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Engineering Design of Rock Slope Reinforcement Based on Non-Linear Joint Strength Model

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SYNOPSIS: Optimum dimensioning of bolts or anchors for the reinforcement of slopes in jointed rock masses requires compatible strength-deformation data, for both the rock joints and the reinforcing elements. Most types of rock joints behave in non-linear fashion and, thus, realistic modeling can have serious implications in the design, both from the economical and the technical standpoints. This paper will present, briefly, the principles of a constitutive model of joint shear behaviour and a method for optimum bolt or anchor design. The implications of non-linear joint behaviour will be demonstrated with numerical examples. Finally, a case study of slope stabilization, in which the method was adopted, will be reported.

INTRODUCTION

Statics show that the minimum tensioning force for the support of a rock mass resting on an inclined plane, requires, if moments are neglected, an optimum angle of installation (β) w.r.t. the failure plane, given by:

$$\tan \beta = \frac{1}{F} \times \tan \varphi$$  \hspace{1cm} (1)

where $\varphi$ is the friction angle along the contact interface and $F$ is the safety factor. Depending upon the desired stiffness of the reinforcing system to be installed, the choice of the design value of $\varphi$, must be made in accord with the amount of shear deformation, which the reinforcing element would be capable of tolerating.

In some rock slope engineering problems, it may prove advantageous to allow a certain amount of deformation, thus dissipating a portion of the excavation induced shear stress. In addition, shearing may also initiate an efficient self-draining process within the rock mass, effected by the opening of dilating joints. By implication, the designer should be able to quantify the changes in shear behaviour at corresponding stages of joint deformation.

The non-linear constitutive model for the shear behaviour of joints reported by Bandis et al. (1981), Barton & Bandis (1982), Barton et al. (1985), and Barton and Bandis (1987), offers the basis for a convenient method for bolt design. The method has been described by Barton and Bakhtar (1983) and Bandis et al. (1985) and is based on a concept of "mobilized" friction, by which the shear strength available at various stages of shear deformation, can be quantified.

NON-LINEAR MODEL OF JOINT SHEAR BEHAVIOUR

The shear behaviour of a singly jointed rock block is largely determined by the effective normal stress ($\sigma_n$) and the length of the joint ($L$). The variations observed in the shear properties of different joint types are attributed to differences in the geometric and strength properties of the joint surfaces, i.e. roughness, aperture, wall strength and basic friction.

Four key-indices are required, to fully model the shear behaviour of an unfilled joint:

1. the Joint Roughness Coefficient (JRCo), a dimensionless number ranging from 0 for planar-smooth to 20 for undulating - rough joints;
2. the Joint Compressive Strength (JCSo), which is the uniaxial compressive strength of the rock material at the joint wall;
3. the Angle of Residual Friction ($\varphi_r$) or Basic Friction ($\varphi_b$) - if the joint is completely fresh, and
4. the Mechanical Aperture (Eo).

Simple index tests have been devised to measure JRCo (tilt, pull or push tests), JCSo (Schmidt hammer tests), $\varphi_r$ or $\varphi_b$ (combination of tilt and S.H. testing) and Eo (flow tests in the field or the lab). The subscript (o) is used to denote the joint length (Lo), which was index tested. Details of the measuring techniques appear elsewhere (Barton and Choubey, 1977, Barton et al., 1985). Extrapolation of the measured indices to field scale (length Ln), require appropriate scaling conversions:

$$JRC_n \approx JRC_o \times \frac{L_n}{L_o}$$ \hspace{1cm} (2)

$$JCS_n \approx JCS_o \times \frac{L_n}{L_o}$$ \hspace{1cm} (3)

The peak shear strength (Tpeak) of a joint can be predicted from Barton's (1973) criterion:

$$T(peak) = \frac{\alpha_n \times \tan [JRC_n \times \log_{10}(\frac{JRC_n}{\alpha_n}) + \varphi_r]}{F}$$ \hspace{1cm} (4)
The latter is attained at a peak shear displacement, \( \Delta h \) (peak):  
\[
\Delta h(\text{peak}) = \frac{\ln 500}{JRCn} \left[ \frac{\text{JRC}_n}{\ln \left( \frac{JRC_{\text{peak}}}{\alpha} \right)} \right] - 0.33 \quad \ldots (5)
\]

The shear strength mobilized at any given displacement, \( \Delta h \) can be expressed by:

\[
T(\text{mob}) = \alpha_r \tan \left[ \sqrt{\ln \left( \frac{\text{JRC}(\text{mob})}{\alpha_r} \right)} \right] + \phi_r \quad \ldots (6)
\]

or \( \Phi(\text{mob}) = \text{JRC}(\text{mob}) \times \log_{10} \left( \frac{\text{JRC}_{\text{peak}}}{\alpha_r} \right) + \phi_r \quad \ldots (7) \]

The model illustrated in Fig. 1 simulates the following fundamental features of joint shear behaviour:

(i) mobilization of the basic frictional resistance, upon initiation of shear
(ii) the amount of initial shear for roughness mobilization is scale dependent (\( \approx 0.3 \Delta h_{\text{peak}} \))
(iii) dilation begins when roughness is mobilized
(iv) peak shear strength is reached at \( \frac{\text{JRC(\text{mob})}}{\text{JRC(peak)}} = 1.0 \) and \( \frac{\Delta h}{\Delta h_{\text{peak}}} = 1.0 \).
(v) the peak shear displacement, \( \Delta h_{\text{peak}} \), corresponds to 1% of the joint length.
(vi) the residual state (\( \text{JRC(mob)} = 0 \)) is reached after large shear displacements (\( \Delta h/\Delta h_{\text{peak}} = 100 \)).

Dilation can be modelled utilizing an expression quite similar to (4):

\[
d\theta(\text{mob}) = \frac{1}{2} \text{JRC(mob)} \times \log_{10} \left( \frac{\text{JCS}}{\alpha_r} \right) \quad \ldots (8)
\]

The increase of the mechanical aperture \( (E_0) \) associated with dilation can be calculated from:

\[
5E_0 = 5(\Delta h) \times \tan d_{\theta} \text{ (mod)} \quad \ldots (9)
\]

Numerical examples of model application are given in Fig. 2.

BOLT DESIGN BASED ON NON-LINEAR JOINT MODEL

Barton and Bakhtar (1983) suggested a graphical solution for optimum bolt design, utilizing normal stress and scale dependent values of \( \Phi \) (mobilized). The technique essentially combines the appropriate "mobilized" strength envelope, with a conventional force diagram.

From the T-\( \Delta h \) plots in Fig. 2a, values of JRC(mob) can be back-calculated at any shear displacement, \( \Delta h, \text{e.g.} \approx 4.0 \text{mm (at peak), 20.0mm and 80mm.} \) Those values of JRC(mob) are then used to derive the corresponding mobilized strength envelopes. Combination of the latter with the force polygon for a simple slope problem is demonstrated in Fig. 3. It is seen, that the frictional resultants \( R_x, R_y, R_z, \) which are perpendicular to the minimum bolt forces \( T_1, T_2, T_3, \) intersect the envelopes at different levels of normal stress. The design \( \Phi(\text{mob}) \) value corresponds to an effective normal stress, which incorporates the normal component of the force \( T \), in addition to the forces \( N, V \) and \( U \).
The engineering implications in bolt/anchor design, if the non-linearity in joint shear behaviour is neglected, can readily be demonstrated. In the hypothetical problem illustrated in Fig. 4, it is assumed that the rock mass structure favours translational sliding along a potential failure surface, consisting of segments with variable inclination. The slope system can be analyzed assuming transfer of loads by superposition from the active to the passive blocks. Joint strength indices were assigned to the failure surface, consisting of segments with variable inclination of the bottom segment of the failure surface. Joint strength indices were assigned to the failure surface.

A computer programme containing a subroutine for the JRC-JCS model was used for the analyses. The computing procedure was to calculate the normal force component acting on each segment of the failure surface and, then, determine the normal stress dependent values of friction (φ) through an iterative procedure.

The calculations gave for dry slope :

Safety Factor (SF) = 1.06
Predicted φ (Bottom) = 45°
φ (Top) = 49°
φ (Middle) = 38.5°

In a simplified approach, a constant value of φ = 40°, equal to the mean inclination of the potential failure surface, might be adopted. For the water pressures assumed in Fig. 4, an external anchoring force, T, is required for stability. It can be shown, that the inclination, w.r.t. horizontal of a minimum force (T) for safety factor, (SF) is given by:

\[
\tan b = \frac{(\tan \theta_{\text{bot}} \div \text{SF}) - \tan \theta_s}{(\tan \theta_{\text{bot}} \div \text{SF}) \tan \theta_s + 1} \quad \ldots \quad (10)
\]

where \( \theta_s \) = inclination of the bottom segment of the failure surface.

For the case of design based on peak strength and safety factor of 1.3:

Fig. 4. Comparison of calculated anchor forces for a hypothetical slope problem, utilizing the non-linear JRC/JCS model and Coulomb's linear concept of joint behaviour.
For the case of design based on ultimate strength, the following were assumed:

- $h = 100$ mm (slope deformation)
- $L_n = 1.0$ metre (in-situ block length)
- JRC(mob) BOT $= 5.5$
- JRC(mob) TOP $= 5.5$
- JRC(mob) MIDD $= 3.5$

non-linear model in $JRC(mob)$

The calculated anchor force for $SF=1.3$ and water conditions as originally:

- $T = 37.0$ MN/m
- $b = -12^\circ$

If we assumed that the normal drawdown GW curve was lowered by 20% due to self-draining, then:

- $T = 27.5$ MN/m
- $b = -21^\circ$

Finally, if the deformed slope was assigned a conservative residual friction angle $\phi=30^\circ$, then:

- $T = 44.0$ MN/m
- $b = -21^\circ$

The above examples indicate a conservative over-estimation of $T$ between 25 and 40%, depending on the mode of joint behaviour and the conditions assumed.

CASE STUDY OF ROCK SLOPE REINFORCEMENT

Background information.

In November 1985 and during the excavation of a new slope face in a drydock at Stavanger, Norway, problems of instability were encountered in the form of translational sliding failures (Figure 5). The slope (0.0-12.0 in height and ~50.0 in length) had a major functional role, bearing the foundation loads of a back-filled sheet-pile wall, which acted as sea-water barrier. Figure 6 presents a vertical section along the final slope line, as appeared in the "good for construction" plans.

Fig. 5. General view of preexisting slope and new excavation (R. H. half).

INVESTIGATION OF THE FIELD CONDITIONS

Rock material

The rock type was a slightly to moderately weathered phyllite, with well-developed foliation. The uniaxial compression strength ranged between 50-60MPa (\(\uparrow\)) and 20-30MPa (\(\downarrow\)).

Rock mass structure

The direction/amount of dip of the foliation was $\vec{N}B^\circ - 95^\circ / 40^\circ - 45^\circ$. Foliation joints of similar orientation appeared frequently, spaced 0.5-1.0m apart and with lengths of up to several metres. No other systematic jointing was observed.
Occasional stress relief joints were of no practical importance, due to the relatively gentle dip. Scarcely distributed subvertical joints were also found.

**Stability conditions**

It was evident that the persistent foliation joints could provide potential failure planes. Several of them "daylighted" at the slope face. Considering the climatic condition in the area and a maximum water head of 16.0 m, the slope could be expected to sustain significant hydraulic loading. Severe seepage was observed at several locations at the slope face. The part of the slope in need of reinforcement comprised the new excavation (between section A-A and D-D) and a potentially unstable block at the middle part (section E-E) as indicated in Fig. 5.

**EVALUATION OF JOINT SHEAR STRENGTH**

The exposed failure plane at the rightmost end of the slope (Fig. 5), was quite accessible for direct measurements of the surface roughness. Detailed line-profiling along section 2-2 in Fig. 8(a) gave the trace presented in Fig. 9.

![Fig. 8](image)

**Fig. 8 (a)** Persistent joint surface along which translational failure took place.

**Fig. 8 (b)** Conditions assumed for back-analysis of failure.

The value of JRCn characterizing a particular surface, represents an approximate measure of the roughness amplitude (a) divided by the length (Ln) of the profile. From analyses of a large volume of data, Barton (1982) arrived at the nomogram of Fig. 10.

The length of profile in Fig. 9 is Ln=11.0 m and a=150 mm. Then, a value of JRCn 6-7 is predicted from the nomogram. The independently derived JRCn value was used to back-calculate the drained friction angle, which was mobilized along the failure plane, as shown in Fig. 8:

$$\psi' = JRCn \times \log_{10} \left( \frac{JCS}{\gamma} \right) + \Phi r \quad (11)$$

where:
- JRCn = 6 Mpa
- JCSn = 30 MPa
- \(\Phi r\) = 22° - 24°
- W = 650 KN/m
- \(\sigma_{on}'\) = 0.032 MPa
- Ln = 12.0 m
- Zw = 6.0 m

**Fig. 10. Joint roughness characterization according to the amplitude of asperities and length of profile. (Barton, 1982).**
The calculated value of $\phi=40^\circ-42^\circ$ clearly complied with the conceptual limit equilibrium condition of $\phi_{\text{hy}}$ (inclination of failure surface) for planar sliding.

Several more foliation joint planes $\approx 1.0$ metre in length were profiled and characterized according to the nomogram in Fig.6. The conclusion from the field observations was that foliation joints invariably contained undulating features of various sizes. Wavelengths ranged from a few cm's to $>5.0$ m and the amplitudes from some mm's to $>10$ cm. In general, high JRC values (15-18) were assigned to joint lengths up to 1.0 m. The latter represented the JRCo and Lo value for scaling extrapolations to JRCn according to eqn(2).

### STATIC ANALYSES-PRESTRESS LOAD CALCULATIONS

A typical slope section (B-B according to Fig.6) is contained in Fig.11, with all information concerning the assumed distribution and calculation of forces.

Table 1 summarises the geometrical data, the joint plane indices, the operating $\phi$ values and the calculated prestress loads for limit equilibrium at the corresponding sections A-A, etc. (see Fig.6 for positions of analyzed sections). The angle of anchor installation w.r.t. horizontal was $25^\circ$ (not optimum, but imposed from practical constraints and the fact that most of the holes had already been drilled). The tensioning force was in each case determined graphically, by locating the common intersection between the vectors of force $T$ and frictional resultant $R$, and the appropriate strength envelope (also refer to previous Fig.3).

For purposes of comparison, the $T$ values calculated for constant $\phi=43^\circ$ have been included in Table 1. It is worth noting, that, despite the same angle of anchor installation,

<table>
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<tr>
<th>Input</th>
<th>A-A</th>
<th>B-B</th>
<th>C-C</th>
<th>D-D</th>
<th>E-E</th>
</tr>
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<td>12.5</td>
<td>10.5</td>
<td>8.5</td>
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<td>12.0</td>
</tr>
<tr>
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<td>6.5</td>
<td>8.5</td>
<td>11.5</td>
<td>-</td>
</tr>
<tr>
<td>$b(m)$</td>
<td>7.6</td>
<td>5.5</td>
<td>4.5</td>
<td>3.5</td>
<td>-</td>
</tr>
<tr>
<td>$z(m)$</td>
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<td>5.6</td>
<td>7.6</td>
<td>10.6</td>
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</tr>
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<td>45°</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\phi_f$</td>
<td>-</td>
<td>85°</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

#### Table 1. Input parameters and calculated anchor prestress loads at limit equilibrium.

**Fig. 11. Diagrammatic illustration of analyzed cross-section and of the assumed forces distribution.**

significant differences in the prestress loads were found, particularly for the shortest slope sections.

As expected, the optimum anchor force ($T$ vector to frictional resultant) gave lower $T$ values for a range of $\phi$ angles from $-7^\circ$ to $+3^\circ$ (the minus symbol indicates angle above the horizontal).

### DESIGN OF THE BOLT REINFORCEMENT

For the reasons already referred to, a rock bolting system was designed to secure the lower
art of the slope. at the final excavation line. Limiting equilibrium analyses were conducted for the slope sections A-A to D-D, assuming a tension crack at the zone of potential relaxation and perpendicular to the failure plane, as illustrated in Fig. 12(a).

Depending upon the geometry of the slope section, the calculated minimum bolting forces at equilibrium ranged between 510 KN/m (A-A) and 64 KN/m (D-D). The corresponding design values were between 45° and 54° and the optimum installation angles β were 0°-90° (see Fig. 12b).

Gally-grouted, untensioned bolts of 250 KN capacity (φ24mm) were recommended for installation in a pattern with similar lateral and vertical spacing (2x2 m). A total of > 50 bolt units were installed (Fig. 13). Drainage holes were also drilled during bolt installation, horizontally spaced at 5-6 metres and drilled at 0 different levels.

Finally, the photograph in Fig. 14 shows the anchor reinforced unstable block in the middle of the slope (section E-E in Fig. 5). Since completion of the reinforcing measures, the operation has presented with no further problems.

![Fig. 13. Rock slope face at the final excavation line reinforced with bolts.](image)
![Fig. 14. Anchor reinforced unstable block in section E-E.](image)

**CONCLUDING REMARKS**

1. The choice of linear or non-linear model for the shear behaviour of joints, has significant implications upon the design of reinforcing systems for unstable rock slopes.

2. A constitutive non-linear model based on simple indices (JRC, JCS, fβ), can be used to calculate the optimum angle of installation for a minimum prestress load.
3. The model predicted value for $\omega$ depends upon the normal stress, the scale of joints and the amount of shear deformation. The effects of all applied forces can be allowed for by using either a graphical solution or an iterative numerical technique for relatively complex failure surface geometry.

4. The reported case study of slope reinforcement revealed good agreement between model predicted and back-calculated friction values of a failed slope section. Comparisons between conventional anchor design and the suggested method, indicate potential over- or under-loading, if non-linearity is ignored.

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REFERENCES


