs, respectively [14].

Assumption 4: The gradient of the approximation error satisfies $|\partial \varepsilon_{ci}(x_i(k+1))/\partial x_i(k+1)| \le \overline{\varepsilon}_{cM}$ in Ω for $1 \le i \le N$.

Next the stabilizer design is introduced.

IV. Actor-critic Stabilizer Design

The central theme of employing an actor-critic design is to use parametric structures, such as neural networks (NNs), to approximate the cost function and optimal control law by assuming that the local states are available for measurement.

A. *The Critic Network Design*

The objective of the optimal control law is to stabilize the system (1) while minimizing the cost functions (20). Since the cost function (20) is analytically not available it will be approximated by a NN as provided in (22). Consequently, by employing the subsystem states, the cost function $J_i'^*(x_i(k))$ is approximated by

$$\hat{J}_i'^*(x_i(k)) = \hat{W}_{ci}^T(k) \phi_i(x_i(k)) \quad (24)$$

where $\hat{W}_{ci}$ is the estimated weight matrix of the target W_{ci}. Note that the function $\vartheta_{1i}(.)$ (and $J_i^*(x(k))$) in (20) cannot be approximated due to unavailability of overall state vector.

Now we construct the critic network by augmenting the individual vectors, we obtain

$\mathbf{W}_c = diag(W_{c1}, \ldots, W_{cN})$, $\hat{\mathbf{W}}_c(k) = diag(\hat{W}_{c1}(k), \ldots, \hat{W}_{cN}(k))$

$$\mathbf{\Phi}(k) = \begin{bmatrix} [\phi_1(k)\ \ \phi_1(k-1)\ \ \phi_1(k-p)] \\ \vdots \\ [\phi_N(k)\ \ \phi_N(k-1)\ \ \phi_N(k-p)] \end{bmatrix}, \text{ and } \boldsymbol{\varepsilon}_c(k) = \begin{bmatrix} [\varepsilon_{c1}(k)\ \ \varepsilon_{c1}(k-1)\ \ldots\ \varepsilon_{c1}(k-p)] \\ \vdots \\ [\varepsilon_{cN}(k)\ \ \varepsilon_{cN}(k-1)\ \ldots\ \varepsilon_{cN}(k-p)] \end{bmatrix}$$

where $p+1$ is the number of past values from the previous time steps. The overall cost function is computed as

$$\hat{\mathbf{J}}(x(k)) = \hat{\mathbf{W}}_c^T(k) \mathbf{\Phi}(x(k)) \quad (25)$$

where $\hat{\mathbf{J}}(x(k)) = [\hat{J}_1'^*(x_1(k)), \ldots, \hat{J}_N'^*(x_N(k))]^T$. Now define the critic error as

$$\mathbf{E}_c(k) = \mathbf{Q}(k-1) + \hat{\mathbf{W}}_c^T(k) \Delta\mathbf{\Phi}(x(k)) \quad (26)$$

where $\Delta\mathbf{\Phi}(k) = \mathbf{\Phi}(x(k)) - \mathbf{\Phi}(x(k-1))$ (27)

and $\mathbf{Q}(k) = [\overline{Q}_1^T \ldots \overline{Q}_N^T]^T$ where $\overline{Q}_i = [Q_i(x_i(k)) + u_i(x_i(k))^T R_i u_i(x_i(k)) \ \ldots \ Q_i(x_i(k-p)) + u_i(x_i(k-p))^T R_i u_i(x_i(k-p))]$ for $1 \le i \le N$. Then, the critic error dynamics become

$$\mathbf{E}_c(k+1) = \mathbf{Q}(k) + \hat{\mathbf{W}}_c^T(k+1) \Delta\mathbf{\Phi}(k+1) \quad (28)$$

By selecting the critic weight update law as [16]

$$\hat{\mathbf{W}}_c(k+1) = \Delta\mathbf{\Phi}(k+1)\left(\Delta\mathbf{\Phi}^T(k+1)\Delta\mathbf{\Phi}(k+1)\right)^{-1} \times \left(\alpha_c \mathbf{E}_c{}^T(k) - \mathbf{Q}^T(k)\right) \quad (29)$$

the critic error dynamics becomes

$$\mathbf{E}_c(k+1) = \alpha_c \mathbf{E}_c(k) \quad (30)$$

where $\alpha_c = diag(\alpha_{c1},\ldots,\alpha_{cN})$ with α_{ci} being a design constant for $1 \le i \le N$. From (20) and (22) we obtain

$$\begin{aligned}\mathbf{Q}(k) &= -\mathbf{W}_c{}^T\mathbf{\Phi}(k+1) + \mathbf{W}_c{}^T\mathbf{\Phi}(k) - \boldsymbol{\varepsilon}_c(k+1) + \boldsymbol{\varepsilon}_c(k) - \mathbf{V}_1(k) \\ &= -\mathbf{W}_c{}^T\Delta\mathbf{\Phi}(k+1) - \Delta\boldsymbol{\varepsilon}_c(k+1)\end{aligned} \quad (31)$$

where $\Delta\boldsymbol{\varepsilon}_c(k+1) = \boldsymbol{\varepsilon}_c(k+1) - \boldsymbol{\varepsilon}_c(k) - \mathbf{V}_1(x(k))$ and

$$\mathbf{V}_1(x(k)) = \begin{bmatrix} [\vartheta_{11}(k) \;\; \vartheta_{11}(k-1) \; \ldots \; \vartheta_{11}(k-p)] \\ \vdots \\ [\vartheta_{1N}(k) \;\; \vartheta_{1N}(k-1) \; \ldots \; \vartheta_{1N}(k-p)] \end{bmatrix}.$$

Define weight estimation error to be $\widetilde{\mathbf{W}}_\mathbf{c} = \mathbf{W}_\mathbf{c} - \hat{\mathbf{W}}_\mathbf{c}$. By using (28), (29) and (31), it can be concluded that

$$\widetilde{\mathbf{W}}_c^T(k+1)\Delta\mathbf{\Phi}(k+1) = -\alpha_c\left(\mathbf{Q}(k-1) + \hat{\mathbf{W}}_c^T(k)\Delta\mathbf{\Phi}(k)\right) - \Delta\boldsymbol{\varepsilon}_c(k+1) \quad (32)$$

Then, by shifting the time step in (31) to obtain $\mathbf{Q}(k-1)$ and substituting it in (32) , we obtain

$$\Delta\mathbf{\Phi}^T(k+1)\widetilde{\mathbf{W}}_c(k+1) = \alpha_c\Delta\mathbf{\Phi}^T(k)\widetilde{\mathbf{W}}_c(k) + \alpha_c\Delta\boldsymbol{\varepsilon}_c(k) - \Delta\boldsymbol{\varepsilon}_c(k+1) . \quad (33)$$

As a result, the dynamics of the weight estimation errors can be obtained as

$$\begin{aligned}\widetilde{\mathbf{W}}_c(k+1) &= \alpha_c\Delta\mathbf{\Phi}(k+1)(\Delta\mathbf{\Phi}^T(k+1)\Delta\mathbf{\Phi}(k+1))^{-1} \times \\ &\left(\Delta\mathbf{\Phi}^T(k)\widetilde{\mathbf{W}}_i(k) + \Delta\boldsymbol{\varepsilon}_c(k)\right) - \Delta\mathbf{\Phi}(k+1)(\Delta\mathbf{\Phi}^T(k+1)\Delta\mathbf{\Phi}(k+1))^{-1}\Delta\boldsymbol{\varepsilon}_c(k+1)\end{aligned} \quad (34)$$

The dynamics of the weight estimation errors in (34) will be well-defined provided the matrix $\Delta\mathbf{\Phi}^T(k)\Delta\mathbf{\Phi}(k)$ is invertible which will be shown next by using the following Lemma.

Lemma 1. *Let $u_i(.)$ be an admissible control for system (1) for $1 \le i \le N$. It can be shown that $\Delta\phi_i(x_i(k+1)) = \{\phi_{i\ell}(x_i(k+1)) - \phi_{i\ell}(x_i(k))\}_1^L$ is basis.*

B. The Action Network Design for Stabilization

The action networks in each subsystem generate the nearly optimal control inputs in order to minimize the cost function (16). The action network for subsystem 'i' is defined such that, by using (22), it estimates the nearly optimal control policy (21) as

$$u_i^*(x(k)) = \xi_i(x_i(k+1)) - \frac{1}{2}R_i^{-1}\overline{g}_i(x_i(k))^T\,\partial\overline{\vartheta}_{1i}(x(k+1))/\partial x_i(k+1) \quad (35)$$

where

$$\xi_i(x_i(k+1)) = -\frac{1}{2}R_i^{-1}\overline{g}_i(x_i(k))^T\left(\frac{\partial\phi_i^T(x_i(k+1))}{\partial x_i(k+1)}W_{ci}(k) + \frac{\partial\varepsilon_{ci}(x_i(k+1))}{\partial x_i(k+1)}\right),$$

which is a function of $x(k)$ (since $x_i(k+1)$ is a function of $x(k)$). In the optimal decentralized control presented herein, the term $x_i(k+1)$ is a function of entire state vector $x(k)$ which is unavailable for measurement and prevents generating the optimal policy in (21). Thus, the dependency of the critic network derivative as well as $\overline{\vartheta}_{1i}(x(k+1))$ in (35) to the overall system state vector $x(k)$ must be carefully considered. Moreover, an appropriate modification of the control input through an additional (feedforward) term $F_i(x(k))$ in (23) is considered such that the stability analysis can be performed in the presence of the interconnection terms.

Next, the following Lemma introduces the dependency of the optimal policy (21) to the interconnection terms.

Lemma 2. *Consider the nonlinear interconnected system (1) where the subsystem optimal policies are given in (35). Then, the optimal policy (35) can be represented as $u_i^*(x(k)) = \overline{\sigma}_i(x_i(k)) - \overline{\beta}_{in}\Delta_i(x(k)) + \overline{\sigma}_i'(x(k))$ where $\overline{\sigma}_i'(x_i(k))$ is a function of subsystem states x_i, $\overline{\beta}_{in}$ is a constant, and $\overline{\sigma}_i(x(k))$ is assumed to be small value for $1 \le i \le N$.*

Proof. Proof is performed by using Taylor series expansion and is omitted due to page constraints. ■

Assumption 5: In the compact set Ω, let the term $\sigma_i'(x(k))$ be $\sigma_i'(x(k)) = \overline{\sigma}_i'(x(k)) + 0.5R_i^{-1}\overline{g}_i(x_i(k))^T\left(\partial\overline{\vartheta}_{1i}(x(k+1))/\partial x_i(k+1)\right)$ and bounded above as a function of the interconnection terms such that $|\sigma_i'(x)| \le \sigma_{i0}'(x) + \sum_{j=1}^N \mu_{ij}|\Delta_j(x)|$, where $|\sigma_{i0}'(x)| \le \sigma_{i0M}'$, with σ_{i0M}' and μ_{ij} are positive constants for $1 \le i \le N$.

Note that only $\overline{\sigma}_i(x_i)$ in $u_i^*(x(k))$ can be approximated by the action NN (i.e. $\overline{\sigma}_i(x_i) = W_{ai}{}^T(k)\psi_i(x_i(k)) - \varepsilon_{ai}(k) + F_i(x_i(k))$, while the others have to be explicitly considered in the proof. Thus, rearranging the terms in $u_i^*(x(k))$, we obtain

$$\begin{aligned}&W_{ai}{}^T(k)\psi_i(x_i(k)) - \varepsilon_{ai}(k) + F_i(x_i(k)) + \overline{\beta}_{in}(x_{i0})\Delta_i(x(k)) \\ &+ \overline{\sigma}_i'(x(k)) - \xi_i(x_i(k+1)) + \frac{1}{2}R_i^{-1}\overline{g}_i{}^T(x_i(k))\partial\overline{\vartheta}_{1i}(x(k+1))/\partial x_i(k+1) = 0\end{aligned} \quad (36)$$

This step is important while analyzing the action NN error dynamics. Next, the nearly optimal controller (23) is approximated by a NN as

$$\hat{u}_i(x_i(k)) = \hat{u}_{io}(x_i(k)) + u_{iF}(x_i(k)) = \hat{W}_{ai}{}^T\psi_{ai}(x_i(k)) + F_i(x_i(k)) \quad (37)$$

where $\hat{W}_{ai}$ is the estimated weight matrix of target weights W_{ai} while $F_i(x_i(k))$ is defined as

$$F_i(x_i(k)) = -g_i(x_i(k))^{-1}[0\;\lambda_i]^T z_i(k) \quad (38)$$

The feedforward term (38) will improve the stability in the presence of the interconnection terms by reducing the stability bound and will be small provided the regulation error is small.

Define the weight estimation error for the action NN as $\widetilde{W}_{ai} = W_{ai} - \hat{W}_{ai}$. Then the action error become

$$e_{ai}(k+1) = \hat{W}_{ai}{}^T(k)\psi_i(x_i(k)) + F_i(x_i(k)) + \frac{1}{2}R_i^{-1}\overline{g}_i(x_i(k))^T\frac{\partial\phi_i(x_i(k+1))}{\partial x_i(k+1)}\hat{W}_{ci}(k) \quad (39)$$

Subtracting (36) from (39) yields

$$\begin{aligned}e_{ai}(k+1) &= -\widetilde{W}_{ai}{}^T(k)\psi_i(k) + \varepsilon_{ai}(k) - \sigma_i'(x) - \overline{\beta}_{in}(x_{i0})\Delta_i(k) \\ &-\frac{1}{2}R_i^{-1}\overline{g}_i(x_i(k))^T\frac{\partial\phi_i(x_i(k+1))}{\partial x_i(k+1)}\widetilde{W}_{ci}(k) - \frac{1}{2}R_i^{-1}\overline{g}_i(x_i(k))^T\frac{\partial\varepsilon_i(x_i(k+1))}{\partial x_i(k+1)}\end{aligned} . \quad (40)$$

Also, define the action NN weight update law as

$$\hat{W}_{ai}(k+1) = \hat{W}_{ai}(k) - \alpha_{ai} \frac{\psi_i(x_i(k)) e_{ai}(k+1)}{\psi_i^T(x_i(k))\psi_i(x_i(k))+1} \qquad (41)$$

which renders the following weight estimation error dynamics

$$\widetilde{W}_{ai}(k+1) = \widetilde{W}_{ai}(k) + \alpha_{ai} \frac{\psi_i(x_i(k)) e_{ai}(k+1)}{\psi_i^T(x_i(k))\psi_i(x_i(k))+1}. \qquad (42)$$

C. Filtered Regulation Error and Stability Analysis

By using the subsystem dynamics (1) and control input (37), the filtered error dynamic defined in (3) is given by

$$r_i(k+1) = f_i'(x_i(k)) + \Delta_i(x(k)) + g_i(x_i(k))u_i'^*(x_i(k)) - g_i(x_i(k))\widetilde{W}_{ai}^T(k)\psi_{ai}(x_i(k)) - g_i(x_i(k))\varepsilon_{ai} \qquad (43)$$

where $f_i'(x_i(k)) = f_i(x_i(k)) + [0\ \lambda_i]^T z_i(k)$.

In this part we show that the nonlinear discrete-time interconnected system (1) along with controller (37), critic network (24), NN weights, the filtered error and weight estimation error (43) and (42) of the individual subsystems are bounded, even in the presence of the unknown interconnection terms $\Delta_i(x)$ for $1 \le i \le N$. A Lemma is introduced before the Theorem.

Lemma 3. *Consider the large-scale interconnected system (1). Suppose that there exist ideal action NN weights* W_{ai} *for* $1 \le i \le N$ *which provide the nearly optimal controller (23) for system (1). Let's* $B_{fi}(x) = \left\| f_i' + g_i u_i'^* + \Delta_i \right\|^2$ *Then* $\left| B_{fi}(x) \right| \le \varepsilon_{Bfi}$ as $k \to \infty$ *for* $1 \le i \le N$ with ε_{Bfi} being a small positive value.

Theorem 2: *Consider the nonlinear interconnected discrete-time system given by (1). Let* $u_i(x_i(k))$ *be an initial admissible control input for the ith subsystem of the nonlinear interconnected discrete-time system for* $1 \le i \le N$. *Let the Assumptions 1 through 5 hold and that the initial conditions for system (1) are bounded in the compact set* Ω. *Let the weight tuning for the critic and action networks be provided by (29) and (41), respectively. Then, the critic error (26), the action error (39), and regulation error* $r_i(k)$ *along with the weight estimation errors of the critic and action NNs for* $1 \le i \le N$ *are all uniformly ultimately bounded (UUB) for all* $k \ge k + T_0$. *In addition,* $u \to u^* + \varepsilon_b$ with ε_b being a positive constant.

Proof: Omitted due to space considerations. ■

V. Conclusions

In this paper, both a decentralized state feedback and an optimal controller were introduced for interconnected nonlinear discrete-time system. For the state feedback controller, the internal dynamics, interconnection terms and the input gain matrix are considered unknown while for the optimal controller, the internal dynamics and interconnection terms are considered unknown. Novel update laws developed in this work render uniform ultimate boundedness result.

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