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An Extended Kalman Filter (EKF) Approach on Fuzzy System Optimization Problem

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Abstract – **Optimizing the membership functions of a fuzzy system can be viewed as a system identification problem for a nonlinear dynamic system. Basically, we can view the optimization of fuzzy membership functions as a weighted least-squares minimization problem, where the error vector is the difference between the fuzzy system outputs and the target values for those outputs. The extended Kalman filter algorithm is a good choice to solve this system identification problem, not only because it is a derivative-based algorithm that is suitable to solve the weighted least-squares minimization problem, but also because of its appealing *predictor-corrector* feature for nonlinear system model. In this paper, we present an extended Kalman filter approach to optimize the membership functions of the inputs and outputs of the fuzzy controller. The effect of the measurement noise covariance R on the convergence of the fuzzy controller is also investigated. Experimental results show that the optimized fuzzy controller achieves significant improvement on performance. In addition, the smaller the measurement noise covariance R is, the faster the optimized fuzzy controller would converge.**

1. INTRODUCTION

The performance of a fuzzy system depends on both its rule base and its membership functions. Given a rule base, the membership functions can be optimized in order to obtain the best performance from the fuzzy system. Several methods have been proposed to solve this problem. Jacomet created a penalty function and applied it in his optimization algorithm [1]; Nakamura adopted numerical optimization techniques to obtain the optimal values of fuzzy membership function parameters such that the performance measure is minimized [2]; a heuristic method was presented by Tao [3]; Wu and Chen presented a new fuzzy learning algorithm based on the α-cuts of the equivalence relations and the α-cuts of fuzzy sets to construct membership functions [4]. Several derivative-based algorithms are also proposed in [5][6].

Kalman filter is a powerful mathematical tool for stochastic estimation from noisy sensor measurements. It is proposed by Rudolph E. Kalman, who described a recursive method for the discrete data linear filtering problem [7]. The extended Kalman filter algorithm resembles that of a *predictor-corrector* algorithm for solving numerical problems [8]. It makes an approximation of the system states, called the *a priori* estimate, which is used to predict the measurement that is about to arrive. This estimate is adjusted by the actual measurement, and thus obtains the *a posteriori* estimate.

Then the *a posteriori* estimate is used to predict the measurement at the next time step, and becomes the *a priori* estimate for the next time step. This recursive nature is one of the very appealing features of the Kalman filter. It recursively conditions the current estimate on all of the past measurements, and generally converges in a few iterations. A Kalman filter that linearizes about the current mean and covariance is referred to as an extended Kalman filter (EKF). The essential idea of the extended Kalman filter was proposed by Stanley F. Schmidt, and it has been called the 'Kalman-Schmidt' filter [9].

We present an extended Kalman filter approach to optimize the membership functions of the inputs and outputs of the fuzzy controller. The performances of the fuzzy system before and after the optimization are compared, as well as the membership functions. We also investigated the effect of the measurement noise covariance R on the convergence of the fuzzy controller, which has not been explored previously in the existing publications.

2. FUZZY MEMBERSHIP FUNCTIONS AND ITS PARAMETERS

Consider a fuzzy system that uses correlation-product inference. Assume that the membership functions of the input and output are symmetric triangles. The initial rule base and membership functions are constructed on the imprecise basis of experience, and trial and error.

We denote the centroid, lower half-width and upper half-width of the ith fuzzy membership function of the jth input by c_{ij}, b_{ij}^-, b_{ij}^+, respectively [10]. The degree of membership of a crisp input x in the ith category of the jth input is specified as follows:

$$F_{ij}(x) = \begin{cases} 0 & x < b_{ij}^- \\ (x - b_{ij}^-)/(b_{ij}^+ - b_{ij}^-) & b_{ij}^- \le x \le b_{ij}^+ \\ (c_{ij} - x)/(c_{ij} - b_{ij}^+) & b_{ij}^+ \le x \le c_{ij} \\ 0 & x > c_{ij} \end{cases} \quad (1)$$

Similarly, for a single output fuzzy system, we denote the centroid and half-width of the jth fuzzy membership function of the output by γ_j and β_j, respectively. For a two inputs and one output fuzzy system, the fuzzy output

is mapped into a crisp value using centroid defuzzification [11]:

$$\text{crisp output} = \frac{\sum_{j=1}^{n} m(\gamma_j)\gamma_j \beta_j}{\sum_{j=1}^{n} m(\gamma_j)\beta_j} \qquad (2)$$

where n is the number of fuzzy output sets. The fuzzy output function $m(\gamma)$ is computed as follows:

$$m(\gamma) = \text{fuzzy output function} = \sum_{i,k} m_{ik}(\gamma) \qquad (3)$$

where $m_{ik}(\gamma)$ is defined as the consequent fuzzy output function when (input 1 $\in$ class i) and (input 2 $\in$ class k).

$$m_{ik}(\gamma) = w_{ik} m_{oik}(\gamma) \qquad (4)$$

$m_{oik}(\gamma)$ is the fuzzy function of the consequent that is activated when (input 1 $\in$ class i) and (input 2 $\in$ class k). And w_{ik} is the activation level of that consequent.

$$w_{ik} = \min[f_{i1}(input1), f_{k2}(input2)] \qquad (5)$$

Since the fuzzy membership functions are triangles as assumed, derivative–based methods can be used to optimize the centroid and half-widths of the input and output membership functions. Consider an error function given by

$$E = \frac{1}{2N} \sum_{q=1}^{N} E_q^2 \qquad (6)$$

$$E_q = \hat{y}_q - y_q \qquad (7)$$

where N is the number of training samples, y_q is the target value of the fuzzy system, and $\hat{y}_q$ is the output of the fuzzy system [12]. We can optimize E by using the partial derivatives of E with respect to the centroids and half-widths of the input and output fuzzy membership functions. The detailed derivation formula can be found in [13].

3. THE EXTENDED KALMAN FILTER (EKF)

This section briefly outlines the extended Kalman filter algorithm. Consider a nonlinear finite dynamic model given as follows [14]:

$$x_k = f(x_{k-1}, k-1) + w_k \qquad (8)$$

$$w_k \sim N(0, Q_k)$$

with a nonlinear measurement model:

$$z_k = h(x_k, k) + v_k \qquad (9)$$

$$v_k \sim N(0, R_k)$$

where the vector x_k is the state of the system at time k, the random variables w_k and v_k represent the process and measurement noise, respectively. z_k is the measurement vector, f($\cdot$) and h($\cdot$) are nonlinear vector functions of the state, and Q_k is the process noise covariance, R_k is the measurement noise covariance. Assume that the initial

state x_0 and sequences $\{w_k\}$ and $\{v_k\}$ are white, Gaussian and independent from each other with

$$E(x_0) = \bar{x}_0 \qquad (10)$$

$$E[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T] = P_0 \qquad (11)$$

$$E(w_k) = 0 \qquad (12)$$

$$E(w_k w_l^T) = Q\delta_{kl} \qquad (13)$$

$$E(v_k) = 0 \qquad (14)$$

$$E(v_k v_l^T) = R\delta_{kl} \qquad (15)$$

where E($\cdot$) is the expectation operator, δ_{kl} is the Kronecker delta, which is interpreted as:

$$\delta_{kl} = \begin{cases} 0 & k \neq l \\ 1 & k = l \end{cases}$$

The problem addressed by the extended kalman filter is to find an estimate $\hat{x}_{k+1}$ of x_{k+1} given z_j (j = 0,1,...,k).

If the nonlinearities in (8) and (9) are sufficiently smooth, the system can be approximated as

$$x_{k+1} = F_k x_k + w_k + \phi_k \qquad (16)$$

$$z_k = H_k^T x_k + v_k + \varphi_k \qquad (17)$$

$$\text{where } F_k = \frac{\partial f(x)}{\partial x}\bigg|_{x=\hat{x}_k} \qquad (18)$$

$$H_k^T = \frac{\partial h(x)}{\partial x}\bigg|_{x=\hat{x}_k} \qquad (19)$$

$$\phi_k = f(\hat{x}_k) - F_k \hat{x}_k \qquad (20)$$

$$\varphi_k = h(\hat{x}_k) - H_k^T \hat{x}_k \qquad (21)$$

It can be shown that the desired estimate $\hat{x}_k$ can be obtained by the recursion

$$\hat{x}_k = f(\hat{x}_{k-1}) + K_k[z_k - h(\hat{x}_{k-1})] \qquad (22)$$

$$K_k = P_k H_k (R_k + H_k^T P_k H_k)^{-1} \qquad (23)$$

$$P_{k+1} = F_k (P_{k-1} - K_k H_k^T P_{k-1})F_k^T + Q_k \qquad (24)$$

where K_k is the Kalman gain, and the P_k is the state estimation error covariance matrix.

4. KALMAN FILTER TRAINING OF THE FUZZY MEMBERSHIP FUNCTIONS

The use of Kalman filter training for the membership parameters of a fuzzy estimator was introduced by Simon for motor current windings [15]. It gives a straightforward representation of fuzzy estimator structure.

The optimization of fuzzy membership functions can be viewed as a weighted least-squares minimization problem, where the error is the difference between the fuzzy system outputs and the target values for those outputs. We use z to denote the target vector for the fuzzy system outputs, and h(k) to denote the actual outputs at the kth iteration of the training.

$$z(k) = [z_1(k) \cdots z_L(k)]^T \qquad (25)$$

$$h(k) = [h_1(k) \cdots h_L(k)]^T \qquad (26)$$

where L denotes the number of outputs of a fuzzy system.

Let's consider a 2-input, one-output fuzzy system. One input has μ fuzzy sets, the other input has v fuzzy sets, and the output has k fuzzy sets. In order to cast the membership function optimization problem in a form suitable for Kalman filtering, we let the membership function parameters constitute the state of a nonlinear system, and let the output of the fuzzy system constitute the output of the nonlinear system to which the Kalman filter is applied. We denote the centroid, lower half-width and upper half-width of the ith fuzzy membership function of the jth input by c_{ij}, b_{ij}^{-}, b_{ij}^{+}, respectively, and we denote the centroid and the half-width of the ith fuzzy membership function of the output by γ_i and β_i, respectively. The state of the nonlinear system can be adapted as

$$x = [b_{11}^{-} b_{11}^{+} c_{11} \cdots b_{\mu 1}^{-} b_{\mu 1}^{+} c_{\mu 1} b_{12}^{-} b_{12}^{+} c_{12} \cdots b_{v2}^{-} b_{v2}^{+} c_{v2}$$

$$\beta_1 \gamma_1 \cdots \beta_k \gamma_k]^T \qquad (27)$$

The nonlinear dynamic system to which the Kalman filter can be applied is as follows:

$$x_{k+1} = x_k + w_k \qquad (28)$$

$$z_k = h(x_k) + v_k \qquad (29)$$

where z_n can be seen as the target output of the fuzzy system, and $h(\hat{x}_k)$ is the actual output of the fuzzy system given the current membership function parameters. And then we can apply the Kalman recursion interpreted in (22) - (24).

5. IMPLEMENTATION AND EXPERIMENTAL RESULTS

Consider a dynamic process plant [16] given as follows:

$$M\dot{v} = -(\alpha v + \beta v^3) + k_e * \theta \quad - \quad M \quad * \quad 9.8 \quad * \quad \sin(\text{grad}) \qquad (23)$$

where M = 800 kg, α = 100N/(m/sec), β = 10^{-2}N/(m/sec)^3, and k_e is 4000 Newtons. v is the velocity of the plant, θ is the throttle position, and grad is a variable of the outside world that is liable to vary. The plant is expected to maintain the velocity at 70 m/s if the variable grad has a 15 percent positive increase. The problem is how to let the velocity and the throttle position of the plant reach the velocity at 70 m/s as fast as possible, and converge to this value unless there is another change.

A two inputs and one output fuzzy controller is designed by defining the error as the reference speed minus the measured speed, and implementing the rule base shown in Table 1. The rule base has five membership functions for each of input1, input2, and the output. So μ ,v and k in (27) are all five. Since each membership function of an input has three parameters (i.e. centroid, lower half-width and upper half-width) to determine, and each membership function of the output has two parameter (i.e. centroid and

half-width) to determine. Thus the fuzzy controller has a total of 40 parameters to be determined.

TABLE 1 FUZZY RULE BASE (ROWS = DIFFERENCE, COLUMNS = CHANGE IN DIFFERENCE)

	NL	NS	Z	PS	PL
NL	NL	NL	NL	Z	PS
NS	NL	NS	NS	Z	PS
Z	NS	Z	Z	Z	PS
PS	NS	Z	PS	PS	PL
PL	NS	Z	PL	PL	PL

NL = Negative Large, NS = Negative Small, Z = Zero, PS = Positive Small, PL = Positive Large.

We use the Intel Pentium IV processors up to 2.2GHz, and 1GB memory PC to do the experiments. We simulate the fuzzy controller in MATLAB for 100 s with an update rate of 0.25 s, so N in (6) equals to 400. The Kalman filter method is implemented to optimize the membership function parameters of the controller's inputs and output. Fig. 1 shows the training error of the extended Kalman filter during the optimization of the membership functions. We can see that it finally converged to 0.0008216 at the 30th iteration.
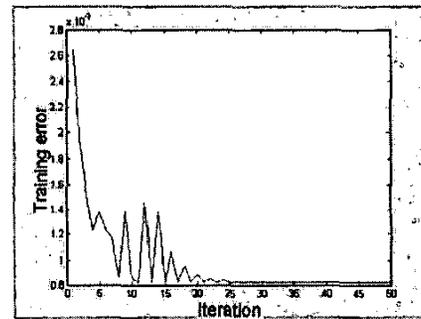

Fig. 1 Training error

In addition, we compare the membership functions of the inputs and the output before and after the optimization. The membership functions of input1, input2, and the output before optimization are shown in Fig. 2 (a), Fig. 2 (b), and Fig. 2 (c), respectively. The membership functions of input1, input2, and the output after optimization are shown in Fig. 3 (a), Fig. 3 (b), and Fig. 3 (c), respectively.
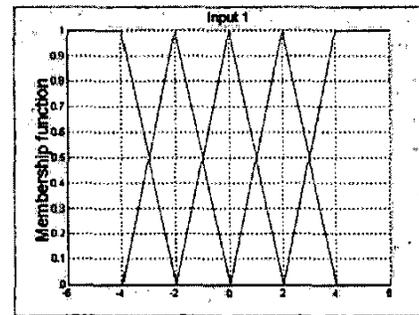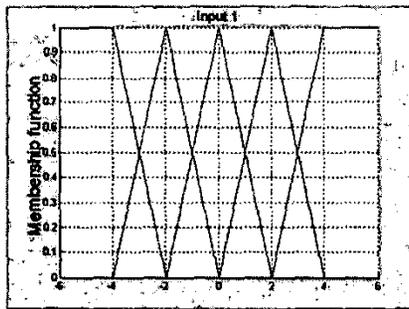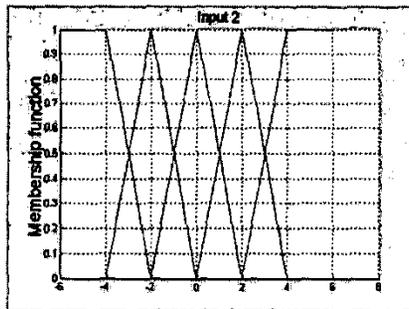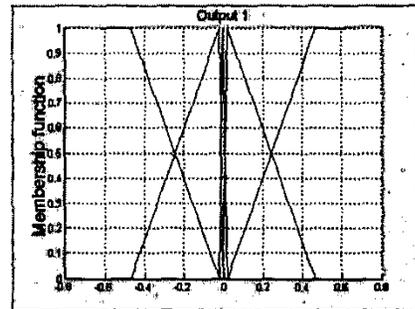

(a)
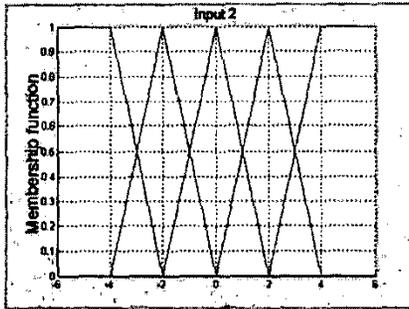
(b)
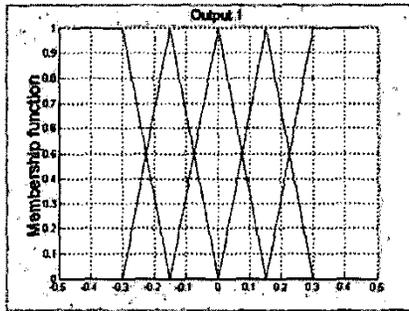
(c)

Fig.2 (a) The membership function of input1 before the optimization. (b) The membership function of input2 before the optimization. (c) The membership function of the output before the optimization.

(a)

(b)

(c)

Fig.3 (a) The membership function of input1 after the optimization. (b) The membership function of input2 after the optimization. (c) The membership function of the output after the optimization.

The comparison of the training data of the velocity between the nominal fuzzy controller and the optimized controller with the EKF is shown in Fig. 4. The blue solid line represents the velocity of the nominal fuzzy controller, and the red dashed line represents the velocity of the optimized fuzzy controller. At time t = 0, the variable *grad* has a 15 percent positive increase, so both curves dropped drastically in the next three seconds. However, the controllers attempt to maintain their velocities at 70 m/s, thus the curves began to oscillate until they converge to the desired value. From Fig. 4, we can see that the optimized fuzzy controller converges much faster than the nominal fuzzy controller.
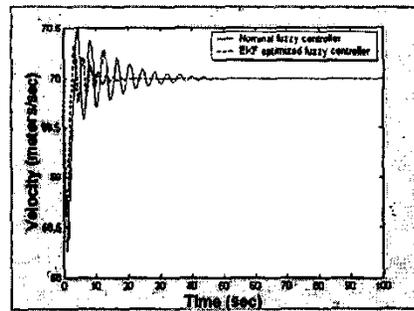

Fig. 4 Comparison of the training data of the velocity between the nominal fuzzy controller and the optimized controller.

Similarly, we can observe the comparison of the training data of the throttle position between the nominal fuzzy controller and the optimized controller, as shown in Fig. 5. The blue solid line represents the throttle position of the nominal fuzzy controller, and the red dashed line represents the throttle position of the optimized fuzzy controller. From Fig. 5, we can see that the optimized fuzzy controller converges much faster than the nominal fuzzy controller.
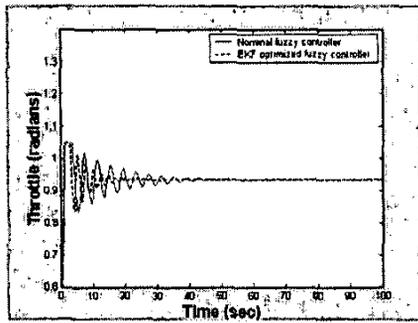
Fig. 5 Comparison of the training data of the throttle position between the nominal fuzzy controller and the optimized controller.

Furthermore, we conducted three experiments to investigate the effect of measurement noise covariance, R on Kalman filter performance, and thus the optimized fuzzy controller's convergence. In the first simulation, we set the measurement noise covariance at $R = 1e - 2$, and then train the extended Kalman filter. And then we applied the optimized membership functions parameters to the fuzzy controller. The performance of the fuzzy controller is evaluated by the convergence speed. Fig. 6 (a) depicts the results of this first simulation.
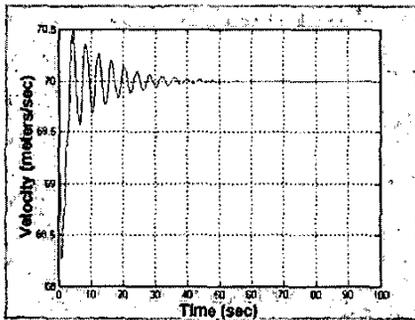

Fig. 6 (a) The response of the optimized fuzzy controller after we train the EKF with $R = 1e - 2$.

In the second simulation, we decrease the parameter R to $1e - 6$. Fig. 6 (b) shows the response of the optimized fuzzy controller after we train the EKF with $R = 1e - 6$. By comparing Fig. 6 (a) and Fig. 6 (b), we can see that the convergence in Fig. 6 (b) is faster than that in Fig. 6 (a).

In the third simulation, we decrease the parameter R to $1e - 8$. Fig. 6 (c) shows the response of the optimized fuzzy controller after we train the EKF with $R = 1e - 8$. By comparing Fig. 6 (b) and Fig. 6 (c), we can see that the convergence in Fig. 6 (c) is faster than that in Fig. 6 (b).

By comparing Fig. 6 (a) through Fig. 6 (c), we conclude that the smaller the measurement noise covariance R is, the faster the fuzzy controller converge.
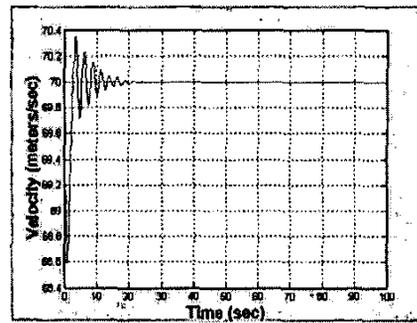

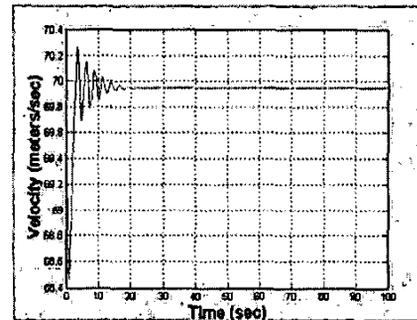Fig. 6 (b) The response of the optimized fuzzy controller after we train the EKF with $R = 1e - 6$.

Fig. 6 (c) The response of the optimized fuzzy controller after we train the EKF with $R = 1e - 8$.

6. CONCLUSION

The power of the Kalman filter has led it to its wide applications in technologies and industries. This paper demonstrates that the extended Kalman filter (EKF) provides an efficient solution to the optimization of fuzzy membership function for both the inputs and output of the fuzzy controller. In our approach, after casting the fuzzy system to a nonlinear system, to which the extended Kalman filter will be applied, the derivative-based EKF approach is then carried out to optimize the membership functions. An appropriate fuzzy rule base is designed. The experimental results are satisfactory. It shows that the optimized fuzzy controller has improved its performance greatly on the fast convergence for both the velocity and the throttle position. The training error of the EKF converges to 0.0008216 at the 30th iteration, which presents the EKF's accuracy and efficiency. We also investigate the effect of the measurement noise covariance R on the convergence of the fuzzy controller to which the extended Kalman filter is applied. It shows that the smaller the value of R is, the faster the optimized fuzzy controller converges.

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