A special investigation in concrete steel and application to square set system

Horace Alonzo Hand
A SPECIAL INVESTIGATION
IN CONCRETE STEEL
AND APPLICATION TO SQUARE BENT SYSTEM

BY
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A SPECIAL INVESTIGATION

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SQUARE SET

SYSTEM IN MINING.

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MISSOURI SCHOOL OF MINES, ROLLA, MO.

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A SPECIAL INVESTIGATION IN CONCRETE STEEL
AND APPLICATION OF SAME TO SQUARE
SET SYSTEM IN MINING.

This investigation has been prompted, first, by a desire to become more familiar with concrete-steel in its general application to engineering construction, second, by the inevitable necessity of a substitute in the near future for lumber in mining construction.

In first case, a thorough review has been made of principles and practice in the construction of concrete-steel structures in general, including bridges, water towers, reservoirs, viaducts, etc., as found in current engineering journals, which is the most up-to-date source of information available.

Special attention has been given to best practice in reinforcing concrete to resist forces of bending, shearing, tension and crushing.

Investigation of Column to Replace Members of Square Set System.

Member designed to carry load of timber post 10" X 10" X 7'
(in clear)

Area section Post = 100 sq. inches.
Crushing strength timber (average) post = 2500 p.s.i. per sq. in.
Total crushing strength of post = 25000#
As the safe working stress of concrete is 250# per square inch, and that of wood 350#, it is apparent that weight alone will prohibit the use of concrete without steel or iron reinforcement.

(Note in first part of investigation the working maximum crushing strength of 1000# per square inch was taken for wood as 350# is average. The later investigations were made on that basis.)

TO DETERMINE APPROXIMATE AREAS OF CONCRETE AND CAST IRON NECESSARY.

On account of the weight of concrete, a post 7-1/4' in length must be limited to 25 to 30 square inches in section.

Section 5" X 5" = 25 square inches.
Crushing strength of concrete in section = 250 X 25 = 6250#
Crushing strength of timber = 100 X 1000 = 10,000#
Amount to be carried by reinforcement = 93,750#
Crushing strength cast iron = 16,000# per sq. in.
Area cast iron necessary = \( \frac{93,750}{16,000} = 5.86 \) sq. in.

INVESTIGATION TO DETERMINE SECTION HAVING MOST ECONOMIC DISTRIBUTION OF STEEL.

As concrete must entirely cover steel, and section is limited (in a solid post) to 5" X 5" the section of steel (whatever shape) cannot be more than about four inches.
COMPARISON OF SQUARE AND ROUND HOLLOW SECTIONS OF CAST IRON.

Diameter cast iron shell, round = 4"
Dimensions cast iron shell, square = 4 X 4
Let \( l \) = length of column in feet
Let \( d \) = external diameter of circle, or least side of rectangle in inches

Then

Ultimate crushing strength in pounds per square inch
(See "Carnegie", Page 140, Edition '02)

Round columns (square bearing) = \( \frac{20000}{1 + \left( \frac{121}{121} \right)^2} \)

Rectangular columns (square bearing) = \( \frac{20000}{1 + \left( \frac{121}{121} \right)^2} \)

Assuming length of post = 9'

Then

Circular section = 40,510#

Rectangular section = 51,040

(See 3-a and 3-b)

INVESTIGATION OF COLUMN MADE UP OF FOUR ANGLES EMBEDDED IN CONCRETE ANGLES, PLACED IN CORNERS OF 7" SQUARE SECTION FLANGES INWARD. Angles chosen for trial are four No. 8-56 Carnegie. Combined area = 6.24 square inch.

\[ I \text{ about gravity axis} = 4(0.54 \text{ plus } 1.58 \times 0.84^2) = 53.03 \]

\[ R = \sqrt{\frac{I}{\text{area}}} = \sqrt{\frac{53.03}{6.24}} = \sqrt{8.54} = 2.9" \]

\[ P = \frac{1.5}{L/\left(1/R\right)^2} \text{ for cast iron both ends.} \]

\[ \text{Fixed} = \frac{1}{5000} \]
4. Therefore 
\[ P = \frac{6.24 \times 10000}{1 + \frac{1}{5000} (27/2.8)^2} = 90,000\# \]

The objection to separate angles is that they should be bound together in some way that is not easily made in one casting. For this reason it is preferable to use channels instead of angles.

DESIGN OF SECTION WITH CHANNELS.

From a discussion of economic section for compression members, page 288, Frame Structures, Johnson and Turnear, (Eighth Edition), it is shown that a section made up of channels placed back to back is most economic section.

A combination of two 3" with two 4" channels, in form of rectangle, back to back, will first be considered, same to be inbedded in concrete, making external dimensions of column 6.16" X 4.3".

Area section gross = 6.16 X 4.3 = 26.432 sq. in.
Area Steel \[ 2 \times \frac{2}{76} = 3.52 = 3" \] channel \[ 2 \times 1.55 = 3.10 = 4" \] channel, Total area = 6.62"

Crushing strength (working stress)

Steel \[ 2 \times 20000 = 40000\# \]
Channels \[ 2 \times 1470 = 29400 \]
Total = 69400

Concrete at 250# per sq. in. = 4267
Total for column = 74667#
At this point in the investigation it was noted that a column of solid concrete in section could be supplanted by section having concrete surround channels, but leaving section within channels hollow. This distribution of concrete has two advantages over solid section:

(1) While not increasing the net area of concrete, increases moment of inertia of section.

(2) All the concrete being on the outside of iron affords best protection from acid waters, which is by no means a small consideration in mine construction.

At this point was also noted that the 4" angle while having less area has greater strength than 3" angle, hence column is finally made up of four 4" angles surrounded by a shell of concrete making a hollow section (See plate 1).

DETAIL ANALYSIS OF SECTION AND FINAL COMPARISON
WITH WOODEN COLUMN.

Crushing strength

Steel = 4 X 20200 = 80,800#  
Steel # 20.72 X 250 = 7,180#  
Area Steel = 4 X 1.55 = 6.20 sq. in.  
Area Concrete =  

Weight steel per linear ft = 4 X 5.05 = 21#  
Weight concrete " " = 10.1#  
Total weight column " " = 51.1#  
Weight column 7-1/4' long = 370 #  
"eight dry oak stick 10" X 10" X 7-1/4' = 515 #
Cost of iron in post 7-1/4' long @ .02 per lb, for
casting of iron = $3.044

Cost of concrete in post 7-1/4' long @ $5.00 per
cubic yard = .80

Total cost of post = $3.84

Cost of timber stick 10" X 10" X 7-1/4' @ $25.00
per thousand = 1.88

DESIGN OF SQUARE SET JOINT.
(See Plates 1, II, III, and IV.)

To assemble joint:

The post member B (see Plate III) having been set, and cavities in ends of members D and E having been filled with concrete (the cavities just mentioned are blocked by a 1/4" board resting on shoulders moulded in cast iron, at a distance of 3/8" from end of piece, not shown on plates. The sides of hollow cast iron from these shoulders to end of member are cast very rough so as to make a bond with concrete.) at this juncture, if conditions of ground would indicate that joint would be subjected to torsion or tension, a steel rod 1/2" X 5" with ends bent at right angles may be imbedded in concrete of joint in such manner that one-half of rod will be imbedded in concrete of other member when joint is complete, thus forming a very strong reinforced concrete joint. Or steel may be omitted, if in the judgment of the engineer the concrete will be sufficient, which will be the case if compression only is to be dealt with.

Member D and E are now brought together, all concrete
of ends having received wash of Portland cement mortar, care being taken that reinforcements have square and true bearing.

Next Member C and A (See Plate IV) may be placed. The steel rod used to connect these members shall be 11" in length, and ends bent at right angles, and must be placed at right angles to connecting rod of D and E, and at time when D and E are brought together.

Member F (See Plate IV) may now be placed. If rod reinforcements are used, the first rod set should be in post Member L, and should be 1/8" X 13", and ends bent at right angles. All concrete surfaces are to have Portland cement mortar wash before being brought together.

CASTINGS FOR REINFORCEMENTS.

Reinforcements must be obtained as cheap as possible. The force to be provided for will be compression mainly. If nature of ground is such as to cause torsion on members, cast iron will prove very unsatisfactory, and steel channel should be used.

The primary idea in this investigation was to utilize the scrap iron of district in making cheap castings. As iron on remelting in cupola absorbs sulphur from fuel, sulphur increases percentage of combined carbon, making casting hard and brittle, and increases crushing strength. For ordinary castings this brittleness is a serious objection, and is prevented by decreasing combined carbon by additions of silicon in form of ferro-silicon.
**Composition ferro-silicon suitable for foundry:**

<table>
<thead>
<tr>
<th>Element</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron</td>
<td>67.16</td>
</tr>
<tr>
<td>Graphite Carbon</td>
<td>1.50</td>
</tr>
<tr>
<td>Combined Carbon</td>
<td>1.24</td>
</tr>
<tr>
<td>Sulphur</td>
<td>0.03</td>
</tr>
<tr>
<td>Manganese</td>
<td>3.36</td>
</tr>
<tr>
<td>Silicon</td>
<td>-10.55</td>
</tr>
<tr>
<td>Phosphorus</td>
<td>0.04</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.00</strong></td>
</tr>
</tbody>
</table>

The writer would suggest the following as to composition of casting.

In cupalo a composition corresponding to No. 3 pig will be aimed at, and so regulated by use of ferro-silicon:

**Composition No. 3 Pig:**

<table>
<thead>
<tr>
<th>Element</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron</td>
<td>62.33</td>
</tr>
<tr>
<td>Carbon, combined and graphitic</td>
<td>3.15</td>
</tr>
<tr>
<td>Silicon</td>
<td>1.00</td>
</tr>
<tr>
<td>Sulphur</td>
<td>0.03</td>
</tr>
<tr>
<td>Phosphorus</td>
<td>0.85</td>
</tr>
<tr>
<td>Manganese</td>
<td>1.71</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.00</strong></td>
</tr>
</tbody>
</table>

If torsion is to be dealt with, the No. 3 Pig will be cast and cooled so as to retain large per cent of graphitic carbon, thus insuring toughness. If compression only, castings will be cooled rapidly, thus increasing carbide of iron and reducing graphitic, giving casting a high crushing strength.
INVESTIGATION OF STEEL SHELL FILLED WITH CONCRETE.

Let \( l \) = length of steel shell in inches.
Let \( t \) = thickness of steel shell in inches.
Let \( F \) = working pressure per sq. in. concrete = 250#
Let \( F_s \) = steel = 16000#
Let \( F_1 \) = horizontal component of \( F \).
Let \( R \) = radius concrete cylinder in inches
Let \( d \) = diameter
Let \( S \) = working tensile stress steel = 16000#
Let \( \beta \) = angle between plane of shearing stress and plane of \( F \).

Consider \( F \) as acting in line of axis of cylinder, then its component, considering solid material in direction of \( R \) will be zero.

Now consider the cylinder filled with water, then with \( F \) acting along axis of cylinder. The pressure on a plane normal to \( R \) will be equal to \( F \).

Now, suppose the cylinder filled with smooth, round grains of sand or shot, the particles will then be acted upon in same manner as particles of water, in so far as the friction of sand grains will permit. On account of this friction, the unit pressure on a surface in the sand, perpendicular to \( R \), will not be \( F \), but some force \( F_1 \). Now \( F_1 \) being less than \( F \) can equal \( F \times \text{cosine of some angle} \). Now by comparison of angles of
pyramids resulting from crushing tests upon granite, limestone and sandstone, in granite specimens angle beta (β) (see Plate V Fig. 2.) ranges from 20° to 70° with and average of 30°.

In limestone beta (average) = 70°
In sandstone beta (average) = 80°

It is noted that the angle corresponding to XXXX beta decreases as crushing strength decreases; that is, the less resistance there is to movement particle upon particle the greater will be $F_1$.

But now the crushing strength of concrete = 2500# per square inch, which is one-half that of sandstone. This will tend to decrease beta.

It is to be noted in test of concrete beams that cracks which first appear due to shearing, make an angle corresponding to beta, of approximately forty-five degrees, which is probably close to the value of beta in a cylinder of concrete; that is, upon a plane normal to a line making an angle of forty-five degrees in case of concrete, there will be a force, $F$, which is same as $F$ which is normal to cross section of cylinder (see Plate V).

Now, resolving $F$ into horizontal component $F_1$

Then $F_1 = F \cos. \beta = F \cos. 45°$

$= 250# \times 0.7071 = 176.8#$ per sq. in.

Now, reasoning in same manner from water to sand, from sand to concrete, it is evident that the surface upon which $F_1$ acts is $d X \angle$ Total pressure tending to burst steel cylinder = $F_1 \times d \times \angle$

The quality of steel which resists this bursting is
tension \( (S) \) over area \( 2XtXz \)

Therefore

\[ F_1 \times d = 2XtXz \]

But

\[ F_1 = \cos \beta \]

Then

\[ F \cos \beta \times d = 2XtXz \]

For concrete, \( 176.9 \# R = t \times z \)

From equation 1, \( F = \frac{2XtXz}{d \times \cos \beta} \)

\[ = \frac{t \times z}{R \cos \beta} \]

For concrete, \( \cos \beta = .7071 \)

For steel \( S = 16,000 \# \)

Then \( F_1 = \frac{t \times 16,000}{.7071R} = \frac{28837.\times t}{R} \)

which is the allowable working stress per square inch on concrete. Thus it is seen that the allowable pressure per square inch on concrete post varies as thickness of steel shell and inversely as \( R \). Now the strength which has been derived from steel, so far, is by virtue of tensile strength; it also had a compressive strength of 16,000\# per square inch.

Area steel \( = 3.1416 \left( d + \frac{t}{2} \right) t = 3.1416 \left( \frac{4Rt + \frac{t^2}{4}}{2} \right) \)

Total resistance steel \( = 16,000 \times 3.1416 \times \left( \frac{4Rt + \frac{t^2}{2}}{2} \right) \)

\[ = 100714. \times R \times t \times \frac{45143}{t^2} \]

Total allowable stress on column, so far as concrete
and tension in steel are concerned, would be \( \frac{22627 \times 5.14 \times R^2}{R} \)
\[ = 22627 \times 5.14 \times R \]
\[ = 22627. t \times 5.14 \times R \]
\[ = 22627. t \times 3.14 \times R \]
\[ = 22627. t \times 7.14 \times R \]
\[ = 22627. t \times 14.3 \times R \]
\[ = 22627. t \times 25143 \times R \]
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Then total strength of column = 22627 \( X \) 7.14 \( X \) \( R \) \( X \) \( t \)
plus 100714.3 \( X \) \( R \) \( X \) \( t \) plus 25143 \( t^2 \)
\[ = 22627. t \] plus 171997 \( R \) \( t \) plus 25143 \( t^2 \)

DESIGN OF COLUMN.

In design the load will be known then by assuming \( R \) and solving quadratic \( t \) is determined.
MATHEMATICAL COMPARISON OF SECTIONS.

Consider circular annules

Let \( d \) be outer diameter

Let \( d_1 \) be inner diameter

Then assuming area is 12 square inches, and that the area is product of mean circumference, and \( (d - d_1) = 1 \) in.

Then mean circumference = 10 inches

Then \( d^2 = \frac{12}{3.1416} \approx 3.82 \)

Then \( d \approx 4.32 \)

Then \( d_1 \approx 3.3 \)

\[
1 = \frac{3.1416}{64}(d^4 - d_1^4) = 0.049 (342.2 - 110.8) = 10.956
\]

\[
R = \sqrt{\frac{12}{\text{area}}} = \sqrt{\frac{10.956}{12}} = \sqrt{0.926}
\]

Consider hollow rectangle

Area = 12 sq. in.

\[
b_1 = 4''
\]

\[
b_4 = 2''
\]

\[
d_1 = 4''
\]

\[
d_4 = 2''
\]

\[
1 = \frac{12}{12} \left( b_1 d_{12}^4 - b_4 d_4^4 \right) = 20
\]

\[
R = \sqrt{\frac{20}{12}} = \sqrt{1.66}
\]

But in column formulae \( P = \frac{2\sigma}{1 + \frac{1}{R}} \)

We see that \( R \) is directly proportionate to \( R^2 \), and \( \sigma \) varies as \( P \). That is, in this case, that the permissible pressure per square inch upon section of hollow rectangle is to pressure on
circular annules as 1.66 is to .016. Therefore hollow rectangle will be chosen as section.

Comparison of column made up of four 4" channel bars, a hollow cast iron post (see Carnegie, page 149) to carry same load of 2,500 lbs -

Area four channels = 6.2 sq. in.
Area round column = 14.1 sq. in.
Weight round column = 47.96 #
Weight channels per post = 21 #
Safe load channels = 30,000 #
Safe load round columns = 85,000 #

The external diameter round columns equal six inches, and thickness of shell equal seven-eighths inch. It is to be noted that while round cast iron column has over one hundred per cent more metal, its crushing value is but about six per cent greater.