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ON DESIGN OF PROFILED SHEETS WITH VARYING CROSS SECTIONS

by

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ABSTRACT

A short review is given on the effective width concept and about its use for determining of the moment capacity of sheets. A modified method, taking into account the varying flexural rigidity along the spans, is described and illustrated on examples carried out on first generation panels.

ON THE EFFECTIVE WIDTH CONCEPT

Since publications of T. Kármán (Kármán, 1932) and G. Winter (Winter, 1947) design of slender cross sections, e.g. profiled sheets is carried out by the well known effective width concept.

The 1989's version of the European design code (Eurocode N°3 Design of Steel Structures, Part 1 — General Rules for Buildings) and its Annex A: Cold–Formed Thin–Gauge Members and Sheeting like the AISI Specification, gives the formula for the effective width of simply supported plates under uniform compression, as follows:

$$b_{ef} = \rho\, b_p$$

where

$$\rho = 1 \qquad \text{if } \lambda_{pd} \leq 0.673$$

$$\rho = (1 - 0.22/\lambda_{pd})/\lambda_{pd} + 0.18(\lambda_p - \lambda_{pd})/(\lambda_p - 0.6) \qquad \text{if } \lambda_{pd} > 0.673$$

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In the formulae above:

$$\lambda_{pd} = 1.052\, b_p/t\sqrt{\sigma_c/(k_\sigma E)}$$

$$\lambda_p = 1.052\, b_p/t\sqrt{f_y/k_\sigma E)}$$

$\sigma_c =$ stress in the compression element computed on the basis of the effective design width
$b_p =$ notional plane width
$t =$ thickness of the sheet
$k_\sigma =$ plate buckling coefficient (usually 4.0 is considered)
$E =$ modulus of elasticity of steel
$f_y =$ yield point used for design

The main difference between the AISI Specification (AISI, 1986) and the EC3–Annex A is in definition of the so called "basic width" as it has been pointed out by J.Rondal (Rondal, 1989) and B.Verőci (Verőci, 1990). Fig.1 shows the two different approaches: it can be mentioned that rounding of corners may be ignored by EC3 if $r \leq 5t$ and $r/b_p \leq 0.15$ where $r =$ the inner radius. The EC3 approach results conservative solution in the effective width and section moduli to the American Specification.

It is evident that, due to the fact that it is a function of the compression stress, in most of cases the effective width is a subject of iteration. One can also state that the effective width may vary in the compression flange and the web along the span due to the varying bending moment line.

The design value of the bending moment shall be determined as follows:

$$M_d = W_{ef}\, f_y/1.1$$

where
$W_{ef} =$ elastic section modulus of the effective section calculated with the extreme compression or tension fiber at f_y (EC3 – Annex A allows utilization of inelastic reserve capacity of the tension zone)
$1.1 =$ safety factor used on the resistance side (EC3 accepts the LRFD concept)

The uniformly distributed load over unit area values determined from the above bending moments used to be tabulated together with the ones obtained taking into account deflection limits. Usual parameters of such tables compiled for a particular profile can be as follows: thickness of sheet, number of spans (1, 2 and 3 used to be considered), position of sheet and span. It should be mentioned that design values determined by the load–bearing criterion sometimes are subject of reduction due to the bending moment – support reaction interaction problem at internal supports of multispan sheets (EC3 prescribes definitely checking of this phenomenon for profiled sheets).

Let us summarize briefly on a two-span-sheet the usual way used for getting particular values of the load table mentioned above. The bending moment and shear force lines under U.D.L. are shown in Fig.2. The load capacity is governed by the design value of the bending moment determined at the internal support and calculated as follows:

$$\text{from} \quad M_{\max} = qL^2/8 \qquad q_d = 8M_d/L^2$$

where L = span , q_d = design value of the load capacity.

It is usually not dominant but the sheet should also be checked at the midspan vicinity using the respective section modulus.

Authors criticize that practice when moments of inertia and section moduli are tabulated without giving any commentary about the level of σ_c.

The design strength with respect to web crippling for one unstiffened sheeting web is as follows (clause A 4.4.2.2 of EC3 – Annex A):

$$R_d = 1.5\,t^2\sqrt{f_y E}(1 - 0.1\sqrt{r/t}) \cdot (0.5 + \sqrt{0.02 l_a/t}) \cdot (2.4 + (\phi/90)^2)$$

where l_a = bearing length, ϕ = web inclination ($45^o \leq \phi \leq 90^o$).

At internal supports the load capacity may be limited by interaction of combined bending and support reaction, therefore the following conditions shall be satisfied (see also Fig.3):

$$M/M_d \leq 1 \qquad \text{when} \qquad R/R_d \leq 0.25$$
$$M/M_d + R/R_d \leq 1.25 \qquad \text{when} \qquad 0.25 < R/R_d \leq 1$$

where M and R = bending moment and support reaction, respectively;
M_d and R_d = design value of bending capacity and support reaction capacity with respect to web crippling, respectively.

If $R/R_d > 0.25$ then the full moment capacity cannot be utilized, i.e. the design value of U.D.L. should be determined from a reduced bearing capacity. Taking into account the fact that the bending moment and the support reaction are linear function of U.D.L. this capacity reduction can be carried out following the way shown in Fig.3: the point representing the limit state of the sheeting at an internal support should be moved along the line between the origo and A to A', i.e. onto the interaction "curve".

With consideration to deflection limits, U.D.L. values may also be determined (usually using ideally elastic material behavior). The latter values are compared to unfactored (working) loads while the bearing values to the factored ones (as we have mentioned above EC3 follows the LRFD concept). It is beside the point but authors remark that Hungary is the country where the LRFD concept at first has been used in the world in the Code for Railroad Bridges (1951); this concept is used in Code for Building Steel Structures since 1957.

MODIFIED METHOD FOR CALCULATION OF MULTISPAN SHEETS

Authors have underlined in the first chapter of this paper that the effective width may vary in the compression parts of the section along the spans due to the varying bending moment line. In other words, the flexural rigidity of profiled sheets varies along the span(s) and this circumstance does affect the moment distribution (and the shear forces one, as well). The only question is whether it is worthwhile to take into account this effect.

It is not very difficult to compile a computer programme running on PC–AT-s which can consider the above mentioned phenomenon, i.e. the profiled sheet may be replaced by a member divided into equal length elements with, in the most general case, different flexural rigidity due to local buckling.

The design values of the load capacity, determined either on load–bearing or on deflection limit conditions, can be calculated by successive approximation.

The calculation may be started with a constant rigidity beam and a load value determined from an approximative formula. Using the ordinates of the bending moment line obtained, moment of inertia of every bar element can be determined usually on the way of iteration. Having had the modified moments of inertia, the new internal forces (bending moments and shear forces) are calculated on a beam with varying cross sections. Using the new bending moment ordinates usually modified moments of inertia can be found for the bar elements. This procedure may be ended if change in the load value after an iteration step is less than the prescribed limit. Consideration of special conditions like web crippling can be made after having the "final" result.

Our experience with the programme shows that using of L/10 long bar elements ensures the necessary accuracy, increasing of bar elements rapidly increases the running time without having a new class of accuracy. In order to decrease the time consumption, convergence control should be used mainly for deflection calculations and special attention should be paid to the symmetry of the section and the beam, as well.

EXAMPLES

The method described above is tested by a series of sections shown in Fig.4. The aim of these calculations is to point out the differences between the results of a "traditional" calculation and the suggested one. These sections without any stiffeners belong to the group of first generation ones: such sections are generally rolled up to about 3 inches (75 mm) depth in Europe; sometimes they have stiffeners in the wide flanges.

The layout shown in Fig.4, where the narrow flange is at the bottom, is called positive, the opposite one is negative. The steel properties used in our calculations have been f_y= 36.26 ksi (250 N/mm^2) and E= 30457 ksi (210 000 N/mm^2). As for deflection limits, L/300 and L/200 have been considered.

Results of the calculations carried out on the example sheets using the programme and the so called traditional method are listed in Table 1 in US Customary and in Table 2 in SI-units for two– and three-span-sheets with equal spans of 10' (3.04 m). For a particular case, three values are given: the first is determined by the strength condition, the two other ones are based on the above deflection limits. The first number (a) is obtained using the programme, the second value (b) is result of a conventional calculation and the third one (c) expresses the difference between the two values in percentage taking as basis the "conventional" result (positive if the first value is greater).

It is worthwhile to mention that, due to the not very high h/t ratio of the web, reduction in width is needed only for the compression flange of the section.

Let us separately evaluate the two types of values involved in the tables: first those ones obtained on the **ultimate limit state,** i.e strength are subject of our investigation.

In case of **positive layout** the wide flange in the span is under compression: as a consequence of this fact the flexural rigidity at an internal support is greater than in the span so that the maximum bending moment at the support increases to the constant rigidity case. Taking into account that design is governed by the support section capacity, such consideration of moment redistribution **reduces** the load capacity to a traditional calculation.

In case of **negative layout** the behavior of a multispan sheet is teh opposite: the load capacity is greater than that of obtained by using the conventional method. The reason of such results is that reduction of the flexural rigidity at an internal support due to the bottom wide flange buckling redistributes the moments, i.e. the moment at the internal support are decreasing while they are increasing at midspan: this circumstance causes an **increase** of the U.D.L. resisted by the sheet.

A similar duality can be found evaluating the results obtained on **deflection limit conditions**. In case of **positive layout** deflection is lesser to a conventional calculation due to increasing support moment, i.e. results based on the usual method are on the **safe side**. On the contrary, decreasing of the support moment causes increasing of deflection at midspan in case of **negative layout**: a conventional calculation is **unsafe** to the above described computer aided design.

In order to make more visible the tendency of the above results, in Fig.5 change of the effective moment of inertia is plotted on the base of the Fig.3 bending moment line for a two-span-sheet with "D" type section.

Authors mention that the above values obtained on strength condition should be reduced due to moment–support reaction interaction: its range is between 5% and 9%. These results are not involved in the tables because they influence the results obtained by the two methods practically to the same extent.

Evaluating the results, it is doubtless that a number of values have been obtained belong to not practicable cases, e.g. some results determined on deflection limits for two-span-sheets in negative layout where the high values cannot be utilized (we mention once more that in the case of a partial safety factor design **unfactored** loads have to be used for deflection calculations). However, many values of the tables can be utilized and the range of their deviation from the ones obtained by the conventional calculation in some cases seems to be considerable and, which is the most important in our view, the sign of the deviation may be different: a so called conventional calculation gives results being **sometimes on the safe, sometimes on the unsafe sides**. Having such a software it is not very difficult to use the above idea for tabulating of sheet load capacity data.

CONCLUSIONS

Using PC–AT-s, it is not very difficult to obtain load data for profiled sheets considering change of the effective width(s) of the section, i.e. varying moment of inertia along the spans. Such calculations can describe in some cases the behavior of the sheets more efficiently than the traditional ones which use constant flexural rigidity. The method presented above seems to be useful mainly for first generation profiles, i.e. for sheets without stiffeners where considerable reduction of compression plate width(s) may take place using the effective width concept.

Examples carried out on a series of sheets show that results of traditional calculations are sometimes on the safe, sometimes on the unsafe side and the range of the deviation frequently is large enough for taking into consideration.

It is obvious that this method is not for individual calculations but for those ones which are typical for sheets when limit loads over unit area of a particular section are tabulated in cases of different number of spans, sheet positions, thicknesses and spans.

APPENDIX - NOTATION

b_{ef}	effective design width of uniformly compressed elements
b_p	notional plane width
E	modulus of elasticity of steel
f_y	yield point used for design
ϕ	web inclination ($45^o \leq \phi \leq 90^o$)
h	design height of the web
k_σ	plate buckling coefficient (usually 4.0 is considered)
l_a	bearing length
$M, M_d, M_{\max}$	calculated, design and maximum (at internal supports) value of bending moment, respectively
q_d	design value of U.D.L.
r	inner radius of the sheet
R, R_d	calculated and design (with respect to web crippling) value of support reaction, respectively
ρ	factor of reduction used for determining of compression plate effective width
σ_c	stress in the compression element computed on the basis of the effective design width
t	thickness of the sheet
W_{ef}	elastic section modulus of the effective section calculated with the extreme compression or tension fiber at f_y (inelastic reserve capacity of the tension zone is not utilized in our calculations)

APPENDIX - REFERENCES

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TABLE 1

Calculated design loads over unit area in lbs/sq.ft

Nº of spans	Layout	Condition	Type of section A a	A b	A c	B a	B b	B c	C a	C b	C c	D a	D b	D c
2	+	Strength	45.43	44.74	+1.5	40.31	40.54	–0.6	35.90	36.78	–2.4	29.30	30.77	–4.8
		L/300	43.09	42.54	–4.4	33.94	34.07	–0.4	31.52	30.81	+2.3	27.19	25.75	+5.6
		L/200	51.19	52.99	–3.4	47.54	47.33	+0.4	43.76	42.50	+2.9	37.47	35.28	+6.2
	–	Strength	45.43	44.74	+1.5	41.27	39.43	+4.7	37.47	34.96	+7.2	31.33	28.34	+10.5
		L/300	43.09	42.54	–4.4	34.23	37.01	–7.5	32.48	35.67	–9.0	28.89	32.37	–10.8
		L/200	51.19	52.99	–3.4	48.64	51.84	–6.2	45.68	49.71	–8.1	40.00	44.80	–10.7
3	+	Strength	56.06	55.91	+0.3	49.83	50.67	–1.6	44.49	45.97	–3.2	36.40	38.45	–5.3
		L/300	28.03	28.53	–1.8	27.07	26.78	+1.1	24.92	24.27	+2.7	21.26	20.32	+4.6
		L/200	41.40	41.65	–0.6	38.16	37.34	+2.2	34.86	33.61	+3.7	29.55	27.97	+5.7
	–	Strength	56.06	55.91	+0.3	50.57	49.29	+2.6	45.66	43.71	+4.4	37.93	35.44	+7.0
		L/300	28.03	28.53	–1.8	27.78	28.47	–2.4	26.75	27.76	–3.6	24.27	25.46	–4.7
		L/200	41.40	41.65	–0.6	40.27	40.81	–1.3	38.39	39.18	–2.0	34.25	35.38	–3.2

1 lb/sq.ft = 47.87789 N/m^2

TABLE 2

Calculated design loads over unit area in N/m^2

N° of spans	Lay-out	Con-di-tion	Type of section A			B			C			D		
			a	b	c	a	b	c	a	b	c	a	b	c
2	+	Strength	2175	2142	+1.5	1930	1941	−0.6	1719	1761	−2.4	1403	1473	−4.8
		L/300	1697	1776	−4.4	1625	1631	−0.4	1509	1475	+2.3	1302	1233	+5.6
		L/200	2451	2537	−3.4	2276	2266	+0.4	2095	2035	+2.9	1794	1689	+6.2
	−	Strength	2175	2142	+1.5	1976	1888	+4.7	1794	1674	+7.2	1500	1357	+10.5
		L/300	1697	1776	−4.4	1639	1772	−7.5	1555	1708	−9.0	1383	1550	−10.8
		L/200	2451	2537	−3.4	2329	2482	−6.2	2187	2380	−8.1	1915	2145	−10.7
3	+	Strength	2684	2677	+0.3	2386	2426	−1.6	2130	2201	−3.2	1743	1841	−5.3
		L/300	1342	1366	−1.8	1296	1282	+1.1	1193	1162	+2.7	1018	973	+4.6
		L/200	1982	1994	−0.6	1827	1788	+2.2	1669	1609	+3.7	1415	1339	+5.7
	−	Strength	2684	2677	+0.3	2421	2360	+2.6	2186	2093	+4.4	1816	1697	+7.0
		L/300	1342	1366	−1.8	1330	1363	−2.4	1281	1329	−3.6	1162	1219	−4.7
		L/200	1982	1994	−0.6	1928	1954	−1.3	1838	1876	−2.0	1640	1694	−3.2

$1\ N/m^2 = 0.020886$ lbs/sq.ft

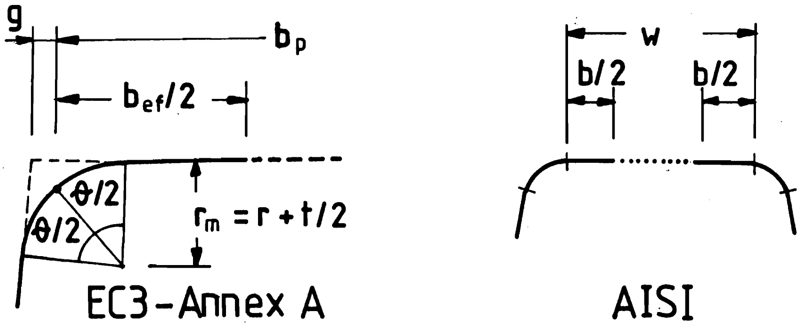


Fig. 1 Definition of the "basic width"

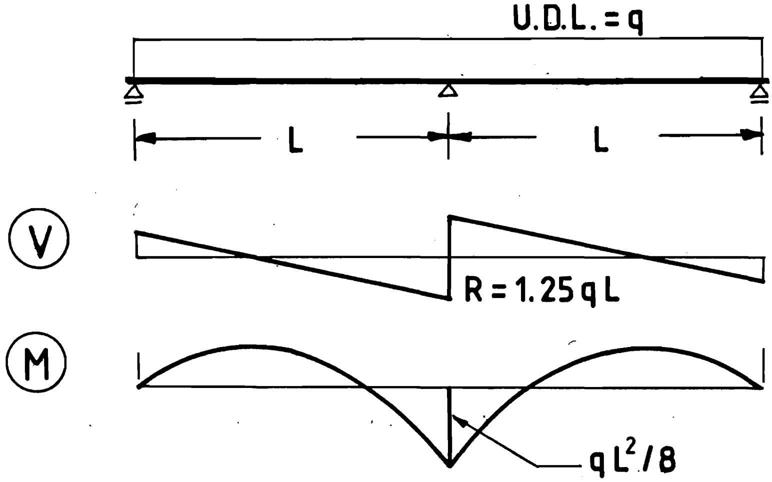


Fig. 2

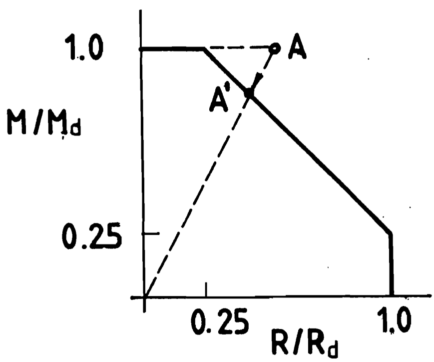
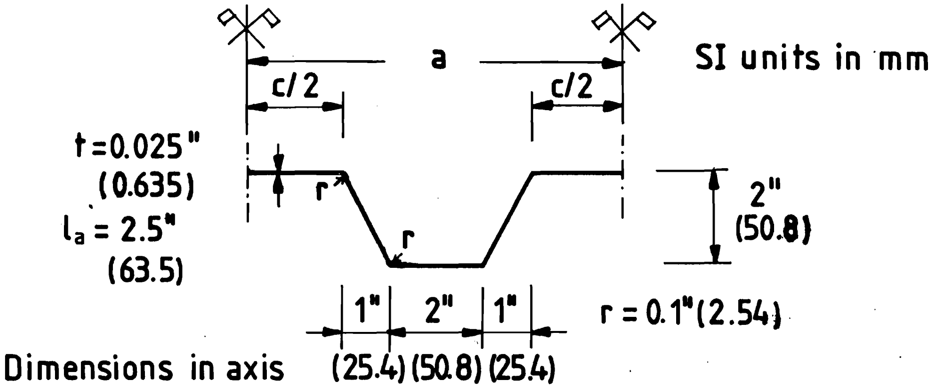


Fig. 3 Moment-support reaction interaction (EC3-Annex A)



Type	a		c	
A	6"	(152.4)	2"	(50.8)
B	7"	(177.8)	3"	(76.2)
C	8"	(203.2)	4"	(101.6)
D	10"	(254.0)	6"	(152.4)

Fig.4 Dimensions of the example sheets

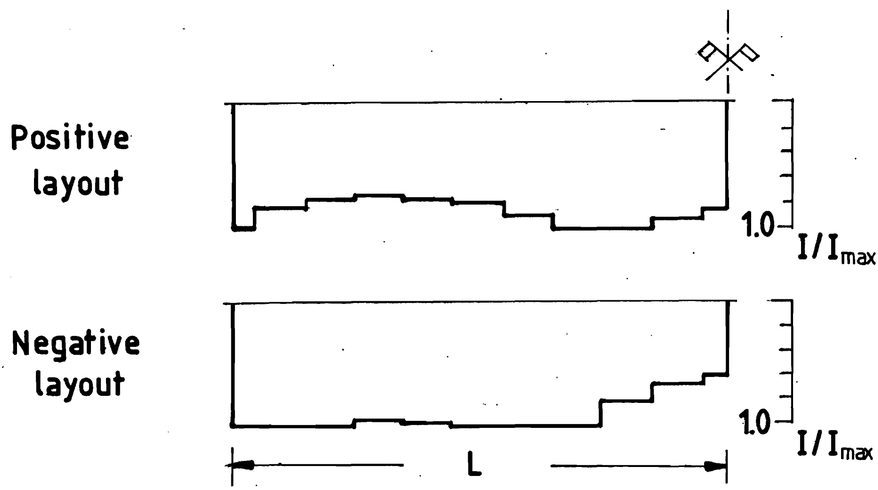


Fig.5 Variation of the moment of inertia