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A Detailed Examination of Interactive Buckling in Plain Channel

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A DETAILED EXAMINATION OF INTERACTIVE BUCKLING IN PLAIN CHANNEL SECTIONS

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Abstract

Plain channel section columns behave substantially differently under fixed end conditions than they do under pin end conditions, and the "wandering neutral axis" can have a large effect on the pin end case. An existing analytical approach to the pin-end case is outlined, and the implications suggested by this approach for the fixed-end case are briefly described. To investigate the veracity of these implications a detailed finite element investigation was made into fixed-end plain channel columns. Points of interest are the considerations of the wandering neutral axis and the assumption of an effective length, LE equal to one half of the column length. To assist in the clarity of the findings much of the finite element analysis is concentrated on completely elastic columns with minimal imperfections.

Introduction

The compressional behaviour of thin-walled columns which are subject to local buckling has been considered in design specifications for many years now. In the case of mono-symmetric sections, such as channels, local buckling involves displacement of the effective neutral axis from its original position. Since design specifications tend to consider columns as pin-ended over an appropriate effective length then at least some cold formed steel specifications deem that the neutral axis movement should be taken into account. This may result in highly

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conservative design for columns which have restrained ends, as there is some dubiety regarding the effects of neutral axis movement in such cases. Under true pin-end laboratory conditions the effective shift of the neutral axis is easy to discern from the subsequent growth in out-of-plane deflections and reduction in load capacity, but with more realistic practical connections the situation can be substantially different. However, there is another aspect of the effects of local buckling in channel type members which is not fully taken into account in many specifications, namely the effects of directionality of the bending resistance of locally buckled columns. Sections which are largely composed of unstiffened elements, such as angles and plain channels, are substantially more highly resistant to flexural buckling which causes compression of the supported edges than to buckling which causes compression of the free edges. Furthermore, after buckling locally the flexural stiffness in the first case is substantially greater than in the second case, which in exacerbates the directionality effect.

Design specifications tend to examine the local effects and the overall buckling effects separately, and then combine these using the Perry-Robertson approach or some such method. In such an approach it is considered that local buckling reduces the axial stiffness of the cross section and increases the stresses for a given load thus inducing early yield and failure. For many of the cross sections in use, given the limitations on cross sectional slenderness of unstiffened elements this leads to reasonable results. However the fact that the effects of local buckling on the flexural stiffness are not taken directly into account could, in the future given the continually increasing yield strengths of steels, lead to problems. It is the aim of this paper to examine these effects in the case of elastic members in which the influence of yield does not blur the picture.

Effects of local buckling on the cross-sectional properties of plain channels.

The behaviour of plain channels in compression has been examined experimentally and theoretically for well over sixty years at the present time. Similarly the interaction of local and overall buckling of a number of different cross sections , initially doubly symmetric sections, has been researched for over fifty years. Almost all of the earlier research considered pin-ended columns. A method of approach initiated by the second author for the interaction behaviour of pin-ended plain channel columns, and extended by a number of researchers to deal with other types of sections, is briefly described now to demonstrate the effects of local buckling on cross-sectional behaviour of a plain channel.

Consider the short cross-section shown in Figure 1, which has a length equal to that required to produce the minimum local buckling load. The load, P, is applied at some distance 'd' from the web. On applying local buckling analysis the critical load, P_{CR}, can be determined. On taking the analysis further the effects of local buckling on the axial and flexural stiffness can be evaluated in terms of the post-buckling tangent axial stiffness reduction E*/E, the tangent flexural stiffness

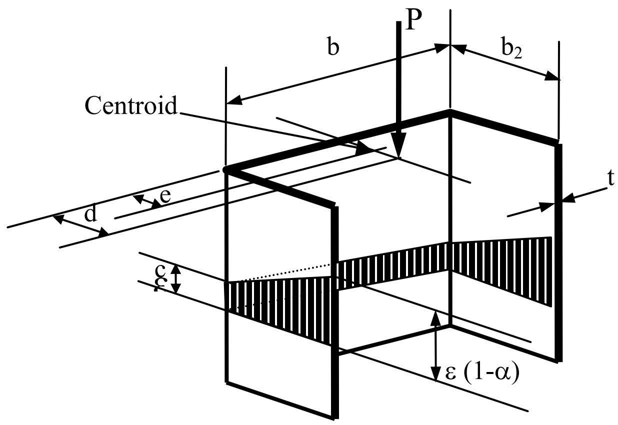


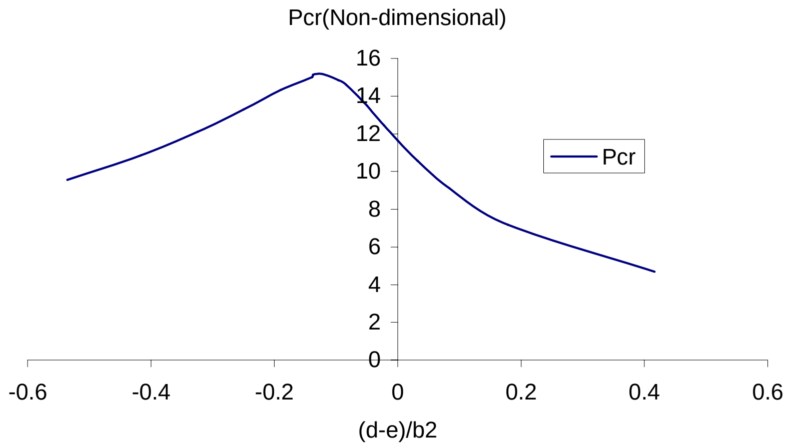
Figure 1. Short length of channel under axial load

reduction I*/I, and the effective neutral axis position e*, measured from the web. All four of these quantities vary with the position of P, and indeed the value of P. In the particular case of a plain channel with web width twice the flange width the flange width the variation in buckling load with load position 'x' is shown in Figure 2 and the post buckling tangent effects are shown in Figure 3.

In the post-buckling range the moment M about the tangent effective neutral axis position can be specified in terms of the load, load position and the effective and initial neutral axis positions by the expression

$$M_{e*} = P(d - e*) + P_{KR}(e* - e) \tag{1}$$

where P_{KR} is the critical load to cause buckling if applied through the centroid. Note that if the load is applied through the centroid, i.e. d=e, then when $P=P_{KR}$ the moment becomes zero.



From Figure 2, it may be observed that the local buckling load (plotted non-dimensionally, with dimensional value given by the equation in the figure)

Figure 2. Variation in buckling load with load distance from centroid

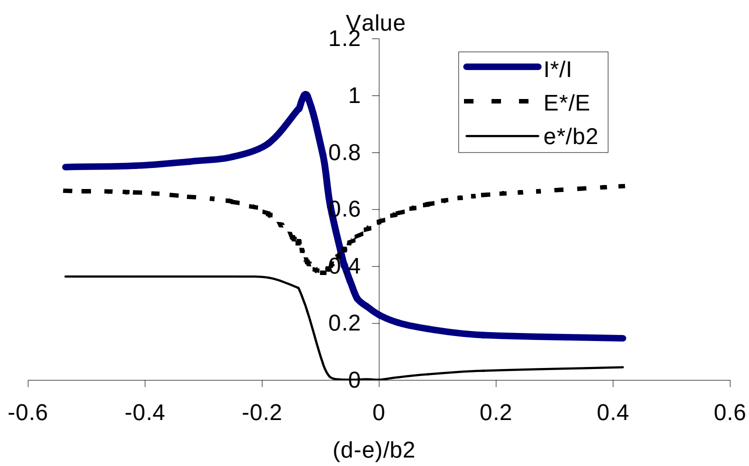


Figure 3. Variation in tangent post-buckling properties with eccentricity

increases if the load is applied slightly eccentrically towards the web until it reaches a maximum value before decreasing with increasing eccentricity. If the load is applied eccentrically towards the flange free edges, however, the local buckling load decreases continuously. From Figure 3 it may be observed that tangent flexural stiffness is very low for flexural buckling to cause compression on the flange free edges, and the tangent effective neutral position moves very close to the web for positive eccentricity of the loading line. These conditions still remain for slightly negative loading until a rapid increase in tangent flexural rigidity with increasing negative eccentricity occurs culminating in the post-buckling flexural stiffness becoming equal to the pre-buckling value then reducing as negative eccentricity increases. The load eccentricity at which the maximum tangent flexural stiffness is reached is coincident with that at which the maximum buckling load arises, and this behaviour is accompanied by a similarly abrupt change in the effective neutral axis position.

The tangent flexural rigidity for negative eccentricity reaches a steady value of approximately 75% of the pre-buckling flexural rigidity while for positive eccentricity the corresponding value is about 14%. This, combined with the fact that for positive eccentricity the neutral axis movement is close to 25% of the flange width while for negative eccentricity the neutral axis movement is only about 12% of the flange width suggests that for this cross section buckling in the positive direction has a much greater effect than buckling in the negative direction. If the eccentricity is taken to $\pm$ infinity then pure moment loading results and the value of the buckling moment is 2.681 Et^3 for buckling to cause compression of the free edges and 8.426 Et^3, more than three times greater, to cause compression of the web. An interesting aside here is that the buckling moment is independent of the flange width but only on the thickness. This was also observed for lipped channel beams by Rhodes (1967) for lipped channels.

It should be mentioned here that these postbuckling properties apply immediately after buckling, and gradually change. Far into the post-buckling range I*/I changes abruptly from its minimum negative curvature value of 0.75 to its minimum positive curvature value (0.14) at a load eccentricity of -0.12 b2.

Now if it is assumed that the properties derived from examination of a single local buckle can be considered as pertaining to an infinitesimally long element of a column then using the relationship $M_{e*} = EI*\dfrac{d^2y}{dz^2}$, where z is the longitudinal direction, together with equation (1), a differential equation can be set up to determine the column behaviour for a pin ended channel member. An example of the results obtained from the solution of this equation is shown in Figure 4 for a channel of web/flange ratio 2:1 subject to a load applied eccentrically towards the web. The load eccentricity is 0.0595 times the flange width towards the web, which unduces a compressive web stress twice the stress at the flange free edges in the pre-buckling range. Load-out of plane deflection paths are shown for columns with various ratios of local to overall buckling loads. Low values of P_{CR}/P_E indicate short columns while high values indicate long columns. The full lines indicate the actual paths predicted from analysis while the dotted lines indicate alternative paths in the opposite direction which would occur if buckling in the predicted paths did not take place.

As may be observed, for short columns local buckling changes the direction of the deflections so that these buckle in the positive direction even though the load eccentricity is in the negative direction. This change in direction was also obtained in experiments.

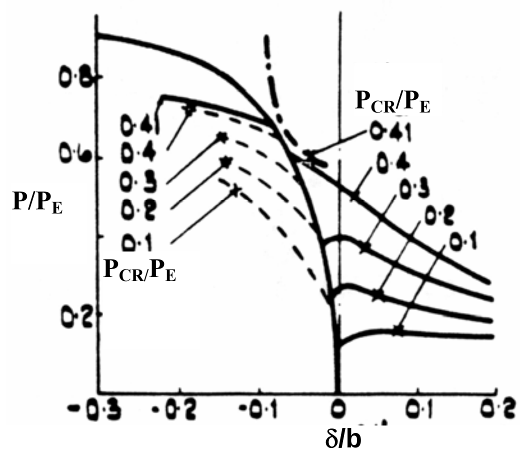


Figure 4. Load-displacement paths for eccentrically loaded pinned columns

Plain channels with fixed ends

Background

It has been argued that in the case of plain channels with fixed ends the fixed ends apply the load in such a way that the loading line varies as the line of internal load resistance varies, so that movement of the neutral axis is counteracted by the effects of the fixed ends. This argument has merit, but is open to question because of the directionality of the flexural rigidity together with the situation that the effective neutral axis, and the line of internal load resistance, moves in opposite directions for different signs of column curvature.

Lim (1987) carried out a series of tests on relatively short plain channel columns with fixed ends with the prime purpose being to examine the effects of neutral axis movement. He used the concept that an effective length of one half of the real length was applicable, together with the design rules of BS 5950, then in draft condition, and came to the conclusion that British Standard rules were adequate without taking the additional moments caused by neutral axis movement into account. Young and Rasmussen (1995) carried out an extensive series of tests on lipped and plain channel columns, also using the concept that the effective length could be taken as half the actual length, and came to similar conclusions, i.e. that the additional moments caused by neutral axis movement could be discounted.

These conclusions are perfectly valid, and is not at all directly evident what the internal moment system for a fixed ended column is, as this is dependant on the end connection moments. The hypothesis which is questioned in this paper is the validity of the assumption that the effective length can be taken as one half of the actual length. It must be confessed that Lim used this assumption on the advice of the second author of this paper, who has since had second thoughts. The sections examined by Young and Rasmussen are considered here in examination of this hypothesis, as the details of the tests and member properties have been detailed admirably.

Initially it was considered that the set-up of a differential equation system derived on the basis of equation (1) could be used to assess the validity of the L_E equals L/2 assumption, and such a system was set up, which indeed indicated that this assumption was invalid. It was then realised that such a proof was open to the criticism that since the approach used assumed a-priori that directional behaviour was to be expected, then this type of behaviour could not fail to be predicted. To avoid any such bias in the solution method the ANSYS finite element package was used to examine the behaviour of fixed end channels.

Finite element Analysis - Elasto-plastic conditions

As mentioned previously, columns examined experimentally by Young and Rasmussen were analysed, and attention was focussed on plain channels having nominally 96mm webs, 48mm flanges, 1.5mm thickness and 450 N/mm^2 yield strength.

Initially, a direct examination of the variation of column strength with overall column length was carried out to ascertain the agreement between ANSYS and the experimental results. In nonlinear ANSYS analysis it is necessary to prescribe some degree of imperfection to provide a means whereby buckling may be initiated. To do this, for a series of different lengths of column eigenvalue ANSYS eigenvalue analyses were carried out, the first local buckling mode was selected and small imperfections in this mode were applied for the subsequent nonlinear analysis. Only local buckling imperfections were used, and the ANSYS model had no imperfections in flexural or torsional-flexural modes. The magnitude of the maximum local imperfections was set at 1/10 of the material thickness, i.e. 0.1mm. The ultimate loads obtained are compared with the experimental results of Young and Rasmussen in Figure 5. The agreement is quite good, with the finite element results slightly overestimating the experimental results. All of the finite element results showed failure due to combined local and flexural buckling, while the experimental findings were that combined flexural and flexural torsional behaviour resulted for the two longest columns. In the case of non-locally-buckled columns torsional-flexural buckling is the dominant elastic buckling mode over much of the range, and it is evident that locally buckling promotes subsequent flexural buckling to a greater extent than it promotes torsional-flexural buckling.

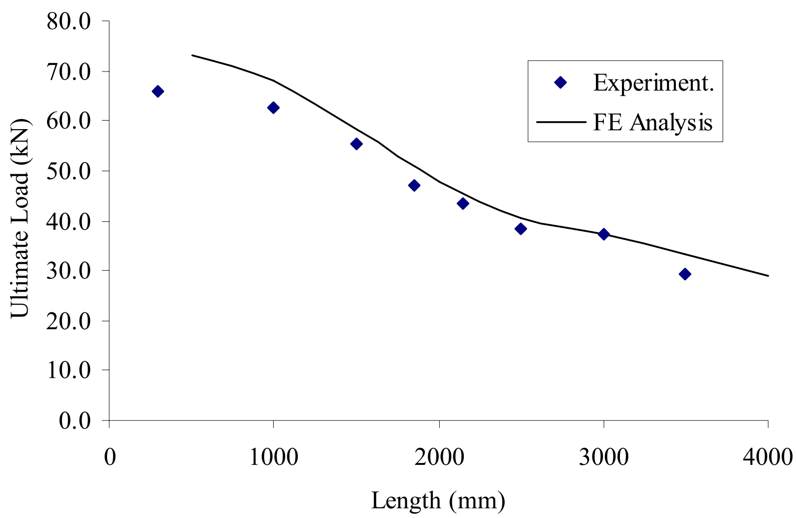


Figure 5. Comparison of finite element and experimental failure loads.

In assessing the comparison it should be noted that in general the experimental material thickness was slightly less than the nominal value, the experimental imperfections were greater than the nominal imperfections and the experimental yield strength was greater than the nominal value. The results are sufficiently close to give confidence in the applicability of the ANSYS package to model the actual behaviour of plain channels with fixed ends.

Detailed finite element results for fully elastic columns.

The main objective of this examination is to examine the hypothesis that L_E=L/2. To obtain the clearest picture of this it was decided that any effects which may interfere with this picture should be minimised. Since the effects of imperfections can cause variations in the buckling mode and since yielding can cause localised effects which may be misinterpreted these effects were minimised. Local imperfections were set to the smallest values which could be used without causing difficulty in ANSYS analysis, all other imperfections were set to zero and the yield strength was set to an extremely large value, 20000 N/mm^2, in the subsequent analyses.

The variation in ultimate loads with variation in overall length for these members is shown in Figure 6, together with the results for the 450 N/mm^2 case and the classical buckling load for a fixed ended channel

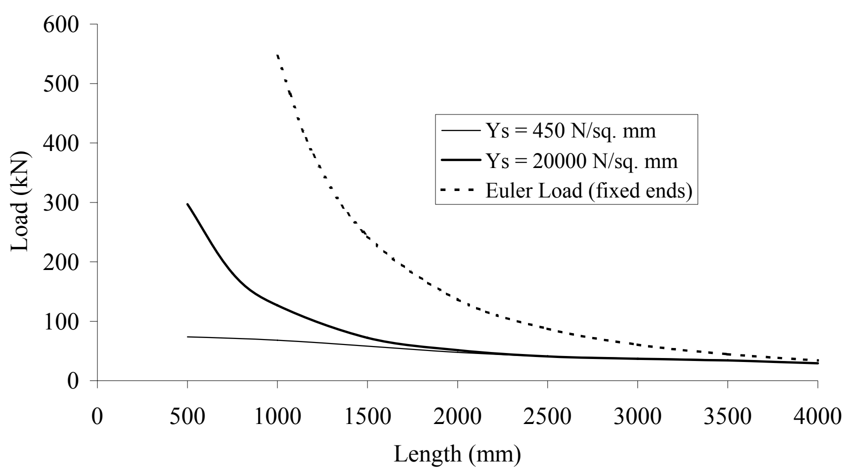


Figure 6. Ultimate load versus length for plain channels with fixed ends

This figure indicates that for lengths greater than 2000 mm The completely elastic columns behave identically to the elasto-plastic columns. In the case of completely perfect columns the Euler load and local buckling load are the same for a column length of 3780 mm, and for shorter columns local buckling occurs before overall flexural buckling, so that the figure is almost completely concerned with columns for which local buckling precedes Euler buckling.

It may be observed from the graph that the ratio of Euler load to ultimate load is very large for very short columns and decreases to a very small value at a length of 4000 mm. For columns with no imperfections the Euler load and ultimate load would be the same at this length. However since the region in which Euler and local buckling have much the same value is very sensitive to imperfections then even the very small local imperfections have the effect of reducing the ultimate load slightly below the Euler load.

To obtain further information on the elastic buckling behaviour for long and short columns detailed examinations of two different column lengths, 1000 mm and 3500mm, are now presented.

Figures 7 and 8 show the ANSYS generated load-end displacement curves for 1000 mm long and 3500 mm long columns.

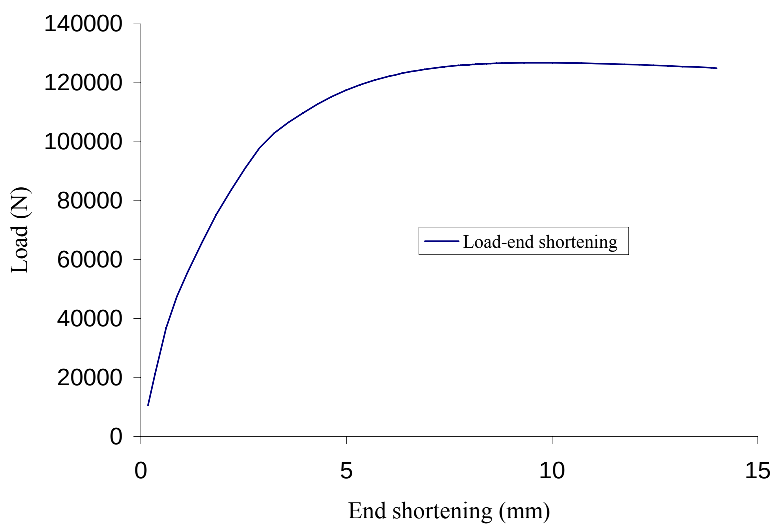


Figure 7. Load-end shortening curves for 1000 mm long channel

The 1000 mm long channel buckles locally at a load substantially below the ultimate load, and the load-shortening curve loses stiffness gradually until the ultimate load is attained at a value of 126.85 kN before a further slight decrease to a minimum value slightly less than this. The maximum stress attained at any point throughout the loading cycle was 3718 N/mm^2, significantly below the yield stress prescribed so that failure was completely due to elastic buckling.

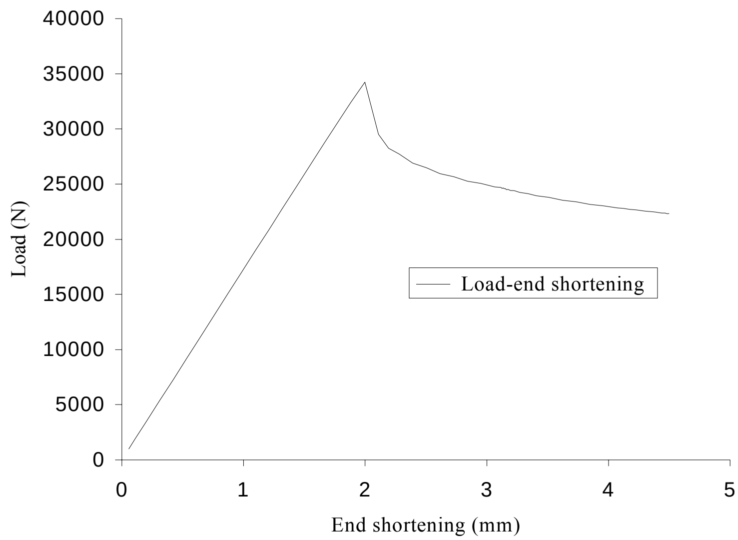
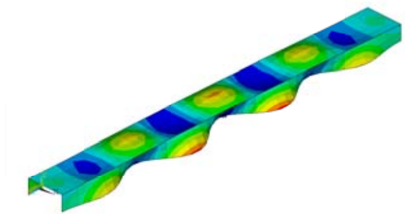


Figure 8. Load-end shortening curve for 3500 mm long channel

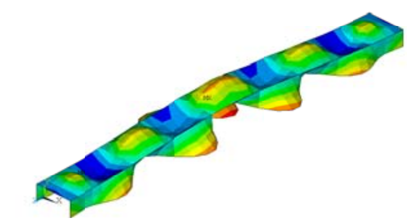
The behaviour of the 3500 mm long channel is completely different. At the onset of local buckling, at a load of 34.25 kN (slightly reduced from the theoretical value of 36.1 kN due to the imperfections), failure ensued immediately and the load rapidly reduced to a value of 22.3 kN at the final displacement step. The maximum stress at any point throughout the load cycle was 1025 N/mm^2, significantly below yield.

The ratio of the failure load to the Euler load for the 1000 mm long column was 0.232, while the ratio of failure load to Euler load for the 3500 mm long column was .768, i.e. substantially different. This is largely because local buckling initiated failure immediately for the longer column. Of greater significance is the ratio of the minimum value to which the load reduced after failure to the Euler load, which for a simply supported column would be equivalent to I*/I. This ratio was 0.228 for the 1000 mm column and 0.5 for the 3500 mm column. On examination of a 500 mm long column this ratio is reduced to 0.136, which is very close to I*/I, as mentioned earlier, for a heavily buckled pin-ended column.

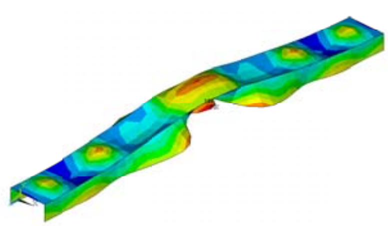
Since Euler buckling depends upon the tangent flexural stiffness and there should not be such a great deal of difference in the tangent flexural stiffnesses of the columns examined, a question arises as to how these substantial differences arise. Certainly if the $L_E/L=0.5$ hypothesis is correct then they could not. Reasons for the differences can be postulated from examination of deflected form diagrams obtained from the ANSYS analysis. Figure 9 shows the growth of deflections and stresses for the 1000 mm long column. At a load of 55 kN, more than 50%



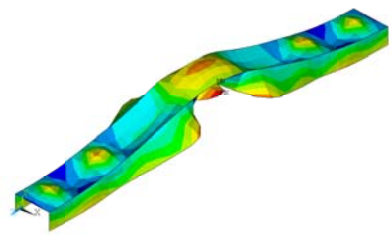
Load:- 55.02 kN



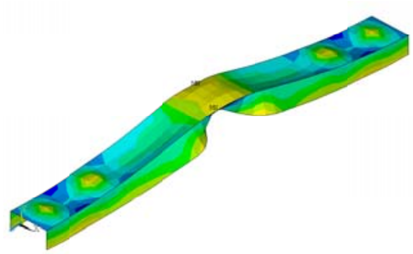
Load:-92.77 kN



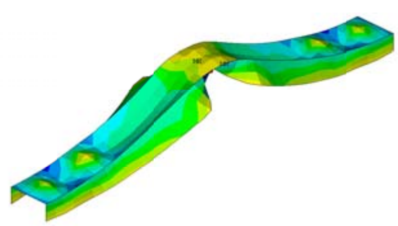
Load:- 111.3 kN



Load:- 126 kN



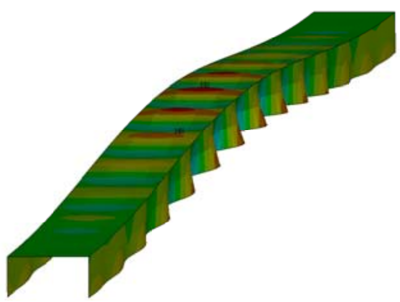
Load:- 126.8 kN Ultimate



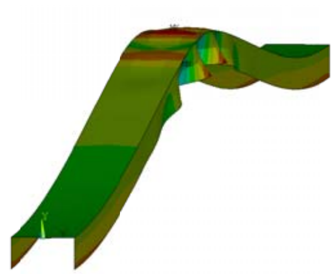
Load 124.9kN-Post-ultimate

Figure 9. Deflection and stress patterns at various loads - 1000mm column

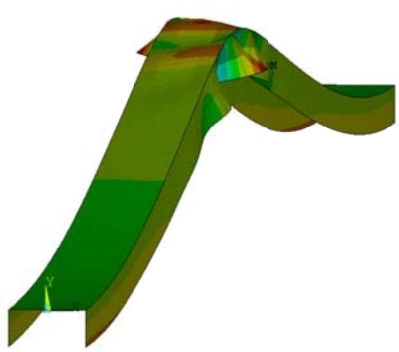
greater than the local buckling load, the local buckles are fairly regular along the column. As the load increases this changes, at 93 kN the local buckles near the centre are greater than those near the ends and at a load of 111 kN a process of localisation has begun, in which all curvature causing compression of the flange free edges (say positive curvature) takes place over a single buckle at the column centre, and the curvature is in the opposite direction over the rest of the column. As the load increases the central buckle becomes more localised while at areas away from the centre the local buckles due to curvature in the opposite direction are evident. The displacements are shown magnified, but nevertheless it appears very likely that this central localised buckle area may be undergoing further degradation of its flexural stiffness due to the gross changes in cross section. After the ultimate load is attained the localisation continues and at the last load step shown fully 80-90% is undergoing negative curvature, with local buckling corresponding to this type of curvature.



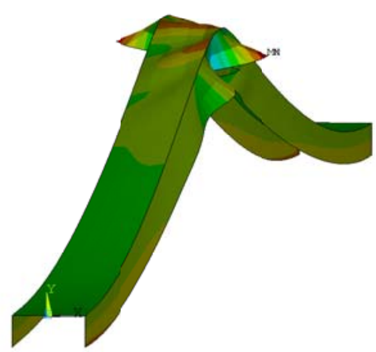
Load:- 34.25 kN -Ultimate



Load:- 29.51 kN



Load:- 24.69 kN



Load:-22.31 kN

Figure 10. Deflections and stress patterns at various loads - 3500mm column

The variation in deflected forms for a 3500mm long column are shown in Figure 10. In this case local buckling initiates failure immediately and the load drops rapidly. All the forms shown are for ultimate and post-ultimate loading, as very little was evident prior to this point.

Immediately after local buckling, at a load of 34.25 kN ripples are evident along almost all of the column. As the end displacement is increased the load drops and the portions under negative curvature become unbuckled. Referring back to Figure 3, the non-dimensional buckling load plotted in this figure gives the buckling load in kN if multiplied by a factor of approximately 3.1, giving a maximum value of about 47 kN. As the load reduces further the unbuckling can even spread to part of the positive curvature range, for example at a load of 22 kN (P_{CR} ~7 in Figure 3) all of the negative eccentricity range shown and a fair part of the positive curvature range is unbuckled.

Furthermore this unbuckling has the effect of preventing full localisation of the buckling in the positive direction. Three full buckle wavelengths, covering a length of about 750 mm are evident even at the last load step. It is interesting to observe that a degree of twist is evident in the locally buckled region at the last load step.

Observations from the detailed ANSYS results

The main reason for eliminating plasticity from the detailed examination was to ensure that localisation and unbuckling was not attributable to this effect, as it is well known that yielding can produce these effects. The fully elastic analysis showed that these effects were indeed obtained elastically, and this is due to the different buckling and postbuckling properties for different curvature directions.

For the 1000 mm column a very large degree of localisation occurred which undoubtedly changed the properties of the localised area due to the gross deformations occurring there. If we assume that the two end sections, comprising 85% of the column length, may be thought of as a pin-ended column of this length with flexural rigidity EI* 75% of EI due to local buckling under negative curvature, as indicated in Figure 3, then simple analysis suggests reduction in the overall buckling load to 0.26 of its classical value, which is in reasonable agreement with the computed result of 0.23.

The examination of the 3500 mm column showed that elastic "unbuckling" is a realistic concept. This is likely to occur in the main after a column has reached its ultimate load, but is also possible during reversal of curvature from positive to negative because of the greater resistance to local buckling for negative curvature. If we assume that the central, positively buckled part constitute a column with flexural rigidity EI*=0.14 EI and the two end portions combine to make a column of flexural rigidity EI then for simultaneous buckling of both columns lengths of both columns would be 0.272L and 0.728L, with a reduction of the overall buckling load to 0.53 of the Euler value, close to the actual value of 0.5 obtained from the ANSYS analysis

Conclusions

For the particular cross section of channel examined, over the range examined, it is clear that the LE=L/2 assumption, while being a reasonable design approximation, does not really hold true. Different lengths of positively curved and negatively curved sections of columns are clearly evident, and using previously derived analytical evaluations of the buckling and post-buckling properties gives realistic assessment of the behaviour. The directional behaviour of the buckled columns and the concept of "unbuckling" are well illustrated by the ANSYS results.

While time constraints prevented a detailed examination of very short columns the very close correlation of the percentage reduction in column capacity (i.e. to 13.6%) with I*/I for positive curvature (i.e. to 14%) for the 500mm column suggests that for short columns the LE=L/2 hypothesis may well be more accurate. In this range the axial loading is very far into the post-buckling range, the deflections are very small and so the "load eccentricity" in both positive and negative directions is small, in which case I*/I is similar for curvatures in both directions, (see Figure 3 and related comments).

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