Differential signalling in PCBs: modeling and validation of dielectric losses and effects of discontinuities

R. Araneo  
Chen Wang  
Xiaoxiong Gu  
S. Celozzi  
James L. Drewniak  
Missouri University of Science and Technology, drewniak@mst.edu

Follow this and additional works at: http://scholarsmine.mst.edu/faculty_work  
Part of the Electrical and Computer Engineering Commons, and the Materials Science and Engineering Commons

Recommended Citation  
http://scholarsmine.mst.edu/faculty_work/484

This Article - Conference proceedings is brought to you for free and open access by Scholars' Mine. It has been accepted for inclusion in Faculty Research & Creative Works by an authorized administrator of Scholars' Mine. For more information, please contact weaverjr@mst.edu.
Differential Signalling in PCBS:
Modeling and Validation of Dielectric Losses and Effects of Discontinuities

R. Araneo (1), Chen Wang (2), Xiaoxiong Gu (2), S. Celozzi (1), J. Drewniak (2)

(1) Electrical Engineering Department
University of Rome “La Sapienza”
Rome, Italy

(2) Department of Electrical Engineering
University of Missouri Rolla
Rolla, USA

Abstract — This paper focuses on differential signal transmission above ground planes with gaps, taking into account the dielectric and conductive losses of the substrate. An equivalent lumped-circuit is proposed and its suitability is investigated by comparing the obtained numerical results with the measured data. Furthermore the differential to common mode conversion of the waves, while crossing the gap, is theoretically analyzed and experimentally verified.

I. Introduction

The operational frequencies of digital systems have increased at such a rapid pace over the past few years, that nowadays high performance printed circuit boards (PCBs) have to deal with digital signals whose spectral content can reach tens of gigahertz. Together with the increase of operational frequencies, system complexity is also growing [1], so that the propagation of high-speed signals with acceptable distortion and loss has become critical.

In this framework, this paper focuses on the effects on high-speed signals that need to cross split planes acting as a signal reference. This specific but common design situation represents quite a broad class of problems in the study of signal integrity (SI). In practical layouts, slots in printed circuit board planes are often introduced for functionality, e.g. to isolate noisy or sensitive circuits or to separate different logic levels. Since any gap constitutes a discontinuity, SI and EMI issues may arise so that it is mandatory that a careful analysis at the preliminary design stage be done.

Improvement in the SI and EMI performance can be achieved by means of differential signalling, since the amount of current density carried by power and ground planes is substantially reduced [2,3]. However, the available budget for critical design parameters is continuously decreasing due to the need of faster clock rates, which have the consequence of enhancing the effects of losses associated with the presence of real, dispersive dielectrics, e.g. FR-4, which have been demonstrated to be very important in the analysis of signal propagation on printed circuit boards.

II. Dielectric Modeling

In this work, a full wave Finite Difference Time-Domain (FD-TD) code is used for prediction of the effects that various kinds of discontinuities have on differential signal transmission in PCB structures [4]. Furthermore, the dispersive and dissipative behaviour of the substrate is accurately modelled and simulated by means of a Debye model, and predictions are validated by measurements on test boards. Finally an equivalent lumped circuit of the discontinuity is extracted and validated.
marching explicit equations for the update of the unknown
electric field \( E^{n+1} \) at each time step. In the past, the accuracy
of the different schemes was often investigated by
considering the wave propagation in an homogeneous half-
space. However, in this way, the overall accumulated and
propagated error is estimated and not only that related to
different schemes. In the following, a new method aimed at
assessing the error – at each step – due to the FD scheme,
either of the RC-type or ADE-type, is presented. The method
is based on the introduction of an auxiliary integral equation
whose error characteristics are the same of the original
problem.

In the time domain, the constitutive relation between
the electric flux density \( D(t) \) and the electric field \( E(t) \) is in
the form of a convolution integral

\[
D(t) = \varepsilon_0\varepsilon_r E(t) + \varepsilon_0 \int_0^t \chi(t-\tau)E(\tau)\,d\tau
\]

where \( \chi(t) \), the susceptibility function, is known,
the integral constitutive relation

\[
\chi(t) = \frac{\varepsilon_s - \varepsilon_r}{\tau} e^{-t/\tau} U(t)
\]

where \( \varepsilon_s \) and \( \varepsilon_r \) are respectively the static and the high
frequency relative permittivity, \( \tau \) is the so called relaxation
time, and \( U(t) \) is the Heaviside function which vanishes for
negative values of \( t \), and is unity for positive \( t \).

In order to investigate the accuracy of different algorithms,
an auxiliary integral equation is considered. Assuming the
curl of \( H(t) \) to be known, and by applying a central-
difference scheme to the time derivative of \( D(t) \), and a time
average for \( E(t) \), the following integral equation results

\[
\varepsilon_0 \varepsilon_r \left( \frac{E^{n+1} - E^n}{\Delta t} \right) = \varepsilon_0 \int_0^{(n+1)\Delta t} \chi(t-\tau)E(\tau)\,d\tau - \int_0^{(n)\Delta t} \chi(t-\tau)E(\tau)\,d\tau + \sigma \left( \frac{E^{n+1} + E^n}{2} \right) = \nabla \times H^{n+1/2},
\]

which is solved in terms of the unknown electric field \( E^{n+1} \).

The curl of \( H \) may be evaluated analytically if the electric
field is assumed to be known, only for this purpose.
The comparison between the numerical value \( E^{n+1} \) obtained by
(3), and the exact one \( E[(n+1)\Delta t] \), used to impose
analytically the curl of \( H \), allows the numerical errors due to
both the time discretization and the approximate evaluation
of the convolution integral to be accessed. According to the
choice of the numerical algorithm, different time-marching
explicit equations hold for the update of the unknown
electric field.

The RC approaches compute directly the convolution
integral. In order to avoid the storage of the complete time
history of the electric field, an efficient recursive summation
is adopted requiring the storage of a few previous values of
the electric field, depending on the number of poles of the
material. The convolution integrals are approximated by
means of an assumption on the time behavior of the electric
field. The electric field is assumed piecewise constant
(PCRC) [6], or piecewise linear (PLRC) [7], between the
discrete times at which it is calculated, or it is assumed to
have a constant dependence over each time segment \( \Delta t \)
centered around \( E^n \) (PC2RC) [8]. It has been demonstrated
that the PCRC scheme is first-order accurate whereas the
PLRC and PC2RC ones are second-order accurate.

On the other hand, the ADE method substitutes the
integral constitutive relation (1) with a time domain auxiliary
differential equation [9], which links the electric flux density
\( D(t) \) or the polarization vector \( P(t) \) to the electric field. The
ADE is time stepped in synchronism with Maxwell’s curl
equation, thus yielding a consistent coupled system of
second-order accurate equations.

Figures 1 and 2 show a comparison between the time
evolution of the errors due to the above algorithms for the
case of an FR-4 medium and a simple double exponential
driving the electric field having \( |E| = 1, \alpha_1 = 10^9 \text{ s}^{-1}, \alpha_2 = 10^2 \text{ s}^{-1} \) and considering \( \Delta t = 0.5 \text{ ps} \).

In Fig. 1, the curl of the magnetic field is analytically
evaluated from the known electric field as previously
explained, and the error is evaluated on the numerically
computed electric fields \( E^{n+1} \). In Fig. 2, the error is evaluated on the numerically computed \( D^n \), having analytically
evaluated \( D \) in (1). It is evident that the ADE scheme is less
accurate than PLRC for the case of a Debye medium. The
values considered for the dielectric material were \( \varepsilon_s = 4.3, \varepsilon_r = 4.1, \tau = 0.033 \text{ ns}, \sigma = 200 \text{ \mu S/m} \). It is worth noting that the
first method shows the global error computed on the electric
field by the chosen algorithm, taking into account both the
integration of \( E \) and the time derivative of \( D \), whereas the
second one shows only the error computed in the integration
of (1), without pointing in evidence the real error of the
global algorithm. In fact, ADE, which could appear better
than the PLRC-PC2RC, looking at Fig. 1, is not better
indeed, since the global error computed on \( E \) is higher.

Furthermore, a numerical error associated with all
previous schemes in late time was observed. In all the
algorithms, the updating equations for the electric field can
be put in the standard form
\[ E_{n+1} = a_1 E_n + a_2 (\nabla \times H)^{n+1/2} + a_3 \Psi^n \quad (4.a) \]

\[ \Psi_{n+1} = b_1 E^{(n)} + b_2 (\nabla \times H)^{n+1/2} + b_3 \Psi^{(n)} , \quad (4.b) \]

where the second equation derives from the RC or the ADE, and the coefficients \( a_i \) and \( b_i \) can be obtained according to the chosen scheme. For late times, it has been observed that after \((\nabla \times H)^{n+1/2}\) reduces to zero, the set of homogeneous equations (4) do not converge to the null solution, which is the exact one, but to an asymptotic solution \((E^\ast, \Psi^\ast)\). For example, in the case of the PLRC scheme applied to the Debye medium \((a_1 = 0.99926, a_2 = 0.01376, a_3 = 0.24381, b_1 = 4.52075E-5, b_2 = 3.12121E-7, b_3 = 0.98496)\), it can be verified that the scheme converges \((n>15,000)\) to the asymptotic values \(E^\ast = -8.26804E-7\) and \(\Psi^\ast = -2.48648E-9\), thus yielding to an electric field which does not decay to zero.

**III. Simulation model and results**

The test configuration is shown in Fig. 1, where a microstrip trace pair above a ground plane with a slit is depicted.

**Fig. 1** - Numerical error computed on \(E^\ast\) due to different FDTD schemes for the case of a Debye medium.

**Fig. 2** - Numerical error computed on \(D^\ast\) due to different FDTD schemes for the case of a Debye medium.

**Fig. 3** - Cross-section (a) and top-view (b) of the considered configuration.

First, a simple differential configuration without a gap (geometric dimensions: \(H = 16\) cm, \(W = 9\) cm, \(L = 12.0\) cm, \(h = 1.5\) mm, \(w = d = 1\) mm) is investigated with a constant permittivity model and a lossy dispersive one in order to illustrate the necessity to include losses in the study of high frequency signal transmission. In fact, from Fig. 4, which shows the comparison between the \(S_{21}\) in the lossy case \((\varepsilon_s = 4.3, \varepsilon_r = 4.1, \tau = 0.033\) ns and \(\sigma = 0.2E-4\) siemens/m) and in the lossless one \((\varepsilon = 4.11,\) permittivity at the central frequency \(5.025\) GHz) for the previous configuration fed by a differential signal, it is evident the importance of accounting for losses above 2 GHz.

**Fig. 4** - Comparison between the frequency spectra of \(S_{21}\) for the lossy and the lossless case.
The S-parameters of several structures, with and without slit on the reference plane are calculated and experimentally validated to show the effect of the gap on signal transmission. In Figs. 5 and 6 $S_{21}$ and $S_{41}$ parameters for the configurations with and without the slit are shown. Both the results simulated by means of the FDTD method and the data measured on a test board are reported. In the case of the split ground plane it is evident that the crosstalk between the two strips increases due to propagation of a slotline mode along the slit. Moreover the transmission along the single strip decreases, since there is a partial reflection of the traveling wave while crossing over the gap. The FDTD results agree favorably with the measurements in the frequency range of interest. The geometric dimensions of the board and the dielectric parameters of the substrate are equal to that of the previous example but with a gap of 2 mm placed at H/2.

![Fig. 5 - Comparison between the frequency spectra of $S_{21}$ for the cases with and without slit. Both the FDTD results and the measurements are reported.](image1)

![Fig. 6 - Comparison between the frequency spectra of $S_{41}$ for the cases with and without slit. Both the FDTD results and the measurements are reported.](image2)

![Fig. 7 - Comparison between the frequency spectra of $S_{11}$ for the cases with and without slit. Both the FDTD results and the measurements are reported.](image3)

![Fig. 8 - Comparison between the frequency spectra of $S_{12}$ for the cases with and without slit with a differential feeding between port #1 and port #3. Both the FDTD results and the measurements are reported.](image4)
extension in the calibration. A translation of the reference plane of 1.1 cm has been done on all the four ports, in order to take into account the effect of SMA jacks.

IV. Equivalent lumped circuit model

Over the years, many different numerical techniques have been developed to analyze microstrip discontinuities. Several modeling tools currently on the market for computer aided design of microwave circuits use equivalent S-parameters models for the subparts of the structures under study, since it is quite common to perform the analysis in the frequency domain. In the design of high-speed digital circuits, both time and frequency domain calculations are used. For example, in digital applications, the most generally used computer aided circuit simulator is the well-known SPICE. Since it works in the time domain, it requires conventional equivalent circuits with lumped parameters that can be conveniently inserted into the program. In view of this, in the following, an equivalent lumped circuit of the microstrip-slotline transition is extracted and validated by a comparison with the data obtained both via FDTD and measurements.

One approach to model a slot in the ground is by determining the effective excess inductance caused by the partial absence of a ground plane [2-7]. Since a slot in the ground plane is a discontinuity for the return current, it can be modeled as an extra inductance in the ground path. In the case of two signal lines that cross the same ground slot, since they share part of the ground inductance, when one signal line switches, its return current flows through the ground inductance and a potential transient develops in the return path at the edges of a slot. However, for split grounds, the effective inductance is a function of frequency, making the time domain simulation with circuit simulators such as SPICE cumbersome. Furthermore the inductance depends on the relative location of signal lines with respect to the slot so that it has to be recalculated whenever the layout changes.

A second more accurate and general way to model such a discontinuity is to consider the ground slot as a transmission line, which in microwave circuit design is denoted a slotline [10]. Without entering into the physics of the conversion from the microstrip mode to the slotline mode, and vice versa, which can be found in [11], the proposed equivalent lumped circuit is shown in Fig. 9. The excess transverse capacitances $C_x$ model the effects of the fringing fields which are present between the signal line and the edges of the reference plane. The excess longitudinal inductance $L_x$ takes into account the effect of the partial absence of the ground plane beneath the stripline. Furthermore the signal conversion from stripline mode to slotline mode is modeled by means of the current-controlled current source $I_x$ (SPICE model $F$). The reciprocal mode conversion is modeled by the voltage-controlled voltage source $V_x$ (SPICE model $E$).

![Fig. 9 - Equivalent circuit for the two microstrip lines that cross a split reference plane.](image-url)
The characteristic impedances $Z_0$ of the signal microstrip and $Z_\pi$ of the ground slotline are calculated by analytical formulas that can be found in [1,10]. The excess inductance and capacitance at the crossovers can be obtained by three-dimensional static analysis [12]. The values of the controlled sources are derived from the S-parameter characteristics of the structure obtained via full wave FDTD simulation, by applying a Genetic Algorithm (GA) [13]. Considering only the frequency range from 500 MHz to 5 GHz, the GA views the S-parameter information between ports #1 and #4 as the specification and searches for the values of the equivalent circuit that matches the specifications in the best way. In this optimization process, the GA marches through the population of potential equivalent circuits toward the best solution, by repetitive simulations of the circuit.

The comparison among the measured data, the full wave FDTD results and the equivalent lumped circuit simulation are shown in Fig. 10. As it is evident, a good agreement is obtained over most of the frequency range, though there are discrepancies at low and high frequencies.

![Fig. 10 - Comparison among measured data, FDTD and SPICE simulations.](image)

**V. Conclusions**

The work herein presents modeling of differential signaling in printed circuit board microstrips crossing slots in ground planes. A full wave analysis is carried out by applying the Finite Difference Time Domain Method, which allows the accurate simulation of the dispersive behaviour of the real substrate. Accurate measurements were made to corroborate the FDTD results. In all cases, generally good agreement was obtained, which confirms the suitability of FDTD for modeling printed circuit structures with dispersive dielectrics and discontinuities. Furthermore a SPICE-like equivalent circuit for this kind of structure is extracted and its parameters values are evaluated in part numerically, and in part resorting to published analytical formulations. The latter allows a consistent speed-up in the overall computational procedure giving the possibility to analyze multiple crossovers.

**References**


