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Analysis and Design of Continuous Sandwich Beams

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ANALYSIS AND DESIGN OF CONTINUOUS SANDWICH BEAMS

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Abstract

Sandwich panels are composed of two thin face layers and a lightweight core between them. Panels are used to carry large bending moments and axial forces, which capacity is reduced, if imperfections, for example initial deflections or transverse loads, appear in the face layers. At intermediate supports the panel is stressed by a high bending moment and, in addition, by a concentrated lateral support reaction. The strength against the simultaneous bending moment and support reaction depends on the bending stiffness and on the bending and buckling strength of the face and on the compressive strength of the core. The paper studies factors having influences on the behaviour and on the failure modes of multispan sandwich panels. Also proposals to estimate the strength at the serviceability and at the ultimate limit states are presented.

1. Introduction

Typical sandwich panels used in building industry consist of two thin face sheets and a well insulating lightweight core between them. The faces are made of flat or profiled steel, aluminium or other metal sheets. Wood and gypsum based boards are also used as face materials. Usual core materials are the structural foams like polyurethane and polystyrene with their many modifications. A new core material is the structural mineral wool with its benefits against the fire. The metal faces can be assumed to follow isotropic material models. The properties of the usual core materials vary considerably in different directions. Typical core materials can be modelled only approximately by isotropic material models.

Sandwich panels used in building industry are typically beam type structures. Therefore, the properties in the directions of the depth and the span have the most important effects on their static behaviour and the knowledge of those properties is sufficient in solving the most problems in the practical design work. In facades and inside walls the panels are often applied as simply supported beams, whose behaviour and failure modes are well known. Because of the absolute requirements for the water tightness and the benefits during the manufacturing, transportation and erection, the roofs are designed to reach from the ridge to the eaves with one panel length. The capacity of single span panels is usually not enough to carry the roof loads. Therefore, a structural system with intermediate supports has to be used. There are several analytical and numerical methods for the calculation of the bending moment and the shear force diagrams and the deflections of multispan sandwich beams supported by point supports without taking the influencies of the finite widths of supports into account. The static continuity produces a new interaction failure mode in multispan panels. The support reaction disturbs the membrane stress state of the compressed face and causes imperfections, which reduce significantly the bending moment capacity of the panel. Unfortunately, the real failure modes at the intermediate supports are not examined in the current design procedures. The procedures base on experiments or on a reduced bending capacity at intermediate supports. Under these circumstances it is important to make analytical and numerical studies with physically valid structural models and evaluate the influencies of the different factors on the interaction failure mode, and then finally, verify the models experimentally for the use in the practical design work.

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2. Behaviour of sandwich beams at intermediate supports

2.1 Design equation for the lower face layer

At an intermediate support a sandwich panel is loaded by a bending moment and a lateral support force. Negative bending moments and positive support reactions are caused by the self weight, by a snow and wind pressure and a positive temperature difference between the lower and upper face. Positive bending moments and negative support reactions are caused by a wind suction and a negative temperature difference (Fig. 1).

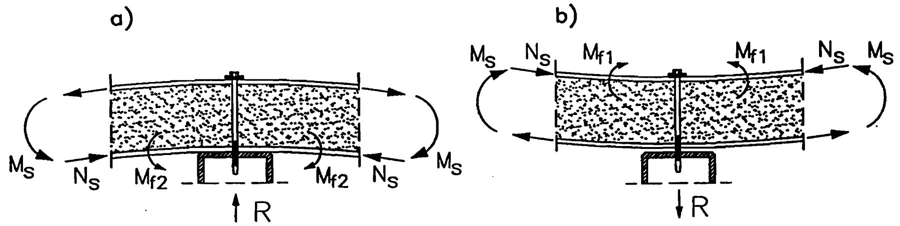


Fig. 1. Axial and bending stress resultants at an internal support of a continuous sandwich panel.
a) Panel is loaded by a negative bending moment and a positive support reaction and b) by a positive bending moment and a negative support reaction.

A bending moment causes axial tensile and compressive stresses and a load acting transversely against the panel local bending stresses in the thin flat faces of a sandwich panel. In the design these two stress components can be superposed and the result finally compared with the yield stress of the face material (1). In fact, the calculation procedure is more complicated because of the geometrically nonlinear dependence of the bending stress σ_R on the axial compressive stress σ_{S2} .

$$\sigma_{S2} + \sigma_R = \frac{M_S}{eA_{f2}} + \frac{M_R}{W_{f2}} \leq f_y \tag{1}$$

Stresses due to the bending moments in a profiled face layer can no more be assumed to be constant over the depth of the face, but they change in a linear way. This is because of the face bending moments M_{f1}, M_{f2} in addition to the moment M_S in the sandwich part of the cross section. The face bending moments are caused by the curvature of the panel w'' and by the nonvanishing bending stiffnesses of the faces themselves B_{f1}, B_{f2}. The additional bending stress component σ_{f2} in the profiled face shall be added in the design equation (1) for thin faces.

$$\sigma_{S2} + \sigma_{f2} + \sigma_R = \frac{M_S}{eA_{f2}} + \frac{M_{f2}}{W_{f2}} + \frac{M_R}{W_{f2}} \leq f_y \tag{2}$$

In the profiled lower face there exists a geometrically nonlinear interaction between the two first stress components (σ_{S2} , σ_{f2}) caused the bending moments M_S and M_{f2} and the third component (σ_R) caused by the lateral load. This contribution is noteworthy mainly on sandwich panels with thin flat faces.

Bending stiffnesses of typical lightly profiled faces are often very small and having only negligible influence on stresses. The lightly profiled faces can thus be designed on the same assumptions used for panels with thin flat faces.

2.2 Beam-column model of the lower face at internal support

A face layer loaded simultaneously by compressive stresses and a lateral load can be modelled as a beam-column, which is supported on a continuous foundation (Fig. 2). The behaviour of a slender elastic beam-column on an elastic foundation can be covered by the following differential equation

$$B_{f2}w'''' + N_S w'' + p(x) = q(x) \tag{3}$$

where $N_S = M_S / e$ is the axial compressive load in the lower face, $p(x)$ the reaction force caused by the foundation and $q(x)$ the lateral load. The prime denotes differentiation with respect to x-coordinate.

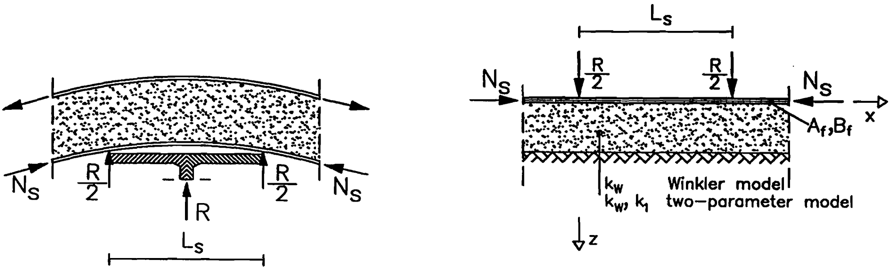


Fig. 2. The lower face is modelled as a beam-column, which is continuously supported by the core and loaded by an axial compressive load and a lateral load on the support.

Function $p(x)$ represents the response of the foundation and it depends on the choice of the foundation model. Well known and widely used is the Winkler's foundation model, in which $p(x)$ is assumed to be proportional to the deflection w through the stiffness parameter k_W . The Winkler's model (4) is able to take into account the compressive stiffness of the foundation, only. In a two parameter model (5) the term $-k_1 w''$ is added to the foundation response function. With the second term it is possible to include the shear stiffness of the core in the foundation model. The distribution of displacements in the depth direction of the core has also a strong influence on the properties of the foundation. The often used distribution function in the local buckling studies is exponential ($\Phi(z) = e^{-kz}$) with a decay factor k regulating the decrease of displacements. In addition to the Winkler's and the two parameter models, several other models can be found in the literature. The most complicated ones of them base on the stress and strain analysis of a two dimensional elastic half space.

$$p(x) = k_W w \tag{4}$$

$$p(x) = k_W w - k_1 w'' \tag{5}$$

2.3 Wrinkling stress of a face layer

The compression strength of a thin face is limited by a local buckling, known as a wrinkling failure mode. The wrinkling stress depends on the bending stiffness of the face and on the stiffness of the core layer. In the wrinkling analysis often a known exact value for the bending stiffness of the face can be used. But, the properties of the core layer have to be described in advance by a foundation model. This has a significant influence on the wrinkling stress and also on stresses caused by the lateral support load. The development of suitable mathematical expressions to describe the compressive and shear stiffnesses of the core layer is therefore a very important task in the work.

By solving the first eigenvalue, $N_{S,cr} = N_w$, of the homogeneous part of the differential equation (3) the buckling stress of the beam-column can be found. If the core is described by a two parameter foundation model, the following expression to the wrinkling stress can be written,

$$\sigma_{w,2} = \frac{N_{w,2}}{A_{f2}} = \frac{1}{A_{f2}}\left(2\sqrt{k_W B_{f2}} + k_1\right) \tag{6}$$

The simplest choice for the first foundation coefficient k_W is $k_W = E_S / e$, which corresponds to the linear decrease of the displacements from *v(x,z=0) = w(x)* to *v(z=e) = 0* with the depth of the core e. From (6) it is easy to see the interdependence between the first and the second foundation parameter and the wrinkling stress.

The wrinkling stress of a beam-column based on the complete elastic half space foundation model is

$$\sigma_{w,elastic} = \frac{C_1}{A_f}\sqrt[3]{E_S G_S B_f} \tag{7}$$

where the parameter $C_1 = 3\left(\frac{2(1-\nu_S)^2}{(1+\nu_S)(3-4\nu_S)^2}\right)^{1/3}$

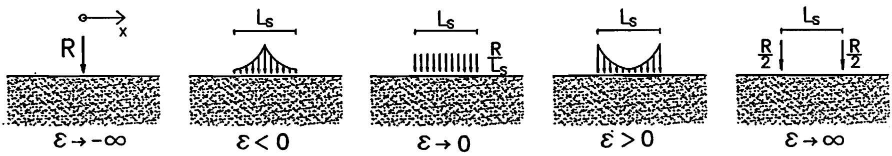
The coefficient C_1 depends on Poisson's ratio ν_S of the core and has the minimum value of $C_1 = 1.805$, when $\nu_S = 0.12$. In the design recommendations /ECCS 1991/ the value of C_1 is reduced because of initial imperfections in the core and the face. The design value for the wrinkling stress is given by the expression

$$\sigma_{w,ECCS} = \frac{0.95}{A_f}\sqrt[3]{E_S G_S B_f} \tag{8}$$

2.4 Interaction between the bending moment and support reaction

The support pressure distribution between the typical substructures and sandwich panels is poorly known. Different theoretical distributions have been presented in the literature. An interesting way to describe the pressure distribution is the generalized function developed by Thomsen /*Thomsen 1992*/. In the function the parameter regulates the pressure distribution. The integral of the function over the support width ($-c \le x \le c$) is independent of the parameter ε and results in the constant value of the total support reaction R. The properties of the function are illustrated in Fig. 3, in which the two extreme cases for

$\varepsilon \to -\infty$ and $\varepsilon \to \infty$ can be found. When the support plate is very flexible, for example a slender open thin-walled section, the support reaction concentrates in the web plane of the profile. On the other hand, when the support plate is very rigid, for example a concrete beam, the support reaction divides into the two point loads located at the edges of the support. For practical design purposes it would be interesting to find out experimentally support pressure distributions between typical substructures and sandwich panels.



$$q(x) = \frac{R\varepsilon}{L_s(1-e^{-\varepsilon})} e^{-\varepsilon\left|\left|\frac{2x}{L_s}\right|-1\right|} \; ; \quad |x| \le \frac{L_s}{2}$$

Fig. 3. Support pressure distributions formulated using a generalized function.

When the support pressure distribution $q(x)$ is determined, the local deflection and the bending moment of the lower face can be solved using the equation (3). The distribution consisting of two loads located at the edges of the support plate is often chosen. In the usual applications the support width is so large, that there is no interaction between the two loads ($R/2$ + $R/2$) located at the opposite edges. The following expressions for local deflection and bending moment of the lower face can be written assuming the origin to locate in one of the loading points */Hetenyi 1946/*. In the derivation of the equations the two parameter foundation model (5) has been used.

$$w(x) = \frac{R}{4k_w}\frac{\Lambda^{-2}}{\alpha_0\beta_0} e^{-\beta_0 x}(\alpha_0 \cos\alpha_0 x + \beta_0 \sin\alpha_0 x) \tag{9}$$

$$M_R(x) = \frac{R}{8}\frac{1}{\alpha_0\beta_0} e^{-\beta_0 x}(\alpha_0 \cos\alpha_0 x - \beta_0 \sin\alpha_0 x) \tag{10}$$

where $\Lambda = \sqrt[4]{\frac{4B_{f2}}{k_w}}$ is a characteristic length and (11)

$$\alpha_0 = \sqrt{\sqrt{\frac{k_W}{4B_{f2}}} + \frac{N_S - k_1}{4B_{f2}}} \quad \text{and} \quad \beta_0 = \sqrt{\sqrt{\frac{k_W}{4B_{f2}}} - \frac{N_S - k_1}{4B_{f2}}} \tag{12a,b}$$

The maximum bending moment can be found at origin,

$$M_{R,\max} = \frac{R\Lambda}{8}\sqrt{\frac{N_{w,2} - k_1}{N_{w,2} - N_S}} = \frac{R\Lambda}{8}\sqrt{\frac{\sigma_{w,2} - k_1 / A_{f2}}{\sigma_{w,2} - \sigma_{S2}}} \tag{13}$$

Using the maximum bending moment value (13), the design equation (1) for the thin lower face now be written in the form

$$\sigma_{S2}+\frac{R\Lambda}{8W_{f2}}\sqrt{\frac{\sigma_{w,2}-k_1/A_{f2}}{\sigma_{w,2}-\sigma_{S2}}}\leq f_y \tag{14}$$

Dividing the equation finally by f_y gives the form

$$\alpha\frac{\sigma_{S2}}{\sigma_{w,2}}+\frac{R}{R_R}\sqrt{\frac{\sigma_{w,2}-k_1/A_{f2}}{\sigma_{w,2}-\sigma_{S2}}}\leq 1 \tag{15}$$

$$\text{where } \alpha=\frac{\sigma_{w,2}}{f_y} \text{ and } R_R=\frac{8W_{f2}f_y}{\Lambda} \tag{16), (17}$$

If the wrinkling stress of the face is calculated using the one parameter foundation model, the design equation (15) obtains the form

$$\alpha\frac{\sigma_{S2}}{\sigma_{w,W}}+\frac{R}{R_R}\frac{1}{\sqrt{1-\frac{\sigma_{S2}}{\sigma_{w,W}}}}\leq 1 \tag{18}$$

$$\text{where } \alpha \text{ and } \sigma_{w,W} \text{ are } \alpha=\frac{\sigma_{w,W}}{f_y} \quad \text{and} \quad \sigma_{w,W}=\frac{2\sqrt{k_w B_{f2}}}{A_{f2}} \tag{19, 20}$$

An additional failure is the yielding or crushing of the core below the face layer. The compressive and shear stresses of the core can be expressed by the formulae

$$\sigma_{Sc}=k_w w(x) \quad \text{and} \quad \tau_S=G_S w(x)' \tag{21, 22}$$

which have the maximum values

$$\sigma_{Sc,\max}=\frac{R}{4}\sqrt{\frac{k_w}{A_{f2}}}\frac{1}{\sqrt{\sigma_{w,2}-\sigma_{S2}}} \tag{23}$$

$$\tau_{S,\max}=\frac{RG_S}{4A_{f2}}\frac{\sqrt{2}\,e^{-\vartheta}}{\sqrt{(\sigma_{w,2}-k_1/A_{f2})(\sigma_{w,2}-\sigma_{S2})}} \tag{24}$$

$$\text{where } \vartheta=\frac{\arctan(\theta)}{\theta} \quad \text{and} \quad \theta=\sqrt{\frac{\sigma_{w,2}+\sigma_{S2}-2k_1/A_{f2}}{\sigma_{w,2}-\sigma_{S2}}} \tag{25a,b}$$

If the one parameter foundation model is used, the expressions of core stresses simplify in the form

$$\sigma_{Sc,\max} = \frac{R}{4}\sqrt{\frac{k_w}{A_{f2}}}\frac{1}{\sqrt{\sigma_{w,W} - \sigma_{S2}}} \tag{26}$$

$$\tau_{SW,\max} = \frac{RG_S}{4A_{f2}}\frac{\sqrt{2}\; e^{-\vartheta_w}}{\sigma_{w,W}\sqrt{1-\frac{\sigma_{S2}}{\sigma_{w,W}}}} \tag{27}$$

where $\vartheta_W = \dfrac{\arctan(\theta_W)}{\theta_W}$ and $\theta_W = \sqrt{\dfrac{\sigma_{w,W}+\sigma_{S2}}{\sigma_{w,W}-\sigma_{S2}}}$ (28a,b)

Failure criteria for multiaxial stress states of typical core materials are not very well known. The maximum compressive and shear stresses are usually simply compared with the corresponding experimental strengths, only. In the European Recommendations */ECCS 1991/* two models are given for the evaluation of support reaction capacity. The first one base on uniformly distributed stresses on the mid height of the core (29), (Fig. 4a). The second model takes into account the capability of the lower face to distribute the support reaction to a larger area (30), (Fig. 4b). In the models the influence of axial stresses of the face is studied. The calculated compressive stresses are compared directly with the experimental compressive strengths.

$$R_{RS} = (L_S + 0.5e) f_{Sc} \tag{29}$$

$$R_{RS} = \frac{4 f_{Sc} \Lambda_W}{\left[1 + e^{-\lambda}(\cos\lambda_S + \sin\lambda_S)\right]} \tag{30}$$

where f_{Sc} is the compressive strength of the core and

$$\lambda_W = \frac{L_S}{\Lambda_W} = \sqrt[4]{\frac{E_S}{4eB_{f2}}}\, L_S \tag{31}$$

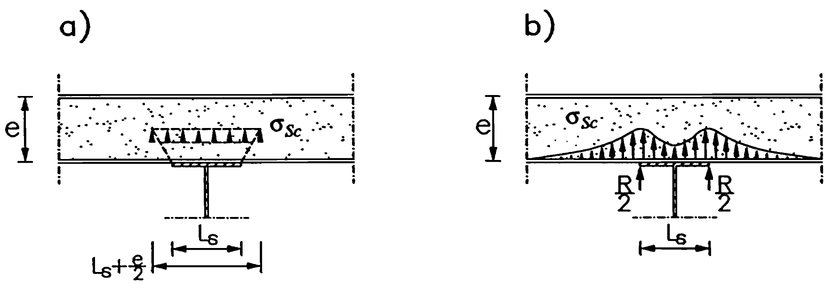


Fig. 4. Support pressure distributions used in the evaluation of support reaction capacity. a) Uniform pressure on middle depth of the core and b) pressure distribution, when the capability of the lower face to extend the support reaction area in the core is taken into account /ECCS 1991/.

2.5 Distribution of global stress resultants

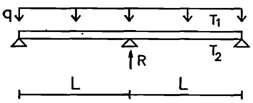
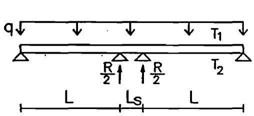
The sandwich panels and supports are assumed to be in contact at points or along lines, when the global stress resultants and the deflections are calculated. In fact, the supports have always a finite width, which has influences not only on the failure modes at the support but on the global stress resultants, also. The finite support width reduces effectively the high peak of the negative bending moment on an intermediate support and decreases also shear forces in some loading cases. A relatively simple closed form solution for the negative bending moment at an intermediate support can be written for two span sandwich panels with equal spans, if the support reaction is assumed to consist of two line loads at the distance of the support width (Table. 1). The real bending moment and shear force distributions of the other multispan static systems, for example nonequal two span systems or three or four span systems, can most easily be solved numerically.

Two different assumptions can be made about the location of the end supports. If the static system at the end supports in calculations is not changed, the negative bending moment area at the mid support increases and the positive bending moment in the span decreases correspondingly (Fig. 5a). If the end support is assumed to consist of one point load at the inner edge of the support, the locations of the zero bending moments remain practically the same (Fig. 5b). The widely used reduction of the negative bending moment for thin-walled structures (32) in not a good approximation to either of the two calculated static systems.

$$\Delta M_S = 0.25\, R\, L_S \qquad (32)$$

When evaluating the influence of the finite support width, the sign of the support pressure resultant ($R/2$) has to be taken into account. If the analysis yields a negative reaction at an edge of the support, that reaction should be released and the computations made again with a new static model */Heinisuo 1988/*.

Table 1. Bending moments and support reactions at the middle support of two span thin face sandwich panels loaded by a uniform load q and a temperature difference $\Delta T = T_2 - T_1$ between the lower and upper face. The length of the sandwich beam is ($L+L$), if the support reaction at the intermediate support is described by one line load, and ($L+L_S+L$), if the support reaction at the intermediate support is described by two line loads at the edges of the support plate. In the table $\lambda = L_S / L$, $k = 6B_B / L^2 S$, $S = e\,G_S$ and $\vartheta_T = \alpha_T\, \Delta T / e$.

Static system of sandwich beam	Minimum bending moment M_S at internal support	Total support reaction R at internal support
	$-\frac{1}{2+k}\left[\frac{1}{4}qL+3\vartheta_T B_S\right]$	$\frac{1}{2+k}\left[\frac{5+2k}{16}qL+\frac{6\vartheta_T B_S}{L}\right]$
	$-\frac{\lambda+1}{2+3\lambda+k}\left[\frac{1-\lambda+\lambda^2}{4}qL+3\vartheta_T B_S\right]$	$\frac{\lambda+1}{2+3\lambda+k}\left[\frac{5+5\lambda+2k}{2}qL+\frac{6\vartheta_T B_S}{L}\right]$

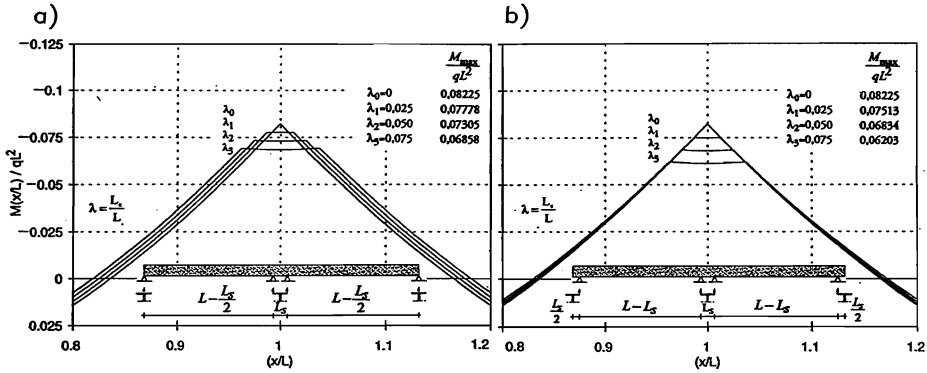


Fig. 5. Bending moment distributions at the mid support of a two span sandwich panel with thin facings and equal span widths. a) The end supports consist of one line load at the ends of the panel and b) the end supports consist of one line load at the distance of $L_S/2$ from the end of the panel.

2.6 Postbuckling capacity of a face layer under compressive and bending stresses

In the design calculations the wrinkling failure is assumed to be of a brittle type. When the wrinkling stress is reached, the compressed face is assumed to loose the axial loading capacity completely. And further, the panel looses the bending capacity at the wrinkling failure point without any postbuckling capacity.

The behaviour of a thin face layer has been studied by a numerical example (Fig. 6). In the example, a beam is supported by a Winkler's foundation. The beam is loaded by an axial compressive load and by a constant lateral load at the mid point of the beam. Both the beam and the foundation follow an ideal elastic plastic material model. The calculated wrinkling stress of the beam is $\sigma_{w,W} = 215.9MPa$ and the ultimate lateral load capacity $F_{y,u} = 13.2N$.

The results show the strong dependence between the compressive strength and the imperfection, which in this case is the constant lateral load. The compressive strength of the beam is reduced from the wrinkling stress level to one fourth, when the constant lateral load is increased from zero up to three fourth of the lateral load capacity. With imperfections the stress deformation curve becomes smoother compared with the linear behaviour of a beam loaded by an axial load, only. A large lateral load yields a low compressive strength and early permanent deformations. The beam with imperfections keeps a relatively higher axial load carrying capacity after the ultimate stress. Finally, all the calculated stress deformation curves tend asymptotic to the same stress level far in the postbuckling phase. The results of the calculated example indicate a strong reduction in the strength due to the imperfections but also a noticeable axial load capacity after the ultimate stress level.

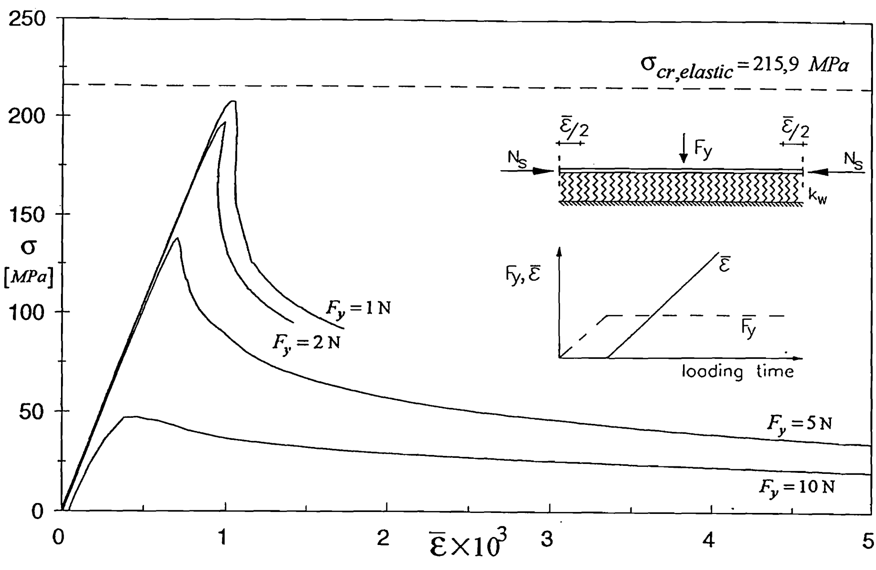


Fig. 6. Influence of a lateral load on the axial load capacity of a beam on an Winkler's foundation. The cross section of the beam is 1 x 1 mm^2, Young's modulus 210 GPa and the yield stress 320 MPa. The foundation coefficient has the value of $k_w = 0.666 MPa / mm$ and the yield stress of 0.2 MPa.

3. Design at the serviceability limit state

3.1 Positive support reaction

At the serviceability limit state the stresses in any part of the panel have to stay below the corresponding yield stress or another limit stress, which can yield permanent deformations. To fulfil the requirement the methods to analyse the multispan panels statically and, in addition, the calculation models to determine the resistances against the different failures have to have a strong physical background and a good agreement with experimental results.

The important additional factors having influencies on the stresses at intermediate supports are the finite support width and the distribution and the intensity of the support pressure. The width of the support and the distribution of the support pressure change the maximum bending moment value M_S . The intensity of the support pressure together with the axial stresses of the face increases more than proportionally the bending stresses of the face and the compressive and shear stresses of the core. Other important factors, used already in the current design models, are the bending stiffness of the lower face and the compressive strength of the core. Both of them are essential parameters in the formulae for the support reaction capacity R_{RS}. The bending stiffness has a great influence on the wrinkling stress value ($\alpha = \sigma_w / f_y$), also.

dimensional stress field and the dependence is nonlinear. Evaluation of the two dimensional stress state and its influence on the compressive strength of the upper face is a demanding task even when the face sheet is flat and the core materials are isotropic. When the face is profiled and the core material is anisotropic, the calculated results are very approximate. In that case experiments are the only way to reach reliable results.

According to the experiments and computed examples, the pull through tensile failure mode is a local mode in the upper face. The diameter of the failed area depends on the face sheet thickness and on the compressive stiffness of the foundation. Based on that remark a calculation model for the bending capacity can be developed, in which the failed local parts of the face are removed and the bending capacity is evaluated using the remaining effective cross section (33), (Fig. 8). The model gives a simple tool to evaluate positive bending capacity but has, however, to be verified experimentally for different combinations of the face and core layers.

$$M_{SR} = b_{eff}\, t\, \sigma_w\, e \qquad (33)$$

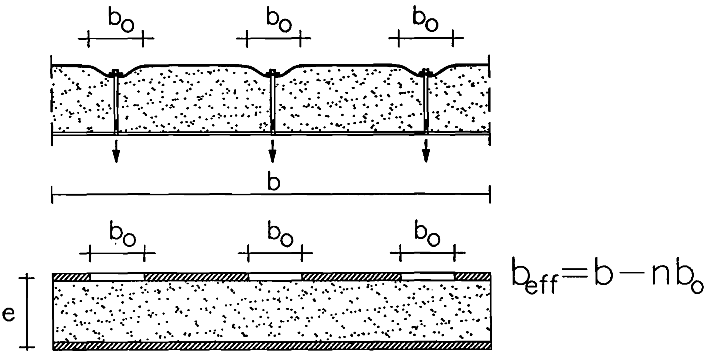


Fig. 8. Evaluation of the positive bending resistance of a sandwich panel at the point of through going screw connection on the basis of unfailed effective width in the upper face.

The new design procedure for the serviceability limit state is illustrated in Fig. 7. Typical interaction curves are drawn using both the simple Winkler's one parameter foundation model and the two parameter model. The limiting support reaction capacity caused by the core (R_{RS}) and the limiting compressive strength of the face given by the design recommendations are also shown in the example. In fact, the capacity R_{RS} is not constant but depends on the axial stress of the lower face (23, 26). The iterative design procedure borders a safe design area, which should quarantee the avoiding of the permanent defomations.

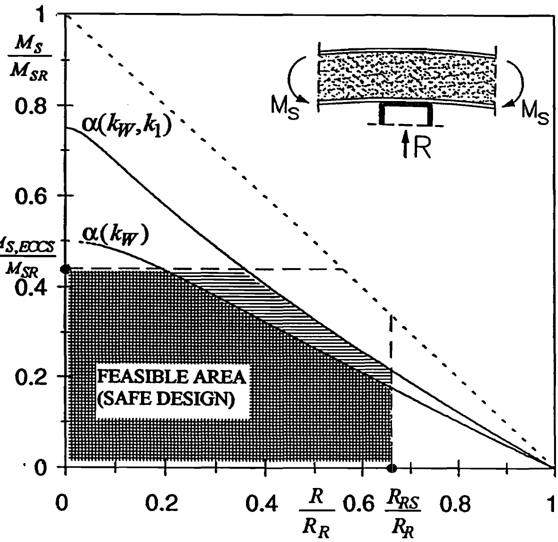


Fig. 7. Interaction diagrams between support reaction and bending moment at the serviceability limit state using the Winkler's model $\alpha(k_W)$ and the two parameter foundation model $\alpha(k_W, k_1)$.

3.2 Negative support reaction

Against the loads due to the wind suction the panels are typically fixed with screw fasteners going through the panels to the supports. The fasteners can fail in three ways, at least. The fastener itself can fail in tension, the fastener can be pulled out from the support plate or the head of a fastener can be pulled through the panel. The last case yields a failure mode in the upper face of the panel, also. The pull through failure mode in the upper face is initiated far before the ultimate fastener load and constitutes a strong local imperfection in the face. The imperfection reduces the axial loading capacity of the upper face and further, the positive bending moment capacity of the panel.

Negative support reaction creates a local bending and tensile stress field in the upper face near the fastener. The stress state is two dimensional and it has to be added to the global, nearby one dimensional stress field of the sandwich beam. The axial stresses of the upper face depend to some extent also on the two

4. Design at the ultimate limit state

At the ultimate limit state the load carrying capacity of the panel has to cover the maximum possible load. Permanent deformations are allowed to take place in the structure at the ultimate limit state. The calculation models have to correspond to the physical behaviour of the structure and they have to be well verified experimentally at least in the cases, where plastic capacity of a thin-walled structure is utilized.

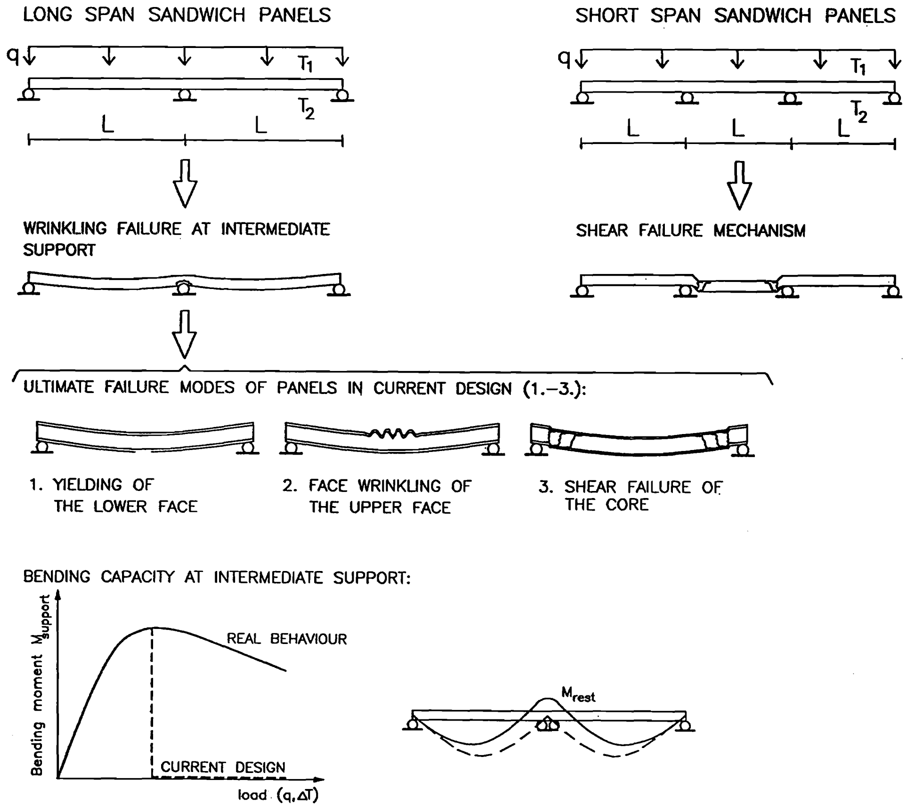


Fig. 9. Usual failure modes of sandwich panels are the buckling failure of a face or the shear failure of the core. The first failure mode defines the static calculation model at the ultimate state design in the current design methods.

The first failure mode of long and medium span length panels is usually the buckling failure at intermediate support due to the interaction between the bending moment and support reaction. The structure fails, when the second buckling failure in a span or a shear failure in the core takes place. Because the wrinkling failure mode is assumed to be of a brittle type, the panel at the ultimate limit state is assumed to consist of single span panels in series with negligible bending capacity at the supports. In practice, a wrinkled compressed face has still some bending capacity left, which is worth to utilize in an economic design procedure. Use of the remaining bending capacity at supports leads to an iterative design process, for which, on the basis of carefully investigated and verified failure modes, effective computer programs have to be developed.

The first failure mode of short span panels is usually the shear failure near the intermediate support. The plastic shear capacity of typical core materials has not been studied strongly yet neither utilized in the current design. The calculated first shear failure load is assumed to be the ultimate load of the panel. For the development of design methods of multispan panels it is important to try to define the frontiers, where the failure mechanism changes. To do this a more profound analysis about the material and structural behaviour of panels is needed.

5. Conclusions

The present paper focuses and analyses the questions arising in the design of continuous multispan sandwich panels. To develope tools for practical design work further analytical and numerical studies and especially experimental results are needed. On the basis of the models to evaluate the behaviour and the capacity at the serviceability and ultimate limit states, the following remarks and conclusions can be done:

• A procedure to take into account the interaction between the negative bending moment and the positive support reaction at the serviceability limit state is based on the theoretical model of a beam-column resting on a continuous elastic foundation. Experimental verifications are still needed before the use of the procedure in practice due to initial imperfections and stresses in cold formed composite members.

• The most of the core materials are strongly anisotropic. Therefore, the parameters for the foundation models have to be proved case by case.

• The support pressure between a substructure and a sandwich panel is assumed to consist of two line loads at the edges of the support plate. The real support pressure distribution depends on the stiffness of the support plate and is different, if the support profile is, for example, a slender Z-profile or a closed tubular section. A close determination of the support pressure distribution is worth of a study.

• Instead the wrinkling stresses based on the equations derived, experimental wrinkling stresses can also be used in design equations. Experimental values are especially recommended in the cases of complicated lightly profiled face layers.

• Bending moment capacity at point of through going connectors is proposed to be based on an effective width approach, in which the uneffective failed widths loaded by screw heads are excluded of the panel width in the calculations. The uneffective width is influenced by profiles of the face and compressive stiffness and anisotropy of the core. The width has to be defined experimentally.

• The finite width of the support plate and the flexibility of the screw connections have influencies on the global stress resultants. It is usefull to take the effects into account in the design calculations.

• The ultimate loading capacity of a long and medium span continuous sandwich beam is influenced by the remaining bending resistance at the intermediate supports. The capacity is not utilized in the current design work because of unsufficient experimental verification.

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References

Chong, Ken, P. 1993. Structural analysis and design of sandwich panels with cold-formed steel facings. Thin-Walled Structures 16(1993), pp. 199-218.

Davies, J.M. 1993. Sandwich Panels. Thin-Walled Structures 16(1993), pp. 179-198.

European Convention for Constructional Steelwork (ECCS). 1991. Preliminary European Recommendations for Sandwich Panels, Part I Design. ECCS Publication No 66, 1991.

European Convention for Constructional Steelwork (ECCS) & International Council for Building Research, Studies and Documentation (CIB). 1993. Preliminary European Recommendations for Sandwich Panels with Additional Recommendations for Panels with Mineral Wool Core Material, CIB Report, Publication No 148, 1993.

Frostig, Y. & Baruch, M. 1990. Bending of sandwich beams with transversely flexible core. AIAA Journal 28(1990)3, pp. 523-531.

Hassinen, P. & Martikainen, L. 1993. Behaviour and design of multispan sandwich panels at intermediate supports. CIB W56 Annual Meeting held in Mainz in June 7, 1993. 6p.

Heinisuo, M. 1988. An exact finite element technique for layered beams. Computers and Structures 30(1988)3, pp. 615-622.

Hetenyi, M. 1946. Beams on elastic foundation. The University of Michican Press, 214 p. + app. 41 p.

Martikainen, L. 1993, Behaviour and design of continuous sandwich panels at intermediate supports. M.Sc. Thesis, Helsinki University of Technology, 142 p. + app. 39 p. (in Finnish).

Schulz, U. 1993. Zum Zwischenstützbereich von Sandwichelementen bei Mehrfeldträgersystemen (Intermediate support area of sandwich panels for multi-span girder systems). Stahlbau 62(1993)10, pp. 295-304.

Thomsen, O.T. 1992. Further remarks on local bending analysis of sandwich panels using a two parameter elastic foundation model. Aalborg University, Institute of Mechanical Engineering, Report No. 40, 30 p.

Appendix A - Notations

A_f	area of face layer per unit width
A_{f2}	area of lower face per unit width
E_s	modulus of elasticity of the core
G_s	shear modulus of the core
I_s	second moment of area of sandwich part of cross section per unit width
I_1, I_2	second moment of area of upper and lower face layer per unit width
$B_{f1} = E_f I_{f1}$	flexural rigidity of upper face per unit width
$B_{f2} = E_f I_{f2}$	flexural rigidity of lower face per unit width
L	span of sandwich beam
L_s	width of the support of sandwich beam
M	bending moment
M_S	bending moment in sandwich part of cross section
$M_{f1} = B_{f1} w''$	bending moment of the upper face layer per unit width
$M_{f2} = B_{f2} w''$	bending moment of the lower face layer per unit width
M_R	local bending moment of lower face caused by support reaction R
ΔM_S	reduction of bending moment at the support
N_S	compressive force of the lower face caused by the bending moment M_S
$N_{w,W}$	local buckling load of the lower face, one parameter foundation model
$N_{w,2}$	local buckling load of the lower face, two parameter foundation model
R	support reaction
R_R	support reaction capacity based on the strength of the lower face
R_{RS}	support reaction capacity based on the strength of the core
$W_{f1} = I_{f1}/e_i$	section modulus of the upper face per unit width
$W_{f2} = I_{f2}/e_i$	section modulus of the lower face per unit width
e	distance of centroids of upper and lower face
f_{Sc}	compressive strength of the core material
f_y	yield stress of face material
k_w	foundation coefficient of Winkler's foundation model
k_1	second foundation coefficient in two parameter foundation model
Λ	characteristic length
α	relation between wrinkling stress and yield stress
λ	relation between support width and span
λ_S	relative support width
ν_S	Poisson ratio of the core material
σ	axial compressive stress in face layer
σ_{f1}, σ_{f2}	bending stresses of faces caused by the moments M_{f1}, M_{f2}
σ_R	local bending stress in lower face caused by the support reaction R
σ_w	wrinkling stress of face layer
$\sigma_{w,2}$	wrinkling stress of face layer based on two parameter foundation model
$\sigma_{w,elastic}$	wrinkling stress of face layer based on elastic half space model
$\sigma_{w,ECCS}$	wrinkling stress of face layer given in ECCS Recommendations
$\sigma_{w,W}$	wrinkling stress of face layer based on Winkler's foundation model
σ_{s2}	axial compressive stress in the lower face caused by the moment M_s
σ_{sc}	compressive stress in the core
τ_s	shear stress in the core
w	local deflection of lower face of a sandwich panel