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A METHOD FOR ESTIMATING NON-STATIONARY VARIATION OF SOIL RIGIDITY DURING STRONG MOTIONS

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ABSTRACT

This paper provides a method for estimating the non-stationary variations of rigidity and damping of soils related intimately to the non-linear earthquake responses of ground. This method uses the concept of the "Complex Envelope" to inversely estimate soil rigidity and damping from the time histories of stress and strain induced in soils. It is quite different from the conventional method, which uses graphical technique for the stress-strain orbit, in the point of quantitatively estimating the time-dependent variations of soil rigidity and damping. The validity of the method is first discussed using theoretical derivations and numerical simulations. It is then applied to the down-hole array records of strong motions obtained at Port Island during the 1995 Kobe Earthquake, Japan. The analyzed results showed that some extreme reductions of soil rigidity occurred in the superficial layers during the mainshock of the earthquake, depending strongly on strain behaviors.

INTRODUCTION

It is well known that soft soils behave non-linearly during strong motions. Such non-linear behaviors of soft soils have a close relation with the reduction of rigidity modulus and increase of damping ratio. These dynamic variations of soil rigidity and damping have been so far dealt with as stationary in time, represented by the equivalent linear method of response analysis. Actually, however, soil rigidity and damping vary in a time-dependent manner because motions incident to soils are substantially non-stationary. Thus we need to make clear the non-stationary characteristics of soil rigidity and damping based on strong-motion records and to introduce them to earthquake response analysis. Until now, the dynamic variations of soil rigidity and damping have been obtained graphically by means of the stress-strain orbit, but such a graphical method fails to follow complicated patterns of orbit. This means that a more sophisticated method is necessary so as to be applicable even to complicated stress-strain behaviors. In line with this necessity, we have been working on a new method for estimating the non-stationary variations of soil rigidity and damping using vertical-array observation systems of strong motions [Kamiyama and Yoshida, 2000]. Our method uses basically the "Complex Envelope" for the time histories of stress and strain, which are obtained numerically from observed strong-motion records, to quantitatively estimate the non-stationary variations of soil rigidity and damping. In this paper, we focus on the theoretical validity of our method and discuss the strain-dependent variations of soil rigidity and damping obtained from the strong-motion records due to the 1995 Kobe Earthquake, Japan.

THE COMPLEX ENVELOPE METHOD

Assuming that the ground motions are due mainly to the vertical propagation of S wave and the soils have the non-viscous and hysteric damping [Ishihara, 1996], the motion equation is given as a simultaneous form:

$$\rho(z)\frac{\partial u^2}{\partial t^2}=\frac{\partial \tau(t,z)}{\partial z} \tag{1}$$

$$\tau(t,z)=G(t,z)\{1+i2h(t,z)\}\gamma(t,z) \tag{2}$$

in which ρ(z) is the density, u(t,z) is the displacement, τ(t,z) is the shear stress, γ(t,z) is the shear strain, G(t,z) is the rigidity modulus, h(t,z) is the damping ratio, t is the time and z is the depth.

In principle, equations (1) to (2) enable us to inversely estimate the rigidity modulus and damping ratio, should the stress and strain be given. The most difficult issue in such inverse estimates of G(t,z) and h(t,z) is that they are expressed by the imaginary number system as shown in equation (2). Therefore it is necessary to find some skills overcoming this difficulty. The "Complex Envelope" [Farnbach, 1975] is available to solve the difficulty because it transfers a time history into its corresponding imaginary number system with a reasonable manner. The Complex Envelope is defined as an imaginary number system in which the real part consists of a time history while the imaginary part stems from the histories' Hibert transform. For example, the complex envelope for the stress τ(t,z) and strain γ(t,z) are, respectively, given as follows:

$$T(t,z)=\tau(t,z)+iH[\tau(t,z)] \tag{3}$$

$$\Gamma(t,z)=\gamma(t,z)+iH[\gamma(t,z)] \tag{4}$$

in which H[ ] is the Hilbert transform.

By substituting equations (3) and (4) instead of τ(t,z) and γ(t,z) into equation (2), we can obtain

$$h(t,z)=\frac{1}{2}\tan\{\varphi(t,z)-\psi(t,z)\} \tag{5}$$

$$G(t,z)=\frac{1}{\sqrt{1+(2h(t,z))^2}}\frac{|T(t,z)|}{|\Gamma(t,z)|} \tag{6}$$

in which

$$T(t,z)=|T(t,z)|\exp\{i\varphi(t,z)\} \tag{7}$$

$$\Gamma(t,z)=|\Gamma(t,z)|\exp\{i\psi(t,z)\} \tag{8}$$

The above method can be applied to the stress-strain relations obtained either at laboratories or in-situ. Especially it is quite effective to the stress-strain data in-situ if they were obtained from an array observation system of ground motions.

THEORETICAL VALIDITY OF THE METHOD

The concept of the Complex Envelope was applied a-priori to derive a method for estimating the non-stationary variations of soil rigidity and damping. In the following, we provide the theoretical validity of the method using the harmonic motions of stress and strain.

Assume that a harmonic-motion shear stress with an amplitude of σ_a and circular frequency ω_0 is loaded to a soil element:

$$\sigma(t)=\sigma_a\cos(\omega_0 t) \tag{9}$$

If the shear rigidity modulus G_0 and damping ratio h_0 constitute the soil element, its strain then responds as follows:

$$e(t)=\frac{\sigma_a}{G_0\sqrt{1+(2h_0)^2}}\cos\{\omega_0 t-\tan^{-1}(2h_0)\} \tag{10}$$

We now estimate inversely the soil rigidity modulus and damping ratio from the time histories of stress and strain in equations (9) and (10) using the Complex Envelope method. Since the Hilbert transform for a cosine function is expressed by a sine function, the Complex Envelopes $\sum(t)$ and $E(t)$ for $\sigma(t)$ and $e(t)$ are each given in the following forms:

$$\sum(t)=\sigma_a\cos(\omega_0 t)+i\sigma_a\sin(\omega_0 t) \tag{11}$$

$$E(t)=\frac{\sigma_a}{G_0\sqrt{1+(2h_0)^2}}\cos\{\omega_0 t-\tan^{-1}(2h_0)\} + i\frac{\sigma_a}{G_0\sqrt{1+(2h_0)^2}}\sin\{\omega_0 t-\tan^{-1}(2h_0)\} \tag{12}$$

According to equations (5) and (6), we can obtain $h(t,z)$ and $G(t,z)$ as follows:

$$h(t,z)=\frac{1}{2}\tan\left[\omega_0 t-\{\omega_0 t-\tan^{-1}(2h_0)\}\right] = h_0 \tag{13}$$

$$G(t,z)=\frac{1}{\sqrt{1+(2h(t,z))^2}}\frac{\sigma_a}{\dfrac{\sigma_a}{G_0\sqrt{1+(2h_0)^2}}} = G_0 \tag{14}$$

That is, the above theoretical derivations in case of a harmonic motion indicate that equations (5) to (6) give an exact solution to the inverse analysis of soil rigidity and damping. This means that the Complex Envelope method enables to determine numerically and exactly the rigidity modulus and damping ratio from a harmonic load experiment at laboratory instead of the conventional method in which they are estimated graphically in terms of the stress-strain locus.

Table 1 Condition for synthetic stress and strain.

No.	rigidity modulus (kpa)	damping ratio
1	52020	0.2
2	52020	0.4
3	52020	0.6
4	70000	0.2
5	90000	0.2
σ_a amplitude(kpa)		50
T Period(sec)		1
sampling time		0.01 sec

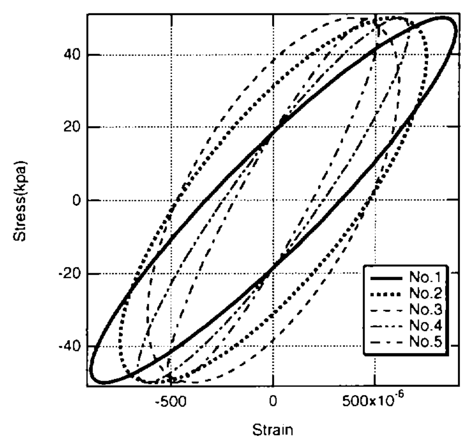

Fig. 1 Orbits of synthetic stress and strain.

In order to further show the validity of our method, we carried out numerical simulations using synthetic stress and strain given by equations (9) to (10). The conditions for the synthetic stress and strain are listed in Table 1 and their orbits are displayed in Fig. 1. Figure 2 shows one example of these time histories of stress and strain. We obtained numerically the rigidity modulus and damping ratio from such time histories as shown in Fig.2 using the Complex Envelope method. The numerically analyzed results are shown in Fig. 3. Figure 3 indicates that the original rigidities and damping ratios are exactly reproduced in the Complex Envelope results without any fluctuations because of their harmonic motions, manifesting the validity of our method.

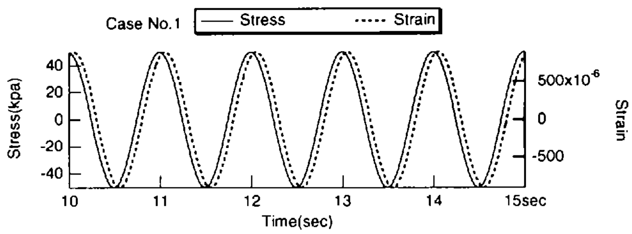

Fig.2 Example of the time histories of stress and strain.

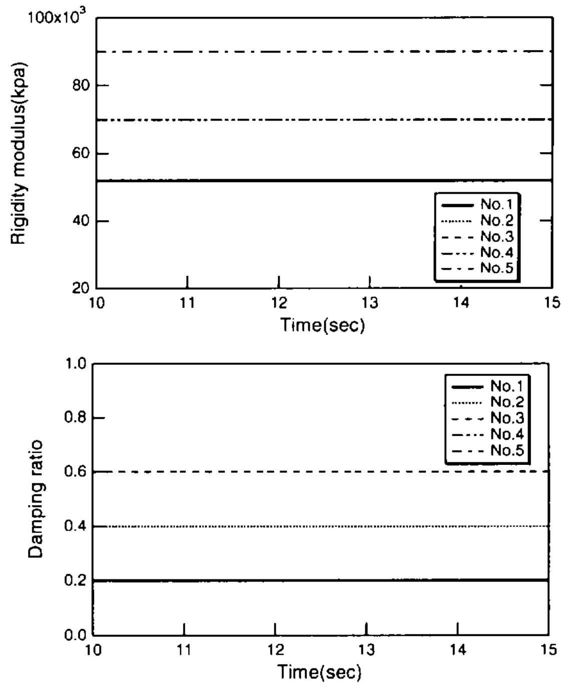

Fig.3 Inversions of rigidity moduli and damping ratios.

NON-STATIONARY VARIATIONS OF RIGIDITY AND DAMPING BY VERTICAL ARRAY OBSERVATIONS

The Complex Envelope method has been applied to some strong-motion records obtained by vertical-array observation systems. In this paper, we show a typical result derived from the strong-motion records obtained at Port Island during the 1995 Kobe Earthquake, Japan. In particular, we place here emphasis on the relation between the non-stationary variations of rigidity and the strain time-history. The Port Island system, which is featured by its down-hole array observation, obtained strong-motion records both during the main-shock and some after-shocks of the Kobe event. Liquefaction phenomena were witnessed extensively at Port Island, indicating non-linear responses of the ground. Therefore, the Port Island records are quite effective for the sake of investigating the dynamic variations of soil rigidity and damping.

Figure 4 shows the array profile of the observation system at Port Island. As shown in Fig.4, the observation system consists of tri-axial accelerometers (the NS, EW and UD components) located at 0m, 16m, 32m and 83m depths. It recorded strong motions during the main-shock of the Kobe

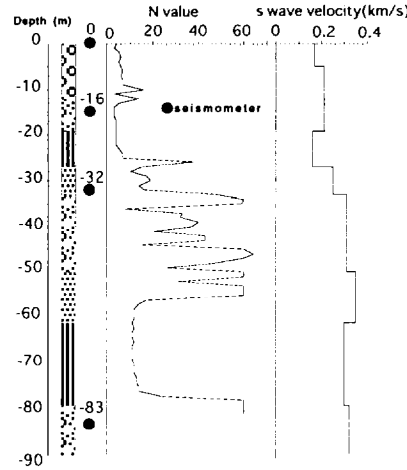

Fig.4 Profile of array observation system at Port Island.

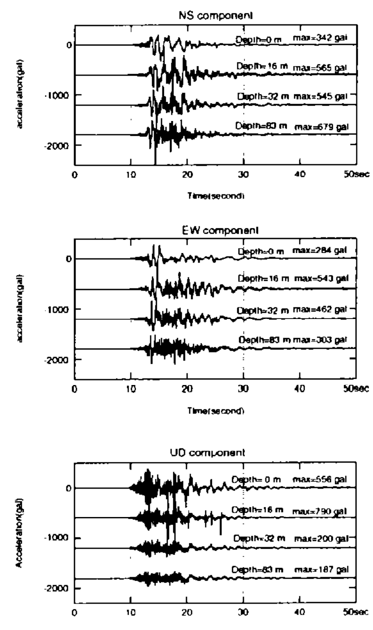

Fig.5 Strong-motion records during the mainshock.

Earthquake. In addition, it also obtained motion records during some after-shocks of the event. It is expected to detect a difference in the degree of non-linear response between the main-shock and after-shocks because they differ remarkably from each other in their motion levels. Figures 5 and 6 show the strong-motion records obtained during the main-shock and a representative aftershock, respectively. In the main-shock records, the horizontal ones at 0m depth differ considerably in the wave forms from those at 16m, 32m, and 83m depths whereas the after-shock records demonstrate less difference in their wave forms. This indicates that the horizontal motions near the surface were due highly to the non-linearity of soils, especially during the main-shock.

We performed the cross correlation analysis as well as the spectral ratio analysis against these array strong-motion records [Kamiyama and Yoshida, 2000]. These analyses made it clear that non-linear responses occurred with remarkable reduction of rigidity and increase of damping, especially, at

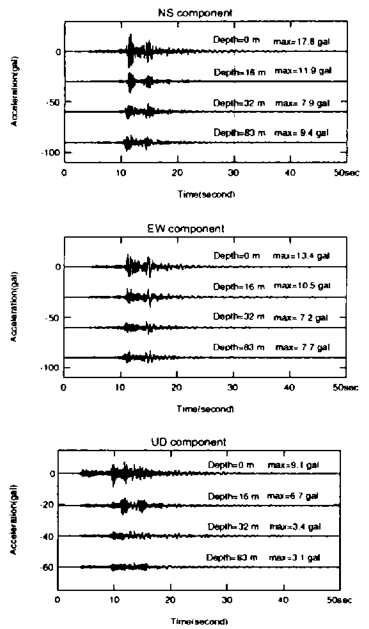

Fig.6 Strong-motion records during an aftershock.

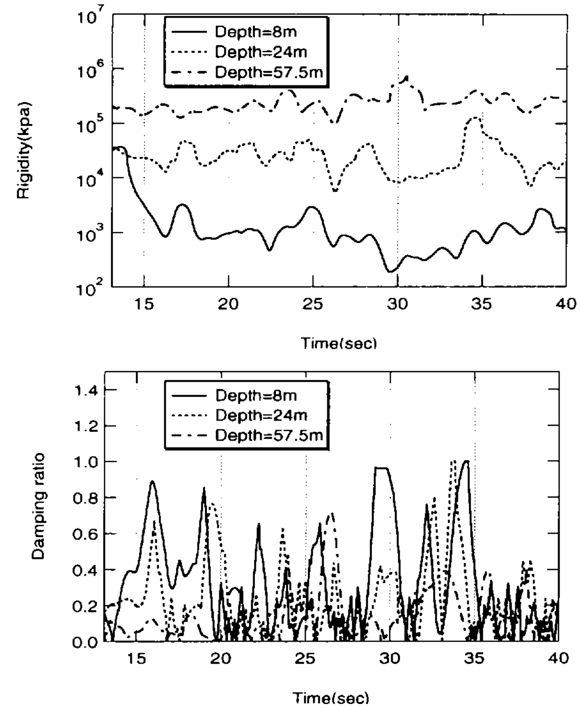

Fig.7 Variations of rigidity and damping obtained at each depth in the NS component.

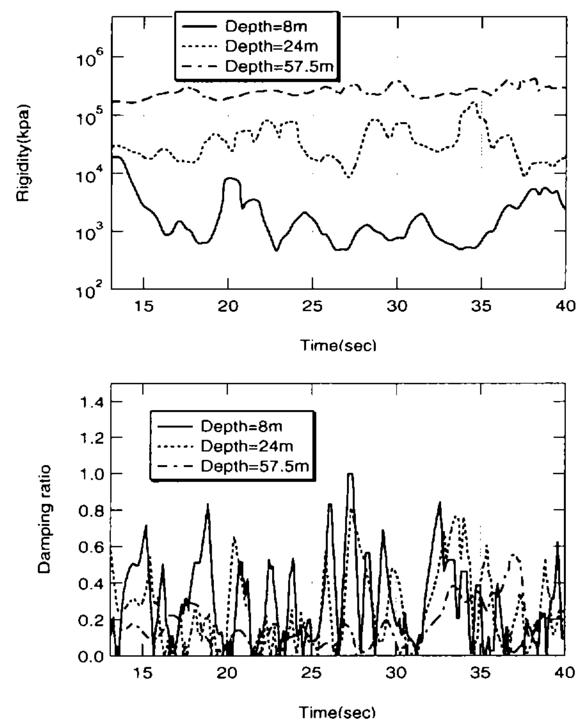

Fig.8 Variations of rigidity and damping obtained at each depth in the EW component.

shallow depths of the ground. However, the time-dependent variations of these parameters were not available in the analyses. We then applied the Complex Envelope method to the array records to further obtain the time-changing information of soil rigidity and damping. The shear stress $\tau(t,z)$ and shear strain $\gamma(t,z)$ were first numerically obtained at the mid-depth between each accelerometer in Fig.4, assuming that the acceleration and displacement vary both in a linear manner between the accelerometers' points. When obtaining the stress and strain, the observed acceleration records were treat through a band pass filter to avoid numerical errors that result from the "spatial ailiasing" due to the array layout. In this paper, the filter's band was set to be 0.1 to 1.5 Hz for the main-shock's records and 0.5 to 3.0 Hz for the after-shocks' records. Following the estimates of stress and strain, the non-stationary variations of h(t,z) and G(t,z) were next obtained according to equations (5) and (6). Equations (5) to (6) show that a phase relation estimates these variations, it is therefore possible that they fluctuate too rapidly in time. To avoid spurious fluctuations, the resultant variations were smoothed by the moving average method with a time window of 1.0 sec.

Figure 7 shows the variations of rigidity modulus and damping ratio estimated at the mid-depth between each accelerometer from the NS component's records. Similar variations were obtained from the EW component's records as shown in Fig.8. These variations of rigidity and damping are plotted for a time interval from 13.1 sec to 43.0 sec, regarded to be principally the S-wave portion. It was confirmed that these variations are well compatible with the soil rigidity moduli and damping ratios that were deduced as a stationary value from the cross correlation analysis [Kamiyama and Yoshida, 2000]. Figures 7 to 8 show that the time-varying characteristics of G(t,z) and h(t,z) are remarkable at the depth of 8 m in contrast to the less variations of them at the other depths. Especially, the reduction of soil rigidity at 8m-depth occurred with a factor of about 50 in a short time-interval of about 5.0 seconds.

These variations could be related closely with the stress-strain behaviors induced in soils. Figures 9 and 10 show the stress-strain orbits estimated at each depth, respectively, in the NS and EW components. As expected from the variations of soil rigidity and damping, the stress-strain orbits at 8m-depth show

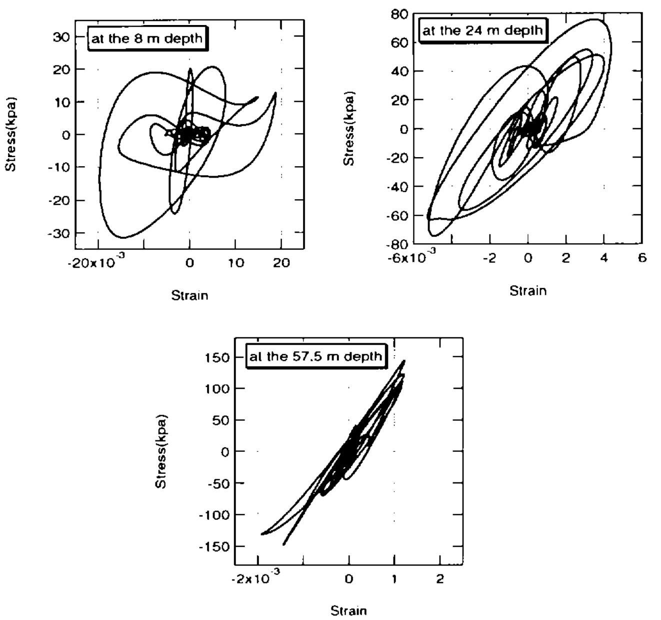

Fig.9 Stress and strain orbits at each depth in the NS component.

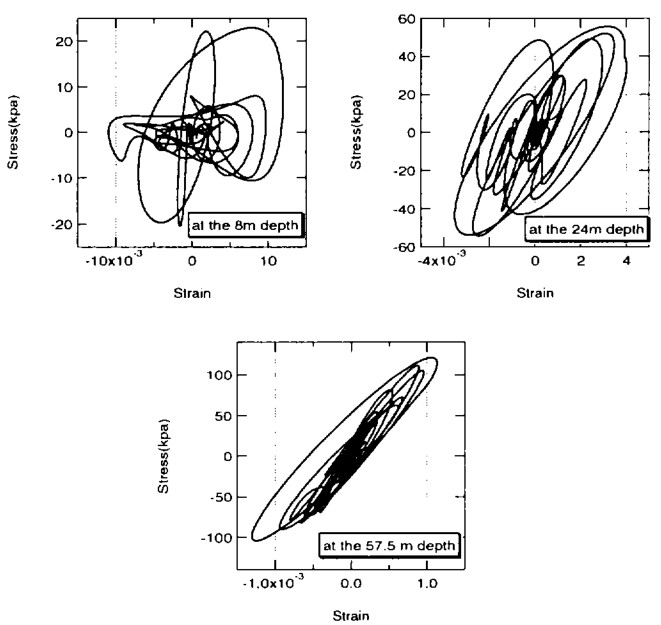

Fig.10 Stress and strain orbits at each *depth in the EW component.*

quite complicated and irregular features whereas the orbits at the deeper points, in particular, at the depth of 57.5 m vary rather regularly in both directions of NS and EW. Such complicated orbits as at 8m-depth are beyond the conventional method, which use a graphical technique to estimate the variations of soil rigidity and damping. In contrast, the Complex Envelope method is able to follow quantitatively even such a complicated orbit. We next made an attempt to correlate these variations of soil rigidity and damping to strain behaviors, that is, the strain-dependent variations of rigidity and damping were revealed from the Port Island records.

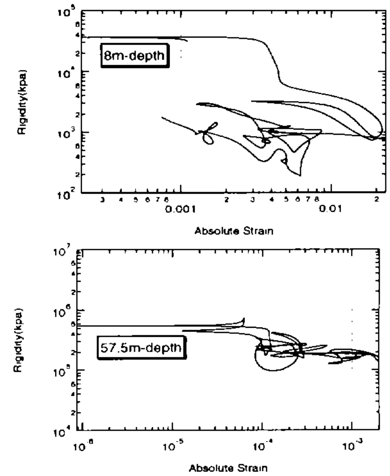

Fig.11 Strain dependent rigidities in the NS component.

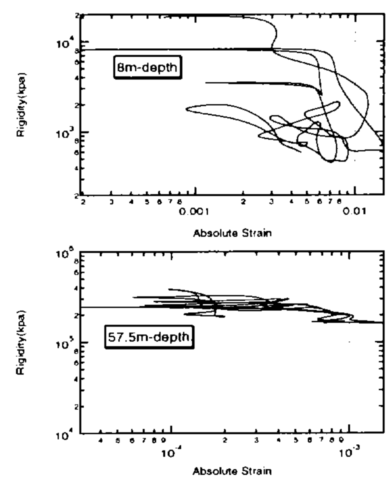

Fig.12 Strain dependent rigidities in the EW component.

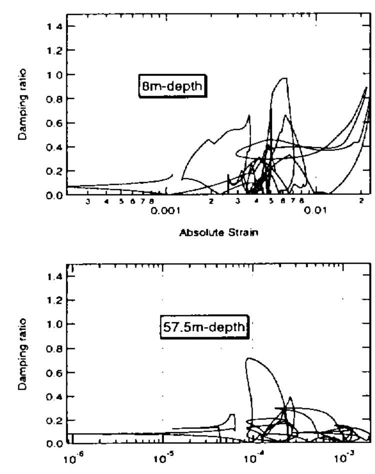

Fig.13 Strain dependent damping in the NS component.

In order to do so, we first obtained the envelopes of the strain time histories and then related them to the variations of soil rigidity and damping in time. Figures 11 and 12 show the

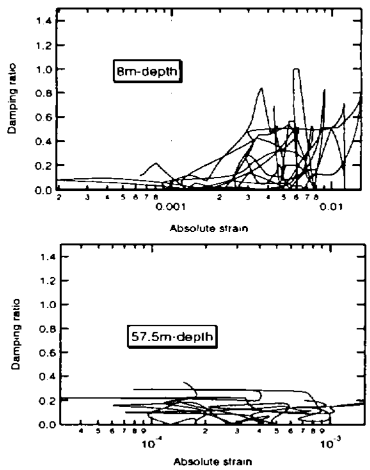

Fig.14 Strain dependent damping in the EW component.

traces in time for the rigidity-strain relations at the depths of 8 m and 57.5 m, respectively, in the NS and EW components. Similar traces for the damping-strain relations are plotted in Figures 13 and 14. On the other hand, Figures 15 and 16 display the rigidity ratios dependent on strain in which the maximum values of each rigidity variation were used as the divisor for the ratio. In Figures 15 and 16, similar relations obtained by a laboratory experiment for the soils sampled at 19m- and 74m-depths at the Port Island site are also plotted in comparison. Figures 15 and 16 indicate that the reductions of the rigidity ratio during the strong motions depend more strongly on strain and their degrees of reduction are much greater than the ones of the laboratory experiments. Especially, the differences between the strong-motion data and laboratory data at the depth of 8 m are remarkable in both components of NS and EW. This may suggest that soil rigidity could decrease greatly depending on strain when the soils liquefy.

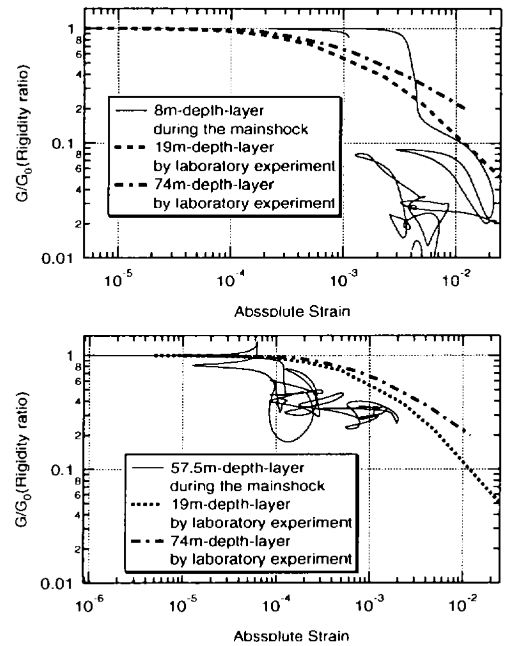

Fig.15 Rigidity ratios against strain (NS component).

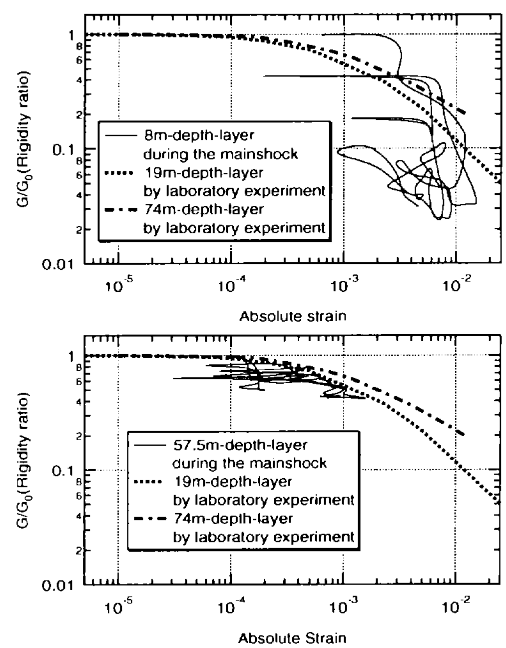

Fig.16 Rigidity ratios against strain (EW component).

CONCLUDING REMARKS

The Complex Envelope method is appropriate and useful for estimating the non-stationary variations of soil rigidity and damping related with non-linear earthquake responses of ground. Its theoretical validity was presented together with numerical simulations for harmonic motions. It was also applied to the down-hole array records obtained at Port Island during the 1995 Kobe Earthquake. The analyzed results showed that extreme reductions of rigidity, depending strongly on strain, occurred in the superficial layers during the main-shock of the earthquake. Such extreme variations of rigidity and damping in time, which are beyond the applicability of the conventional method due to graphical technique, were quantitatively clarified based on the Complex Envelope method.

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