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Predicting the Maximum and Distribution of Displacements on Liquefaction-Induced Lateral Spreads

Alan F. Rauch
The University of Texas at Austin, Austin, TX

James R. Martin II
Virginia Polytechnic Institute and State University, Blacksburg, VA

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Alan F. Rauch
The University of Texas at Austin
Austin, Texas 78712 USA

James R. Martin, II
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061 USA

ABSTRACT

Lateral spreading is the mostly horizontal movement of gently sloping ground due to liquefaction in shallow soil deposits. To assess the seismic hazards related to lateral spreading, estimates are needed of the maximum potential ground movement at these sites. One approach to this problem is to predict, using empirical models, the mean and standard deviation of the displacement magnitudes across the surface of a lateral spread. Then, using a probability density function, the maximum likely deformation at the site can be predicted with a suitable degree of conservatism. In the analysis described here, probability density functions are studied for modeling the variation in horizontal displacements measured in twenty-nine case studies of lateral spreading. The quality of fit between the measured displacements and the normal, lognormal, and gamma distributions are evaluated using statistical goodness-of-fit tests. The results show that the gamma distribution provides a good representation of the variation in displacement magnitudes across a slide area. Moreover, the 99.5 percentile of the gamma distribution is found to yield reasonable, conservative estimates of maximum horizontal movement. Using this approach, with appropriate percentiles of the gamma distribution, maximum likely movements can be estimated in a rational, probabilistic manner.

INTRODUCTION

Historically, tremendous damage in large-magnitude earthquakes has resulted from liquefaction and lateral spreading. Depicted schematically in Fig. 1, *lateral spreading* is defined as the finite, lateral displacement of intact soil blocks on mild slopes (< 5%) resulting from the liquefaction of shallow, underlying soil deposits. As defined here, lateral spreading does not refer to other liquefaction failures that produce lateral ground deformations, such as deep-seated flow failures, slumping of embankments, and the outward rotation of earth retaining walls.

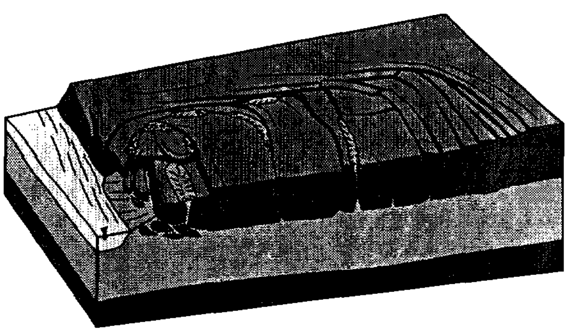

Fig. 1. Schematic depiction of a lateral spread.

For buried pipeline networks, assessments of potential damage from liquefaction-induced lateral spreading have been hampered by the "lack of a means to estimate the location, magnitude and distribution of ground movements" (Honegger 1992). Current engineering practice relies on empirical relationships to predict horizontal displacements (Glaser 1994). Empirical equations are available for estimating displacements at specific locations (Bartlett and Youd 1995), average displacements (Rauch and Martin 2000), and maximum likely displacements (Youd and Perkins 1987) on lateral spreads.

However, new methods are needed to relate mean and maximum displacements as well as predict deformation patterns on lateral spreads (Ballantyne 1994; Honegger 1994). Given the typical lack of detailed subsurface data at a potential lateral spread, precise forecasts of deformation patterns are probably not feasible in most situations. In general, more reliable predictions of the average and maximum potential displacements that may occur at a given site would be useful.

In this paper, data from sites in Japan and California are used to investigate how well probabilistic distributions can model the variation in displacement magnitudes on a lateral spread. This work suggests that empirical predictions of the mean and standard deviation of displacements on a lateral spread, together with the gamma distribution, can be used to represent the entire deformation field. This approach then allows for the prediction of maximum horizontal displacement with an appropriate level of conservatism.

CASE STUDIES

To investigate patterns of surface deformation, published studies of lateral spreads were examined. Eliminating sites with fewer than ten measured displacement vectors, the 29 case studies listed in Table 1 were compiled for this study. Most of these lateral spreads occurred in Japan where horizontal displacements were determined mainly from aerial photographs taken before and after an earthquake. At three sites in California, displacements were determined mostly from offsets in street curbs and other reference points. The number of observations, mean, standard deviation, and maximum of the reported horizontal displacements are given in Table 1. The case studies are documented more fully in Rauch (1997).

Table 1. Case studies of lateral spreading.

Location (year)	Slide No.	Horizontal Displacements (m) # Meas.	Mean	Std. Dev.	Max
Fukui, Japan (1948)	6	24	1.96	0.84	4.00
	7	25	1.89	0.99	4.30
	8	36	1.69	0.71	3.40
	9	24	1.56	0.65	3.69
Niigata, Japan (1964)	25	14	3.75	2.45	9.25
	26	75	3.94	2.97	11.81
	27	24	3.76	1.94	8.72
	28	38	2.08	1.21	6.49
	29	46	4.21	1.98	8.82
	30	26	4.78	2.64	10.15
	31	37	1.22	0.41	2.07
	32	72	2.34	1.01	4.65
	34	16	0.98	0.64	2.16
	35	22	4.59	2.66	10.55
	37	63	3.23	1.55	6.46
	38	66	4.74	2.10	8.34
	39	84	2.76	1.43	7.64
San Fernando, California (1971)	40	26	1.02	1.19	3.69
	41	79	0.90	0.58	1.82
Imperial Valley, California (1979)	43	33	1.40	1.19	4.24
Noshiro, Japan (1983)	45	28	1.47	0.66	2.92
	46	34	1.46	0.43	2.72
	47	59	1.58	0.83	4.01
	48	57	1.26	0.42	2.65
	49	187	1.55	0.58	3.25
Shiribeshi-toshibetsu River Valley, Japan (1993)	109	11	1.38	1.06	3.91
	110	17	0.68	0.35	1.40
	116	11	0.67	0.27	1.10
	117	13	1.36	0.90	3.39

At these sites, the magnitude of horizontal displacement vectors measured at specific locations varied with relative position across the slide. Larger displacements tended to occur in the central area of a lateral spread, or near a free face along the toe, with smaller displacements found along the sides and head of a slide. Displacements also varied significantly with location across a site due to local changes in surface slope or subsurface soil conditions.

In the analyses that follow, the measured displacement vectors are assumed to be evenly dispersed across the surface of each lateral spread. While this is not rigorously true for every case study in Table 1, the available displacement vectors at these sites do tend to be located across the entire slide surface including the center, head, toe, and sides. Hence, the magnitude of measured displacement vectors can be treated as a variable that may follow some probability density function.

Histograms of the measured displacements were generated for each case study; five of these histograms are shown in Fig. 2. Inspection of all 29 histograms revealed that many are skewed somewhat toward the smaller magnitudes. This tendency is strongly evident in Fig. 2 for Slide No. 8 and less so for Slide No. 49. This trend could result from anomalously large displacements, but the larger vectors were often found in groups at these sites. Indeed, the skewness of these histograms is consistent with the tendency to observe the largest displacements in smaller, central areas on the surface of a lateral spread, or in zones close to a free face.

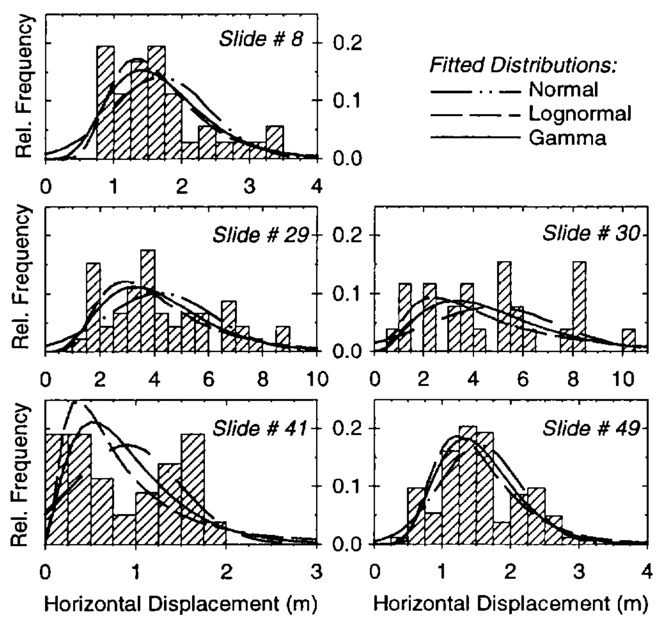

Fig. 2. Histograms of measured displacements with fitted statistical distributions.

FIT OF PROBABILITY DENSITY FUNCTIONS

Three statistical distributions (normal, lognormal, and gamma distributions) were considered for representing the observed histograms of measured displacements. The probability density functions of these distributions, which are defined in Table 2, were fit to the displacement histograms using the mean and standard deviation of the measured displacement magnitudes. The relationships between the sample statistics and the parameters of each distribution are given in Table 2.

Table 2. Candidate statistical distributions considered (Scheaffer and McClave 1990).

Probability density function, f(x)	*Parameters*
Normal distribution: $f(x)=\frac{1}{\sigma_x\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x}\right)^2}$	μ_x = mean of sample x; σ_x = standard deviation of sample x
Lognormal distribution (x > 0): $f(x)=\frac{1}{x\sigma_{\ln x}\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln(x)-\mu_{\ln x}}{\sigma_{\ln x}}\right)^2}$	$\mu_{\ln x}$ = mean of sample ln(x); $\sigma_{\ln x}$ = standard deviation of sample ln(x)
Gamma distribution (x ≥ 0): $f(x)=\frac{x^{\lambda-1}}{\beta^{\lambda}\Gamma(\lambda)}e^{-\frac{x}{\beta}}$ where: $\Gamma(\lambda)=\int_0^{\infty}u^{\lambda-1}e^{-u}\,du$	$\lambda=\frac{\mu_x^2}{\sigma_x^2}$; $\beta=\frac{\sigma_x^2}{\mu_x}$

In Fig. 2, the fitted normal, lognormal, and gamma distributions are plotted on the histograms of the measured displacements. While the normal distribution is symmetric about the mean value, both the lognormal and gamma distributions are non-symmetric and can better represent the apparent skew of the histograms. More significantly, both the lognormal and gamma distributions are defined only for positive values of displacement, whereas the normal distribution extends to values less than zero. Since displacements are positive by definition, this suggests that the normal distribution is not a good choice for modeling the distribution of displacement magnitudes.

STATISTICAL TESTS FOR GOODNESS-OF-FIT

While Fig. 2 yields a rough indication of how well each probability distribution represents the observed histograms, statistical *goodness-of-fit tests* give a more objective measure of how well a particular distribution fits the data. Such tests are based on a null hypothesis that the sample data is taken from a larger population that follows a given mathematical distribution. If this hypothesis is accepted at a given significance level, then we can believe that the statistical distribution fits the sample data. The higher the significance level at which the hypothesis is accepted, the more confident we can be that a distribution fits the data. Goodness-of-fit testing is discussed by Conover (1971), D'Agostino and Stephens (1986), and Scheaffer and McClave (1990).

The chi-square test, perhaps the most familiar goodness-of-fit test, is based on the difference between a histogram of the sample data and a given probability density function. For non-discrete data, the chi-square test requires an arbitrary grouping of the sample data into histogram cells; the selection of these cell limits has a direct impact on the results of a chi-square test. Consequently, the chi-square test is not preferred for testing goodness-of-fit with continuous data (D'Agostino and Stephens 1986).

More powerful goodness-of-fit tests for continuous data are based on *empirical density functions* (EDFs), which represent the cumulative frequency of the observed data. The displacement histograms in Fig. 2 are re-plotted in Fig. 3 as EDFs and overlain with the cumulative density functions (CDFs) for each of the three candidate statistical distributions. Small vertical departures between the EDF and CDF in Fig. 3 indicate a good fit between the data and a particular distribution.

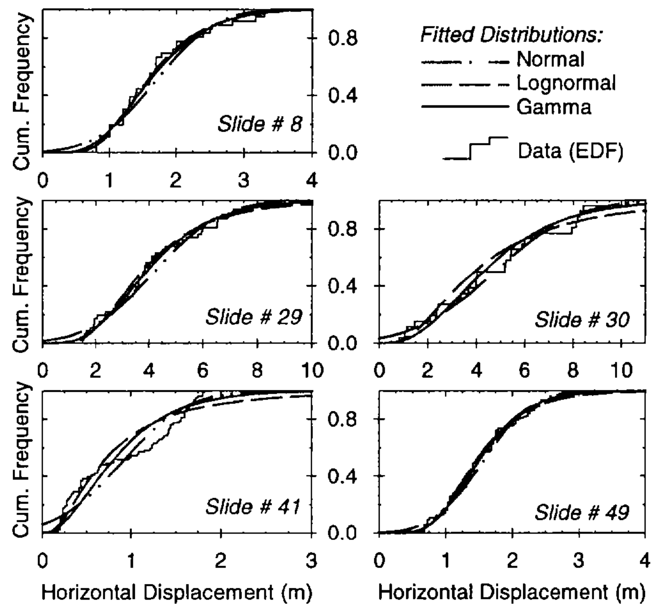

Fig. 3. Empirical density functions of measured displacements with fitted statistical distributions.

For this analysis, two goodness-of-fit tests based on EDF statistics were chosen. The Kolmogorov-Smirnov "D" test is based on the single, maximum vertical offset between the EDF and CDF. In equation form:

$$D=\max\left(\left|F(x_i)-\frac{i-1}{n}\right|,\left|F(x_i)-\frac{i}{n}\right|\right) \tag{1}$$

where n is the sample size, x_i is the sample data arranged in ascending order, and $F(x_i)$ is the cumulative density function at x_i for the statistical distribution under consideration. The Cramér-von Mises "W^2" test, which is computed from the departures between the EDF and CDF over the full range of the sample data, is defined by:

$$W^2=\frac{1}{12n}+\sum_{i=1}^{n}\left(F(x_i)-\frac{2i-1}{2n}\right)^2 \tag{2}$$

W^2 is usually considered to yield a more powerful goodness-of-fit test than D (D'Agostino and Stephens 1986). For both test statistics, smaller values indicate a closer fit of the hypothesized distribution to the sample data.

To test the hypothesis that a certain distribution fits the sample data, critical values of the test statistic are needed for a selected level of significance (α). When the parameters for the population distribution are estimated from sample data, as done in this analysis, these critical values depend on the distribution tested. Appropriate critical values of D and W^2 for testing the normal and lognormal distributions are given by Stephens (1974), while critical values of W^2 for testing the gamma distribution are given by D'Agostino and Stephens (1986).

To test the fit of a hypothesized distribution to the displacements on a lateral spread, the following procedure was followed:

1. Arrange the measured displacements in ascending order.
2. Compute the mean and standard deviation of the data and fit the chosen distribution using the parameters in Table 2.
3. Compute the cumulative density function for the hypothesized distribution at each data point.
4. Compute the test statistic, D or W^2.
5. If the computed test statistic is less than the critical value at $\alpha = 2.5\%$, conclude that the hypothesized distribution fits the data to a 2.5% level of significance. This implies a 97.5% confidence that the fit of a given distribution has not been erroneously rejected.

In a strict sense, acceptance in a goodness-of-fit test indicates only that the given distribution is a reasonable approximation of the population from which the sample was taken.

RESULTS OF GOODNESS-OF-FIT TESTS

Goodness-of-fit tests, based on the D and W^2 statistics, were performed for the normal, lognormal, and gamma distributions using data from 23 of the lateral spreads listed in Table 1. The six case studies with fewer than 20 measured displacement vectors (judged to be the minimum sample size needed) were not considered in this particular analysis.

The number of case studies for which each distribution function was judged to fit the sample data is listed in Table 3. To determine if the displacements from a given lateral spread fit a given distribution, values of D and W^2 were computed and compared with critical values (for a level of significance of α = 2.5%) of D and W^2. If the computed test statistic was less than the critical value, the result was interpreted to mean that the distribution fits the data.

Table 3. Number of case studies out of 23 considered where a given distribution fits the measured displacements to a significance level of $\alpha = 2.5\%$.

Test Statistic	Distribution		
	Normal	Lognormal	Gamma
Kolmogorov-Smirnov D	13	15	--*
Cramér-von Mises W^2	10	15	19

* *critical D values are not available for the gamma distribution*

In general, the results of these tests are mixed and none of the three distributions tested were accepted for all cases. However, the more powerful W^2 test gives a positive result (at α=2.5%) for the gamma distribution in 19 of 23 cases. By the same criteria, the lognormal and normal distributions are accepted in 15 and 10 cases, respectively. Therefore, this goodness-of-fit test suggests that, for the majority of the lateral spreads investigated, horizontal displacements follow the gamma distribution.

The D and W^2 statistics can also be used in another way, to rank the fit of the three candidate distributions to the sample data. Smaller values of D or W^2 indicate a closer match between the EDF of the data and the CDF of a given distribution. For each case study, the statistical distribution that best fits the observed displacements yields the lowest value of D or W^2. These rankings based on D or W^2 are summarized in Table 4 as the number of case studies for which each of the three candidate distributions gave the best and second-best fit to the data.

Table 4. Number of case studies out of 23 considered where a given distribution was found to give the best or second-best fit to the measured displacements.

Test Statistic	Rank of fit to data	Distribution		
		Normal	Lognormal	Gamma
Kolmogorov -Smirnov D	best	8	8	7
	2nd best	0	8	15
Cramér-von Mises W^2	best	7	10	6
	2nd best	1	8	14

Considering the results in Table 4, no single distribution emerges as the best fit for the majority of the lateral spreads studied. On the other hand, the gamma distribution is the first or second choice (based on both D and W^2) in the greatest number of cases. Also, the normal distribution yields the best or second-best fit in only eight cases; that is, the normal distribution gives the worst match in two-thirds of the cases. This clearly shows that the normal distribution is not the best choice for representing the pattern of displacements on a lateral spread.

The goodness-of-fit tests for the three distributions considered give fairly mixed results, which may be related to the fairly small number of measured displacements in the available case studies. However, the results of this analysis suggest that the gamma distribution gives the best fit for the greatest number of lateral spreads.

MAXIMUM DISPLACEMENT

Given a prediction of the mean and standard deviation of the expected surface movements, probabilistic distributions can be used to estimate the maximum likely displacement. As demonstrated later in an example, the maximum displacement predicted in this way corresponds to a selected percentile of the expected distribution in displacement magnitudes. To gain some insight into what is an appropriate percentile for this application, the case study data were investigated further.

For each lateral spread in Table 1, the mean and standard deviation were used to define normal, lognormal, and gamma distributions that represent the range in displacement magnitudes measured across the surface of each slide. Using the cumulative density functions, maximum displacements were then predicted as the 99.0, 99.5, and 99.9 percentile values of each distribution. In Fig. 4, histograms of the resulting error (difference between the predicted and observed maximum displacement) are shown for each distribution and percentile level.

From Fig. 4, it appears that 99.5 percentile predictions from the normal and gamma distributions yield reasonable, conservative estimates of maximum displacement. That is, the maximum displacement is over-predicted by less than 2 meters for most of the cases and is under-predicted in only a few cases. More significantly, the lognormal distribution tends to produce several excessively large predictions of the maximum displacement (error > 4 m even at the 99.0 percentile). This indicates that the lognormal distribution is a poor choice for estimating maximum displacement on a lateral spread.

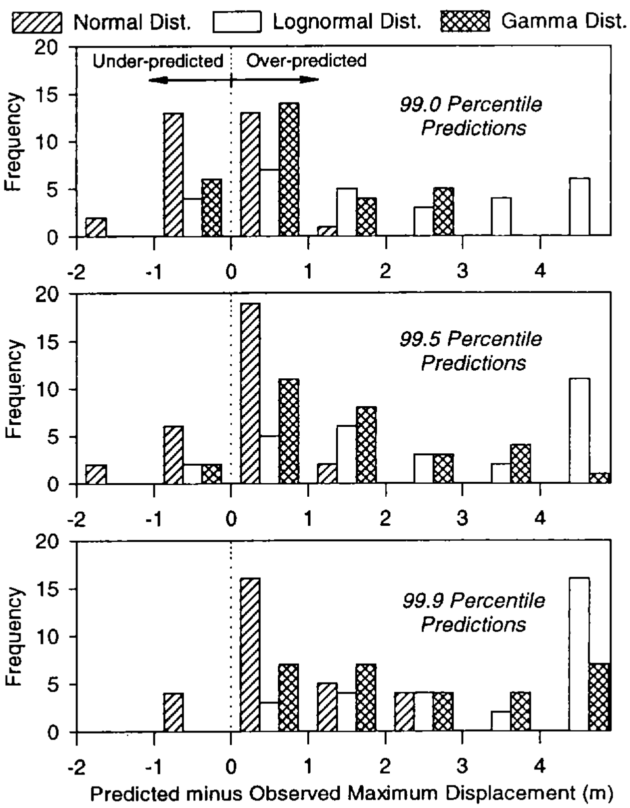

Fig. 4. Errors in the maximum displacement predicted for each lateral spread using statistical distributions.

EXAMPLE APPLICATION

To demonstrate how this approach could be used, consider Slide No. 37 from Table 1. The maximum displacement at this site can be predicted in the following manner:

1. This lateral spread occurred during a moment magnitude (M_W) 7.6 earthquake at a distance from the fault rupture (R_f) of about 12 km. Based on recorded ground motions in the area, the site was subjected to a peak horizontal acceleration (A_{max}) of about 0.17 g and the duration of strong shaking (T_d) was about 19 sec.
2. Based on these input parameters, the average horizontal displacement can be predicted using the "Regional" component of the EPOLLS model (Rauch and Martin 2000):

$$D_R = \frac{613M_W - 13.9R_f - 2420A_{\max} - 11.4T_d}{1000} \quad (3)$$

$$Avg\_Horz = (D_R - 2.21)^2 + 0.149 \quad (4)$$

   For Slide No. 37, the average horizontal displacement predicted using these equations is Avg_Horz = 2.88 m.
3. For the sites in Table 1, the standard deviation is, on average, equal to about one-half of the mean horizontal displacement. Therefore, for Slide No. 37, the standard deviation could be roughly predicted as 1.44 m, or one-half of the predicted average movement.
4. Using the predicted values of μ = 2.88 m and σ = 1.44 m, the parameters of the gamma distribution can be computed (see Table 2) as λ = 4.00 and β = 0.72.
5. The maximum likely displacement is then predicted to be 7.92 m, corresponding to the 99.5 percentile of a gamma distribution with λ = 4.00 and β = 0.72 (this 99.5 percentile value was computed using a built-in statistical function in a computer spreadsheet application.)

This prediction can be stated more accurately as "99.5% of the horizontal displacements on this lateral spread are expected to be less than 7.92 m."

Note that the largest displacement measured at Slide No. 37 (6.46 m) is significantly smaller than the predicted likely maximum of 7.92 m. Of course, it is possible that larger single displacements did occur at the site, but were not measured. Similar comparisons with other case studies in Table 1 show larger and smaller errors, including both over- and under-predictions of the maximum observed displacement. Such predictions are subject to the combined errors associated with the adequacy of (1) the predicted average displacement, (2) the estimated standard deviation of displacement, (3) the fit of the gamma distribution, and (4) the selection of the 99.5 percentile to represent the maximum displacement.

CONCLUSIONS

In a liquefaction-induced lateral spread, horizontal displacements vary with relative position on the slide mass. Larger movements tend to occur in the central portions of a slide area, or nearer free faces, and are smaller along the sides and head of the lateral spread. Displacements also vary with local changes in soil conditions and surface slope.

Using data from 29 case studies, the normal, lognormal, and gamma distributions were evaluated for representing the pattern of horizontal displacements on lateral spreads. The gamma distribution was found to be the best choice for representing the distribution of displacements on a lateral spread. This conclusion is based on the following:

- Displacements are non-negative and the gamma distribution is defined only for positive values.
- According to the W^2 goodness-of-fit test, the gamma distribution fits the sample data in 19 of 23 cases to a 2.5% level of significance.
- Based on both the Kolmogorov-Smirnov D and Cramér-von Mises W^2 statistics, the gamma distribution yields the best or second-best fit to the sample data in the greatest number of case studies.

Using estimates of the displacement mean and standard deviation, predictions of the maximum likely displacement can be made using statistical distributions. Conservative, yet reasonable, estimates of the maximum displacement were obtained at the 99.5 percentile of the gamma and normal distributions. On the other hand, the lognormal distribution tends to give excessively large predictions of maximum displacement and should not be used in this application.

This analysis shows that the variation in displacement magnitudes across a lateral spread can be modeled statistically. Where conventional engineering practice might seek a deterministic prediction of maximum movement on a lateral spread, this study suggests an alternative approach where the maximum likely deformation is estimated in a probabilistic framework. In this approach, predictions of the mean and standard deviation of displacements, coupled with the gamma distribution, can be used to forecast the maximum deformation with a degree of conservatism appropriate for a given project.

ACKNOWLEDGMENT

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