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A Non-Isotropic Model for Mobile-to-Mobile Fading Channel Simulations

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A NON-ISOTROPIC MODEL FOR MOBILE-TO-MOBILE FADING CHANNEL SIMULATIONS

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Abstract—**Accurate modeling of the mobile-to-mobile fading channel is critical not only to physical layer transceiver design but also to the design and performance of link and network layers. A two-dimensional non-isotropic scattering model is developed in this paper, which adopts the von Mises probability density function for the angle of departure surrounding the transmitter and the angle of arrival surrounding the receiver. This model includes the isotropic scattering channel and the base-to-mobile channel as special cases. An efficient computer simulation model is also developed to generate the non-isotropic scattering channel impulse responses. Statistical properties of the non-isotropic mobile-to-mobile channels are analyzed in comparison to the isotropic channels and the base-to-mobile channels.**

I. Introduction

Mobile-to-mobile fading channels have found increasing applications in mobile ad hoc networks and dedicated short range communication systems, where direct communication between a mobile transmitter and a mobile receiver is required. Such mobile-to-mobile wireless systems differ from the conventional cellular radio systems where the base station is fixed in location and only the mobile user is moving. Though the received signal envelope is Rayleigh distributed under the rich multipath and non-line-of-sight (LOS) conditions, significant differences are found in their statistical properties due to 1) the mobility of both transmitter and receiver, 2) the angular distribution of the scatterers surrounding the transmitter and receiver.

In the literature, the two-dimensional isotropic scattering model is often assumed for base-to-mobile [1] and mobile-to-mobile channels [2]. Various computer simulation models [4] – [10] are also proposed for efficient simulation of the isotropic scattering channel. The models proposed in [4] – [8] are for base-to-mobile channels and those in [9] and [10] are for mobile-to-mobile channels.

In many real world scenarios, however, non-isotropic scattering is often experienced by the mobile transmitter and the mobile receiver. For base-to-mobile channels, it has been shown that non-isotropic scattering around the mobile station is often the case [11]–[13], especially in dense urban and indoor environments. Various non-uniform probability density functions (pdf) have been discussed for the Angle of Arrival (AoA) of the mobile receiver, including Gaussian pdf [13], quadratic pdf, LaPlacian pdf, cosine pdf [12], and von Mises pdf [11]. It has been shown that small variation in non-isotropic scattering environment can lead to large difference in the channel statistical properties. These properties greatly affect the design and analysis of the wireless systems, not only on the physical layer but also on the link and network layers [14].

In this paper, we extend the Akki and Haber's mobile-to-mobile channel model to non-isotropic scattering fading channels by adopting the von Mises pdf for both the Angle of Departure(AoD) and the AoA. This pdf is suggested in [11] for base-to-mobile channels because it can approximate many other non-uniform pdf and can provide mathematical convenience for analysis. We analyze the autocorrelation function and Doppler spectrum density under such non-isotropic condition for mobile-to-mobile channels and show the significant effects of various parameters on the statistical properties. We also develop an efficient computer simulator to generate the channel impulse responses of non-isotropic scattering mobile-to-mobile channels. The proposed simulator is a generalized tool which can also simulate the base-to-mobile and/or isotropic scattering channels as its special cases under a unified framework.

II. MOBILE-TO-MOBILE FADING CHANNEL DESCRIPTION

A. The Akki and Haber's Reference Model

Akki and Haber proposed a statistical model for the mobile-to-mobile Rayleigh fading channel. For a frequency flat fading channel, the baseband equivalent channel impulse response is given as a complex fading envelope:

$$\begin{aligned} g(t) &= \sum_{n=1}^{N} A_n \exp\left[j2\pi t(f_{d_1}\cos\alpha_{1n} + f_{d_2}\cos\alpha_{2n}) \right. \\ &\quad \left. + j\psi_n\right] \qquad (1) \\ &= \sum_{n=1}^{N} A_n \exp\left[j2\pi t\left(\sum_{i=1}^{I} f_{d_i}\cos\alpha_{in}\right) + j\psi_n\right], \end{aligned}$$

where $j = \sqrt{-1}$, N is the number of propagation paths, f_{d_i}'s are the maximum Doppler frequencies due to the motion of the transmitter and the receiver, respectively. The random variables $\alpha_{i,n}$ are the angle of departure (AoD) and the angle of arrival (AoA) of the n-th path with reference to the velocity vectors of the transmitter and the receiver, respectively. And ψ_n is the random phase uniformly distributed on the circle, denoted as $U[-\pi, \pi)$. The amplitude A_n are chosen to normalize the power with $A_n = \sqrt{2/N}$. The difference between the mobile-to-mobile fading channel and the Clarke's model for the base-to-mobile fading channel is that the total Doppler frequency of each path of the mobile-to-mobile channel is the sum of the Doppler frequencies induced by the transmitter and the receiver.

Assume that α_{1n}, α_{2n} and ψ_n are independent for all n. For sufficiently large N, the central limit theorem guarantees that the real part and the imaginary part of $g(t)$ are zero-mean Gaussian and independent. Thus the envelope $|g(t)|$ is Rayleigh distributed.

In the literature, omnidirectional antennas and isotropic scattering are generally assumed meaning that α_{1n} and α_{2n} are also uniform on $[-\pi, \pi)$. Base on this assumption, the statistical properties of the mobile-to-mobile channel have been derived [2], [3] and are cited here for convenience. The auto-correlation function $R_{gg}(\Delta t)$ of the fading impulse response $g(t)$ is

$$\begin{aligned} R_{gg}(\Delta t) &= \frac{1}{2}E[g(t)g^*(t-\Delta t)] \\ &= J_0(2\pi f_{d_1}\Delta t)J_0(2\pi f_{d_2}\Delta t) \qquad (2) \end{aligned}$$

where $E[\cdot]$ is the statistical expectation operator, $*$ denotes the complex conjugate, and $J_0(\cdot)$ is the zero-order Bessel function of the first kind.

The corresponding Doppler power spectrum $S(f)$ is given as

$$\begin{aligned} S(f) &= \int_{-\infty}^{\infty} R_{gg}(\Delta t)e^{-j2\pi f\Delta t}d\Delta t \\ &= \frac{1}{\pi^2\sqrt{f_{d_1}f_{d_2}}}K\left[\sqrt{\frac{(f_{d_1}+f_{d_2})^2 - f^2}{4f_{d_1}f_{d_2}}}\right] \qquad (3) \end{aligned}$$

where function $K(\cdot)$ is the complete elliptic integral of the first kind [16].

In many real world scenarios, however, non-isotropic scattering is often experienced by the mobile transmitter and the mobile receiver. For base-to-mobile channels, it has been shown that non-isotropic scattering around the mobile station is often the case [11]–[13], especially in dense urban and indoor environments. Various non-uniform probability density functions (pdf) have been discussed for the AoA of the mobile receiver, including Gaussian pdf [13], quadratic pdf, LaPlacian pdf, cosine pdf [12], and von Mises pdf [11]. It is argued [11] that the von Mises pdf is favorable because it can approximate many other non-uniform pdf and can provide the mathematical convenience for analysis. Therefore, we also adopt the von Mises pdf for the AoD and AoA in mobile-to-mobile fading channels.

B. The von Mises non-isotropic Scattering

The von Mises pdf plays an important role in the circular statistics [15]. Also known as the circular normal distribution, it describes a "normal" distribution on a circle with period 2π, analogous to that of the normal distribution on a line. It can approximate the cardioid, wrapped normal, wrapped Cauchy distributions very well. The pdf of the von Mises AoD or AoA $\alpha_{i,n}$ is given as:

$$p_\alpha(\alpha) = \frac{\exp[\kappa\cos(\alpha-\mu)]}{2\pi I_0(\kappa)}, \quad \kappa \geq 0, \qquad (4)$$

where $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind, μ is the mean direction of the AoD or AoA, and κ is the concentration parameter which controls the width of the scatterers. We shall denote the pdf as $\alpha \sim M(\mu, \kappa)$. Figure 1 shows the probability density function of $M(0, \kappa)$ with different values of κ. If $\kappa = 0$, then the von Mises pdf reduces to the uniform distribution. If $\kappa = \infty$, the scatterer becomes a deterministic path with an impinging angle μ.

When the AoD and the AoA are non-uniform, the statistical properties (2) and (3) are not valid since they

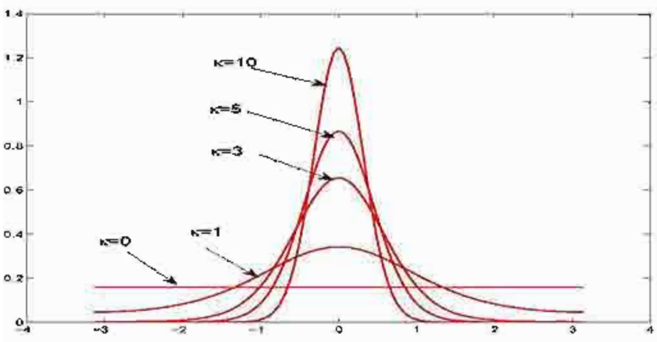


Fig. 1. The von Mises probability density function of non-isotropic scatterers.

are derived from uniform pdf. We now derive the autocorrelation function of the fading envelope $g(t)$ as the following theorem.

Theorem 1: Assume that the AoD is $\alpha_{1n} \sim M(\mu_1, \kappa_1)$ and the AoA is $\alpha_{2n} \sim M(\mu_2, \kappa_2)$, then the autocorrelation function of the complex fading envelope $g(t)$ is given by

$$R_{gg}(\Delta t) = \prod_{i=1}^{2} \frac{I_0\left(\sqrt{\kappa_i^2 - 4\pi^2 f_{d_i}^2 \Delta t^2 + j4\pi\kappa_i f_{d_i} \Delta t \cos\mu_i}\right)}{I_0(\kappa_i)} \quad (5)$$

where $\prod$ denotes the multiplication operator.

Proof: : From the definition of the autocorrelation function, we have

$$\begin{aligned} R_{gg}(\Delta t) &= \frac{1}{2} E_{\alpha,\psi}\left[g(t) g^*(t-\Delta t)\right] \\ &= \frac{1}{N} E_{\alpha,\psi}\left\{\sum_{n=1}^{N} \exp\left[j2\pi t \left(\sum_{i=1}^{I} f_{d_i} \cos\alpha_{in}\right) + j\psi_n\right]\right. \\ &\quad \left. \cdot \sum_{m=1}^{N} \exp\left[-j2\pi(t-\Delta t)\left(\sum_{i=1}^{I} f_{d_i}\cos\alpha_{im}\right) - j\psi_m\right]\right\} \\ &= \frac{1}{N}\sum_{n=1}^{N} E_\alpha\left\{\exp\left[j2\pi\Delta t \left(\sum_{i=1}^{I} f_{d_i}\cos\alpha_{in}\right)\right]\right\}. \quad (6) \end{aligned}$$

Since α_{1n} and α_{2n} are independent random variables with pdf defined in (4), we have

$$\begin{aligned} & E_\alpha\{\exp[j2\pi\Delta t f_{d_i}\cos\alpha_{in}]\} \\ &= \frac{1}{2\pi I(\kappa_i)} \int_{-\pi}^{\pi} \exp[\kappa_i \cos(\alpha-\mu_i) + j2\pi f_{d_i}\Delta t \cos\alpha] d\alpha \\ &= \frac{I_0\left(\sqrt{\kappa_i^2 - 4\pi^2 f_{d_i}^2 \Delta t^2 + j4\pi\kappa_i f_{d_i}\Delta t\cos\mu_i}\right)}{I_0(\kappa_i)}. \quad (7) \end{aligned}$$

This leads to (5). ∎

The Doppler spectrum of the non-isotropic scattering mobile-to-mobile channel is also different from that in (3). Unfortunately, the closed form solution for the $S(f)$ is difficult and numerical integration has to be used to compute the Doppler spectrum. Examples of the autocorrelation function and Doppler spectra are shown in Fig. 2 and Fig. 3, respectively.

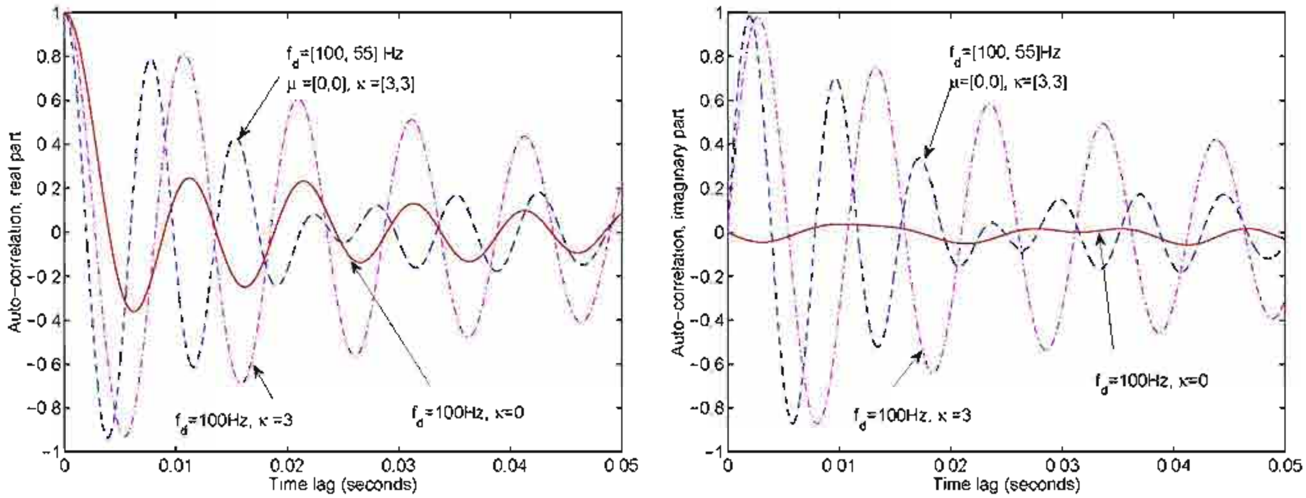
Fig. 2 shows the autocorrelation functions of several fading channels. With on-isotropic scattering, the autocorrelation of the mobile-to-mobile channel is similar to that of the base-to-mobile channel with slowly decaying real and imaginary parts. When $\kappa_1 = \kappa_2 = 3$, and $\mu_1 = \mu_2 = 0$, the mobile-to-mobile channel de-correlates faster than the base-to-mobile channel because higher total maximum Doppler shift is experienced in the mobile-to-mobile channels. If $\mu_1 = \mu_2 = \pi$, then the opposite is true. The base-to-mobile isotropic scattering channel shows that the imaginary part of the autocorrelation function is close to zero, as those reported in [5], [7].

The Doppler spectra of the mobile-to-mobile non-isotropic scattering channel is quite different from those of isotropic scattering channels. Figure 3 shows that the non-isotropic scattering channel often exhibit one-sided Doppler shifts while the isotropic scattering channels always have symmetrical Doppler spectra with respect to the carrier frequency.

III. Simulation models

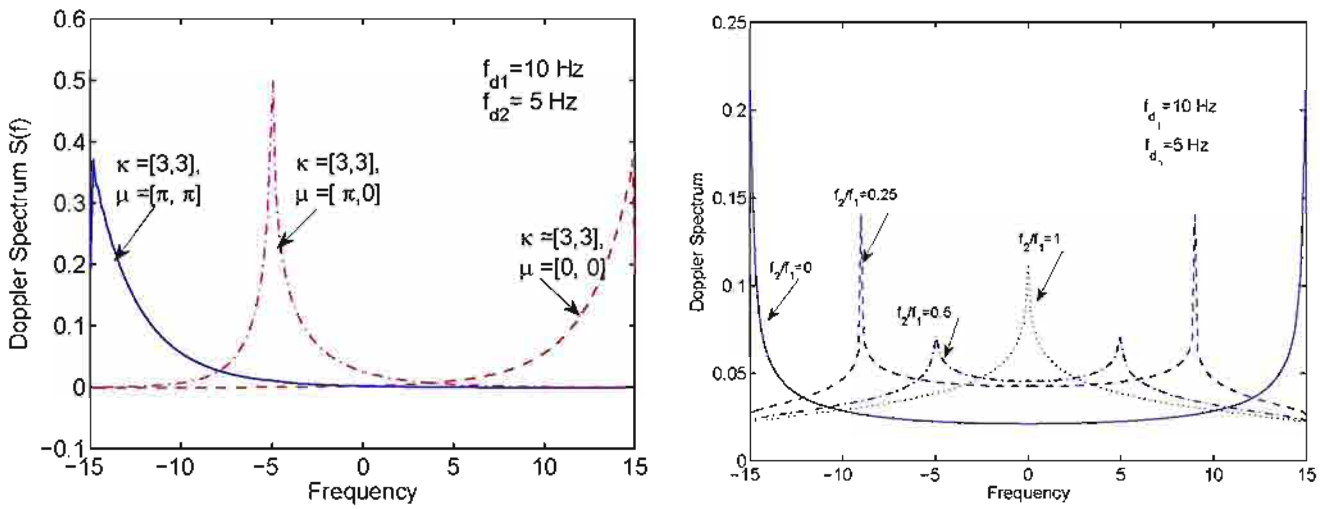
Several computer simulation models have been reported in the literature for isotropic scattering mobile-to-mobile channels. Unfortunately, they can not be used directly for non-isotropic scattering channels. For example, the sum-of-sinusoids method presented in [8] exploits the property of uniform AoD and AoA by a independent double-ring approach, It achieves high efficiency and excellent match in statistical properties for the isotropic scattering channel but can not be applied to non-isotropic scattering mobile-to-mobile channels. The line-spectrum filtering method proposed in [9] may, in principle, be adopted for non-isotropic scattering channels. But it has high complexity and severe performance deficiencies. It is not practical even for isotropic scattering channels because, as pointed out in [8], the numerical integration required by the line-spectrum filtering method makes the implementation not reconfigurable for different Doppler frequencies. With non-isotropic scattering channels, Doppler spectrum exhibits more variations due to different pdf parameters and Doppler frequencies. Therefore the line spectrum filtering method is not a good choice.

To simulate the non-isotropic mobile-to-mobile channel, a straightforward method is to use (1) directly



(a) Real part of $R_{gg}(\Delta t)$ (b) Imaginary part of $R_{gg}(\Delta t)$

Fig. 2. The autocorrelation functions of the non-isotropic mobile-to-mobile channels model. Solid line – base-to-mobile isotropic scattering channel, Dash-dotted line – base-to-mobile non-isotropic scattering channel, Dashed line – mobile-to-mobile non-isotropic scattering channel.



(a) Non-isotropic scattering channel (b) Isotropic scattering channel

Fig. 3. Doppler Power Spectra of mobile-to-mobile fading channels: comparison of isotropic and non-isotropic scattering fading.

with a finite N and independent von Mises random angles. This generally requires a large value N on the order of a couple of hundreds. We propose here a better simulation model that takes advantage of the symmetrical property of the von Mises pdf and reduces the number of sinusoids to $M = N/2$. By mapping $\alpha_{i,n} = \mu_i + \beta_{i,n}$, $\alpha_{i,n+1} = \mu_i - \beta_{i,n}$ and $\psi_n = \phi_n + \phi'_n$, $\psi_{n+1} = \phi_n - \phi'_n$ in (1), we derive the simulation model as follows:

$$g(t) = g_c(t) + jg_s(t) \tag{8a}$$

$$g_c(t) = \sum_{n=1}^{M} B_n(t) \cos[2\pi(C_{1,n} + C_{2,n})t + \phi_n] \tag{8b}$$

$$g_s(t) = \sum_{n=1}^{M} B_n(t) \sin[2\pi(C_{1,n} + C_{2,n})t + \phi_n] \tag{8c}$$

where

$$C_{i,n} = f_{d_i} \cos\mu_i \cos\beta_{i,n}, \quad i = 1, \cdots, I. \quad (8d)$$

$$S_{i,n} = f_{d_i} \sin\mu_i \sin\beta_{i,n}, \quad i = 1, \cdots, I. \quad (8e)$$

$$B_n(t) = \sqrt{\frac{2}{M}} \cos[2\pi(S_{1,n} + S_{2,n})t + \phi'_n]. \quad (8f)$$

The random variables are independent for all n with the following distributions

$$\beta_{1,n} \sim M(0, \kappa_1), \quad \beta_{2,n} \sim M(0, \kappa_2)$$
$$\phi_n \sim U[-\pi, \pi), \quad \phi'_n \sim U[-\pi, \pi)$$

The von Mises random variables $\beta_{i,n}$ are generated by the acceptance-rejection algorithm detailed in [15]. A large N is needed to ensure that the simulated AoD and AoA are approximately von Mises distribution. Increasing N will reduce the variation of the statistical properties of each trial and improve the simulation accuracy. The choice of the parameters κ_i and μ_i has little effect on the simulation accuracy.

The simulation accuracy is evaluated by comparing the simulated fading envelope and autocorrelation function with the theoretical results. The parameters used in the example are $N = 144$, $\kappa_1 = \kappa_2 = 3$, $\mu_1 = 0$, and $\mu_2 = \pi/2$. Ten trials each with one million samples ware used yielding the total of 10 million samples. The envelope of the simulated channel impulse response approaches Rayleigh fading distribution, as shown in Fig. 4, with the histogram matching the theoretical pdf. Figure 5 shows that the real part of the autocorrelation function of the ensemble average of ten trials is very close to the theoretical one. The imaginary part of the autocorrelation function, not shown here due to page limitations, is also very close to the theoretical one.

Another advantage of the proposed simulator is its high efficient and low computational complexity. The number of operations required for each simulated sample is

- the number of sine or cosine operations: $3.5N + 4$;
- the number of multiplications: $4N$;
- the number of additions: $3N$.

With this complexity, it can be easily implemented on a personal computer. It takes less than 30 seconds to generate 100 million samples.

Although the proposed simulator is designed for generating channel impulse responses for non-isotropic mobile-to-mobile fading channels, it can also be used to simulate isotropic mobile-to-mobile fading channels, isotropic base-to-mobile channels, and non-isotropic base-to-mobile channels. These channels are the special cases of the proposed channel simulator and can be easily implemented by changing the parameters within a unified framework. For example, setting $\kappa_1 = \kappa_2 = 0$ will result in the isotropic mobile-to-mobile fading channel; Setting $I = 1$ in (1) and (8a) will yield the impulse responses for base-to-mobile fading channels (isotropic or non-isotropic).

The Doppler spectra and autocorrelation functions of several simulation examples are shown here to

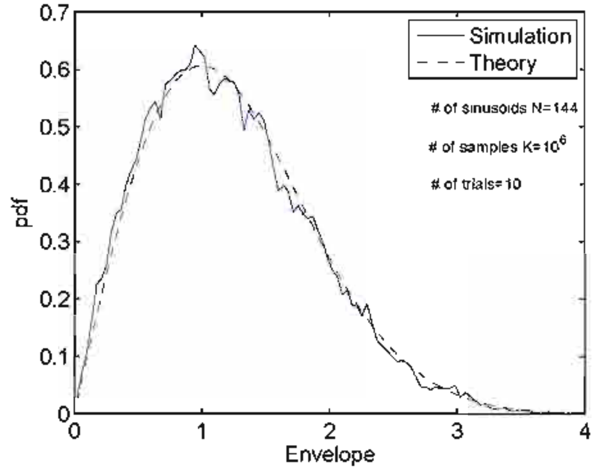


Fig. 4. Distribution of the simulated fading envelope samples versus the theoretical Rayleigh pdf.

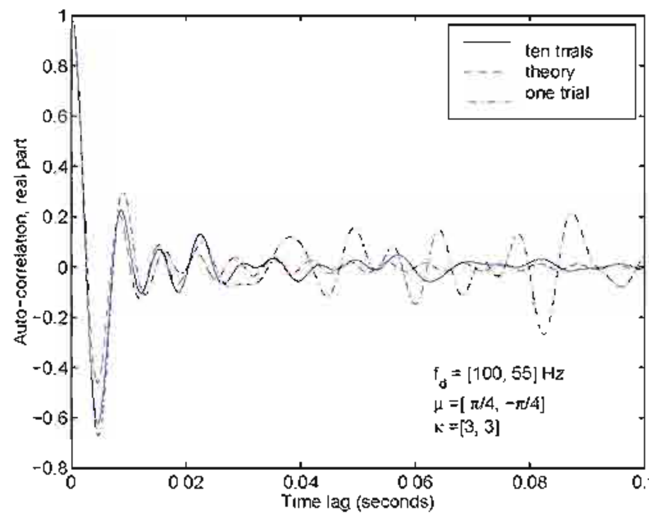


Fig. 5. The autocorrelation functions computed by theory and simulations indicating the effectiveness of the proposed simulation model.

demonstrate the different statistical properties of these channels. Fig. 6 shows the Doppler spectra of several base-to-mobile channels simulated by the proposed model with $I = 1$. The isotropic scattering channel is obtained by setting $\kappa = 0, \mu = 0$ and it reproduces the result as those in [5], [7]. Comparing to the non-isotropic scattering channels with $\kappa = 3$, one-sided Doppler spectrum is often resulted except $\mu = \pi/2$. These results are similar to the ones reported in [11].

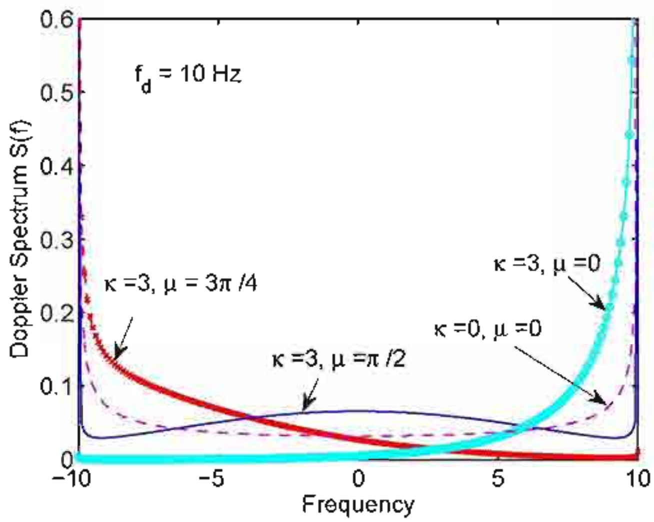


Fig. 6. Doppler Power Spectra of the base-to-mobile fading channels simulated by the proposed model. Only one maximum Doppler shift $f_d = 10$ Hz is used. For isotropic scattering channels, $\kappa = 0$; For non-isotropic channels, $\kappa > 0$.

The autocorrelation functions of two mobile-to-mobile channels are shown in Fig. 7. Both the real part and imaginary part show that the channel with maximum Doppler frequencies 100 Hz and 55 Hz decorrelates much faster than that with maximum Doppler frequencies 10 Hz and 5 Hz.

IV. Conclusion

A two-dimensional non-isotropic scattering model has been developed in this paper, which adopts the von Mises probability density function for the angle of departure surrounding the transmitter and the angle of arrival surrounding the receiver. This model includes the isotropic scattering channel and the base-to-mobile channel as special cases. An efficient computer simulation model is also developed to generate the non-isotropic scattering channel impulse responses. Analytical derivation and numerical examples are shown to demonstrate the statistical properties of the non-isotropic mobile-to-mobile channels in comparison to the isotropic channels and the base-to-mobile channels.

Acknowledgment

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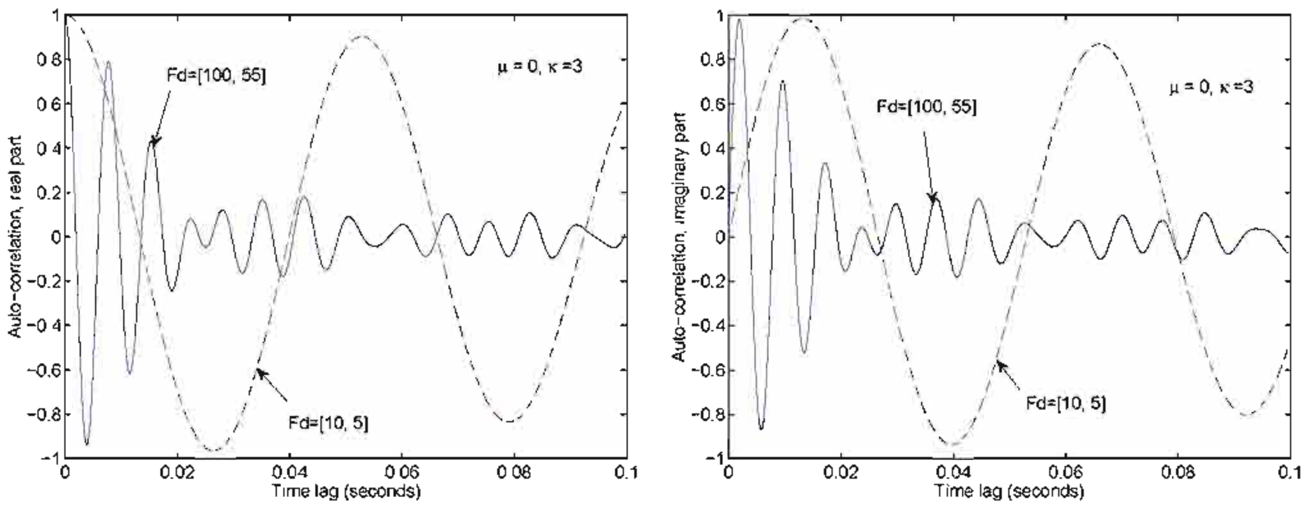

(a) Real part of $R_{gg}(\Delta t)$

(b) Imaginary part of $R_{gg}(\Delta t)$

Fig. 7. The temporal autocorrelation function of the mobile-to-mobile fading channel with non-isotropic scattering