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EVALUATION OF THE SHEAR MODULUS IN MODELS FOR SHALLOW-FOUNDATION DYNAMICS WITHIN THE ELASTIC DOMAIN

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ABSTRACT

The aim of this presentation is to examine the relationship of the equivalent homogeneous shear moduli used in impedance models with stresses under the footing, through the use of scaled models in the centrifuge and an impact loading. The analysis of time and frequency vertical responses of footings reveals that reflections on the boundaries are negligible. The frequency response of a series of circular and square footings is shown to be rather easily-fitted with Wolf's models for foundations on an infinite half-space with reasonably consistent parameters for masses, damping and shear moduli. The damping is nearly constant, yet significantly lower than in a prototype scale with real soil. The mass is fitted with a greater level of scatter. The correlation of shear modulus to the square root of the minimum mean stress appears to be better than that to the square root of the uniform stress under the footing.

1. INTRODUCTION

A major concern in soil-structure interaction is the characterization of the dynamic response of surface foundations. Analytical solutions provide complex expressions even in the simplest situations. Numerical modeling using FEM or BEM leads to lengthy calculations for practical purposes. Derived from more rigorous analytical solutions, the impedance formulation is of great interest herein for its intuitive approach as well as for its connectivity to a modal analysis of the superstructure. Field experimental data have been recorded by many authors. A compilation of these results is presented in the Handbook of Impedance Functions by Sieffert and Cevaer (1992). The frequency dependence of the impedance function makes it difficult to implement in multi-frequency response computing. Both the cone model, developed by Meeks and Wolf (1992), and the "lumped-parameters" models by Wolf (1994) are interesting formulations that provide solutions for the assessment of more complex situations than circular footing. A second operational aspect of these formulations is their potential implementation in computing frequency-dependent dynamic responses. However, such models still require experimental data in order to fit their parameters.

Field experiments aimed at achieving complete control over soil parameters are expensive and difficult to conduct from a technical standpoint. Scaled modeling in the centrifuge enables extensive parametric studies under well-defined conditions. Leguay (1984), Coe *et al.* (1985), Cheney *et al.* (1990), Lenke *et al.* (1991), and Pak and Guzina (1995) have all explored this modeling technique and detailed the range of difficulties encountered: the need for large containers, preferably rectangular, and the need for treating the boundaries with an absorbent material. These constraints were due primarily to the type of footing loading: continuous harmonic or white noise.

In what follows, another improvement is proposed: the use of impact loading. This mean of studying dynamic responses in vertical movement is expected to reduce the pollution by wave reflections on the boundaries of the container, while providing a wide frequency-range response.

The main goal of this paper is to prove that scaled modeling with impact loading is relevant to the testing of dynamic responses of footings. This relevance is demonstrated through both controlling soil behavior and comparing the experimental responses to classical, field-improved prototype models. By benefiting from the possibility of embedding accelerometers in the soil, a second aim of this work is to propose an experimental response to the following question: given the stress dependence of the shear modulus G, one of the models' main parameters, in which location is this parameter to be computed?

2. TESTING PROGRAM

Experimental Set-up

The series of tests have been conducted on the 200 g-ton centrifuge of the Laboratoire Central des Ponts et Chaussées, in France. Large rectangular 1.20 x 0.80 x 0.36-m containers were used. In order to determine the influence of both impact loading and boundary lining, two containers were tested: one with and one without a 2.5-cm coating, with the density of sand being held the same.

The model footings were composed of aluminum cylinders of different diameters or square plates (see Table I) lying on the sand . The response of the footing was monitored by two B&K 4393 accelerometers which, placed on top of the footings, enabled controlling the movements and avoiding rocking.

Table I: Characteristics of the circular footings tested					
Footing #	Mass (kg)	Ø (mm)	prototype mass/50 g	prototype mass/40 g	prototype mass/30 g
1	0.118	60	14.3 T	7.33 T	3.1 T
2	0.237	60	28.6 T	14.7 T	6.2 T
3	0.129	50	16.6 T	8.5 T	3.6 T
Characteristics of the square footings tested					
5	0.046	26x26	5.7 T	2.9 T	1.2 T
6	0.200	52x52	23.7 T	12.1 T	5.1 T

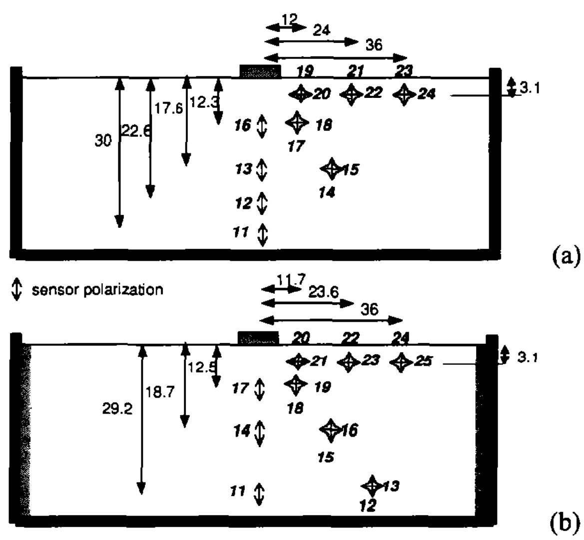

Fig. 1: Location and direction of the accelerometers in the sand (a: container without Duxseal – b: container with Duxseal); dimensions given in cm

The hammer was a simple seesaw supported by a beam over the container, with a PCB 200A2 force transducer at one end and an electromagnetic jack at the other, driven from the command room.

The model soil was a fine dry Fontainebleau sand, rained into the container at a density of 16.30 kN/m^3 (I_D = 0.79). During the raining process, PCB 352A10 accelerometers were placed at different locations, as presented in Figs. 1a and b.

Output data were amplified in the basket and digitized with a Krenz device placed at the centrifuge pivot, before being transferred to the computer via an IEEE connection through the slip rings. All data have been subjected to post-processing.

Test Procedure and Data Processing

Each foundation was tested at different levels of gravity: 30, 40 and 50 g, providing data for a sort of modeling of models. Each test was repeated 5 times in order to both control the consistency of the response and improve the signal-to-noise ratio from embedded accelerometers.

An analysis of the footing response - from experimental data as well as from impedance functions - has been carried out in terms of mobility, i.e. the velocity frequency-response function (FRF):

$$M(f) = \frac{\dot{X}(f)}{F(f)} \qquad (1)$$

where $\dot{X}(f)$ and $F(f)$ are the Fourier transform (FT) of the velocity measured on the footing and the force applied by the hammer, respectively. The velocity FT was obtained from the FT of the accelerometers divided by j.ω.

Model mobility was computed as follows:

$$M(f) = \frac{j\,\omega}{k(\omega) - m\,\omega^2 + j\,\omega\,c(\omega)} \qquad (2)$$

where the functions K and C were those initially proposed by Lysmer (with both K and C independent of ω):

$$K = Kst = \frac{4.G.ro}{1-\nu} \qquad C = \frac{3.4.ro^2}{1-\nu}\sqrt{G.\rho} \qquad (3)$$

with G: shear modulus; ro: radius of the footing; ρ: soil density; and ν: Poisson's ratio. Having proved their irrelevance as regard to damping, we were led to definitively adopting those proposed by Wolf (1994):

$$K(\omega) = Kst\left[1 - \frac{\frac{\omega^2 M_1}{K}}{1+\frac{\omega^2 M_1^2}{C_1^2}} - \frac{\omega^2 M_0}{K}\right] \qquad C(\omega) = Kst\left[\frac{M_1}{C_1}\cdot\frac{\frac{\omega^2 M_1}{K}}{1+\frac{\omega^2 M_1^2}{C_1^2}} + \frac{C_0}{K}\right] \qquad (4)$$

where M_0, C_0, M_1 and C_1 are parameters specific to the type of movement (see Appendix).

3. EXPERIMENTAL RESULTS

Wave Velocities

Wave velocities are the subject of another detailed paper by the same author (Chazelas et al. 2000). Only the primary results need to be recalled herein. The first result of interest is that embedded accelerometer recordings show that there is a vertical gradient of P-wave velocities in the sand, compliant with the law proposed by Iwasaki and Tatsuoka (1977). This result is consistent with the findings of other authors (e.g. Siemer and Jessberger, 1994) for sensors placed vertically under the footing, but also for sensors placed near the surface. This study of wave propagation led to conclude that there was probably a reduction of the density of the sand in the second container due to the prparation process (15.8 kN/m^3 vs. 16.3 kN/m^3). An estimation of shear wave velocities based on the method SASW also suggested that there was a difference on the Poisson's ratio in this container (0.3 vs. 0.22). These values, along with the experimental error, are consistent both with those presented by Pak and Guzina (1994) and with that currently used in the literature (0.25). Other experiments are needed to improve this point.

For the following computations, the density was those cited just above and $\nu = 0.25$. The shear wave velocity in the computations of the parameters of the impedance functions was taken equal to $\sqrt{G/\rho}$ and then contributed to the fitting of the equivalent shear modulus under examination below.

Impact Loading and Container Wall Reflections

Cheney *et al.* (1990) introduced the use of Duxseal in order to limit reflections on container walls. Lenke *et al.* (1991) showed that it was better to use rectangular containers because they scatter the reflections, and that the coating of the container could be limited to just the side-walls. These prescriptions have been adopted for the second container.

Figure 2 presents typical time response of a footing. The velocities given in Table II enable localizing the probable return times of energy reflected by the nearest boundaries. The figure has been drawn using data from the non-coated container; they show that the reflections are of lesser significance in the response because they arrive late, once the main part of the damping has already occurred (the two accelerometers on the foundation have been plotted, phase coherence can be noticed).

Another approach to this problem has been presented in Fig. 3: there is no obvious difference between the FRF computed with the complete signal and that computed with a windowed signal, as shown in Fig. 2 (for instance, a rectangular window limited to the part of the signal before the reflected wave return time).

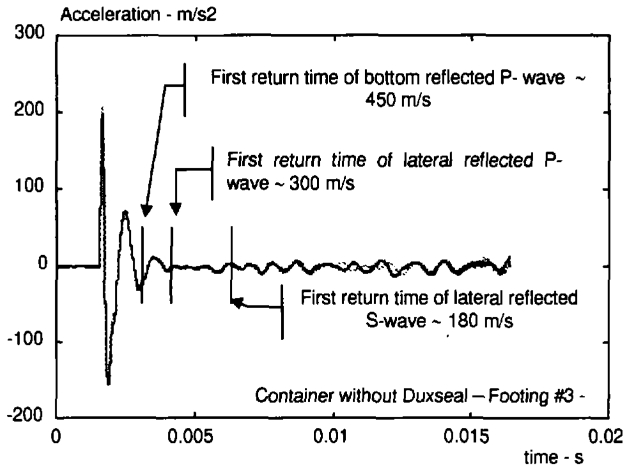

Fig. 2: Time history of a footing response to impact and wave reflection return times

The results are similar for the coated container, with the only difference being that the "noise" at the end of the signal is reduced. The conclusion proposed is that for containers of about 0.8 m in width and for foundations tested alone without a supporting structure, the Duxseal coating is not necessary with impact loading. However, this conclusion should be considered with care: the tests herein only concerned isolated footings. For lesser-damped structures, the interaction with reflections could be complex.

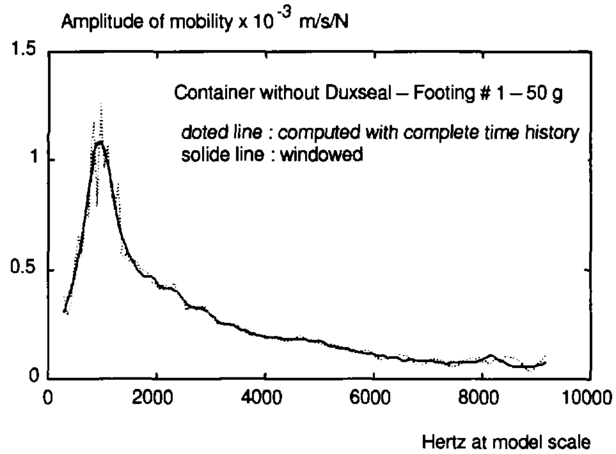

Fig. 3: Comparison of mobility functions commuted with and without a rectangular window limited to the 1,800 first data points (before the vertical P-wave return time)

Impact Loading and Impedance Models

Scaled modeling must still prove its relevance with respect to prototype behavior. The aforementioned works validate a continuous loading regime, but here the loading is of an impact type. Considering that the simple case of circular footings is well-documented at the prototype scale, it has been deemed possible to fit the parameters of these models to the experimental FRF. This fitting process has been carried out at the prototype scale strictly by applying scaling factors.

Wolf's models (1994) were used for the fitting process. The fit parameters here were: the mass, the shear modulus G, and the parameter γo of the damping factor C_0 in equation (4) (see also the appendix). The model, which introduces many parameters and exhibits frequency dependence, is much more likely to fit the experimental data. The first point to be noted is that fitting remains possible both with experimental data for all levels of gravity, translated directly onto the prototype scale by applying classical scaling relations, and with the prototype scale model. The second point is that the parametric values determined by this fitting process are relatively consistent with observation. This point will be discussed in the following section.

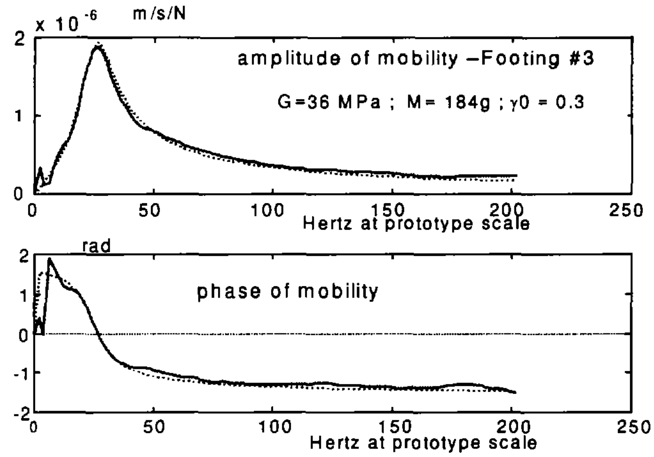

Fig. 6: Fitting a Wolf's model onto experimental FRF for a circular footing (fitting carried out at prototype scale)

Results of the Fitting Process

The fitting process consisted of a least-squares method applied to the complex mobility responses. Table III presents the values of the parameters obtained, translated at the model scale.

The parameter Co was chosen as a fitting variable for two reasons: first, the fit with Lysmer's model revealed the importance of the damping factor; and second, its expression was independent of ν, unlike M_0, M_1 and C_1. For vertical movements, Co=Kst .γ_0.ro/Vs, where $\gamma_{o\,model} = 0.8$. In Table III, it should be pointed out that the values of γ_o are systematically between 0.3 and 0.35. The differential with respect to Wolf's value is probably due to the specificity of the model soil: dry Fontainebleau sand is truly a non-cohesive material. Such is not common in real soils, and neither G nor ν are able to account for this.

Table III –Parameters fitted to Wolf's model

Footing #	Gravity g	G (MPa) cont. #1	G (MPa) cont. #2	Mass cont. #1 g	Mass cont. #2 g	γo
1	30	40	26	110	110	0.31
	40	57	32	121	102	0.33
	50	51	39	139	103	0.35
2	30		38		188	0.30
	40	48	44	259	178	0.32
	50	56	45	196	169	0.35
3	30	32	26	116	108	0.30
	40		35		103	0.33
	50	44	39	112	100	0.30
4	30		32		36	0.32
	40					
	50		40		37	0.31
5	30		31		172	0.31
	40		36		163	0.33
	50	48	42	179	156	0.35

Variations in the G modulus will be examined in the following section: the values lie within a reasonable range. It is obvious that those from container #2 are generally lower than for container #1 coated with Duxseal: it is possible that the raining process was disturbed by the presence of a gauze placed between the sand and the Duxseal. Though the raining parameters were identical, the above discrepancy in the evaluation of ν is likely to be associated with this latter finding.

The mass values are more heavily scattered: about 20% around the true value. This aspect must be improved with additional experimental results.

Analysis of the Fitted Shear Modulus

As could be expected, the fitted values of the shear modulus - an equivalent homogeneous shear modulus - tend to increase with the level of gravity. This finding complies with the behavior of the true shear modulus described by Hardin and Drnevich (1972) and then by Iwasaki and Tatsuoka (1977). Pak and Guzina (1995) proposed a formula for this equivalent homogeneous shear modulus based on the uniform pressure under the footing. This formula is slightly different from that of Iwasaki and Tatsuoka, which was based on the confining pressure of laboratory tests. These relations are of the following form:

$$Gdyn = Cst.f(e)\sigma^n \qquad (5)$$

where f(e) is a function of the void ratio and $0.4 \le n \le 0.5$.

Richard *et al.* (1970) had proposed another relationship. Since response to the soil foundation is stress-dependent and since pressure under the footing is not uniform (even under rigid footings), their proposal referred to the mean stress in the soil at the point where this mean stress is minimized.

This stress and its depth can be computed from the two components of stress in the soil under the foundation: the weight of the earth material (linearly increasing), and the stress induced by the foundation load (decreasing with depth). For reasons of symmetry, the computation has been performed vertically under the center of the foundation. The mean stress due to the weight of the earth is expressed as:

$$\sigma_o = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3), \quad \sigma_2 = \sigma_3 = \frac{\upsilon}{1-\upsilon}\sigma_1 \text{ and } \sigma_3 = \rho.g.z \qquad (6)$$

where ρ is the density and g the gravity.

The stress induced by the foundation load has been computed by integrating Boussinesq's equations of the distribution of stresses within a linear elastic half-space resulting from a point load.

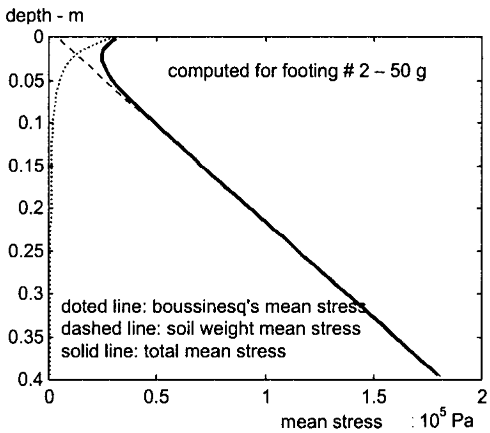

Fig. 7: Evolution of mean stresses under the center of the footing

Fig. 7 shows that in the region just under the footing, the stress is clearly variable: the mean stress varies from 338 kPa to 238 kPa between the point just under the surface and the minimum point.

The two relationships between fitted shear modulus and stress level under the footing have been tested. In Fig. 8, fitted G values have been plotted against $\sigma_u^{0.5}$, where σ_u is the uniform static pressure under the foundations. In Fig. 9, fitted G values are plotted against $\sigma_{min}^{0.5}$, where σ_{min} is the minimum total mean stress computed as above. The entire set of tests, including all footings and levels of gravity, have been combined on these plots. A solid line connects points from the same footing under the different gravity levels. It is clear that of the two relations, the one in Fig. 9 is more satisfying: a global linear trend appears, which is parallel to the line linking the points of each individual footing. The scatter in experimental results prevents further progress in evaluating the exponent. Additional results are necessary.

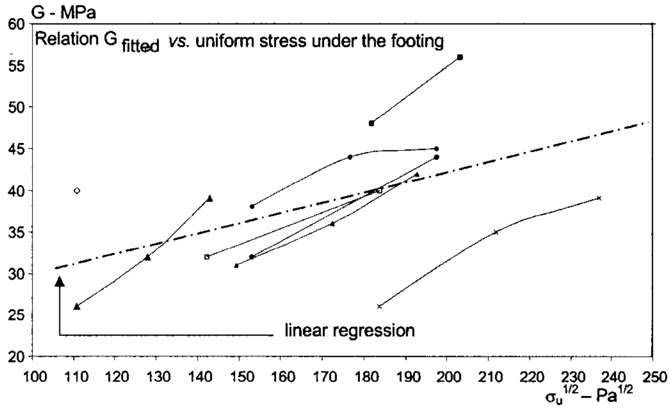

Fig. 8: Fitted values of the equivalent homogeneous shear modulus vs. uniform stress under the foundation

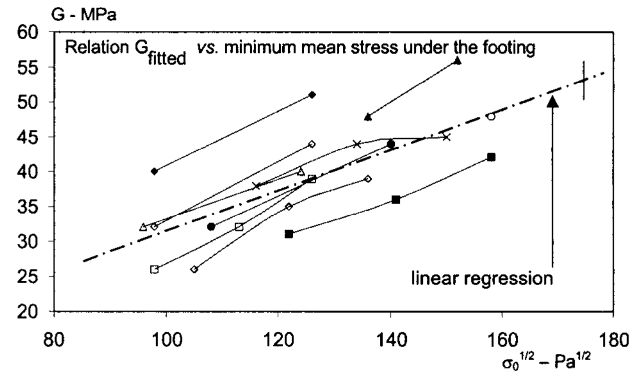

Fig. 9: Fitted values of the equivalent homogeneous shear modulus vs. minimum stress under the footing

CONCLUSION

Impact loading in the centrifuge has been shown to be a relevant method for conducting physical modeling of the dynamic response of footings in vertical movement. The influence of reflected waves on the boundaries of large rectangular containers is negligible. This conclusion should be controlled for lesser-damped systems, such as footing support structures.

The relevance of this method for studying dynamic responses is controlled through the possibility of fitting well-documented analytical models, proposed by Wolf, for circular and square footings. It is remarkable that such fittings are indeed possible - on dry sand - simply by applying scaling factors resulting from the fundamental dynamic equilibrium equation. At this stage, fittings of the various parameters are not all of the same quality: the scatter of values for the chosen damping factor is negligible, whereas that for the mass is about 20%. The fitted shear modulus appears to be gravity-dependent and hence stress-dependent, as expected. The use of dry sand in this kind of modeling exercise probably leads to underestimating the damping factors with respect to current soils.

Embedded accelerometers have enabled confirming both the variation in P-wave velocities with depth and the relation with mean stress. It has given rise to an attempted experimental validation of the following hypothesis: the value of the equivalent homogeneous shear modulus to be introduced into the computation of impedance function models is more strongly related to the minimum mean stress under the footing than to the uniform pressure just under the footing.

AKNOWLEDGEMENT

The author would like to express his gratitude to Chantal Bouhier and Pierre Hascoet for their help in preparing the experiments on which this paper is based. Thanks also to the technical staff at the "Mécanique et centrifugeuse" department of the Laboratoire Centrla des Ponts et Chaussées.

BIBLIOGRAPHY

Chazelas J.-L., O. Abraham, J.-F. Semblat [2000], *Identification of different seismic waves generated by foundation vibration: centrifuge experiments, time analysis and numerical investigations,* Fourth Int. Conf. on Recent Adv. in Geotech. Earthquake Engrg. and Soil Dyn., San Diego, CA, March 26-31, 2001, Paper 9-21.

Cheney J.A. , R.K. Brown, N.R. Dhat and O.Y. Hor [1990]. *Modeling free-field conditions in centrifuge models*. J. of Geotech. Engrg., Vol. 116 (9), pp. 1347-1367.

Coe, J, J.H. Prevost and R.H Scanlan [1985]. *Dynamic stress wave reflections/attenuation: Earthquake simulation in centrifuge soil model*, Erthq. Engrg. and Struct. Dyn., Vol. 13, pp. 109-128.

Hardin, B.O. and V.P. Drnevich [1972]. *Shear modulus and damping ratio in soils: Design equations and curves*, Proceedings ASCE, 98(7), pp. 667-692.

Iwasaki T. and F. Tatsuoka [1977]. *Effects of grain size and grading on dynamic shear moduli of sands*, Soils and Found., Vol. 17 (3), pp. 19-35.

Leguay P., P. Morlier and J. Rivière [1984]. *Essais dynamiques de fondations en centrifugeuse*, Revue Française de Géotechnique, n° 28, pp. 49-54.

Lenke L.R., R.Y.S. Pak and H.Y. Ko [1991]. *Boundary effects of modeling of foundations subjected to vertical excitation*, H.Y. Ko, F.G. McLean (Eds) Centrifuge 1991, Boulder, CO, USA, June 13-14 1991, AA Balkema, Rotterdam, pp. 473-480.

Meeks J.W. and J.P. Wolf [1992]. *Cone models for homogeneous soils*, J. of Geotech. Engrg., Vol. 118 (5), pp. 667-685.

Pak R.Y.S. and B.B. Guzina [1995]. *Dynamic characterization of vertically-loaded foundations on granular soils*, J. of Geotech. Engrg., Vol. 121 (3), pp. 274-286.

Richard F.E., R.D. Woods and J.R. Hall Jr. [1970]. *Vibration of soils and foundations*, Prentice-Hall, Englewood Cliffs, New Jersey.

Sieffert J.G. and F. Cevaer [1992]. *Handbook of impedance foundations: Surface foundations*, Ouest Edition, Nantes, France.

Siemer Th. and H.L. Jessberger [1994]. *Wave propagation and active vibration control in sand,* C.F. Leung, F.H. Lee and T.S. Tan (Eds), Centrifuge 1994, Singapore, August 31 - Sept. 2, AA Balkema, Rotterdam, pp. 307-312.

Wolf J.P. [1994]. *Foundation Vibration Analysis using simple physical models*, Prentice-Hall, Englewood Cliffs, New Jersey.

Appendix: Parameters of Wolf's models in equation (4) for vertical movement (ro radius of the footing)

$$Co = \frac{ro}{Vs}.\gamma o.\frac{4.G.ro}{(1-\nu)}$$

$$C1 = \frac{ro}{Vs}.\gamma 1.\frac{4.G.ro}{(1-\nu)}$$

$$Mo = \left(\frac{ro}{Vs}\right)^2.\mu o.\frac{4.G.ro}{(1-\nu)}$$

$$M1 = \left(\frac{ro}{Vs}\right)^2.\mu 1.\frac{4.G.ro}{(1-\nu)}$$

$\gamma o = 0.8$

$\gamma 1 = 0.34 - 4.3.\nu^4$

$\mu o = 0$

$\mu 1 = 0.4 - 4.\nu^4$