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## A Non Linear Design Model for Continuous Multi-Span Light Gauge Sheeting and Members

by  
Leopold Sokol\*

### ABSTRACT

The calculation of profiled sheeting and members by using the post-critical stage in which a plastic hinge is originated at the internal support is possible provided that the relationships between the support moment, reaction and hinge rotation are known.

There are two alternative procedures of testing for obtaining these data: double-span test or internal support test. In the second one, instead of a global behaviour, only the local relationships between the three above characteristics are determined.

When calculating with the model proposed in Eurocode 3, Part 1.3, the above two different test procedures do not lead to close enough results.

The present paper is aimed at both explaining the reasons of these differences and proposing consequently an improved approach to the behaviour at the internal support.

### 1. - INTRODUCTION

Profiled sheeting and members may be calculated by using the post-critical stage during which a plastic hinge is formed at internal support provided that the relationships between moment, reaction and hinge rotation are known. In Europe, Ref. 1 gives a choice between two alternative testing procedures for obtaining these data: either the double-span test or the internal support test. The first one determines the global behaviour of system. The second one determines the local relationships between the three above characteristics, that are afterward used in the given model of calculation for checking the two following states: *serviceability limit state* and *ultimate limit state*.

When using this model, one may notice that the above two different testing procedures are not equivalent because their results are not close enough. This observation is generally negative as concerns the reliability of the internal support test.

Let us understand the reasons of this discordance by considering a continuous system under uniformly distributed, progressively increasing loading.

As long as this loading is small, all sections are fully effective and the global behaviour of the system is elastic and linear, that means, the deformations are proportional to the load. We will define then this first stage as the *elastic - linear phase*.

Next, the most highly loaded sections become partially ineffective, although no plastic deformations occur. Since the deformations become non linear but remain still elastic, this second stage may be considered as an *elastic - non linear phase*. For members, such as sleeved or overlapped purlins, the non linearity in this stage can occur also because of the semi-rigid behaviour of the connection (clearance of the bolt holes).

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Afterwards, the first plastic stresses and deformations appear in the most highly loaded sections, situated at and near the internal support, because of the effect of interaction between moment and reaction as well as between moment and shear forces. A plastic hinge occurs at the support and the system enters in the third, *plastic phase* which remains, of course, non linear. The property of this hinge is quite different from that of an usual hinge in thick, hot rolled sections. Here the carrying capacity falls down with the rising rotation. When the loading increases, the support moment decreases and the span moments and deflexions increase in amplified proportion.

Finally, when a second hinge is created at the span, collapse occurs by creation of a mechanism.

This model was studied in the previous work reported in Ref. 2, resulted in a proposition of a simplified analytical approach, that can not be furthermore extended for the irregular systems.

The present study is based on the similar theoretical analysis, but results in proposition of a numerical treatment, that is general and valid for all practical cases of continuous systems.

## 2. - TESTING - CALCULATION PROCEDURES PRESENTED IN REF. 1

### 2.1. - CALCULATION PRINCIPLE

According to Ref. 1, the two following states should be verified:

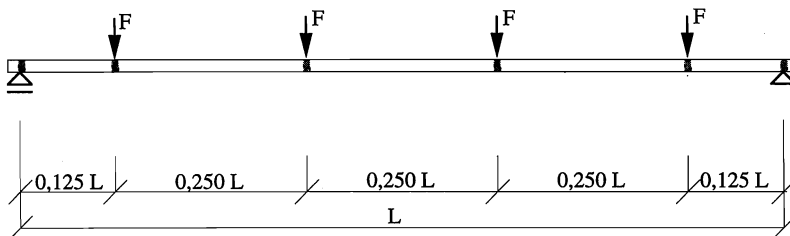
- the *serviceability limit state*, that is reached when:
  - deflection at mid-span reaches its allowed value,
  - the combination of moment and reaction at an internal support of continuous sheeting or member attains 0.9 times its ultimate value,
- the *ultimate limit state*, that is reached when mechanism is created, while the plastic hinges at support and at span occur.

The use of the plastic deformation in the global analysis aimed at determining the ultimate limit state is allowed only when the relationship between the support moment and the corresponding hinge rotation is obtained by testing.

The serviceability limit state (including first and second phases, as described above) is verified by a linear calculation whereas the ultimate limit state (including phase 3) is verified by a non-linear calculation.

### 2.2. - TEST SET-UP

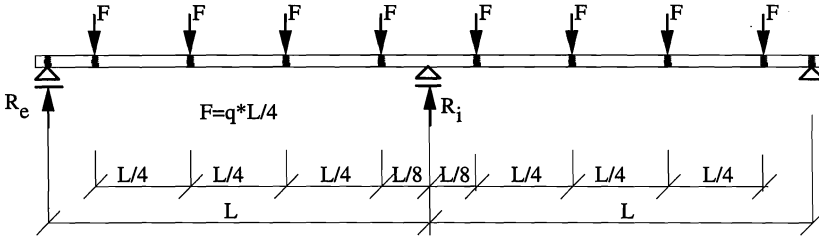
The span moment resistance and the effective flexural stiffness are determined by the single span test, with uniformly distributed load, that can be simulated by 4 line loads (Fig. 1):



**Fig. 1.** Test set-up for single span tests

There are two testing procedures for determining the resistance in the case of two or more spans for a given support width:

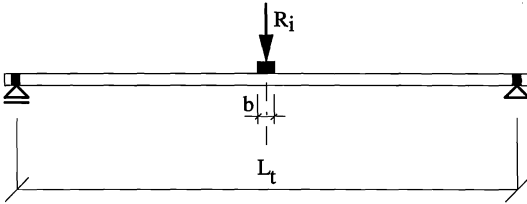
- the double span test, with uniformly distributed load, that can be simulated by 2 or 4 line loads per span (Fig. 2).



**Fig. 2.** Test set-up for double span tests with 4 line loads per span

- the internal support test, with 1 line load applied at the centre of the span, that simulates the internal support reaction  $R_i$  (Fig. 3). The test span  $L_t$  used to give the same Moment/Reaction ( $M/R$ ) ratio as in the case of the double span (Fig. 2), is:

$$L_t = 0.4 L \quad (1)$$



**Fig. 3.** Test set-up for internal support tests

The condition (1) results from the following consideration:

- for a double span (Fig. 2) of a system with perfectly linear behaviour:

$$M_i = \frac{q * L^2}{8} \quad (2)$$

$$R_i = \frac{5 * q * L}{4} \quad (3)$$

$$M_i / R_i = \left( \frac{q * L^2}{8} \right) / \left( \frac{5 * q * L}{4} \right) = \frac{L}{10} \quad (4)$$

- for the equivalent test set-up (Fig. 3):

$$M_i = \frac{R_i * L_t}{4} \quad (5)$$

$$M_i / R_i = \frac{L_t}{4} \quad (6)$$

The equation (1) is obtained comparing (4) and (6) that means that this ratio is valid only for double span linear system.

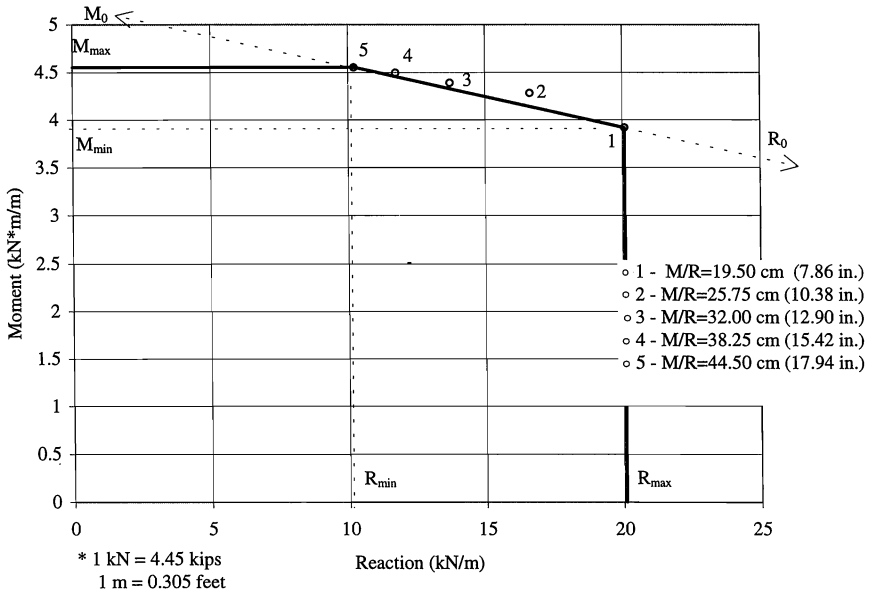
### 2.3 - CALCULATION PROCEDURE

The Moment-Reaction interaction (M-R relation) should satisfy the following relation that we present in slightly different, but more general form than given in Ref 1:

$$C_{M-R} = \frac{M_i}{M_0} + \frac{R_i}{R_0} \leq \Gamma_a \quad M_i \leq M_{\max} \quad R_i \leq R_{\max} \quad (7)$$

where:  $M_0$  and  $R_0$  are the points of intersection of the inclined line (Fig. 3) with the  $M_i$  (moment) and  $R_i$  (reaction) axes, respectively,

$\Gamma_a$  is a safety (reduction) coefficient of the resistance at the support (taken as equal to 0.90).



**Fig. 4.** Internal support test. Moment - Reaction (M-R) interaction

An example of the relationship between the support moment and the corresponding plastic hinge rotation (M- $\theta$  relationship) is shown in Fig. 4.

The points marked by circles in Fig. 1, are the statistically processed maximum design values corresponding to the maximal values reached by the curves in Fig. 5.

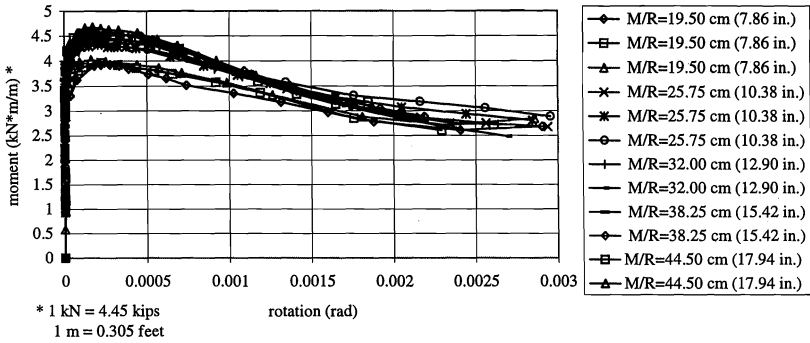


Fig. 5. Example of the moment - rotation relationship ( $M-\theta$ ) for different test span lengths of sheeting

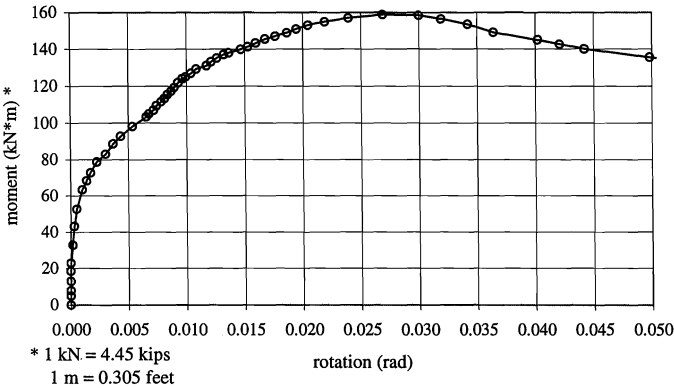


Fig. 6. Example of the Moment - Rotation ( $M-\theta$ ) relationship for a sleeved purlin

The two following limit states are distinguished:

- first, serviceability limit state, treated as elastic and linear phase, governed by eqn (7), up to 0.9 times the combined design resistance of the section above internal support (Fig. 4),
- second, ultimate limit state, treated as plastic (non linear) phase, governed by the  $M-\theta$  relationship (Figs. 5, 6), starting when the plastic hinge is originated at the support and continuing until another plastic hinge occurs in the span.

The design value of  $M$  as a function of  $\theta$  is taken equal to 0.9 times the mean value for all the tests corresponding to the beam span  $L$ .

In other words, the possible non linearity under the limit conditions in the first phase, as described in paragraph 1, is neglected and the area between the end of first and the beginning of the second phase is not considered.

## 2.4 - CRITICAL ANALYSIS OF THE PROCEDURE GIVEN IN REF. 1.

The procedure using the internal support test gives results that often are quite different in comparison to those obtained with the double span test.

The following points of the this procedure seem to lead to inaccurate results :

- As one can see in Figs. 5 and 6, the non linear behaviour of the system may start quite before 0.9 times the ultimate resistance of the section at the internal support. In such a case, the linear calculation presented in Ref. 1 obviously leads to inaccurate results.
- When calculating with taking into account the rotation at the support as a function of the support moment in the elastic - non linear phase, one may notice that when the rotation and the moment progressively increase, owing to the moment redistribution the  $M_s/R_s$  ratio becomes more and more different from the constant value given by (4), valid only for a linear behaviour of system.
- The discontinuity of the moment value, when taken equal to 0.9 times the mean tested value as a function of  $\theta$ , leads to the discontinuity of the equilibrium of the system, just after the ultimate resistance is reached at the support. This can lead to a divergence of iteration.

In view of estimating the differences between the results obtained by applying the two mentioned test procedures, for a current type of sheeting, a test program has been carried out with the sheeting "1000 P" of PAB, consisting of 7 single span tests, 7 double span tests and 5 internal support tests. The results are shown in Table 1.

**Table 1.- Profile 1000 P - Comparison of the double span test with the internal support test using the linear calculation**

Con-figuration	Load (kN/m <sup>2</sup> )*				Ratio calcul./ dble span test	
	non linear calculation		double span test			
n*L (cm)*	deflect.	supp. res.	deflect.	supp. res.		
1	2	3	4	5	2/4	3/5
2*240	2.45	2.60	2.42	2.89	1.01	0.90
2*360	0.73	1.22	0.71	1.34	1.03	0.91

\* 1 kN/m<sup>2</sup> = 0.0186 kips per square foot  
1 cm = 0.394 in.

In the Table 1:

- in the column 1, n is the number and L is the span length,
- the loads given in the columns 2 to 5 correspond to the "serviceability limit state" (see paragraph 2.1),
- the loads given in the columns 2 and 3 are determined by linear calculation, using the "single span tests" and "internal support tests",
- the loads given in the columns 4 and 5 are determined directly by the "double span tests",
- the loads given in the columns 2 and 4 correspond to the limit deflection  $L/200$ ,
- the loads given in the columns 3 and 5 correspond to the resistance (limit value of the combination of moment and reaction) of the section at the internal support.

The comparison of the two different procedures shows that the obtained differences go up to 10 %.

## 3 - PROPOSAL OF AN IMPROVED CALCULATION MODEL

### 3.1 - GENERAL

Let take one graph from Fig. 5, corresponding to a given relation M-R, and present it in a more detailed manner in Fig. 7.

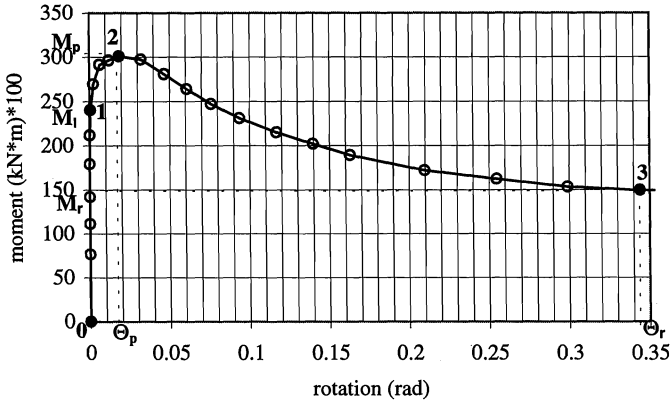


Fig. 7. Internal support test. Moment - rotation graph.

All phases specified in paragraph 1, may be identified in Fig. 7:

- phase 1 (elastic - liner) lasts from point "0" to point "1",
- phase 2 (elastic - non liner) lasts from point "1" to point "2",
- phase 3 (plastic) lasts from point "2" to point "3".

Point "1" corresponds to the linear limit moment  $M_l$  and point "2" corresponds to the limit moment that induces the plastification  $M_p$ .

Note, that the phase 2 may be simulated by an elastic, non-linear hinge over the internal support, that we will call the "equivalent conventional hinge". When calculating with this hinge, in both non linear phases 2 and 3, the  $M/R$  ratio does not keep its initial value (1), because of the redistribution of moment and forces. Simultaneously, the  $M-\theta$  curve, that depends on the  $M/R$  ratio also changes.

Some examples of this variation, calculated with actual characteristics defined by tests, are given in Table 2.

Table 2 - Variation of the equivalent test span for different phases of the structure

	Type of profile	COFRASTRA 40			COFRADAL 60			COFRASTRA 70			1000P		
Phase	L (cm)*	175	225	300	220	275	380	250	325	450	240	300	360
elastic	$(M_l/R_l)/L$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
linear	$M_l/R_l$ (cm)	17.50	22.50	30.00	22.00	27.50	38.00	25.00	32.50	45.00	24.00	30.00	36.00
	$L_{eq}=4M_l/R_l$ (cm)	70	90	120	88	110	152	100	130	180	96	120	144
elastic	$M_p/R_p$ (cm)	14.47	18.70	27.46	22.00	16.93	23.77	19.14	29.69	23.37	22.25	27.00	32.45
non	$L_{eq}=4M_p/R_p$ (cm)	57.9	74.8	109.8	88.0	67.7	95.1	76.6	118.8	93.5	89.0	108.0	129.8
linear	$(M_p/R_p)/L$	0.083	0.083	0.092	0.100	0.062	0.063	0.077	0.091	0.052	0.093	0.090	0.090
plastic	$M_r/R_r$ (cm)	10.70	13.35	17.40	12.20	14.90	19.20	13.20	16.90	23.40	12.70	15.05	17.05
	$L_{eq}=4M_r/R_r$ (cm)	42.8	53.4	69.6	48.8	59.6	76.8	52.8	67.6	93.6	50.8	60.2	68.2
	$(M_r/R_r)/L$	0.061	0.059	0.058	0.055	0.054	0.051	0.053	0.052	0.052	0.053	0.050	0.047

\* 1 cm = 0.394 in.

As one can see, the  $M/R$  ratio may be reduced in the elastic - non linear phase near twice a value defined by (4).



### 3.2 - PRINCIPLE OF THE PROPOSED PROCEDURE CALCULATION

As a consequence of the above observations, an improved procedure of calculation is proposed, where:

- The elastic - non linear phase is explicitly taken into consideration in the verification of the serviceability limit state. To ensure a convenient degree of safety in this state, the 0.9 coefficient is applied not to the combined design resistance at the support (as proceeded in Ref. 1), but to the limit load that induces the combined design resistance.
- In both the elastic - non linear and plastic phases, the  $M-\theta$  relation varies as a function of the changing  $M/R$  ratio. In the numerical calculation, this variation may be accomplished by an iterative procedure, where the  $M-\theta$  relation is adopted in each step as a function of the variable  $M/R$  ratio.
- The moment  $M$  is defined as function of the rotation  $\theta$  and of the  $M/R$  ratio. As for its design value, it is preferable to be taken as a reduced value to which the correction factor obtained for the  $M-R$  relation is applied, instead of the 0.9 constant value.

### 3.3 - RESEARCH AFTER AN ANALYTICAL EMPIRICAL FORMULA FOR THE $M-\theta$ RELATION

A set of tabulated  $M_i$  and of corresponding  $\theta_i$  data values is obtained by tests. These data that may be presented in the form:

$$M_t(\Theta_i) = M_i \quad (8)$$

This form of data is not easy to use in a numerical calculation. For purposes of practical use, it should be easier to calculate directly the  $M$  values from an analytical formula:

$$M_a(\Theta_i) = M_i \quad (9)$$

In general, an approach to (9) may be found with any well known polynomial (such as Legendre, Chebyshev, Hermite) or trigonometric (Fourier) series. However, and especially in case of profiled sheeting and physically continuous purlins, with quite smooth and regular shape of the  $M-\theta$  curve (Figs. 5 and 7), we should expect a very good approximation with the following function :

$$M_a(\Theta_i) = M_1 + \eta_0 \Theta_i + \eta_1 \Theta_i \eta_2 e^{\eta_3 \Theta_i} \quad (10)$$

where:  $M_1$  is the linear limit moment, indicated by the point "1" in Fig. 7,  
 $e$  is the base of natural logarithm,  
 $\eta$  are unknowns coefficients.

The least squares method is used for the best definition of the coefficients  $\eta_j$  ( $j=0,3$ ). They should be determined in such a manner that the error between the exact  $M_t(\Theta_i)$  and approximate  $M_a(\Theta_i)$  values:

$$\varepsilon_i = M_t(\Theta_i) - M_a(\Theta_i) \quad (11)$$

is the smallest one in the whole series of points  $i = 1, n$ . Let take the variance:

$$H = \sum_{i=1}^n \rho_i (\varepsilon_i)^2 \quad (12)$$

as a global error measure. The factored load values  $\rho_i$  allow to take into account, in an arbitrary manner, the importance of the values in various  $i$  points. The variance  $H$  is minimised by making null its first variation, as a function of the  $\eta_j$  coefficients :

$$\frac{\partial H}{\partial \eta_j} = \sum_{i=1}^n 2\rho_i \varepsilon_i \frac{\partial \varepsilon_i}{\partial \eta_j} = 0 \quad j = 0,3 \quad (13)$$

By putting (10), (11) and (12) into (13), we get the following non linear equation system:

$$\left. \begin{aligned} \sum_{i=1}^n \rho_i \varepsilon_i \Theta_i &= 0 \\ \sum_{i=1}^n \rho_i \varepsilon_i M_{ni} &= 0 \\ \sum_{i=1}^n \rho_i \varepsilon_i M_{ni} \ln \Theta_i &= 0 \\ \sum_{i=1}^n \rho_i \varepsilon_i M_{ni} \Theta_i &= 0 \end{aligned} \right\} \quad (14)$$

where:  $M_{ni} = \eta_1 \Theta_i \eta_2 e^{\eta_3 \Theta_i}$

The solution of (14) may be found, for instance, by using the Newton approximation method. The iteration procedure should start by evaluating in a first step the "good" initial values. The parameter  $\eta_0$  may be taken as equal to the tangent of inclination of the straight line linking the points "1" and "3" (Fig. 7):

$$\eta_0 = \frac{M_3 - M_1}{\Theta_{\max}} \quad (15)$$

At the point "2" (Fig. 7):

$$\frac{dM}{d\Theta} = \eta_0 + \eta_1 \Theta_u \eta_2^{-1} e^{\eta_3 \Theta_u} (\eta_2 + \eta_3 \Theta_u) = 0 \quad (16)$$

By taking approximately  $\eta_0 = 0$ , we get:

$$\eta_3 = -\frac{\eta_2}{\Theta_u} \quad (17)$$

$$M_p = M_1 + \eta_0 \Theta_p + \eta_1 \Theta_p \eta_2 e^{\eta_3 \Theta_p} \quad (18)$$

By substituting (17) into (18) we get:

$$\eta_1 = (M_p - M_1 - \eta_0 \Theta_p) \left( \frac{e}{\Theta_p} \right)^{\eta_2} \quad (19)$$

In any point "i" situated between "1" and "3" (Fig. 7):

$$M_i = M_1 + \eta_0 \Theta_i + \eta_1 \Theta_i \eta_2 e^{\eta_3 \Theta_i} \quad (20)$$

By substituting (15), (17) and (19) into (20) we get:

$$\eta_2 = \frac{\ln \frac{M_i - M_1 - \eta_0 \Theta_i}{M_p - M_1 - \eta_0 \Theta_p}}{1 - \frac{\Theta_i}{\Theta_p} + \ln \frac{\Theta_i}{\Theta_p}} \quad (21)$$

Any intermediate  $\Theta_i$  value may be arbitrarily chosen to be substituted into (21), for instance, that situated near the  $\Theta_p/2$  value.

It is interesting to note that the above determined  $\eta_j$  starting values, substituted into (10), may give directly a quite good approach. However, a complete solution of (14) with a satisfactory accuracy should usually be found.

### 3.4 - NUMERICAL APPLICATION OF PROPOSED PROCEDURE

#### Hypotheses:

The mechanical properties of section are defined from "single span" and "internal support" tests, as described in paragraph 2.2, but taking into account the remarks concerning the equivalent test span (see 3.1).

The system is calculated as linear until the "linear limit moment" is reached at the intermediate support (point "1" in Fig. 7), and as non linear above this limit.

At the internal supports:

- the resistance of the section is governed by M-R relation (7),
- the stiffness of support hinge is governed by the M- $\theta$  relationship (8), adjusted by step by step iteration procedure as a function of the M/R ratio (this means that the hinge stiffness is variable)

The final equilibrium of the system is calculated by an iteration procedure, where:

- in the first step, the support moments may be calculated assuming rigid nodes,
- in each one of the following steps, the support moments are recalculated by considering the support nodes as being elastic, non linear hinges, with a stiffness variation as a function of the rotation defined by the M- $\theta$  relationship, corresponding to the M/R ratio. This leads to a double interpolation between two proximate M- $\theta$  curves and between two proximate values of M- $\theta$  curve.

#### Data

A typical data set contains the following characteristics:

- geometric data (number and dimensions of spans L),
- load q,
- second area moment of section I,
- value of the ultimate span moment  $M_s$ ,
- values of the intermediate support moment and reaction resistance  $M_{i,max}$ ,  $M_{i,min}$ ,  $R_{i,max}$ ,  $R_{i,min}$ , that give the M-R relation (as those plotted in Fig. 4),
- value of the end support reaction resistance  $R_e$ ,
- values giving the M- $\theta$  relationship (as those plotted in Fig. 5 to 7).

#### Example of data:

2-span system, with span length:  $L = 4.50$  m  
 Units: daN, cm  
 Element incidences: 1 2 2 3 3 4 4 5  
 Nodal forces - rigid supports: 1 2 8 14  
 Coordinates of nodes X(i) in cm : .00 225.00 450.00 675.00 900.00  
 Coordinates of nodes Y(i): .00 .00 .00 .00 .00  
 Uniformly distributed load:  $q = 1.85$  kN/m \*  
 $M_s = 6.31$  kN\*m/m  
 $M_{i,max} = 4.55$  kN\*m/m  
 $M_{i,min} = 3.92$  kN\*m/m  
 $R_{i,max} = 20.08$  kN/m  
 $R_{i,min} = 10.23$  kN/m  
 $R_e = 13.66$  kN/m

$$I = 70.78 \text{ cm}^4/\text{m}$$

$$\text{Limit deflexion } f = L/200 = 2.25 \text{ cm}$$

$$* 1 \text{ kN/m} = 0.06852 \text{ kips per foot}$$

$$1 \text{ m} = 3.28 \text{ feet}$$

$$1 \text{ cm} = 0.394 \text{ in.}$$

By reason of lack of space, tabulated values of  $M$  and  $\theta$ , corresponding to those plotted in Fig. 5, for various test span lengths, are not given here.

### Example of results

The coefficients of the analytical function (10) found by solving the equations (14) for a family of 2 identical tests with a span test  $L_t = 153 \text{ cm}$  ( $M/R = L_t/4 = 38.25 \text{ cm}$ ) are as follows:

$M_1$	$\eta_0$	$\eta_1$	$\eta_2$	$\eta_3$
3.96E+04	-5.13E+04	1.64E+05	6.90E-01	-2.11E+01

The curve defined by the above coefficients according to the test results is plotted in Fig. 8.

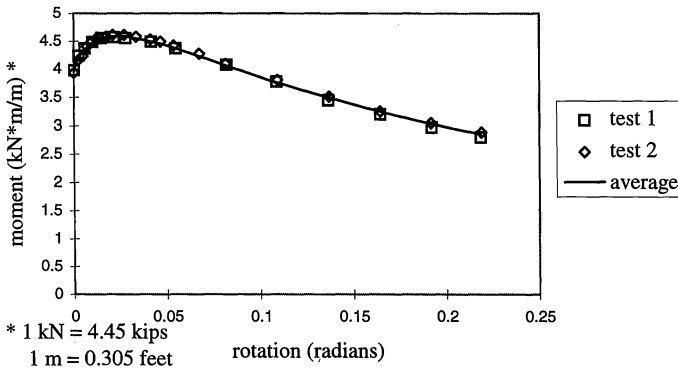


Fig. 8 - Average analytical  $M-\theta$  curve for a family of tests

### Displacements of element ends

No	Left			Right		
	X	Y	Rot.	X	Y	Rot.
1	.000	.000	-.025013	.000	-2.892	.005561
2	.000	-2.892	.005561	.000	.000	.002768
3	.000	.000	-.002768	.000	-2.892	-.005561
4	.000	-2.892	-.005561	.000	.000	.025013

### Forces and moments in elements

No	Node	N (kN)	T (kN)	M (kN*m)
1	1	.000E+00	.318283E+01	0.
1	2	.000E+00	.979671E+00	.247855E+01
2	2	.000E+00	-.979672E+00	-.247855E+01
2	3	.000E+00	.514217E+01	-.440852E+01
3	3	.000E+00	.514217E+01	.440852E+01
3	4	.000E+00	-.979672E+00	.247855E+01
4	4	.000E+00	.979672E+00	-.247855E+01
4	5	.000E+00	.318283E+01	0.

Reactions (kN)

$$R(1) = 3.183$$

$$R(2) = 10.284$$

$$R(3) = 3.183$$

Nodal rotations

Node	Moment	Rotation	Stiffness	M/R
3	44085.2	.00554	.796322E+07	42.866

Internal support reactions:      Moment  $M_i = 4.40852 < M_{i,max} = 4.55 \text{ kN}\cdot\text{m/m}$   
 Reaction  $R_i = 10.284 < R_{i,max} = 20.08 \text{ kN}$

Interaction coefficient:       $C_{M-R} = \frac{M_i}{M_0} + \frac{R_i}{R_0} = 0.973$

Remarks:

- The rotation is not null and the M-R interaction is lower than 1. This indicates that the system is in the elastic - non linear phase.

- The ratio  $M/R = 42.866 \text{ cm} = L / 10.5$ , while in a linear phase it would be equal to  $L/10$ . This demonstrates that when calculating by using an iteration procedure, the M- $\theta$  curve has to be adjusted according to the M/R ratio.

### 3.5 - COMPARISON OF THE RESULTS OF THE IMPROVED CALCULATION AND THE DOUBLE SPAN TEST RESULTS

The results of the improved calculation procedure based on the same test program as that mentioned in paragraph 2.4, are given in the Table 3.

**Table 3.- Profile 1000 P - Comparison of the double span test with the internal support test using the non linear calculation**

Con- figu- ration n*L (cm)	Load (kN/m2)				Ratio calcul./ dble span test	
	non linear calculation		double span test			
	deflect.	supp. res.	deflect.	supp. res.	2/4	3/5
1	2	3	4	5	2/4	3/5
2*240	2.44	2.84	2.42	2.89	1.01	0.98
2*360	0.73	1.36	0.71	1.34	1.03	1.01

\*  $1 \text{ kN/m}^2 = 0.0186 \text{ kips per square foot}$   
 $1 \text{ cm} = 0.394 \text{ in.}$

In the Table 3:

- in the column 1, n is the number of spans and L is the span length,
- the loads given in the columns 2 to 5 correspond to "serviceability limit state" (see paragraph 3.2),
- the loads given in the columns 2 and 3 are determined by the improved, non linear calculation, using the "single span tests" and the "internal support tests",
- the loads given in the columns 4 and 5 are determined directly by the "double span tests",
- the loads given in the columns 2 and 4 correspond to the limit deflection  $L/200$ ,
- the loads given in the columns 3 and 5, correspond to the resistance (limit value of combination of moment and reaction) of the section at the internal support.

Here, the comparison of the two different procedures shows that the obtained results are quite close.

### 3.6 - COMPARISON BETWEEN THE RESULTS OF THE LINEAR CALCULATION AND OF THE IMPROVED NON LINEAR CALCULATION MODEL

In order to estimate the differences between the results obtained with the proposed modified calculation procedure in comparison with those obtained by the calculation method presented in Ref. 1, a large programme of testing of various types of PAB profiles has been carried out. For each type of profile, at least 3 "single span tests" and 22 "internal support tests", with different test spans, has been effectuated. The thus obtained results are shown in Table 4.

Table 4. - Comparison between the results of the linear and non linear calculation based on "internal support tests"

Table 4a. - COFRADAL 60

Con-figuration n*L	Load (kN/m <sup>2</sup> ) *				Ratio	
	Procedure calculation				linear/ non lin.	
	linear		non linear		2/4	3/5
	defl.	resist.	defl.	resist.		
1	2	3	4	5	6	7
2*220	****	4.74	****	6.12	****	0.77
2*275	****	3.17	****	3.72	****	0.85
2*380	1.75	1.75	1.69	1.88	1.04	0.93

Table 4b. - 1000 P

Con-figuration n*L	Load (kN/m <sup>2</sup> ) *				Ratio	
	Procedure calculation				linear/ non lin.	
	linear		non linear		2/4	3/5
	defl.	resist.	defl.	resist.		
1	2	3	4	5	6	7
2*240	2.45	2.89	2.44	3.15	1.00	0.92
2*300	1.25	1.91	1.25	2.13	1.00	0.90
2*360	0.73	1.35	0.73	1.51	1.00	0.89

Table 4c. - COFRASTRA 40

Con-figuration n*L	Load (kN/m <sup>2</sup> ) *				Ratio	
	Procedure calculation				linear/ non lin.	
	linear		non linear		2/4	3/5
	defl.	resist.	defl.	resist.		
1	2	3	4	5	6	7
2*175	****	7.14	****	8.50	****	0.84
2*225	3.90	4.60	3.69	5.55	1.06	0.83
2*300	1.67	2.75	1.65	3.02	1.01	0.91

Table 4d. - COFRASTRA 70

Con-figuration n*L	Load (kN/m <sup>2</sup> ) *				Ratio	
	Procedure calculation				linear/ non lin.	
	linear		non linear		2/4	3/5
	defl.	resist.	defl.	resist.		
1	2	3	4	5	6	7
2*250	****	5.30	****	6.95	****	0.76
2*325	****	3.30	****	3.75	****	0.88
2*450	1.74	1.80	1.69	2.02	1.03	0.89

\* 1 kN/m<sup>2</sup> = 0.0186 kips per square foot

In Table 4:

- in the column 1, n is the number of spans and L is the span length in cm (1 cm = 0.394 in.),
- the loads given in the columns 2 to 5 correspond to "serviceability limit state" (see paragraphs 2.1 and 3.1.1), where:
  - . the values in the columns 2 and 4 correspond to the limit deflection L/200,
  - . the values in the columns 3 and 5, correspond to the resistance (limit value of combination of moment and reaction) of the section over the internal support.
- the loads in the columns 2 and 3 are determined by the improved, non linear calculation, based on "single span tests" and "internal support tests",
- the loads in the columns 4 and 5 are determined by the linear calculation method presented in Ref. 1, based on the same above mentioned tests,
- the loads given in the columns 3 and 5, correspond to the resistance (limit value of the combination of moment and reaction) of the section at the internal support.

The asterisks in the columns 2 indicate that the limit deflection is not attained before the resistance condition is reached.

The results of this comparison indicate that in the serviceability limit state, when calculating with the linear procedure presented in Ref. 1, in a general way, there is tendency to:

- under-estimate the deflection, and
- over-estimate the resistance.

There are differences in deflection values up to 4% and in resistance ones up to 24%. The differences of deflection values indicate that the non linear behaviour starts before the resistance condition at the internal support being reached.

#### 4. - CONCLUSIONS

The behaviour of continuous multi-span light gauge profiles has been studied by taking into account the non linear behaviour, near the limit value of resistance of the section at the internal support, in the elastic phase corresponding to the serviceability limit state.

This approach, which represents a more homogenous integration of the relationships between moment, reaction and corresponding rotation, provides for a better accuracy of calculation and, as a consequence, let the internal support test be still an essential tool for this kind of analysis.

Additionally, an analytical empirical formula for M- $\theta$  relation is defined in the present study, which is particularly well adapted for continuous profiled sheeting elements.

In conclusion, a method for taking into account the non linear behaviour in the serviceability limit state is proposed aimed at improving the reliability of the results of calculation.

#### Appendix .--References

1. "EUROCODE 3 - ENV 1993, Design of Steel Structures. Part 1.3: Cold Formed Thin Gauge Members and Sheeting", February 1996.
2. L. SOKOL "Some Specific Aspects of Elastic-Plastic Behaviour of Profiled Steel Sheeting and Decking", TWS, Bicentenary Conference on Thin-Walled Structures, University of Strathclyde, December 1996.

#### Appendix .--Notations

##### Symbols:

E	Young modulus
f	deflexion
I	second area of section
L	span length
M	support moment
q	transversal load
R	support reaction
$\Theta$	rotation
$\eta$	parameters of analytical empirical equation

##### Subscripts:

e	end support reaction
i	internal support moment, reaction or rotation
l	limit linear value
p	plastification condition
r	residual moment or rotation
s	span
t	test