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Lateral Buckling of Singly Symmetric Beams

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LATERAL BUCKLING OF SINGLY SYMMETRIC BEAMS

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ABSTRACT

General solutions for the elastic lateral buckling moment of singly symmetric sections are studied. The studies include the effect of the location of the load on the section as well as the effect of moment gradients on the lateral buckling moment. Design provisions are outlined for the case of moment gradients.

BACKGROUND

Singly symmetric sections are used frequently as beams or beam-columns in aluminum and cold-formed steel structures. The studies presented here were carried out to develop design provisions for aluminum members. However, the general approach for calculating elastic lateral buckling moment is applicable to steel as well. Some typical members for which the general subject is relevant are shown in Fig. 1.

Lateral buckling of singly symmetric sections has been studied by many researchers. The design approach presented here is for the most part based on the work of these researchers. The results of these studies were simplified for design, and a design approach was developed for the case of varying moment along the span.

Clark and Hill [1960] present a solution for the lateral buckling of singly symmetric sections under a variety of loading conditions.

Peköz [1969] and Peköz and Winter [1969] have studied the lateral buckling of singly symmetric sections under eccentric axial loading. These studies include lateral buckling of singly symmetric sections subjected to linearly varying moments. Various end conditions are accounted for.

Kitipornchai, et al [1986], give an analysis of buckling of singly symmetric I-beams under moment gradient. It is seen in this reference and in Peköz [1969, pages 59-62] that the use of moment gradient correction factor C_b may give grossly erroneous

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results for unsymmetric sections subjected to a moment gradient that causes reverse curvature.

An interesting study on the lateral buckling of singly symmetric I-Beams is presented by Wang and Kitipornchai [1986]. The coefficients given in their paper are included in the design recommendations developed here.

GENERAL SOLUTION

Based on the elastic torsional-flexural buckling theory, Clark and Hill [1960] derive an equation for the lateral buckling of singly symmetric beams bending in the plane of symmetry. This expression also considers the location of the laterally applied load with respect to the shear center. With a slight change in notation, their equation can be written as follows:

$$M_e = C_b A \sigma_{ey}\left[U + \sqrt{U^2 + r_o^2\left(\frac{\sigma_t}{\sigma_{ey}}\right)}\right] \qquad \text{Eq. 1}$$

In the above equation

$$\sigma_{ey} = \frac{\pi^2 E}{\left(\frac{k_y L_b}{r_y}\right)^2} \qquad \text{Eq. 2}$$

$$\sigma_t = \frac{1}{A r_o^2}\left(GJ + \frac{\pi^2 E C_w}{L_t^2}\right) \qquad \text{Eq. 3}$$

$$U \quad C_1 g + C_2 j \qquad \text{Eq. 4}$$

A full cross-sectional area
C_b, C_1 and C_2
coefficients to be taken as discussed below
C_w torsional warping constant of the cross-section
E modulus of elasticity
G shear modulus
g distance from the shear center to the point of application of the load.
I_y moment of inertia of the section about the y axis
J torsion constant

$$j = \frac{1}{2I_x}\left(\int_A y^3 dA + \int_A yx^2 dA\right) - y_o \qquad \text{Eq. 5}$$

L_t effective length for twisting
L_t can be taken conservatively as the unbraced length. If warping is restrained at one end it can be taken as .8 L_b. If warping is restrained at both ends it can be taken as .6 L_b.

$$r_o = \sqrt{r_x^2 + r_y^2 + y_o^2} \qquad \text{Eq. 6}$$

polar radius of gyration of the cross-section about the shear center.

r_x, r_y radii of gyration of the cross-section about the centroidal principal axes

S_c section modulus for the extreme compression fiber for bending about the x-axis

y_o y - coordinate of the shear center

In calculating the section properties, as well as the parameter g, it is essential to use a proper and consistent axis orientation. Equation 1 assumes that the centroidal symmetry axis is the y-axis and bending is about the x-axis. The y-axis is oriented such that the tension flange has a positive y-coordinate. The value of g is to be taken as + when the load is applied directed away from the shear center and - when the load is directed toward the shear center. When there is no transverse load (pure moment cases) g = 0. The orientation of the axes and the cross-sectional notation are illustrated in Fig. 2.

Kitipornchai, et al. [1986] show that for singly symmetric I sections j can be approximated as

$$.45 d_f \left(2\frac{I_{cy}}{I_y} - 1\right)\left[1 - \left(\frac{I_y}{I_x}\right)^2\right] \qquad \text{Eq. 7}$$

In this equation I_{cy} is the moment of inertia of the compression flange, I_x and I_y are the moments of inertia of the entire section about the x- and y-axes and d_f is the distance between the flange centroids or for T-sections d_f is the distance between the flange centroid and the tip of the stem. In a conversation, Dr. John Clark pointed out that when the areas of the compression and tension flanges are approximately equal, j can also be approximated by $-y_o$.

DETERMINATION OF C_b FOR DOUBLY SYMMETRIC SECTIONS

The moment gradient in the span or the unbraced segment is usually accounted for by multiplying the critical moment for the uniform moment case by a factor designated as C_b. The following expression is used in the AISI [1989, 1991] and the AISC [1986] Specifications:

$$C_b=1.75+1.05\left(\frac{M_1}{M_2}\right)+0.3\left(\frac{M_1}{M_2}\right)^2\leq 2.3 \qquad \text{Eq. 8}$$

in this equation M_1 is the smaller and M_2 the larger bending moment at the ends of a laterally unbraced length, taken about the strong axis of the member. The ratio of end moments, M_1/M_2, is positive when M_1 and M_2 have the same sign (reverse curvature bending) and negative when they are of opposite sign (single curvature bending). When the bending moment at any point within an unbraced length is larger than that at both ends of this length, and for members subject to combined axial load and bending moment C_b is to be taken as unity.

A more general expression for C_b being considered for inclusion in the AISC Specification is

$$C_b = \frac{12.5\,M_{MAX}}{2.5\,M_{MAX}+3\,M_A+4\,M_B+3\,M_C} \qquad \text{Eq. 9}$$

where

M_{MAX} absolute value of maximum moment in the unbraced beam segment
M_A absolute value of moment at quarter-point of the unbraced beam segment
M_B absolute value of moment at mid-point of the unbraced beam segment
M_C absolute value of moment at three-quarter-point of the unbraced beam segment

The AISI Specifications [1989, 1991] have provisions for lateral buckling of singly symmetric sections. For bending about the symmetry axis (x-axis is axis of symmetry oriented such that the shear center has negative x-coordinate.) the following equation is given:

$$M_e=C_b r_o A\sqrt{\sigma_{ey}\sigma_t} \qquad \text{Eq. 10}$$

For bending centroidal perpendicular to the symmetry axis the following formula is given

$$M_e=\frac{C_s A\sigma_{ex}\left(j+C_s\sqrt{j^2+r_0^2\frac{\sigma_t}{\sigma_{ex}}}\right)}{C_{TF}} \qquad \text{Eq. 11}$$

where

$$\sigma_{ex} = \frac{\pi^2 E}{\left(\frac{k_x L_b}{r_x}\right)^2} \qquad \text{Eq. 12}$$

In this formula C_s is + 1 for moment causing compression on the shear center side of the centroid and -1 for moment causing tension on the shear center side of the centroid. The factor C_{TF} is to be calculated using

$$C_{TF}=0.6-0.4\frac{M_1}{M_2} \qquad \text{Eq. 13}$$

The expression for $1/C_{TF}$ gives very close results to those obtained using the expression for C_b. The basic difference is the upper limit of 2.3 for C_b.

A comparison of equations 8, 9 and 13 is illustrated in Fig. 3. While all equations agree well above M1/M2 = - 0.25, equation 13 differs significantly with the other two equations at values of less than - 0.25. This difference needs to be considered further, particularly for singly symmetric sections. The principal advantage of equation 9 over 8 is the ability of equation 9 to estimate C_b values accurately for most nonlinear moment gradient cases such as for beams with lateral loading.

DETERMINATION OF C_b FOR SINGLY SYMMETRIC SECTIONS

The application of the C_b factor to singly symmetric sections in the same manner as for doubly symmetric sections has been shown to be very unconservative in certain situations by Kitipornchai, et al [1986]. They show clearly that this is the case with plots such as given in Fig. 4. They have considered unsymmetric I sections, however similar results are expected for other singly-symmetric open sections. The unconservative results arise if the C_b factor is applied to the critical moment determined for the case of larger flange in compression, M_L, when it is possible that somewhere in the unbraced segment the smaller flange may be subject to compression.

Parameters appearing in Fig. 4 are

$$\overline{K}=\sqrt{\frac{\pi^2 E I_y h^2}{4GJL^2}} \qquad \text{Eq. 14}$$

$$\beta=\frac{M_1}{M_2} \qquad \text{Eq. 15}$$

$$\rho = \frac{I_{cy}}{I_y}$$ Eq. 16

The factor m shown in the figure is the same as C_b. Namely,

$$m = \frac{M_e}{M_o}$$ Eq. 17

M_e is the elastic lateral buckling moment for the given moment gradient, M_o is elastic lateral buckling moment for uniform moment.

The curve designated "Equation 6" is plotted using Eq. 8 for C_b except for the upper limit of 2.56.

Single Curvature Cases

It is seen in Fig. 4 and other similar figures in Kitipornchai, et al [1986] that for single curvature cases, namely for M1/M2 less than zero, it is satisfactory to modify the lateral buckling moment for equal end moments through the use of coefficients C_b, C_1 and C_2 except when ρ is less than 0.1. For values of ρ less than 0.1 it appears reasonable to take $C_b = 1$.

The expressions for C_b, C_1 and C_2 for some special cases are given in Wang and Kitipornchai [1986]. The expressions given below are somewhat simplified versions of the ones given in the reference. These expressions are valid for single span, simply supported beams with singly or doubly symmetric sections bent in the plane of symmetry.

a. Uniformly distributed load over the entire span

   $C_b = 1.13$, $C_1 = 0.46$, $C_2 = 0.53$

b. One concentrated load placed at aL from one of the ends of span

$$C_b = 1.75 - 1.6a(1-a)$$ Eq. 18

$$C_1 = \frac{C_b}{a(1-a)\pi^2}\sin^2\pi a$$ Eq. 19

$$C_2 = \frac{C_b - C_1}{2}$$ Eq. 20

   When a = 0.5: $C_b = 1.35$, $C_1 = 0.55$, $C_2 = 0.40$

c. Two concentrated loads placed symmetrically at aL from each end of span

$$C_b = 1+2.8a^3 \qquad \text{Eq. 21}$$

$$C_1 = \frac{2C_b}{a\pi^2}\sin^2\pi a \qquad \text{Eq. 22}$$

$$C_2 = (1-a)\,C_b-\frac{C_1}{2} \qquad \text{Eq. 23}$$

Reverse Curvature Cases

It is seen in Fig. 4 that when M1/M2 is greater than zero, the use of C_b factor, without considering the singly symmetric nature of the section, can give very inaccurate results. A singly symmetric section can have two critical moments that can be significantly different from one another. For a singly symmetric I section, the critical moment when the larger flange is in compression, M_L, can be several times that when the smaller flange is in compression, M_S. If the maximum moment in the span occurs at a section with the large flange in compression and the C_b factor is applied to M_L then the critical moment calculated may be several times the actual critical moment. For open sections such as lipped C sections as shown in Fig. 1, M_L is for the case when compression is on the shear center side of the centroid, and M_S for the case when tension is on the shear center side of the centroid.

Some reverse curvature cases are illustrated in Figs. 5 and 6 . In Fig. 5, if the top flange is the smaller flange and M_{MAX} occurs at a section with smaller flange in compression, the application of the C_b factor M_S to determine the critical moment would give conservative results. This is because in each case, the larger flange is subjected to compression in a part of the span and the actual critical moment is larger than C_bM_s.

If the top flange is the larger flange in Fig. 5, and M_{MAX} occurs at a section with the large flange in compression then determining the critical moment as C_bM_L would be unconservative because the presence of a segment with a smaller flange in compression would lead to a lower actual critical moment. A lower bound to the lateral buckling moment at the end with the smaller flange in compression can be found assuming the moment gradient in the beam as shown in Case 2 of Fig. 6. The lower bound is obtained because it is assumed that throughout the entire span the smaller flange is subjected to compression and the moment varies from zero to the value of the maximum moment that is present in the portion of the span with the smaller flange in compression.

The application of the coefficients C_b, C_1 and C_2 to end moment cases can be demonstrated for the four beams shown in Fig. 6. If the top flange is the smaller flange, the C_b factor can be applied to M_S conservatively in each case. The resulting lateral buckling moments are required to be larger than the actual applied maximum moments.

If the top flange is the larger flange, the C_b factor cannot be applied to M_L conservatively in Cases 3 and 4 without checking to see if a lower lateral buckling moment is possible, due to the fact that over a portion of the beam the smaller flange is in compression. A lower bound to the buckling moment for the case with the smaller flange in compression over a portion of the span can be found by assuming that the smaller flange is subjected to a moment distribution as shown for Case 2 with the small flange in compression.

For Case 3 in Fig. 6 with the smaller top flange, C_b for the actual moment distribution can be computed and applied to M_S and compared with M2. The moment at the end with M1 does not need to be checked.

In summary, C_b can be determined as usual for all cases except when M_{MAX} produces compression on the larger flange and the smaller flange is also subjected to compression in the unbraced length. In this case, the member need also be checked at the location where the smaller flange is subjected to its maximum compression. At that location C_bM_S should be larger than the actual moment. Load and resistance factors or factors of safety need to be taken into consideration in this comparison.

DETERMINATION OF C_1 AND C_2

Values of C_1 and C_2 are given above for some cases. For doubly and singly symmetric sections subjected to a linear variation of moment along the span or in the unbraced segment C_1 is equal to zero. For other variations there are no theoretically obtained values available except for the special cases listed above. For these variations, unless more accurate values are available it appears reasonable to take $C_1 = 1$.

For doubly symmetric sections j = 0, thus C_2 is not needed. For singly symmetric sections, when moments vary linearly between the ends of the unbraced segment $C_2 = 1$. For other variations there are no theoretically obtained values available except for the special cases listed above. For these variations, as pointed out in a conversation by Dr. LeRoy Lutz, it may be reasonable to interpolate between the values given for the special cases and the linear moment case.

SUMMARY AND CONCLUSIONS

A general design procedure for calculating lateral buckling moments of singly symmetric beams has been developed.

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REFERENCES

American Institute of Steel Construction [1986], "Load and Resistance Design Specification for Structural Steel Buildings," September 1, 1986 Edition

American Iron and Steel Institute [1989], "Specification for the Design of Cold-Formed Steel Structural Members," August 19, 1989 Edition with December 11, 1989 Addendum

American Iron and Steel Institute [1991], "Load and Resistance Design Specification for Cold-Formed Steel Structural Members," March 16, 1991 Edition

Clark, J. W. and Hill, H. N. [1960], "Lateral Buckling of Beams," J. of the Structural Division, Vol. 86, No. ST7, July 1960, ASCE, pp. 175-196

Kitipornchai, S., Wang, C. M. and Trahair, N. S. [1986], "Buckling of Monosymmetric I-Beams Under Moment Gradient," J. of the Structural Division, Vol. 112, No. ST4, Apr., 1986, ASCE, pp. 781-799.

Peköz, T. [1969] (with a contribution by N. Celebi), "Torsional-Flexural Buckling of Thin-Walled Sections Under Eccentric Load," Cornell Engineering Research Bulletin 69-1, Cornell University.

Peköz, T. and Winter, G., [1969] "Torsional-Flexural Buckling of Thin-Walled Sections Under Eccentric Load," J. of the Structural Division, Vol. 95, No. ST5, May, 1969, ASCE, pp. 941-1063.

Wang, C. M. and Kitipornchai, S. [1986], "Buckling Capacities of Monosymmetric I-Beams," J. of the Structural Division, Vol. 112, No. ST11, Nov., 1986, ASCE, pp. 2373-2391.

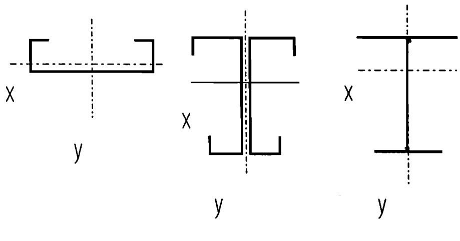

Fig. 1 Singly symmetric sections

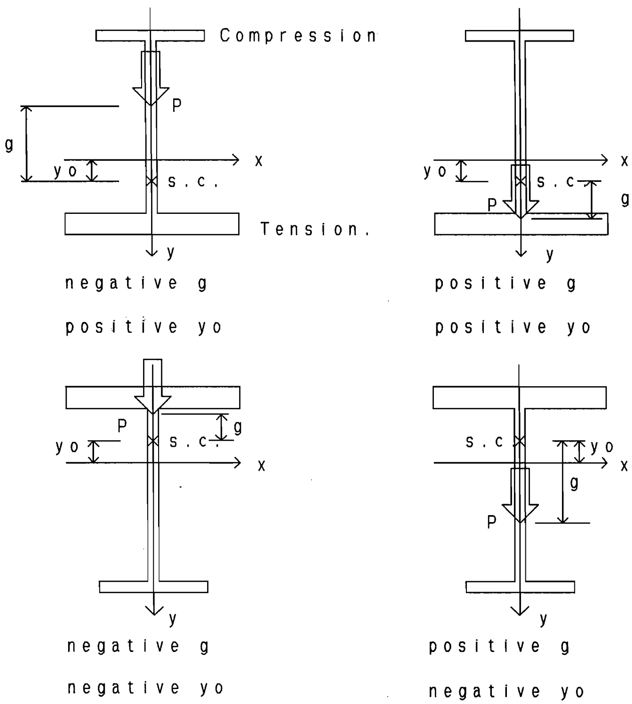

Fig. 2 Orientation of the axes and cross-sectional notation

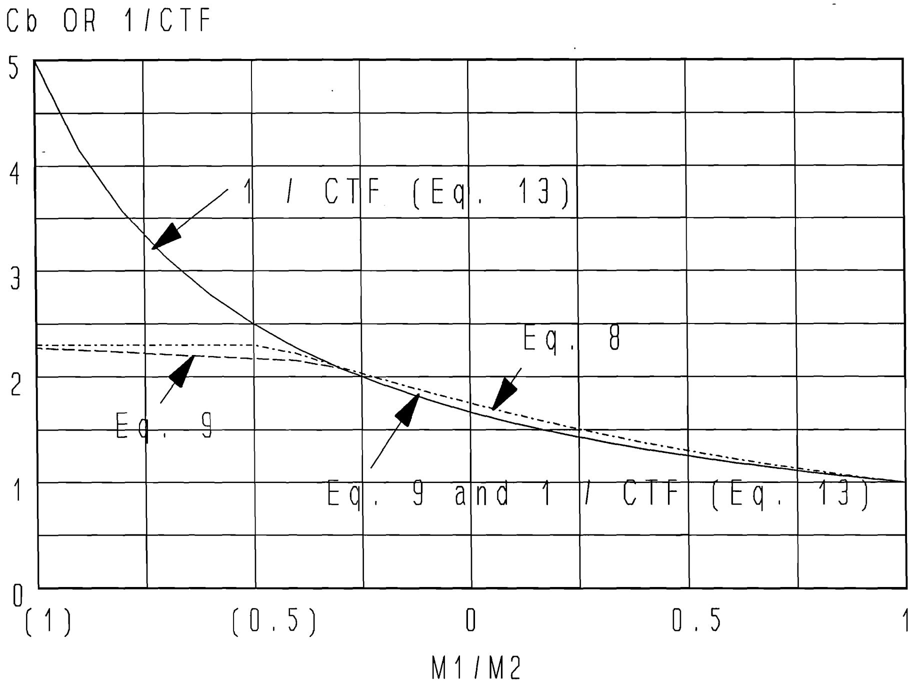

Fig. 3 C_b or $1/C_{TF}$

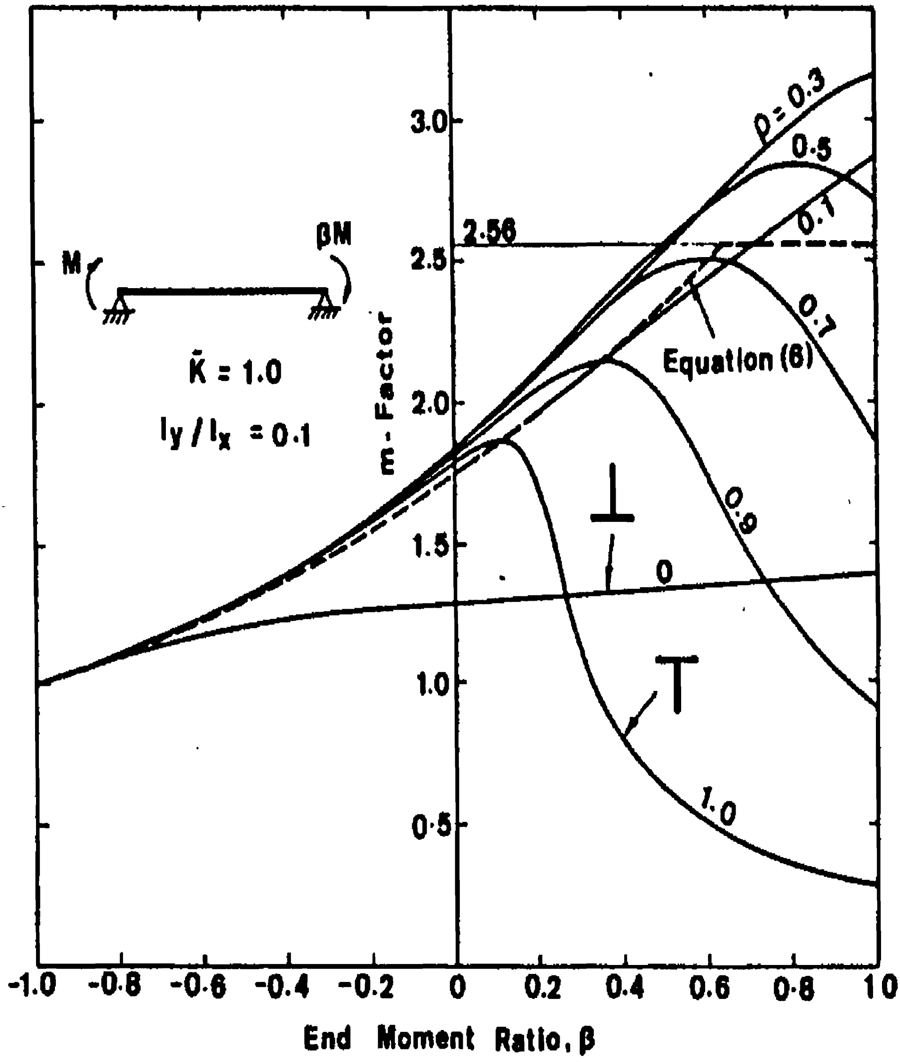

Fig. 4 Coefficient m (= C_b) versus β (= M1/M2) from Kitipornchai, et al [1986]

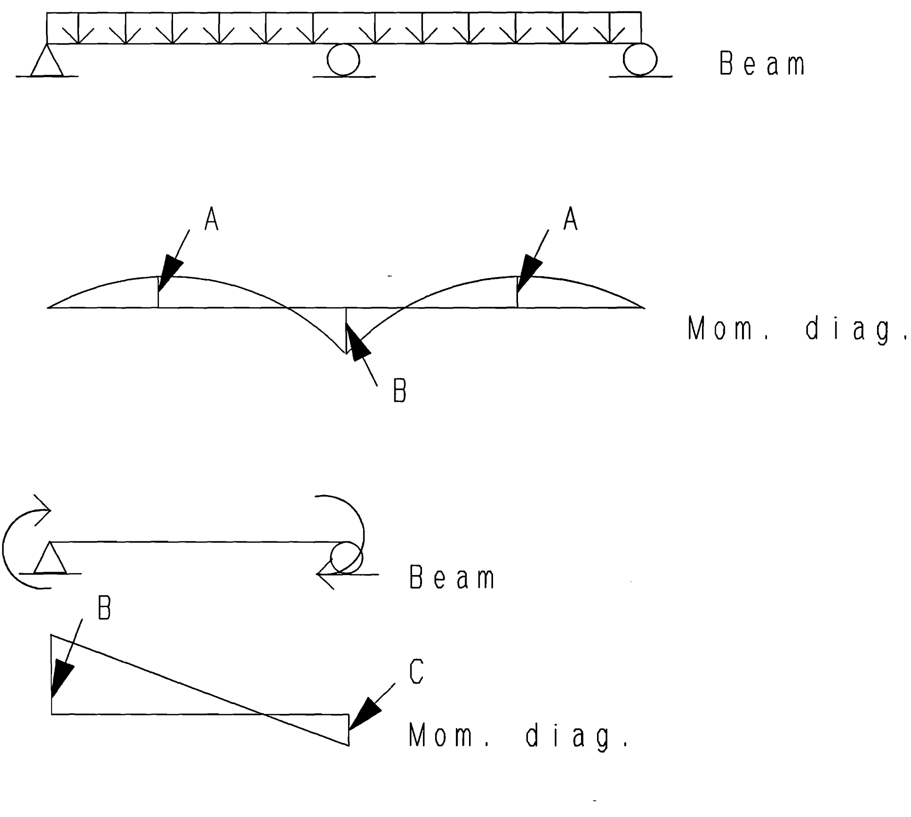

Fig. 5 Beam and Moment Diagram Examples

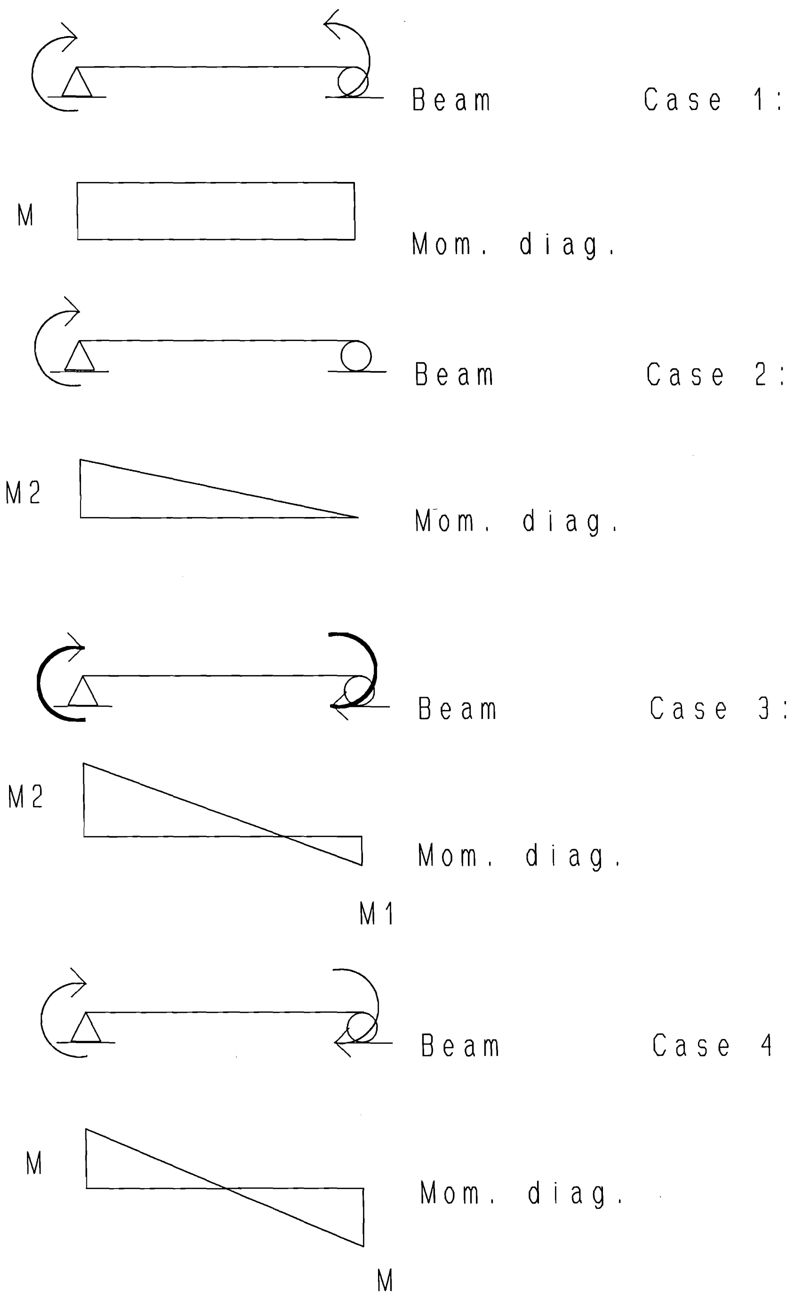

Fig. 6 Beam and Moment Diagram Examples