An experimental investigation of the effects of ambient and heated steady flow and intense sound levels on the response of acoustic filter elements

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AN EXPERIMENTAL INVESTIGATION OF THE EFFECTS OF AMBIENT AND HEATED STEADY FLOW AND INTENSE SOUND LEVELS ON THE RESPONSE OF ACOUSTIC FILTER ELEMENTS

by

RAYMOND H. SCHAFFART, 1940-

A DISSERTATION

Presented to the Faculty of the Graduate School of the UNIVERSITY OF MISSOURI-ROLLA

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ABSTRACT

The primary purpose of this research was to investigate the effects of flow and finite amplitude waves on the response of acoustic elements. A further purpose was to evaluate the effects of elevated temperatures and cross-sectional dimensions on element performance. As an integral part of this work various laboratory techniques were explored for their suitability.

Empirical models for the acoustic impedance of side branch resonators were developed for the two operational modes, i.e., flow past the resonator and finite amplitude wave inputs. A description of the complex variation of the acoustic resistance of the resonator with flow past and variable input level is given. A region characterized by high input levels was found in which the resistance becomes a linear function of the flow Mach number. Empirical factors to account for effects of finite amplitudes and steady flow are presented. In addition to the impedance measurements conducted on the resonators, response measurements were obtained and compared to those generated using the empirical impedance expressions for both cold and heated flow operations. Good agreement was obtained between measured and calculated response parameters, indicating the validity of the developed models.

A flow perturbation technique was employed to analytically describe the acoustic response of area discontinuities in the presence of steady flow. An adiabatic process was used to describe a simple expansion and an isentropic process was used for the simple contraction. Measurements of the response of simple expansion chambers under cold and
heated flow operations agreed well with the response calculated using the developed models. No finite amplitude effects were observed at incident sound levels up to 150 dB. Measurements were also obtained for combinations of resonators and simple expansion chambers and compared well with the generated response. The analytic models and empirical relations developed in this work can be used to design sidebranch resonators and simple expansion chambers to attenuate finite amplitude sound waves superimposed upon a steady flow.

Modifications to an existing laboratory facility at University of Missouri-Rolla are described. These modifications were necessary to enable the experimental portion of this work to be accomplished. An anechoic termination capable of use with flow was constructed and its design, fabrication and evaluation are discussed.
ACKNOWLEDGEMENTS

I would like to take this opportunity to express my appreciation to the many people who aided me in this undertaking. My thanks to Dr. W.S. Gatley for suggesting this project and the National Science Foundation for the necessary funding.

I would like to especially thank Mr. Dick Smith, Mr. Lee Clover, and Mr. Lee Anderson whose help in constructing the laboratory facility and solving the day to day operational problems was sincerely appreciated. My sincere appreciation must also be expressed to my fellow graduate student Mr. C.R. Oboka.

I would be remiss if I did not thank all of the various members of the Mechanical and Aerospace Engineering faculty who took of their time to discuss the thoughts and aims of my research. I am especially indebted to Dr. R.D. Rocke, Dr. L.G. Rhea, and Dr. L.R. Koval.

Three men who are not on the Rolla campus must also be mentioned. They are Mr. Robert Brossemer, Mr. Pat Depew and Mr. Harold Shaw. What instrumentation skill I may possess I owe to their tutelage and only with this knowledge was the experimental work possible.

Finally, I wish to acknowledge and thank my wife Ruth Ann who has persevered with me throughout this work. Without her understanding and encouragement the task would have been impossible.
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NOMENCLATURE

\( A_+ \) = magnitude of the displacement of an acoustic plane wave propagating in +x direction

\( C_f \) = flow coefficient of orifice

\( c \) = propagation velocity of sound in a waveguide

\( c_0 \) = propagation velocity of sound in free space

\( D \) = inside diameter of standing wave tube

\( d_0 \) = resonator neck diameter

\( e \) = base of natural logarithms

\( f \) = frequency, Hz

\( f_c \) = cut off frequency of horn

\( f_n \) = resonant frequency of Helmholtz resonator

\( h \) = scale factor of horn

\( k \) = \( \omega/c \) = wave number

\( L_B \) = backing depth of resonator volume

\( L_c \) = inside length of expansion chamber

\( L_H \) = thermal boundary layer thickness, cm

\( L_V \) = viscous boundary layer thickness, cm

\( l_{eff} \) = effective length of resonator neck

\( l_{eff, NF} \) = effective length for no-flow case

\( l_{eff, V} \) = effective length for flow-past case

\( M, M_a, M_b \) = Mach number of flow

\( P_\pm \) = magnitude of acoustic plane wave propagating in the \( \pm \) direction
\[ P_{\text{max}}^{\text{min}} = p_{\text{max}}^{\text{min}} = \text{acoustic pressure at the maximum of a standing wave minimum} \]

\[ p = \text{effective acoustic pressure} \]

\[ p_b = \text{acoustic pressure transmitted into a branch} \]

\[ p_i = \text{acoustic pressure incident at discontinuity} \]

\[ p_o = \text{pressure at origin; i.e., } x=0 \]

\[ p_r = \text{acoustic pressure reflected at discontinuity} \]

\[ p_t = \text{acoustic pressure transmitted past discontinuity} \]

\[ R = \text{complex reflection factor} \]

\[ R = \text{magnitude of reflection factor} \]

\[ R_B = \text{specific acoustic resistance} \]

\[ R_b = \text{acoustic resistance} \]

\[ R_{bv} = \text{acoustic resistance for flow-past case} \]

\[ R_s = \text{specific acoustic resistance resulting from an oscillatory flow over an infinite plane surface} \]

\[ R_v = \text{specific acoustic resistance of an orifice with flow through the orifice} \]

\[ R_{vis} = \text{acoustic resistance of resonator neck resulting from viscosity effects at boundaries} \]

\[ \text{Rey} = \text{Reynolds number} \]

\[ S, S_a, S_b = \text{cross-sectional areas of tubing} \]

\[ T = \text{magnitude of transmission factor} \]

\[ U = \text{acoustic volume velocity} \]

\[ u = \text{acoustic particle velocity} \]

\[ u_o = \text{acoustic particle velocity in orifice} \]
\( V_t \) = mean flow velocity through an orifice

\( X_b \) = acoustic reactance

\( x \) = location coordinate

\( x_{\text{max}} \) = location of standing wave maximum pressure locations relative to tube termination

\( x_{\text{min}} \) = location of standing wave minimum pressure locations relative to tube termination

\( Z \) = acoustic impedance

\( a \) = attenuation constant for viscous wall effects in a waveguide, cm\(^{-1}\)

\( \alpha_b \) = absorption coefficient of a side branch

\( \beta_1 \) = empirical effective length proportionality constant

\( \beta_r \) = finite amplitude nonlinear coefficient for resonator resistance

\( \gamma \) = ratio of specific heats

\( \gamma_r \) = linear resistance end correction

\( \Delta L \) = reduction of effective length due to flow past resonator neck

\( \delta_1 \) = no-flow effective length end correction

\( \epsilon_1 \) = effective length flow coefficient

\( \epsilon_r \) = flow coefficient for side branch resonator resistance with flow past the neck

\( \lambda \) = wavelength of sound

\( \lambda_o \) = no-flow wave length of sound

\( \eta \) = area ratio

\( \xi \) = acoustic particle displacement

\( \rho_o \) = mass density

\( \omega \) = angular velocity = \( 2\pi f \), radians per sec.
I. INTRODUCTION

In recent years, there has been an ever increasing awareness of the public to its surroundings. An integral part of this new awareness is the demand for quieter products in every field. In response to these public demands, new legislation is being passed almost daily, placing noise restrictions on industrial products, household appliances, and recreational vehicles as well.

The internal-combustion engine is considered by many as a leading offender in terms of noise generation as well as other forms of pollution. For this reason it has become imperative that better muffling elements be designed for use on these engines. However, because of the concern over clean emission prevalent today, the performance of an engine cannot be sacrificed to obtain noise reduction.

In the past, the design engineer has had to rely on what is termed linear acoustic theory. Although this theory adequately defines the response of acoustic elements under controlled laboratory conditions, it is inadequate for the vast majority of noise-control problems encountered in practice. The primary causes for variation from the linear theory are the effects of high-amplitude sound waves, the propagation of sound in a moving medium, and variations in temperature of the gas through which the sound is being propagated.

There are many types of testing procedures used to evaluate the effectiveness of acoustic filters, including mufflers. One of the primary techniques used is the evaluation of a given filter or filter element with pure tone analysis. This entails propagating a
discrete pure tone through the filter and measuring its response. By conducting a set of these tests, the filter response is obtained over a desired frequency range.

Gatley (1)* employed this procedure to design a muffler system for a small refrigeration unit. Other investigators have also used this technique with some success using a combination of theory and an experimental test program. More recently, a similar procedure has been used in the aerospace field to improve the design of acoustical damping devices.

With the previous successes have come new and better design criteria, but there is still much work to be done. It was decided that an area of great interest is the response of acoustic elements which are suitable for systems such as an internal-combustion engine exhaust. Since a large portion of the acoustic energy in this type of system occurs at frequencies below 1000 Hz, this region was selected for investigation. The two basic acoustic filter elements in use today are the side-branch resonator and the expansion chamber; therefore, these two types of elements as well as combinations of them were selected for investigation. It was decided to investigate these basic elements, primarily by experimental pure-tone analysis, to determine their nonlinear response when using finite-amplitude waves and/or a flowing propagation medium. The physical sizing of the elements was selected on the basis of the elements found in many typical silencing systems such as automotive exhaust mufflers.

*Numbers in parentheses refer to listings in the bibliography.
In addition, it was decided to extend the investigation by heating the flow medium. In this way, the temperature effects could be determined as well as an evaluation of the feasibility of laboratory procedures for testing acoustic filters under conditions which, in so far as is possible, approach those encountered in actual use.

If suitable laboratory techniques can be developed they would serve to greatly reduce the cost now required for trial and error testing of prototypes.
II. LITERATURE REVIEW

One-dimensional linear theory has been used for some time for the design of acoustic silencers and silencer elements. However, the design-development cycle for these filters is still very much a "cut and try" procedure. This is due mainly to the fact that no means has yet been devised to account for the deviations from linear theory which occur in practice.

Nonlinear response of elements arise from a variety of effects, namely: finite amplitudes of the acoustic waves, varying temperature, a superimposed flowing medium, and sizing of elements so that some frequencies have wave lengths violating the assumption of plane waves (one-dimensional) used in linear theory.

Another factor which adds to the "black-art" concept of silencer design is the techniques and terminology for describing the response of these filters, e.g., the same term is used interchangeably by different authors to describe different response parameters.

A. Plane-Wave Theory and Its Application to the Analysis of Acoustic Elements

The use of acoustic filter theory for the design of silencer elements has had widespread acceptance for many years. This theory, first advanced by Stewart (2,3), is based on the assumption that a series of plane acoustic waves propagated down a waveguide which is terminated with an acoustic filter is analogous to an electric transmission line terminated with a complex impedance. The acoustical filters are then analyzed by means of lumped-parameter methods using
plane wave theory to describe the sound propagation into and out of the filter elements.

The simplest form of wave motion propagated through a medium is that of the plane waves of sound. Plane waves are a form of longitudinal wave, i.e., the molecules transmitting the wave move back and forth in the direction of propagation. The characteristic property of these waves is that the acoustic pressure, particle displacement, density change, etc., have common phase and amplitude in a plane perpendicular to the direction of propagation.

Plane-wave theory is used to describe the propagation of acoustic waves in a tube, with the restriction that the wave length is "large" compared to the diameter of the tube. This restriction is used to insure that no higher-order modes of vibration are present in the propagated wave. Hartig and Swanson (4) established that the plane wave condition was met if the following relationship applied:

$$\lambda = 0.82D$$  \hspace{1cm} (2.1)

where:

$$\lambda = \text{minimum wavelength of interest}$$

$$D = \text{inside diameter of wave guide}$$

The analytic development of plane wave theory is given in any elementary acoustics text, e.g., Morse (5) and Kinsler & Frey (6) and, therefore, will not be given in detail. The assumptions necessary for the application of this theory are listed now in order to show the limitations of this theory when applied to non-idealized problems.
a) The medium is homogeneous, isotropic, and perfectly elastic (no dissipative forces are present).

b) The effects of gravitational forces may be neglected; therefore, the density and equilibrium pressure of the medium are uniform throughout the medium.

c) The amplitude of the waves are small such that the acoustic pressure is much less than the equilibrium pressure and the incremental density is much less than the ambient density of the fluid.

d) The propagated waves cause adiabatic and reversible compressions and expansions.

With these simplifying assumptions and following the development of Morse (5), it can be shown that the acoustic pressure, particle displacement and particle velocity of waves propagated in a wave guide are described by the following governing partial differential equations:

\[
\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2} \quad (2.2)
\]

\[
\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial^2 \xi}{\partial x^2} \quad (2.3)
\]

\[
\frac{\partial^2 u}{\partial t^2} = \frac{2}{c} \frac{\partial^2 u}{\partial x^2} \quad (2.4)
\]

where:
\[ p = \text{acoustic pressure} \]
\[ \zeta = \text{particle displacement} \]
\[ u = \text{particle velocity} \]
\[ c = \text{propagation velocity of the wave} \]

With the added assumption that only simple harmonic waves will be considered, the method of separation of variables can be applied to Equations (2.2), (2.3), and (2.4), with the resultant expressions for a positive traveling wave

\[ p_+ = p_+ e^{ik(x-ct)} \tag{2.5} \]
\[ \zeta_+ = \Lambda_+ e^{ik(x-ct)} \tag{2.6} \]
\[ u_+ = u_+ e^{ik(x-ct)} \tag{2.7} \]

Similar results are obtained for a wave traveling in the negative direction. These expressions may be combined to give the analytic description of the resultant wave form occurring in a tube when waves of the same frequency traveling in opposite directions are present. For the acoustic pressure, this total expression is given by:

\[ p(x,t) = p_+ e^{ik(x-ct)} + p_- e^{-ik(x+ct)} \tag{2.8} \]

Davis and his colleagues (7) conducted a very extensive investigation of a large number of acoustic filters. These included various types of expansion chambers and resonators both as individual components and in combinations. The theoretical characteristics of the test elements were predicted using plane-wave acoustic theory. The continuous or distributed parameter approach was used for those elements
with physical dimensions precluding the lumped parameter method. The calculated responses were compared to experimentally-determined values. The theoretical response curves were obtained using the assumption that the element was terminated anechoically. (An anechoic termination is one which allows no portion of an incident wave to be reflected back toward the source.) This constraint was incorporated into the experimental test setup by using an absorbent lined diffuser to eliminate reflected waves at the termination of the test element. Very good agreement was obtained for frequencies where the assumption of plane waves was valid.

The standing-wave tube, or impedance-tube method, (1) was used in the experiments of Davis, et al (7). This method is based on the assumption that only plane waves of sound are present in a waveguide, providing, of course, that the restriction of Equation (2.1) is adhered to. Whenever a plane wave traveling down a waveguide encounters a change in cross-sectional area, some of the incident wave is reflected back toward the source as a plane wave and some is transmitted through the discontinuity in the form of a plane wave. Measurements obtained by this method are used to determine "acoustic impedance", "reflection coefficients" and "transmission coefficients" of the terminating element(s).

B. The Concept of Attenuation Coefficient for Plane Waves

The assumption that no dissipative forces exist puts a much larger constraint on the phenomenon of sound wave propagation than is permissible in many applications; however, it is possible to relax this restriction with the concept of an "attenuation coefficient".
This concept is applied when it is assumed that only thermal and viscous boundary layers cause attenuation of a plane wave as it travels down a waveguide.

The initial theory concerning the attenuation in tubes was postulated by Helmholtz in 1863. In it, he attempted to describe the decrease in the propagation velocity due only to viscous effects. Kundt in 1868 experimentally determined that the reduction was larger than was predicted by Helmholtz and suggested that heat conduction could be the cause of this discrepancy. Finally, in 1868 Kirchoff (3) presented the complete theory, accounting for both thermal and viscous effects, for the decrease in the propagation velocity. This was expressed as a formula for the determination of an attenuation constant as follows:

\[ \alpha = \frac{\omega}{C \cdot D} [L_v + (\gamma - 1)L_n] \]  

(2.9)

where:

- \( L_v \) = viscous boundary layer thickness, cm
- \( L_h \) = thermal boundary layer thickness, cm
- \( \gamma = \text{ratio of specific heats } c_v/c_p \)

With this constant, the value of the sound pressure of a plane acoustic wave traveling in a waveguide is given for any location \( x \) by:

\[ p = p_0 e^{-\alpha x} \]

where:

- \( p \) = sound pressure at location \( x \)
- \( p_0 \) = sound pressure at reference location, \( x=0 \)
Many investigations have been conducted to experimentally confirm this constant \((5,8,9)\). It should be noted that accurate experiments are difficult to design because of the small magnitude of this constant. Many of the investigations dealt with the water vapor content of the medium within the waveguide. Beranek (10), after comparing the results of many of these studies, concluded that the attenuation constant was best described by increasing Kirchoff's equation by 15 percent. He gave the numerical evaluation as:

\[
\alpha = 3.18 \times 10^{-5} \left(\frac{f}{r}\right)^{1/2} \text{ cm}^{-1} \tag{2.10}
\]

where:

- \(f\) = frequency in Hz
- \(r\) = radius of tube in cm

Simon (11) stated that although there was a large amount of scatter in his results they tended to group around the value determined from Beranek's expression. Simon used the same waveguide that is employed in the experimental work of this dissertation and therefore the same expression for \(\alpha\) will be used for all non-flow calculations. Since Kirchoff did not implicitly take into account the presence of a steady flow superimposed on the acoustic wave, this value for \(\alpha\) is not valid where there is steady flow through the waveguide.

Ronneberger (12), as part of an experimental investigation of the acoustic properties of changes in cross-sectional area, determined the attenuation constant for his waveguide with flow. He determined that very little change occurred in this constant for low Mach numbers
(M≤0.1) and that for higher Mach numbers (M>0.1) the value was increased up to two to three times the no-flow value. The waveguide used in the present investigation has a diameter which is slightly smaller than those employed by Ronneberger.

C. Analysis Concepts for Determining Response of Acoustic Elements

Impedance is a basic concept in acoustic analysis and can be very useful in interpreting the physical ramifications of test results. This is due to the fact that the various parts of the impedance can be directly related to the physical characteristics of a test element.

There are three basic impedance terms encountered in acoustic analysis work (6,10,13), each having a specific application. They are:

1) The "acoustic impedance".
2) The "specific acoustic impedance".
3) The "radiation (or mechanical) impedance".

The "acoustic impedance" is defined as the complex quotient of the instantaneous sound pressure divided by the instantaneous volume velocity (particle velocity times area) at a given location. This is analogous to the electrical impedance from AC circuit theory in which the sound pressure is analogous to a voltage and the volume velocity is analogous to an electrical current. The term "acoustic resistance" is applied to the real component of the complex impedance. It is associated with the dissipation of energy. The imaginary component of the impedance is termed the "acoustic reactance". The reactance is comprised of two distinct parts. The first part, associated with the effective mass of the medium, is termed "acoustic inertance"
and is analogous to an electrical inductance. The second portion of the reactance is called the "acoustic compliance" and is associated with the elasticity of the medium. It is analogous to an electrical capacitance. The unit of the acoustical impedance is the "acoustic ohm". Acoustic impedance is often used in discussing the transmission characteristics of lumped acoustic elements. Many investigators have employed this technique in analyzing filter systems (5,6,7).

Another useful concept is that of "specific acoustic impedance" which may be thought of as the acoustic impedance on a per unit area basis. It is defined as the complex quotient of the instantaneous sound pressure divided by the instantaneous particle velocity at a given location. If one is dealing with the one dimensional case of a free plane wave propagating in an ideal gas, the specific impedance of the medium is a real quantity with a magnitude equal to the product of the static density and the propagation velocity of the wave. This product is defined as the "characteristic impedance" of the medium. Because it is a real quantity, it is also called the "characteristic resistance" of the medium. In the mks system where density is given in kg/m$^3$ and the acoustic velocity in m/sec, the unit of specific acoustic impedance is the "rayl". Since this is a characteristic property of the medium, it is very useful when discussing the transmission of sound waves from one medium to another. Also, if by some means one can determine the sound pressure of a free traveling wave at a point in the medium, the particle velocity may also be determined for that location from the following relationship:
The "radiation impedance" which is also known as the "mechanical impedance" (5) at a given location is defined as the complex quotient of the instantaneous force divided by the instantaneous particle velocity. It is related to the specific acoustic impedance and the acoustic impedance by a factor of area and area squared, respectively. The radiation impedance is used when discussing the coupling between sound waves and a driving source or driven load (6). The unit of radiation impedance is the "mechanical ohm".

The standing wave tube measurement is well described in the literature, e.g., Beranek (10), and therefore will not be dealt with here. However, it should be pointed out that these measurements in conjunction with other measurement techniques are employed to obtain various acoustic parameters in addition to the impedance of the termination. The "reflection factor" is defined as the complex quotient of the sound pressure reflected from the termination divided by the sound pressure incident at the termination. The magnitude of this quantity is often called the "reflection coefficient". This is used as a means of describing the effectiveness of a filter since the greater the reflection coefficient, the less sound pressure is available to be transmitted beyond the element. The "transmission factor" is defined as the complex quotient of the transmitted sound pressure divided by the sound pressure incident at the termination. Its magnitude is referred to as the "transmission coefficient". It is also used as a measure of the performance of an acoustic filter.

\[ u = \frac{p}{\rho_0 c} \]  

(2.11)
"Transmission Loss" is an energy term which is often used in acoustic analysis. It is a measurement of the apparent energy loss through a filter. It is defined as the quotient of the acoustic energy being propagated into an element divided by the energy being discharged from the element, expressed in decibels. This quantity has also been called attenuation, e.g., in reference 13 attenuation is defined as:

\[
\text{ATTN} = 10 \log_{10} \frac{\text{Incident Energy}}{\text{Transmitted Energy}}
\]  

(2.12)

Since, for the case of plane waves, the energy of a plane wave is directly proportional to the square of the pressure, Equation (2.12) can be written in terms of pressures as:

\[
\text{ATTN} = 20 \log_{10} \frac{P_{\text{incident}}}{P_{\text{transmitted}}}
\]  

(2.13)

It is obvious that the pressure ratio in Equation (2.13) is the inverse of the transmission factor.

D. Nonlinear Acoustic Impedance of Orifices Due to Finite-Amplitude Waves

An assumption of elementary acoustic filter theory is that viscous effects; i.e., dissipation of energy due to viscous forces at cross-sectional area changes, may be neglected. This is a valid assumption for small-amplitude plane waves. However, for finite-amplitude waves this assumption does not hold.

Marino, Bohn, and Garrison (14) suggested, based on the previous work of Ingard (15) and Blackman (16), that nonlinear effects are encountered whenever a sound-pressure level is generated such that
a certain critical particle velocity occurs in the orifice. Garrison experimentally determined a nonlinear correction term to be added to the linear resistance of the orifice; however, no agreement was found between his data and that of Blackman or Ingard. All of these experiments were conducted using the standing-wave method for obtaining acoustic impedance.

Ingard and Labate (17) investigated finite-amplitude effects visually by use of smoke photos. They ascertained that there were four distinct regions associated with increasing acoustic flow through the orifice. These regions were defined by the types of circulation in and about the orifice as follows:

Region 1. A low sound region with stationary circulation, the flow is directed out from the orifice along the axis

Region 2. A region of stationary circulation in which the direction of flow along the axis is toward the orifice, i.e., the reverse of that in Region 1

Region 3. A medium sound intensity region where pulsatory effects are superimposed on circulation of the kind in Region 2

Region 4. A high sound intensity region in which pulsatory effects are predominant, resulting in the formation of jets and vortex rings: the jet consists of a strong airflow through the orifice, signified by a sudden burst of air; this burst appears symmetrically on both sides of the orifice and is made up of pulses contributed by each cycle of the sound wave.

Ingard and Ising (18) examined the nonlinearity of an orifice using a hot wire anemometer to measure the particle velocity within the orifice and measuring the acoustic pressure differential across the orifice in order to obtain its impedance. They concluded that the
transition from the linear to the nonlinear region occurred when
the orifice particle velocity exceeded $\approx 10$ meters per second and
postulated that for the nonlinear region, the specific acoustic
resistance of the orifice could be expressed by:

$$R_\delta = \rho u_o$$  \hfill (2.14)

Garrison (19) confirmed the transition but established the
critical particle velocity to be 60 ft/sec. He determined that it
was necessary to correlate the acoustic resistance with the orifice
flow coefficient as well as with the particle velocity. The correla-
tion was accomplished by establishing an analogy between flow re-
sistance of an orifice and the acoustic resistance of the orifice.

Orifice flow resistance was defined by:

$$\frac{P_1 - P_2}{u_m} \Delta \frac{\rho u_m}{c_f^2}$$  \hfill (2.15)

where:

- $P_1$ = static pressure upstream of the orifice
- $P_2$ = static pressure downstream of the orifice
- $u_m$ = mean velocity within the orifice
- $c_f$ = flow coefficient of the orifice
- $\rho$ = density of the medium in the orifice

The specific acoustic impedance of the orifice was given as

$$\frac{P_1 - P_2}{u_o} \Delta \frac{z}{\rho}$$
where:

\[ P_1 \] = complex acoustic pressure upstream of the orifice

\[ P_2 \] = complex acoustic pressure downstream of the orifice

\[ u_0 \] = acoustic particle velocity within the orifice

The real portion of this impedance was assumed to be analogous to the flow resistance. The empirical expression for the acoustic nonlinear resistance of an orifice with finite-amplitude incident waves was given as follows:

\[ R_B = 0.37 \frac{\rho u_0}{c_f} \] \hspace{1cm} (2.16)

It was further pointed out by Garrison that applying the flow coefficient for the type of orifice used by Ingard and Ising in this expression yields Equation (2.14) which provided additional validity to this work.

In addition to studying the change in acoustic resistance, Garrison also investigated the finite-amplitude effects on the reactance of the orifice. This correlation was made in terms of the effective length of the orifice. The effective length is defined as the length of the column of air within the orifice and extending beyond either side of it which contains the effective mass of air. This mass oscillates with the acoustic wave. It is this effective mass which causes the orifice to have reactance. It consists of the length or thickness of the orifice plus an end correction to account for that portion of the air column which extends beyond either end of the orifice. Garrison concluded that the best expression for the
effective length was the same as that used in the linear resistance region. This expression was:

$$l_{\text{eff}} = t + d_0$$  \hspace{1cm} (2.17)

where:

- $t$ = thickness of the orifice
- $d_0$ = diameter of the orifice

No description of this type of correlation for sudden expansions or contractions could be found in the literature. If a simple relationship such as given for the orifice could be determined for sudden changes in cross-sectional area, it would aid in simplifying their design as components of acoustic filters.

Ronneberger (20), during his studies of the acoustic characteristics of ducts with sudden changes in cross-sectional area, obtained the alternating pressure profile across expansions and also through orifices. These plots were very similar for simple expansions and orifices and indicated the presence of a standing wave in front of the area change and a decaying wave of different phase velocity after the discontinuity. The similarity of these plots implies that if a "loss coefficient", or quantity which describes the losses through the discontinuity, could be obtained for an expansion, then a correlation similar to that performed by Garrison for orifices could be attempted.
E. Investigation of Flow Effects with Change in Cross-Sectional Area

An analytic investigation of the propagation of a pressure pulse in a channel carrying a steady compressible flow was conducted by Powell (21). He was able to determine the reflection factor and transmission factor associated with a change in cross-sectional area using both a multiple reflection approach and a steady state approach.

The multiple-reflection approach applied the method of characteristics to the governing equations and yielded an infinite-series solution for the linear interaction of the reflections of the pulse as it propagated through the area change. Powell gave restrictions which could be applied to the series solution so that only the first term of the series could be used. He also stated that under certain conditions the series solution failed to converge.

The steady-state approach assumed steady isentropic flow throughout the cross-section change. The energy and continuity equations across the area change were written and perturbations in the form of an acoustic pulse were then applied. This approach obviously included the fact that both an incident and a reflected pulse were present upstream from the change. Only a transmitted pulse was present in the downstream region which implied that the change in area was connected to an infinitely long section or an anechoic termination. The reflection factor and transmission factor were obtained by simultaneously solving the perturbed equations for the appropriate ratios. Only first-order terms of the perturbed quantities were retained in this
development. Both analyses yielded identical numerical values. The steady-state solution did not require any restrictions in order to obtain a solution for special cases as did the multiple-reflection approach.

The four specific cases considered in this investigation were:

a) Wave enters with subsonic flow
b) Wave enters against subsonic flow
c) Wave enters with supersonic flow
d) Wave swept in by supersonic flow

The multiple-reflection theory was extended by Powell (22) for the case of sinusoidally varying pressure waves propagated with steady flow. A series solution was again obtained and the validity of using the first two terms was discussed. This investigation indicated that the transmission factor was independent of frequency, while the reflection factor decreased with increasing frequency due to cancellation effects.

Ronneberger (12) applied the multiple-reflection theory of Powell to smooth expansions and contractions. He then compared these calculated values to those he obtained experimentally. Good agreement was obtained between measured and predicted values of the reflection coefficient until "critical Mach numbers" of the flow were reached. These critical values were the Mach numbers at which flow separation occurred for the test section. When separation occurs, flow turbulence and accompanying flow losses are encountered. The acoustic response of a long nozzle consisting of a short contraction, a long straight intermediate section, and a short expansion was also predicted using
Powell's theory. Agreement between measured and predicted values of the reflection coefficient were again quite good up to a Mach number of 0.3 for the incoming flow.

Ronneberger concluded that Powell's theory satisfactorily describes the acoustic characteristics of smooth variations in cross-section if flow losses are negligible. He also ascertained that the ratio of the reflection factor with flow to that with no flow was independent of frequency and dependent only on the Mach number of the incoming flow. Because of the apparatus used in the experimental stage of this study, no information on the change in transmission factor with flow could be obtained for comparison to theory.

The steady state approach was applied by Ronneberger (20) to sudden changes in cross-sectional areas. The continuity, energy, and momentum equations for the flow were written and small perturbations representing the acoustic waves were applied. An additional perturbation term was introduced with the assumption that an entropy wave was generated at the cross-section change and propagated with flow. This perturbation appeared in the density term. A general expression for the reflection coefficient was obtained as a function of the incoming and outgoing Mach numbers of the flow, the pressure differential across the change in area, and the rate of the change of this pressure differential.

A solution was also obtained for the special case of a simple expansion for which it was assumed that the pressure on the face of the expansion was equal to the static pressure of the incoming flow.
Plots of the static pressure were obtained across a series of simple expansions and also across a perforated plate. A transition region existed for each expansion which was independent of the Mach number of the flow. This transition region had a pressure recovery length associated with it defined as the longitudinal distance from the cross-sectional change at which the static pressure of the flow has attained its new equilibrium. The relatively short transition length for the perforated plate allowed a frequency range where the wave-length of the acoustic input was greater than the length to be investigated. The transition length of all the simple expansions tested, and the frequency limits of the test equipment, precluded this type of investigation. Calculated values using the analytic expression were in good agreement with the experimental data for the perforated plate. No such success was attained with the expansions. It was concluded that the developed expression was valid only if the wavelength of the sound remained longer than the static pressure transition length. Above this frequency some new mechanism, undefined at present, becomes dominant in affecting the change in the reflection coefficient, and thus the impedance, of the area change. The experimental data of Ronneberger indicated that the reflection coefficient was strongly influenced by the Mach number of the incoming flow and weakly related to frequency.

F. The Effects of Flow on the Response of Resonators

The effect of flow on the response of acoustic elements was investigated experimentally for the specific case of resonators and arrays of resonators by Garrison (19,23). In his investigations of
the acoustic nonlinearity of orifices, the problem of steady flow through the orifice and steady flow past the orifice were examined. The orifice in these studies was the connecting passage to a resonator. A correlation of the acoustic impedance with orifice particle velocity and the orifice flow coefficient was obtained. This correlation was made using the analogy described earlier for finite amplitude investigations.

The particle velocity within the orifice was determined to be the governing parameter for the acoustic resistance for the case of steady flow through the orifice. When the mean flow velocity through the orifice exceeded the particle velocity generated by the acoustic wave the following empirical relationship was determined:

$$R_v = \frac{\rho v_t}{c_f^3}$$

(2.18)

where:

$$v_t$$ = equivalent mean flow velocity within the orifice.

The expressions developed for finite-amplitude waves apply for mean orifice velocities lower than the static-flow particle velocity.

An assumption that the compliance of the resonator volume did not change for the flow cases and was equal to the compliance resulting from linear theory was introduced. This was necessary in order that the reactance portion of the impedance could be correlated with the effective length of the orifice. Although he observed a slight dependence on the acoustic wave amplitude. Garrison concluded that for high incident sound pressure levels, i.e., 160 db for his experiment, the effective length of the orifice was the same as for the no-flow case.
The Mach number of the flow was determined to be the critical parameter when the case of resonator response with flow past the neck was investigated. The resistance and effective length were empirically determined as linear functions of the Mach number:

\[
R_v = R_{NF} (1+1.9M_p) \\
L_{eff,v} = L_{eff,\text{NF}} (1-1.65M_p)
\]

where:

\[M_p = \text{Mach number of flow past the resonator input}\]

The subscripts NF and V represent the static and flow situations respectively.

A shift in the resonant frequency of the resonators was caused by the change in the effective length and a reduction in the attenuation attained with the resonators was observed. The effectiveness of this type of element was thus greatly influenced by the orifice impedance, which was the coupling for the resonator volume. For sufficiently high flow velocities the attenuation of the test elements became linearly independent of frequency thus effectively destroying the benefit of the resonator in attenuating its design frequency.

The results of Garrison's investigations were successfully incorporated into the design of a series of resonators. The success was confirmed by using these resonators to suppress very high amplitude pressure oscillations (190 db maximum) in a small rocket chamber.

In a recent work on the impedance of a hole in the wall of a flow duct, Ronneberger (24) stated that "the real part of the measured
impedance...increases strongly and linearly with increasing flow velocity." He further stated that the critical parameter influencing the change of the impedance with the flow was the particle velocity within the orifice.

The orifice sizing employed by Garrison was such that the transition length of the static pressure change could be assumed small compared to the wavelengths of the acoustic inputs. This assumption would, according to the work of Ronneberger, allow the changes in transmission loss to be correlated with the static pressure change. This was effectively what the analogy of Garrison allowed.

Investigations using side branch resonators as integral components of mufflers were undertaken by Gosele (25). Changes in the attenuation characteristics of test mufflers with and without steady flow were obtained. As pointed out by Baade (26), the attenuation characteristics appeared to be directly related to the manner in which the acoustic input was coupled to the resonator volume.

G. Conclusions

Linear acoustic filter theory is widely used in the design of silencer and silencer elements. If, therefore, a simple modification of this theory could be obtained to account for the nonlinearities introduced by flow superposition and finite amplitude waves, an improved design approach would evolve.

Although work toward this objective has been accomplished for high frequency (1000-4000 Hz) and small physical sizing, only limited definitive work has been done in the lower frequency range (75-1000 Hz).
This frequency range is of great interest because of its application in areas such as automotive exhaust systems. Pure tone testing and analysis were used for the high frequency work which helped to add credance to the planned use of this method for the present investigations. In addition, the finite-amplitude testing conducted by previous investigators was in substantial agreement for a wide variety of resonator neck diameters. Therefore, it is reasonable to anticipate this type of nonlinearity for resonators of the size proposed for investigation.

Previously reported work indicates that attempting to correlate changes in acoustic response with flow parameters is a reasonable approach to the problem. The work of Powell and Ronneberger should be directly applicable for determining the response of expansion chambers with superimposed flow. In addition, a flow-loss-coefficient approach analogous to the resonator analysis employed by Garrison may be possible.

Once the responses of various elements have been determined, a building block approach such as suggested by Gatley (1) would be feasible. However, the nature of the coupling between individual elements must also be understood before this approach can be expected to be successful.
III. EXPERIMENTAL APPARATUS

The standing wave tube method has been used by many investigators for response measurements of acoustic filters. Simon (11) designed and constructed the two-inch I.D. standing wave tube which was used for the present investigation. However, numerous modifications were necessary to enable experiments to be performed with superimposed flow.

A. Description of the Standing-Wave Tube

The standing wave tube was constructed from a six foot length of 2 1/2 inch O.D. by 2 inch I.D. aluminum tube. A flat was milled for a distance of 60 inches along the top of the tube. A slot, 52 inches long by 1/8 inch wide, was then machined along the center of the flat to allow access to the interior of the tube. Teflon strips, 1/4 inch wide by 0.012 inch thick, were installed on the flat along both sides of the slot to facilitate sealing. The slot was sealed by a 1.0 inch wide continuous spring steel band. An aluminum block was attached to the steel band by two dowell pins, four small machine screws and gasket cement. A one-inch diameter microphone cavity was machined into the block; a probe fabricated from #12 hypodermic tubing extended from the cavity into the tube. One end of the probe passed through the steel band into the block, and a cigar shaped body was silver-soldered to the other end. The probe was inserted through the slot and into the interior of the tube until the center of the body coincided with the center of the aluminum tube. The probe body was machined so that the inlet to the microphone cavity was located five body diameters from the leading nose as shown in Figure 1. This
Figure 1. Cross Sectional View of Standing Wave Tube Microphone Block Assembly
A dimension was chosen to reduce the quasi-noise generated by the air flow since it is the location of the idealized stagnation point on the body at which the flow has reattached itself. A sketch of the standing wave tube, steel band, and microphone block assembly is shown in Figure 2. Details of the aluminum tube are presented in Figure 3.

Brass guide rails were used to maintain the microphone block assembly and steel band in position over the slot. The guide rails extended the length of the flat area on either side of the slot and were maintained in place by attachment to eight equally spaced cradle supports. The cradle supports also served to support the aluminum tube and were bolted to a section of steel channel used as the base.

The sealing of the tube was accomplished by pressurizing two lengths of surgical tubing placed immediately over the steel band. The surgical tubing was constrained within brass angles attached to the guide rails are shown in Figure 2. The two pieces of tubing were formed into a loop by attachment to a U-shaped piece of copper tubing. One end of the surgical tubing was attached to a bottle of compressed air via a pressure regulator and the other end sealed with a plugged copper fitting. The steel band, aluminum tube, teflon seals, and surgical tubing were lubricated with high-low temperature aircraft screw grease. The lubrication aided in forming an air-tight seal and helped to reduce the effort required to move the block when the tubing was pressurized. Although previous investigators, e.g., Simon, were able to maintain a seal with only 14 PSIG pressure in the tubing, a
Figure 2. Cross Sectional View of Complete Standing Wave Tube Assembly
Figure 3. Side View Photograph of Standing-Wave Tube
pressure of 28 PSIG was necessary during this investigation with 30 PSIG necessary during maximum flow conditions. At the high sealing pressures it was somewhat difficult to move the block; this provided impetus for the design of a mechanical drive system.

A roller positioned at each end of the flat maintained pressure on the steel band and provided the seal at the end of the slot. After the steel band passed under the rollers it was directed away from the tube and over an aluminum pulley mounted above the tube. Thus each end of the steel band ended above and parallel to the flat at which point they were attached to each other by means of two small bolts. The junction was located so as to prevent the bolts from coming into contact with the guide pulleys and in this way provided a continuous seal for all positions of the microphone block along the tube axis.

B. Air Supply System

Shop air supplied by the University of Missouri-Rolla power plant was available in the Acoustics Laboratory of the Mechanical Engineering Department where the experiments were conducted. The air was supplied to the laboratory via 2-inch piping at 90 PSIG and a nominal temperature of 75°F.

At the exit of the supply line globe valve, the air line was connected to a 150 cfm separator in an attempt to remove any oil and water vapor present in the supplied air. The discharge of the separator was connected to a large high strength plenum type tank. The tank measures approximately three feet in diameter by five feet long.
A lining was installed within the tank to reduce the noise generated by the flow expansion into the tank and to insulate the tank for the experiments with heated flow. The lining consisted of 2-inch thick glass fiber matting held in place by an interior lining of 1/4 inch square wire mesh. A set of six "Hot Watt" strip heaters was installed within the plenum tank to enable the flow medium to be heated to a maximum of 200°F with the maximum flow rate tested (approximately 150 cfm). The heater leads were routed out of the plenum and connected to a 220 volt A.C. supply available in the laboratory.

The discharge of the plenum tank was connected by 2-inch piping to a flange-type orifice flow-metering section. A straight section of pipe of ten diameters long was provided before the orifice and another section of twenty diameters was provided downstream of the orifice. This insured fully developed flow within the metering section. A temperature readout was provided at the entrance to the metering section. The orifice was constructed and the necessary pressure taps located as specified in the ASTM code. A computer program was written and used to determine the volumetric flow rate and the mean flow velocity. The program was incorporated as a subroutine in the data analysis program.

By following the ASTM code and using an appropriate iteration technique an accuracy of ± 2% was obtained for the flow rate.

After the flow was metered the supply was passed through two large expansion chambers. The chambers were approximately two feet in diameter by three feet in length and were filled with fiberglass
except for a 2-inch open flow area. This lining was provided both for insulation purposes as well as to help reduce the flow-generated noise prior to the flow being introduced into the standing wave tube.

The air on leaving the second expansion chamber was introduced into the standing wave tube through a 30-inch straight section of 2-inch I.D. by 2 1/2 - inch O.D. aluminum tubing. Just prior to the entrance into the standing wave tube the temperature and static pressure of the gas were again measured so that any changes in the Mach number of the medium between the flow-metering section and the tube entrance could be determined.

The flow-piping system between the plenum tank and the standing wave tube as well as the termination tubing of the system was insulated using conventional magnesium pipe insulation. The standing wave tube was insulated by using a formable magnesium compound.

C. Mechanical Drive System

As mentioned in Section A, the air pressure necessary to maintain a good seal between the steel band and the standing-wave tube caused the microphone block to be difficult to move. To alleviate this condition and improve measurements with flow, a mechanical drive system for the microphone block assembly was designed and installed.

The drive system consisted of a 5/8-inch acme thread screw shaft which was mounted parallel to and above the brass guide rails and immediately adjacent to the microphone block. The shaft support consisted of three self-aligning bearings mounted on the same support plates used for the steel band pulleys. Two acme thread nuts were installed on the screw shaft and their relative position maintained
by a heavy automotive valve spring installed between them.

An aluminum drive block was machined for attachment of the drive system to the microphone block. The connecting block was installed over the acme thread nuts and constrained to move with the nuts by using a pair of set screws. Two 3/8-inch threaded bolts were inserted through clearance holes in the microphone block and into the drive block so that the drive system was rigidly connected to the probe system. Details of the drive system assembly are shown in Figure 4. A cursor for accurately determining the longitudinal position of the probe was also attached to the drive block.

The cursor was situated directly above a meter stick installed on top of the cradle supports. A cross-hair was scribed on the cursor and located exactly 8.50 cm upstream of the microphone probe center line. The meter stick was accurately adjusted and locked down so that the position of the microphone relative to the end of the standing-wave tube was directly under the cursor cross-hair, i.e., zero of the meter stick was 8.50 cm upstream of the termination. The microphone assembly could be positioned for measurements within approximately 28 cm of the termination.

A five-inch O.D. belt pulley was installed on the end of the threaded shaft located at the standing wave tube termination. The belt drive for this pulley was driven from a set of speed-reducer pulleys installed on an intermediate shaft and another belt drive to a 1/3 Hp electric motor. The motor was a single-phase induction type with an operating speed of 1725 rpm. The starter winding of the motor
Figure 4. Standing Wave Tube Drive System Assembly
was connected to an external switch so that the motor could be operated in either rotational direction thus allowing automatic traversing of the tube in either longitudinal direction. Various speeds were obtainable with the pulley system; however, all operations were made with a reduction of 15:1. This gave a transverse speed of approximately 5 cm/sec for the microphone block. Evaluation showed this to be the maximum speed of operation for which the sound pressure observed when the microphone was traversed by a location, and that observed at the same location with the microphone stationary, were identical.

The position of the microphone block could be adjusted by hand by turning the intermediate drive pulley shaft. Because of the large mechanical advantage this adjustment could be made with great ease and since one revolution of the intermediate shaft represented .07 cm of block travel, accurate position adjustments were greatly improved over the hand adjustment previously used by other investigators.

D. Anechoic-Termination Design and Construction

An anechoic or reflection-free termination to be attached to the discharge of the test elements was required to aid in the analysis of experimental data. This will be discussed in Chapter IV.

The procedure followed in the design of this termination was suggested by Mr. Gale Myers of Carrier Corporation Research Laboratories, Syracuse, New York. It is based on the fact that an optimum match between the acoustic impedance of the standing wave tube and the air of the room is obtained by using a horn having a catenoidal
profile. This conclusion is reached by solving an approximate form of the wave equation as shown by Wood (27) and Morse (5).

Morse (5) states that the cut-off frequency of the catenoidal horn below which no sound is radiated is a function of the scale factor \( h \) and can be obtained from the expression:

\[
\frac{f_c}{c} = \frac{c}{2\pi h}
\]

(3.1)

where \( f_c \) = cut-off frequency, Hz

\( h \) = scale factor, the distance in which the diameter of the horn increases by a factor of \( e = 2.718 \)

Wood (27), on the other hand, states that the horn may be considered reflection free for the above-cut-off frequency only if the diameter of the discharge of the horn approaches \( 4/\pi \) wave lengths for this frequency. This requirement is to insure sufficient area at the exit so that all the sound is transmitted into the air by providing a nearly exact impedance match.

An expansion rate equivalent to a cut-off frequency of 45.5 Hz was used in the design of the anechoic termination. The criterion of Wood could not be achieved because of the physical size which would be involved; the horn would have to be approximately 21.2 feet long with an exit diameter of 18.2 feet.

Myers (28) had determined that by partially filling the horn with an insulating material he could achieve a satisfactory anechoic termination without meeting the exit size requirement of Wood. It
was therefore decided to attempt a similar result for the anechoic termination used in this research.

The horn was fabricated from seven cone sections constructed out of 12-gauge sheet metal. This resulted in an exit diameter of 24.84 inches. A cylindrical section of 24.8 inches I.D. and 18 inches length was attached to the exit end of the caternoidal profile to give a total length of 13 feet. This latter variation was suggested by Myer's results and was found to be necessary to achieve satisfactory performance of the termination. All joints were sealed using a commercial grade high-temperature epoxy adhesive. A collar was installed at the throat of the horn in order to provide easy connection to the standing-wave tube system.

The interior of the horn was filled with glass fiber (approximately 2.5 lb/cubic feet density) wrapped around a two-inch I.D. sleeve of 1/4-inch square wire mesh. The horn lining began at approximately 18 inches from the throat of the horn and continued the entire length to the exit. The glass fiber was contained in the horn by a perforated wooden disk held in the exit plane by a series of wood screws. A cut-away view of the assembly is shown in Figure 5.

A movable support frame was constructed from 2 x 4 beams and a sheet of 3/4" plywood. The horn was suspended at three locations on nylon stringers. The elevation and pitch of the horn were controlled by adjusting these stringers. The support and horn assembly are shown in Figure 6.
Figure 5. Cross Sectional View of Anechoic Termination
Figure 6. Side-View Photograph of Anechoic Termination and Support System
E. Modification of Transmission-Microphone Station

Simon (11) and Gegesky (29) used a pair of fixed microphone stations for the measurement of transmission coefficients. These microphone stations consisted of cylindrical brass sleeves soldered to two-inch O.D. by 1 7/8 I.D. brass tubes. The microphones were held in place by small set screws located in each sleeve.

One of these transmission tubes was used for the present experiment, with several modifications. A probe identical to the one used in the standing-wave tube was constructed and installed in the orifice by using a press-fit collar (Figure 7). In addition, the tapered bottom of the microphone cavity was filled with epoxy in an attempt to make the cavity similar to that used for the standing-wave tube microphone.

During the early test programs it was determined that the set screw which held the microphone in place was exerting pressure on the piezo-electric crystal. This caused non-repeatable data to be generated since a change in this pressure, e.g., when the microphone was removed for system check calibrations and reinstalled, caused a shift in the microphone response which was not predictable. A new assembly was designed and installed so that the microphone could be removed and reinstalled without causing a shift in response.

The holding assembly consisted of a steel cylinder which was inserted over the existing brass sleeve and held in place by a 10-32 screw. The cylinder was bored to provide a slip fit for the microphone. A step was provided similar to the standing-wave tube microphone block so that the microphone face could not come in contact
Figure 7. Cross Sectional View of Transmission Microphone Station
with the bottom of the cavity. A steel collar was machined to the same O.D. as the cylinder and bored for a tight slip fit of the microphone. A step was machined on the lower I.D. of the collar to allow insertion of an o-ring seal. Four holes were drilled longitudinally through the collar to provide clearance for 10-32 assembly screws.

The cylinder was installed on the existing sleeve and locked securely. Modeling clay was used to provide a seal between these two assemblies. Clay was also used to seal the set screw hole previously employed in the mount block. The collar was inserted over the microphone and an o-ring inserted in the seal slot. The microphone was then inserted into the holding cylinder and locked in place using four 10-32 screws. The o-ring held the microphone securely without allowing pressure to be applied to the crystal case. Figure 7 shows the details of the new assembly.

F. Instrumentation System

The microphones used were of the piezoelectric-ceramic type. In this microphone a pressure diaphragm is sealed to a piezoelectric crystal and when pressure is applied to the diaphragm, the stressing of the crystal causes it to produce an electrical signal. The signal is then strengthened by a preamplifier which transmits the signal over relatively long lines. The preamplifier also provides an electrical impedance match with analysis equipment.

A schematic diagram of the analysis system used to obtain the test data is shown in Figure 8. The signal from the microphone was connected to a narrow-band wave analyzer via a switching box. The
Figure 8. Schematic Diagram of Instrumentation System
switching box allowed selection of either of the two microphones employed during the tests. The analyzer was used to filter the acoustic data so that effectively only the magnitude of the imposed acoustic signal was measured.

This was necessary in the case of flow operations because of the quasi-noise generated by the flow. When flow passes over the pressure diaphragm it causes the transducer to generate a spurious output which is a function of the flow velocity. This is the so-called wind noise phenomenon. By using a filter, a pure tone, which may constitute only a small portion of the total signal; i.e., random wind noise plus pure tone, may be measured. Also, when high amplitude sound waves were propagated in the tube there were always harmonics of the primary frequency present.

The input to the wave analyzer was also connected to one channel of a dual-trace oscilloscope to provide a visual monitor of the data. The filtered output of the analyzer was connected to the second channel of the oscilloscope to provide a monitor to insure that the analyzer was performing properly. The scope monitor also served the purpose of insuring that the analyzer's tuneable filter was centered at the primary frequency. This was possible because in this type of analyzer an input signal at the filter center frequency is shifted 180° when passing through the analyzer. The filtered signal was also connected to a precision VTVM so that accurate readings could be obtained of the sound pressure both as a voltage, analogous to pressure, and a decibel level, analogous to a SPL.
The output of the analyzer was simultaneously connected to the input of channel B of the phase meter. Channel A of the phase meter was connected to the oscillator signal after it had been passed through a sound-level meter used as a precision amplifier. The sound-level meter was used only as an amplifier so that the input signal to the phase meter channel A could be maintained at a level of 800 mv ± 200 mv, for improved accuracy in phase measurements.

The oscillator signal was also connected to a Dynakit 40 watt amplifier which in turn was connected to the acoustic driver which was used to generate pure tone signals in the standing wave tube. A maximum amplitude of approximately 153 db re 0.0002 μbar at the entrance to the test elements was obtained for the frequency range 200 to 600 Hz.
IV. EXPERIMENTAL PROCEDURES

There are many different criteria for specifying the effectiveness of acoustic filters, some of which were described in Chapter III. One of the objectives of the present work is to develop a laboratory technique suitable for silencer evaluation under conditions approaching those which are encountered in practice.

Pure tone analysis was considered to be the most appropriate technique for this investigation. Using this procedure, the reflection and transmission characteristics of the test elements could be determined. Once the reflection factor is obtained, the acoustic impedance of the termination can also be obtained (see Appendix A). Therefore, two important design parameters are available from the same test procedures.

A. Probe Response and System Calibration

Previous investigation with the 2-inch I.D. standing wave tube were conducted without a frequency response calibration of the probe. This is not to imply, however, that the data collected were necessarily incorrect.

The reflection factor was obtained by determining the amplitude of the standing wave at two or more well defined locations. If the response of the microphone probe did not change with variation of the incident sound pressure amplitude, no error was encountered. This was true even if the response varied with frequency. It was felt that verification of this assumption should be made. Also, a necessary requirement for the calculation of some of the acoustic parameters
is knowledge of the magnitude of the incident sound pressure. This necessitated obtaining both the frequency and amplitude response of the probe configuration.

For this purpose solid plug was fabricated to fit into the termination end of the standing wave tube. Next, an opening was bored along the longitudinal axis such that a slip fit would occur between the plug and an installed microphone. The face of the microphone was flush with the face of the plug and perpendicular to the axis of the tube. An o-ring groove was machined on the outside of the plug. An o-ring installed in the groove provided the necessary seal when the plug was installed in the standing wave tube. The details of this assembly are shown in Figure 9.

It was assumed that this assembly would act as a solid piston when installed in the waveguide. Thus, a reflection factor of 1.0 was anticipated, meaning that the incident wave was totally reflected.

In the development of the standing wave tube analysis detailed in Appendix A, it is shown that the sound pressure amplitude at a standing wave maximum is described by the following expression:

\[ p_{\text{max}} = A_1 e^{\alpha x_{\text{max}}} + B_1 e^{-\alpha x_{\text{min}}} \]  

(4.1)

Where A, and B, are the magnitudes of the incident and reflected waves respectively. If the reflected wave has a magnitude equal to that of the incident wave, Equation (4.1) is reduced to the following:

\[ p_{\text{max}} = A_1 e^{\alpha x_{\text{max}}} + e^{-\alpha x_{\text{max}}} \]

\[ = 2A_1 \cosh(\alpha x_{\text{max}}) \]  

(4.2)
Figure 9. Cross Sectional View of Piston Microphone Assembly
The attenuation constant \( \alpha \) has a magnitude on the order of \( 10^{-4} \) cm.\(^{-1} \). The maximum position measurement for the tube is 150 cm. Therefore, the magnitude of the quantity \( \alpha x_{\text{max}} \), approaches zero for all maximum positions. This indicates that the standing wave maximums approach a constant value equal to twice the incident sound pressure. The magnitude of the pressure measured at the face of the end-plug \((x=0)\) is also equal to twice the incident sound pressure. This analysis was used as the basis for the microphone-probe response calibration.

In the experiments, the oscillator was set at a desired frequency and the output level increased until a sound pressure level of 140 db was observed at the end-plug microphone. This is the maximum level which could be measured with the microphones that were used.

A standard reflection-factor analysis was made of the standing wave. This procedure is described in Section C of this chapter. The magnitude of the sound pressure at the maximum nearest the termination was obtained and compared to the level at the end-plug. This gave the response of the microphone probe for the given input frequency.

By adjusting the oscillator output level, changes in the probe response with varying acoustic wave amplitudes were determined for the preset frequency. This procedure was repeated for selected frequencies over the range of 75 to 2500 Hz. The amplitude variations observed were considered negligible as they were well within the accuracy of the measurement system (0.5 db). The reflection factor
obtained during the calibration indicated that the microphone and end-plug were analogous to a solid piston as assumed, over the frequency range of interest. The test data are reproduced in Figure 10.

After removal of the end-plug, the transmission tube and anechoic horn were installed on the discharge of the standing wave tube. With this set up, a calibration of the transmission microphone probe was conducted. With the anechoic horn used as the termination it was assumed that variations in the magnitude of the wave at different locations were caused only by the attenuation inherent in the tube. Therefore, the variation of the transmission pressure from the value predicted assuming only viscous losses was a measure of the transmission probe response.

If the termination is not totally anechoic, a standing wave is formed in the tube. The analytic description of this wave is given in Appendix A. An examination of this expression reveals that the wave is repeated every one-half wave length. The phase of the standing wave changes by $180^\circ$ over this half wave length. These facts are used to reduce the potential for error in the calibration.

The oscillator was set at the initial calibration frequency and its output adjusted to the same maximum level used in the previous probe calibration. The standing wave tube microphone was located at the nearest one-half wave length distance from the fixed position of the transmission microphone.

The amplitude of the sound pressure was determined at both microphone locations. In addition, the relative phase between the oscillator and the filtered signals was obtained. The difference in
Figure 10. Reflection Factor Versus Frequency, Solid Piston Termination
the amplitudes gave the relative difference between the two probe configuration responses. However, differences caused by the tube attenuation must be taken into account in this comparison. The difference between the measured phase angles are also compared to the theoretical value. This theoretical value is obtained from:

\[ \phi_{\text{ther}} = n \cdot 180^\circ \]  

(4.3)

where: \( n \) = number of half-wave lengths separating the two microphones.

By combining the relative response between the two probes with the calibration of the standing-wave-tube probe a total response is obtained for the transmission microphone. The total response did not change with varying incident pressure; i.e., the response was independent of amplitude.

The calibrations were used in the data-analysis computer program to correct the test information for the probe responses. Using this correction, the incident sound-pressure levels at the test element entrance could be calculated accurately.

B. Reflection Factor Measurement Procedure

Static, or no-flow, reflection factors were obtained for comparison to linear theory, and also to investigate the possibility of finite-amplitude effects. At the conclusion of the static investigations, an identical procedure was followed with superimposed steady flow. The only additional test data required were those needed to calculate the flow rate.

The test element was installed between the standing-wave tube and the transmission tube-anechoic termination combination as shown in
Figure 11. All joints not permanently sealed were sealed at this time. The oscillator was adjusted to the desired test frequency and an initial output level. The frequency was accurately set using the precision counter which was also used for continuous monitoring throughout the test operation. The drive system was engaged and a visual record of the standing wave measured by the probe microphone was obtained. This was accomplished by recording the filtered analyzer output on the graphic-level recorder. The traverse was extended through two minimums whenever possible.

The traverse mechanism was used to position the microphone probe near the minimum closest to the termination. The location of the minimum was determined using the procedure recommended by numerous earlier investigators (1,11,29). This consisted of determining the locations of equal sound pressure on either side of the minimum. The locations were then read on the meter stick positioned directly below the cursor of the microphone drive block. The position of the minimum was then designated as the mean of the two measurements.

The microphone block was positioned at the minimum using the hand adjustment of the drive system. The minimum sound-pressure level and its location were then recorded. If a second minimum could be found in the measurement section, its amplitude and location were determined in a similar manner.

The minimum pressure locations were used in the following expressions, which is derived in Appendix A for the distance between two adjacent minimums:
Lined Expansion Chambers

Flow Metering Section

Plenum Containing 6 Strip Heaters

Air Filter

Shop Air Supply

Acoustic Driver

Standing Wave Tube

Transmission Tube

Test Element

Anechoic Termination

Figure 11. Schematic Diagram of Test Facility
where: \( \lambda_o \) = wave length in the tube with no flow

\[ M = \text{Mach number of the flow} \]

Thus, if the Mach number is known, the wavelength of the sound source can be found. Also, the maximum of the standing wave is located exactly half-way between the two minimums. This can easily be demonstrated by using the conditions for the locations of the maximums and minimums developed in Appendix A:

\[ x_{\text{max} 1} - x_{\text{min} 1} = \frac{\lambda_o}{4} (1-M^2) \]  

(4.5)

Similarly, if only one minimum is located within the measurement section, the maximum can be located when the wavelength of the sound is known. In order to determine this wavelength, the value of the corrected speed of sound within the waveguide is first determined. The expression for the corrected speed of sound is determined from Kirchoff's equation given in Chapter II. This expression was confirmed by Simon (11) to be applicable for the 2-inch tube used in this investigation. In addition, the two-pressure minimum procedure described above is used routinely to verify this relationship. Beranek's (10) numerical equation for the corrected speed based on Kirchoff's equation is given as:

\[ C_0 = C[1 - \frac{0.579}{D(\pi f)^{1/2}}] \text{ cm/sec} \]  

(4.6)
The corrected wavelength is then computed from the basic relationship between wavelength and propagation velocity, viz.,

\[ \lambda_o = \frac{c_o}{f} \]  

(4.7)

In the experiments, a set of tube extensions were used for the low frequency operations so that all operations were made with a minimum and maximum located in the measurement section. The values and locations of the minimum nearest the tube termination and its adjacent maximum were used as inputs to the computer program for calculation of the reflection factor and the acoustic impedance of the test element. The nearest minimum was used because it could be shown that any error in position measurement causes an error in the reflection factor phase angle, and this error increases markedly as a function of the number of half wavelengths between the pressure minimum and test element.

Previous investigators (19, 23, 26) have pointed out that the major effects of low superposition occur at the resonances of the elements. Therefore, for the side branch resonators tested, the frequency range investigated was from approximately one-half the resonant frequency to twice the resonant frequency. For an expansion chamber, the range of interest was based on the length of the chamber, Lc; specifically, the frequency range for which the sound wavelength is one-half Lc to twice Lc was used.

When the reflection factor was computed, it was assumed that the attenuation constant was given by Beranek's expression, Equation (2.10), for both static and steady flow conditions.
Ronneberger (12), determined that for flows up to Mach number 0.2, no change occurred in the magnitude of the attenuation constant. The maximum flow rate of the present system was below this value. Also, the waveguide in use was comparable to that used by Ronneberger.

C. Transmission Factor Measurement Procedures

Previous investigators (11,29) used a set of matched entrance and exit tubes with fixed microphone stations to perform transmission factor measurements. A new procedure incorporating the standing wave tube and a transmission tube with a fixed microphone station was employed in the present work. This eliminated the necessity of two separate test set-ups and helped reduce the influence of flow parameter variations which could have caused significant errors.

As the final step of the reflection factor measurements, the standing wave microphone was positioned at the maximum pressure location. After the filtered magnitude was obtained, the phase difference between the filtered signal and the amplified oscillator output was recorded from the phase meter. The output of the oscillator was amplified for this measurement so that the phase meter had two inputs of similar voltage. This was desirable since the phase meter works on the principle of detecting the zero crossing of the inputs by using the slope of these signals. By having nearly equal input amplitudes, their slopes were also nearly equal thus increasing the accuracy of the data.

The switch box was used to connect the fixed transmission tube microphone to the input of the analyzer. The amplitude and phase angle of the filtered signal were recorded. The two amplitude and phase
measurements were used as inputs for the analysis computer program.

The analytic description for the transmission factor developed in Appendix A was solved to obtain the desired information. The standing wave maximum was selected to improve the accuracy of this calculation. The maximum pressure amplitude was always greater than the background noise found in the tube. This is not always the case with the minimum or levels in between the maximum and minimum especially for the case of flow (ef. Sec. F, Chapter II). Also, if the standing wave is distorted such as from the presence of harmonics of the test frequency, the effect of this distortion on the magnitude of the standing wave is minimum at the maximum pressure location.
V. SIDE BRANCH RESONATORS

Side branch resonators have been used for many years as components in acoustic filtering systems. The analysis of this device, as first developed by Helmholtz, has been found to satisfactorily describe its acoustic response when operations are conducted in the linear regime (small amplitude sound waves and static or no-flow operations).

Attempts to describe the nonlinear characteristics of the Helmholtz resonator were attempted in 1938 by Sivian. The primary emphasis in these works have been directed at the nonlinearity of the connector when finite amplitude waves are impinging on the resonator. The more recent works of Phillips (30) and Garrison, et al (19,23) have also attempted to deal with the nonlinear effects resulting when the medium of sound propagation is flowing past the resonator.

A. General Theory of a Side Branch

A convenient starting point for developing acoustic filter theory is the study of the transmission of plane acoustic waves through an infinite wave guide which contains a side branch. At the side branch junction there is a change in the acoustic impedance from the characteristic value of $\frac{\rho_o c}{s}$; which produces a reflected wave.

In addition, part of the incident acoustic energy may be transmitted into the branch and dissipated there. Thus, the energy transmitted beyond the side branch and down the waveguide may be reduced by one or both of these mechanisms.
Let an arbitrary side branch be attached to a uniform pipe of cross section $S$ as represented in Figure 12. If a sinusoidally excited plane wave incident on the left hand side of the side branch is given by

$$p_i = A_1 e^{-i(\omega t - kx)}$$  \hspace{1cm} (5.1)

the reflected wave created at the branch can be represented by

$$p_r = B_1 e^{-i(\omega t + kx)}$$  \hspace{1cm} (5.2)

and the transmitted wave by

$$p_t = A_2 e^{-i(\omega t - kx)}$$  \hspace{1cm} (5.3)

If the origin for the fixed coordinate system is located at the branch junction, the pressures produced by each of the waves at this point are given by

$$p_i = A_1 e^{-i\omega t}$$

$$p_r = B_1 e^{-i\omega t}$$

$$p_t = A_2 e^{-i\omega t}$$

Similarly, the wave entering the branch may be represented by

$$p_b = A_b e^{-i\omega t}$$  \hspace{1cm} (5.5)

Since the basic assumption of plane-wave theory is that the cross dimensions of the pipes used, including the side branch, are small compared to the wavelength of the sound, the condition of continuity
Figure 12. Side Branch Junction
of pressure may be expressed as follows:

\[ p_i + p_r = p_b = p_t \]  \hspace{1cm} (5.6)

From the definition of acoustic impedance the volume velocities associated with each of the waves may also be obtained:

\[ U_i = \frac{p_i}{\rho_o c/s} \]
\[ U_r = \frac{p_r}{\rho_o c/s} \]
\[ U_t = \frac{p_t}{\rho_o c/s} \]
\[ U_b = \frac{p_b}{Z_b} \]  \hspace{1cm} (5.7)

where

\[ Z_b = \text{acoustic impedance of the branch} \]

The condition of continuity of volume velocity may now be represented by

\[ U_i + U_r = U_t + U_b \]  \hspace{1cm} (5.8)

Equations (5.6) and (5.8) can now be combined to give the more useful continuity condition of continuity of impedance.

\[ \frac{U_i+U_r}{p_i+p_r} = \frac{U_t}{p_t} + \frac{U_b}{p_b} \]  \hspace{1cm} (5.9)

The left hand side of Equation (5.9) is the inverse of the impedance looking into the junction from the upstream side. By using Equations (5.4) and (5.7) this impedance can be expressed in terms of the
characteristic impedance of the medium and the incident and reflected pressure amplitudes as follows:

\[ Z = \frac{\rho_0 c}{s} \cdot \frac{A_1 + B_1}{A_1 + B_1} \quad (5.10) \]

If the junction is assumed to be terminated by an infinite pipe (or an anechoic termination) the impedance looking away from the junction is merely the characteristic impedance of the medium, i.e., \( \frac{\rho_0 c}{s} \).

Equation (5.9) can now be rewritten in a simpler form using the definition

\[ \frac{1}{Z} = \frac{1}{Z_t} + \frac{1}{Z_b} \quad (5.11) \]

where

\[ Z_t = \text{acoustic impedance of the termination} \]

Combining Equations (5.10) and (5.11) yields the reflection factor of the junction which, as mentioned in Chapter II, is used as a design parameter:

\[ \bar{R} = \frac{B_1}{A_1} = \frac{-\rho_0 c/2s}{\rho_0 c}{2c} + Z_b \quad (5.12) \]

The impedance of the side branch can be written in its two basic components, resistance and reactance.

\[ Z_b = R_b + iX_b \quad (5.13) \]

The energy reflection coefficient in terms of pressure is defined as the square of the reflection factor magnitude.
The energy transmission coefficient is obtained by combining Equations (5.6) and (5.14).

\[ \frac{B}{A}^2 = \frac{\left(\frac{\rho c}{2s}\right)^2}{\left(\frac{\rho c}{2s} + R_b\right)^2 + X_b^2} \]  \hspace{2cm} (5.14)

It can be shown that the energy flux density per unit area, called the intensity of the wave, is the product of the particle velocity and pressure. Integrating over the time period of the sinusoidally-varying wave then yields the average energy present in the wave. Conservation of energy can thus be represented in a ratio form by the following expression:

\[ 1 = \left(\frac{p_r}{p_1}\right)^2 + \left(\frac{p_t}{p_1}\right)^2 + \alpha_b \]  \hspace{2cm} (5.16)

The final term, \(\alpha_b\), of Equation (5.16) is defined as the absorption coefficient of the branch. It is the ratio of the energy absorbed or dissipated in the branch to the energy incident upon the juncture. It is obtained as a function of the impedance of the branch by combining Equations (5.14), (5.15), and (5.16), and is given by

\[ \alpha_b = \frac{\rho c}{\left(\frac{\rho c}{2s} + R_b\right)^2 + X_b^2} \]  \hspace{2cm} (5.17)

The preceding development is general with no restrictions placed on the construction or form of the side branch. It is desirable to
extend the general theory to the specific case of a Helmholtz resonator used as a side branch, which is a configuration commonly used in noise control applications.

If expressions can be obtained for the acoustic resistance and reactance of the resonator, they can be substituted into the previously developed expressions to obtain the system response.

B. Linear Impedance of Helmholtz Resonator

In the following discussion, small-amplitude acoustic waves are assumed. In addition, all of the dimensions of the resonator in question are assumed to be small compared to the wave length of the incident sound. Thus, the case now considered is described as the "linear impedance case".

The resistance portion of the impedance as was pointed out in Chapter II, is a dissipation term. The dissipation in the resonator is mainly due to viscosity and heat conduction losses in the neck. Additional dissipation can be achieved by the use of a material in the resonator cavity or by installing resistive material or elements in the neck. Only those losses incurred by the resonator itself are considered here.

The viscous losses may be thought of as a surface resistance to the flow of the acoustic wave. If the radius of curvature of the aperture is large compared to the viscous boundary layer, the expression developed for the oscillatory flow over an infinite plane surface can be used (15).

\[ R_s = \frac{1}{2} \left( \frac{2\mu \omega}{\eta} \right)^{1/2} \]  

(5.18)
This expression, however, has been shown to be in serious error for the region near the rim of an aperture (15). In this region the radius of curvature can be of the same order of magnitude as the boundary layer thickness. The tangential particle velocity has a large magnitude in this region and since the dissipation is a function of this magnitude squared, the dissipation can be quite large in this area.

The total viscous dissipation at the resonator neck consists of two parts, the losses on the side walls and the losses at the ends. The former is adequately expressed using Equation (5.18) and the latter is a function of the assumed velocity profile in the neck and the geometry of the neck junction. The assumption of a uniform velocity profile across the neck cross-section is most often used. However, Ingard has shown that this assumption leads to a lower value of the so-called end correction applied to the viscous resistance and has empirically determined that the total viscous resistance of a resonator neck is best described by the following expression:

\[ R_{\text{vis}} = 4R_s \left( \frac{t+d}{s_0 d} \right) \]

\[ = (8\rho \mu \omega)^{1/2} \left( \frac{t+d}{s_0 d} \right) \]  \hspace{1cm} (5.19)

where:

- \( t \) = thickness or length of the neck
- \( d \) = diameter of the neck
- \( s_0 \) = cross-sectional area of the neck
Next, it is desired to obtain the acoustic reactance of the resonator. This consists of two parts, namely, the mass reactance of the neck and the compliant reactance of the cavity volume. The mass reactance of the resonator neck is defined by the following expression.

\[ \omega M = \omega \frac{\rho_o l_{\text{eff}}}{s_0} \]  

(5.20)

where:

- \( \omega M \) = mass reactance
- \( \rho_o \) = density of medium in-neck
- \( \omega \) = angular frequency of the acoustic wave
- \( l_{\text{eff}} \) = the effective length of the resonator neck

The effective length is greater than the thickness of the neck. This is due to the fact that the oscillating mass is contained not only in the neck but actually extends slightly beyond both ends of the neck. At low frequencies it is generally assumed (6) that the end correction may be approximated by \( \Delta l = .85r \), where \( r \) = radius of the opening. Using this relationship, the effective length of the neck is given by:

\[ l_{\text{eff}} = t + .85d \]  

(5.21)

The compliant reactance of the resonator cavity is the resistance of the volume to compression by the acoustic wave. It is defined by the following expression:

\[ \frac{1}{\omega C} = \frac{\rho_o c^2}{\omega V} \]  

(5.22)
where:

\[ \omega C = \text{compliant reactance} \]

\[ c = \text{adiabatic speed of sound in the resonator cavity} \]

\[ V = \text{volume of the cavity} \]

With these two expressions, the total reactance of the resonator is now written:

\[ X_b = \omega \frac{\rho o_{\text{eff}}}{s_o} - \frac{\rho o c^2}{\omega V} \]  \hspace{1cm} (5.23)

C. Experimental Measurements of the Effective Length of the Side-Branch-Resonator Necks

1. Finite Amplitude Variations

As mentioned in section B of this chapter, the assumption of small amplitude waves is made when designing resonator elements. Since this is not the case in many normal applications, the preliminary investigations were conducted with a two-fold purpose. First, verification of the linear impedance expressions was to be obtained, and second, the region of finite-amplitude waves was to be investigated to determine its extent and to ascertain if appropriate expressions for the acoustic impedance in this region could be obtained.

Three variable-backing-depth resonators, each with a different neck diameter, were tested, see Table I and Figure 13. By varying the backing depth, a total of sixteen resonator configurations were tested. In addition, a fourth resonator was used during preliminary response testing; however, a mechanical failure occurred in its construction so that no conclusive data were obtained.
Figure 13. Cross Sectional View of Experimental Side Branch Resonator
Table I. Resonator Configurations

<table>
<thead>
<tr>
<th>Resonator Number</th>
<th>Neck Length (in)</th>
<th>Neck Diameter (in)</th>
<th>Cavity Diameter (in)</th>
<th>Backing Depth (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-1</td>
<td>1.00</td>
<td>0.750</td>
<td>5.047</td>
<td>1.67-6.55</td>
</tr>
<tr>
<td>R-2*</td>
<td>1.00</td>
<td>1.250</td>
<td>5.047</td>
<td>1.67-6.55</td>
</tr>
<tr>
<td>R-3</td>
<td>1.00</td>
<td>1.500</td>
<td>5.047</td>
<td>1.95-5.36</td>
</tr>
<tr>
<td>R-4</td>
<td>0.953</td>
<td>0.375</td>
<td>2.000</td>
<td>1.94-5.41</td>
</tr>
</tbody>
</table>

*Destroyed During Testing

Using the expression for the reactance of the resonator previously developed and the definition of resonance as the frequency for which the reactance is zero, the well-known expression for the resonant frequency of a Helmholtz resonator is obtained namely,

\[ f_c = \frac{c}{2\pi} \left( \frac{s}{\frac{\alpha}{V_{\text{eff}}}} \right)^{1/2} \]  

(5.24)

The only unknown or variable in Equation (5.24), once the backing depth is fixed, is the effective length.

Three methods were used to experimentally determine the effective length of the resonator. First, after the resonator was installed between the standing wave tube and the anechoic terminatin, a standard reflection factor test was conducted over the desired frequency range. It can be shown that at resonance the reflection factor has a phase angle of 180° (cf. Appendix A). Thus, by interpolating between the frequencies where the reflection factor phase angles approach 180°, the resonant frequency is determined. Once the
resonant frequency is known, the effective length is calculated using Equation (5.24).

Next, since at resonance the reflection factor phase angle is 180°, it can also be shown that a minimum in the standing wave occurs at exactly one-half wave length from the resonator neck. The standing-wave-tube microphone was positioned one-half wavelength from the center of the resonator neck. The desired frequency was set on the oscillator supplying the acoustic signal and the backing depth of the resonator volume was varied. By monitoring the output of the standing-wave-tube microphone, the backing depth corresponding to a minimum pressure at the one-half-wavelength position was determined and the effective length of the neck computed as before. Since the standing wave tube was not designed for precision impedance measurements, the accuracy of the phase angle and/or the microphone location was subject to possible error. For ±2% accurate impedance calculation, the minimum location must be measureable within ± 0.10 mm. Thus, additional testing was necessary to confirm the above results.

A solid piston was used to terminate the resonator-standing wave tube combination and was installed sufficiently close to the entrance of the resonator so that the shunt resistance of the tube extension could be neglected. The resonator was thus said to terminate the standing wave tube. A microphone was installed through the piston so that it measured the pressure at the entrance to the resonator. A second microphone was installed in the adjustable piston of the resonator so that it sensed the pressure in the resonator volume (see Figure 12). By measuring the pressure differential between the
two microphones and the phase difference of the two signals, the acoustic impedance of the resonator was determined using the expression developed in Appendix E.

There are two criteria which may be used for determining the resonance conditions. These are the conditions that the pressure differential is a minimum, and that the phase difference between the measured pressures is 90° at resonance. The latter criterion was used in this investigation. After a desired frequency was set, the backing depth was adjusted to give a 90° phase difference. The backing depth was then measured and the effective length calculated from Equation (5.24).

As a check on the accuracy of the two previous methods, once the phase difference of 90° was obtained the minimum was located in the standing wave tube. The minimum occurred at the one-half-wavelength position within the accuracy of the measurement system (± 0.05 cm).

The third method of obtaining the effective length was considered to be the most accurate in that no response corrections had to be applied to the recorded measurements. In addition, the two pressure readings could be obtained in a much faster sequence than the standing wave measurements. This served to reduce the potential for error due to drift in the instrumentation system.

The resonators were each tested for a minimum of five frequencies. The incident sound amplitude was varied over the measurable range of the system with a maximum incident sound pressure level of 153.3 dB re 0.0002 bar obtainable for frequencies of 200 to 600 Hz. The effective
length was observed to decrease with increasing amplitude for all cases tested; however, this decrease was not significant when compared to the value obtained at the lowest sound levels measured. The high amplitude value of effective length represented a maximum deviation of 7.1% from the low level value. (175 Hz, resonator R-1) which was within the limits of the measurement system accuracy. Therefore, no correlation with increasing amplitude was attempted.

The values of the effective length were observed to remain essentially constant for a range of incident pressures in all tests. The mean value of this low level effective length was obtained for each resonator using the data from all of the frequencies tested. In addition three expressions in common use for the effective length were determined for comparison with the experimental mean value.

The first expression may be termed the "standard" effective length value. It is derived from the radiation load on a piston in an infinite plane wall and is, according to Kinsler and Frey (6), given by

\[ l_{\text{eff}} = t + 0.85 \, d_0 \]  

(5.25)

where:

\( t \) = neck length
\( d_0 \) = neck diameter

The latter term of the right-hand side of (5.25) is termed the end correction". Only a very limited number of special cases have been solved for this parameter due to the complexity of the development.
The second expression used for comparison is that used by Davis and his colleagues (13). This is, in effect, an empirical relationship developed during their extensive testing program:

\[ l_{\text{eff}} = t + 0.8 \left( \frac{\pi d^2}{4} \right)^{1/2} = t + 0.71 \, d_o \quad (5.26) \]

The third expression used was that found by Garrison in his resonator tests to define the effective length for a high frequency, small physical size resonator used in an array:

\[ l_{\text{eff}} = t + d_o \quad (5.27) \]

Finally, since each of these three expressions involve the neck length plus a proportionality constant, say \( \beta_1 \), times the neck diameter, a correlation of this constant for the experimental mean value of the effective length was obtained for each resonator. The deviations of the mean experimental values from the above expressions were obtained. In addition, the deviation of all experimental values from these values were obtained. A listing of these deviations is given in Table II. Because of the wide variation in the accuracy of the three compared expressions for determining the effective length, the empirically-developed expressions were used for all ensuing work in this investigation. Using the values of \( \beta_1 \) from Table II, the effective length is given by

\[ l_{\text{eff}} = t + \beta_1 \cdot d_o \quad (5.28) \]
Table II. % Deviation of Experimental Effective Length for Three Analytical Models

<table>
<thead>
<tr>
<th>Resonator Number</th>
<th>$l_{\text{eff}}$ mean (Experimental)</th>
<th>Proportionality Constant $\beta_1$</th>
<th>% Deviation Based on $l_{\text{eff}} = t + d_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$t + 0.85d_0$ $t + 0.71d_0$ $t + d_0$</td>
</tr>
<tr>
<td>R-1</td>
<td>1.535</td>
<td>.788</td>
<td>$\pm 5.0$ $-12.9 \pm 3.1$ $-6.87 \pm 7.1$ $-18.5$</td>
</tr>
<tr>
<td>R-3</td>
<td>1.834</td>
<td>.628</td>
<td>$\pm 6.2$ $-24.2 \pm 4.7$ $-11.00 \pm 10.6$ $-26.5$</td>
</tr>
<tr>
<td>R-4</td>
<td>1.399</td>
<td>1.202</td>
<td>$\pm 3.4$ $7.8 \pm 4.9$ $12.50 \pm 3.4$ $-4.7$</td>
</tr>
</tbody>
</table>
2. Variation of the Effective Length with Flow Past the Resonator Neck

At the conclusion of the finite amplitude testing, the flow system described in Chapter III was employed to investigate the effect of a steady flowing medium past the branch junction on the acoustic impedance of the single side branch resonators. With the flow system in use, only the first two methods discussed in the preceding section were possible.

After a desired flow rate was set using the flow-metering orifice, a series of resonance tests were performed using the frequencies employed for the static finite amplitude tests. The frequencies used varied from 100 to 350 Hz. The standing wave tube microphone was located upstream of the junction at a distance equal to one-half the wavelength times the quantity \((1-M^2)\), where \(M\) is the Mach number of the flow assuming a uniform velocity profile having a magnitude equal to the mean velocity for the mass flow rate used. The variation in the microphone location from that used for the no-flow case was due to the fact that the introduction of a flowing medium changes the acoustic propagation velocity. The propagation velocity of the waves traveling with and against the flow is obtained by multiplying the static value by the quantities \((1+M)\) and \((1-M)\) respectively.

The resonator was tuned for resonance conditions by adjusting the backing depth until a minimum sound pressure level occurred at the microphone. A standard reflection factor measurement was then made to obtain the impedance of the resonator. The effective length was calculated using Equation (5.24). This procedure was repeated for a
series of amplitude settings at each of the test frequencies. A total of three flow rates were used with Mach numbers of 0.042, 0.083, and 0.107.

The effective length of the resonator neck was observed to decrease with increasing flow past Mach number. This phenomenon has been observed by a number of investigators and several attempts to correlate the length variation with the flow-past velocity and/or Mach number have been attempted. The vast majority of these investigations have been performed using resonator arrays with the most extensive work being done by Mechel, Mertens, and Schilz (31) and Garrison (19). A comparison of the former work to that accomplished on a single element array (similar to the test elements of this investigation) was conducted by Phillips (30).

Correlation of the effective length change with the flow past Mach number was attempted in two forms. First the change in the total effective length was examined in a manner after Garrison. For this correlation it was assumed that the effective length of the resonator with flow past could be represented by

\[ l_{\text{eff}} = l_{\text{eff}_\text{NF}} (1 - \varepsilon_1 \cdot M) \]  

(5.29)

or,

\[ \varepsilon_1 = [1 - \frac{l_{\text{eff}}}{l_{\text{eff}_\text{NF}}}] \frac{1}{M} \]  

(5.30)

The effective length used for the no-flow case was obtained using Equation (5.28).
The value of $\varepsilon_1$ varied over a considerable range for the various flow rates tested; see Figure 14. The largest variation of $\varepsilon_1$ occurred for Resonator R-3 (1.5 inch neck diameter) with values of 1.13 at 200 Hz and $M=0.042$; and 3.92 at 275 Hz and $M=0.083$. The mean value determined using all the data points obtained was $\varepsilon_1 = 2.19$. This is of the same order of magnitude as found by Garrison for his high frequency resonator arrays. Assuming $\varepsilon_1 = 2.19$ - constant, the maximum deviation of the experimentally obtained effective length from that calculated using Equation (5.29) was approximately 19.08% and was observed for Resonator R-3 with $M=0.083$ and a frequency of 275 Hz.

It was possible to obtain a much lower variation by treating the coefficient as a function of both the flow-past Mach number and the resonator geometry. Thus, by obtaining the mean value of the data points for each resonator at a fixed flow condition and various frequencies, the maximum observed deviation of the experimental values was approximately 14% and for the vast majority of points did not exceed 8%.

This means of describing the effective length with a flow-past medium has two inherent disadvantages. The first is that as the Mach number exceeds the value such that $\varepsilon_1 M$ exceeds the static mass end correction length, the oscillating mass would not occupy the entire resonator neck. Secondly, once a critical Mach number is reached such that $\varepsilon_1 M > 1.0$, Equation (5.29) would yield $l_{eff,V} < 0.0$. There is no physical description possible for a negative effective length and an effective length of zero would imply that you are no longer dealing with the simple oscillator as there is no means to transmit the sound wave to the cavity.
Figure 14. Effective Length Flow Coefficient Versus Flow Mach Number
The second correlation of effective length with flow was accomplished assuming that the change could be accounted for by a reduction in the mass of gas extending beyond the resonator neck. This is represented by

$$l_{eff} = l_{eff_{NF}} - \Delta L$$  \hspace{1cm} (5.31)

or

$$l_{eff} = t + \beta_1 \cdot d_o - \Delta L$$  \hspace{1cm} (5.32)

Thus, if

$$\delta l = \beta_1 \cdot d_o$$

$$l_{eff} = t + \delta l \left[ 1 - \frac{\Delta L}{\delta l} \right]$$  \hspace{1cm} (5.33)

The value of $\Delta L/\delta l$ (called the "end-correction ratio") was obtained for all of the flow-past resonance testing. The mean values for each flow case and each resonator are given in Figure 15. Inspection of these data indicated that for the two smaller diameter resonator necks tested, namely 0.375 in. and 0.750 in., the value of the ratio increased more rapidly as the Mach number was increased. The largest resonator tested did not have the same trend but appeared to be sloping more toward a constant value. The results for the two smaller resonators exhibit trends similar to those obtained by Phillips (30); however, a large difference in the amplitudes was observed. There was a very significant difference in the ratio of the resonator neck diameter to the flow duct diameter between these two investigations; the ratio was several orders of magnitude larger for the present investigation.
Figure 15. End Correction Ratio Versus Flow Past Mach Number
This would indicate a more pronounced possibility of turbulence at the resonator neck which could account for the magnitude variation. In addition, the geometry of the resonator neck entrance due to the variation in duct diameter is quite pronounced and most certainly would have an effect on the flow into the resonator. Ingard and Labate (17) have shown that the changes in resonator impedance are directly related to the flow circulation through the neck.

No finite amplitude effects were observed in the effective lengths obtained during the flow past testing. In addition, operation of the test set-up using a maximum flow gas temperature of 206°F gave comparable results.

D. Experimental Measurements of the Acoustic Resistance of Side Branch Resonators

1. Finite Amplitude Variations

During the effective-length testing, the acoustic resistances of the resonators were calculated from the test data. Again, as with the effective-length measurements, the two-fold purposes of defining the linear and nonlinear expressions for this impedance were manifest. Since the particle velocity and impedance are directly related, and because of previous investigator's successes in relating the nonlinear resistance to this quantity, it was also calculated from this data. The development of the expressions used are given in Appendix C.

The two methods used for calculating the acoustic resistance were the use of reflection factor measurements, and the pressure-phase differential technique discussed in Section C of this chapter. All of these tests were performed with the resonator tuned to the test
frequency; i.e., the resonant and test frequency were identical. As a check on the relative accuracy of the two methods, a set of simultaneous tests was conducted. The deviation of the values obtained in the reflection factor calculations from those obtained by the pressure-phase differential method were computed. The latter method was used as a base because of its inherent accuracy, as discussed in the previous section. This comparison was made since only the reflection factor technique was available for the flow tests. The results of the comparison are shown in Table III.

Table III. Comparison of Acoustic Resistance Using Reflection Factor and Pressure-Phase Differential Methods. Resonator No. R-1

<table>
<thead>
<tr>
<th>Frequency</th>
<th>% Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg.</td>
</tr>
<tr>
<td>150</td>
<td>+2.1</td>
</tr>
<tr>
<td>175</td>
<td>+9.6</td>
</tr>
<tr>
<td>200</td>
<td>+7.1</td>
</tr>
</tbody>
</table>

The pressure-phase differential procedure was used for all static testing of the elements at resonant conditions. The value of the acoustic resistance was observed to be constant until a certain critical particle velocity was attained in the neck. For all operations above this critical particle velocity, the resistance varied as a linear function of particle velocity.

The low-level or "linear" value of the resistance did not agree with that obtained using Equation (5.19). Therefore, it was necessary
to determine if the additional resistance could be accounted for. The mean value of the linear resistance was obtained from a series of measurements for each resonator at each test frequency. Analysis of these values revealed that the resonator neck resistance could be expressed as a function of the square root of the frequency. This functional relationship is identical to the one for the resistance caused from viscous effects along the walls. Thus, it was assumed that the additional resistance represented internal resistance due to roughness of the neck bore and resistance of the acoustic flow due to the neck geometry. The additional resistance was assumed to be a viscous term and it was correlated as an end correction such that the linear resistance could be computed from

\[ R_b = \left(8\rho\mu\omega\right)^{1/2} \frac{t+d}{s_o d_o} \left(\frac{\gamma_r}{s_o} + \frac{\gamma_r}{s_o}\right) \]  

where:

\[ \gamma_r = \left[\frac{R_b \exp t+d}{R_b^o - 1}\right] \left(\frac{t+d}{d_o}\right) \]  

A comparison of the experimental values to those obtained using (5.29) and the appropriate \( \gamma_r \) for each resonator indicated a maximum variation of approximately ± 9.0%. The largest variations was for R-3 which has a very small magnitude of resistance, thus making it the most difficult to measure accurately.

In the nonlinear region, the resistance approached a linear function of the particle velocity in the throat; see Figures 16, 17, and 18. A correlation was obtained using the analogy of Garrison. An empirical coefficient \( \beta_r \) was obtained using the following expression.
Figure 16. Acoustic Resistance at Resonance for
Resonator R-1 Neck Diameter = 0.75 inch

\[ R_b = \beta r \frac{\rho u_o}{s_o} \]
Figure 17. Acoustic Resistance at Resonance for Resonator

\[ R_b (\# M/FT-SEC) \]

A non-linear region is observed in the particle velocity, \( u_0 \) (FT/SEC) plot.

\[ R_b = \frac{\rho u_0}{s_0} \]

\( \rho \) = density, \( u_0 \) = particle velocity, \( s_0 \) = speed of sound.

R-3 Neck Diameter = 1.50 inch
Figure 18. Acoustic Resistance at Resonance for Resonator
R-4 Neck Diameter = 0.375 inch
\[
R_b = \beta_r \cdot \frac{\rho u_o}{s_o}, \quad \text{from which}
\]

\[
\beta_r = \frac{R_b s_o}{\rho u_o}
\]  

(5.36) \hspace{1cm} (5.37)

As an aid in defining the nonlinear region of each of the resonators, the specific acoustic resistance was plotted versus particle velocity; samples are shown in Figures 19, 20, and 21. As the neck diameter increases, the particle velocity at which the nonlinear region is attained is reduced in magnitude. Since there is substantiating evidence that the finite-amplitude-caused nonlinearity of the resistance is associated with the generation of turbulent flow, the Reynolds numbers based on the critical particle velocities were calculated and are listed in Table IV below.

<table>
<thead>
<tr>
<th>Resonator Number</th>
<th>(u_o)-Critical (fps)</th>
<th>Reynolds Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-1</td>
<td>6.0</td>
<td>1950</td>
</tr>
<tr>
<td>R-3</td>
<td>4.5</td>
<td>2930</td>
</tr>
<tr>
<td>R-4</td>
<td>12.0</td>
<td>1950</td>
</tr>
</tbody>
</table>

Phillips (30), in his investigation, determined that the Reynolds number based on the particle velocity attained a magnitude of approximately 2100. He states that, "the transition from linear to
Figure 19. Specific Acoustic Resistance at Resonance for Resonators R-1 and R-4, 150 Hz
Figure 20. Specific Acoustic Resistance at Resonance for Resonators R-3 and R-4, 250 Hz
Figure 21. Specific Acoustic Resistance at Resonance for Resonators R-1, R-3, and R-4, 200 Hz

\[ R_B = \rho u_0 \]
nonlinear resistance seems to be similar to the transition to turbulence in pipe flow."

Since the experimental data obtained by Garrison (19) were available, the Reynolds numbers based on his assumed critical velocity of 60 ft/sec were calculated and were in the range of 1600-2100. Although turbulence generation is a complex function of geometry as well as the Reynolds number of the flow, it appears that (to a reasonable approximation) the Reynolds number may be used as a criterion for establishing the approximate nonlinear region. A Reynolds number on the order of 1800-2200 is suggested.

Using the experimental values of $R_p$, the coefficient $\beta_r$ was calculated for all of the experimental data. A different coefficient was obtained for each of the three resonators. The results of this correlation as well as the results of the linear end-correction calculations are given in Table V below.

<table>
<thead>
<tr>
<th>Resonator Number</th>
<th>Linear Resistance End Correction ($\gamma_{l}$)</th>
<th>Finite Amplitude Nonlinear Coefficient ($\beta_r$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-1</td>
<td>0.715</td>
<td>0.941</td>
</tr>
<tr>
<td>R-3</td>
<td>1.113</td>
<td>0.871</td>
</tr>
<tr>
<td>R-4</td>
<td>1.173</td>
<td>0.834</td>
</tr>
</tbody>
</table>

No means were available to obtain the flow coefficients of the resonator necks in order to perform an analysis similar to that of
Garrison; however, inspection of the results listed in Table V indicate a similar trend. The additional resistance of the neck is given by \( \gamma_r \); therefore, as \( \gamma_r \) increases the neck resistance increases. The nonlinear coefficient was obtained by a best fit approximation to the data for which high magnitude particle velocities were obtained. The point at which the data closely approached this straight line approximation was then selected as the beginning of the nonlinear region. The particle velocity for this point was specified as the critical particle velocity and the coefficient was refined using only the data for particle velocities equal to or greater than this value. Since acoustic resistance and flow resistance are analogous, as the acoustic resistance increases so does the flow resistance, and thus, the orifice coefficient of the neck would increase. If the expression developed by Garrison and given as Equation (2.16) is valid, namely,

\[
R_b = 0.36 \frac{\rho u}{c_f} \frac{s_o}{s_0} \tag{2.16}
\]

then the coefficient \( \beta_r \) corresponds to \( 0.36/c_f s_0 \). It is assumed that the linear resistance end correction \( \gamma_r \) is directly related to the flow coefficient \( c_f \) since both are an indication of viscous losses through the neck. Thus, as \( \gamma_r \) increases, \( c_f \) also increases, thereby causing \( \beta_r \) for the nonlinear region to decrease. This is exactly the trend of the experimental data.


The acoustic resistance of the resonator was obtained during the effective length variation measurements described in Section C-2 of
this chapter. The reflection factor measurements were used to obtain the impedance of the resonator and thus the resistance as well.

Correlation of the change in the resistance with flow to the flow Mach number was attempted by assuming the following functional relationship.

\[ R_{bv} = R_{bNF} (1 + \varepsilon_r \cdot M) \]  

(5.38)

where:

\[ \varepsilon_r = \text{flow resistance coefficient} \]

During preliminary analysis, a large variation in \( \varepsilon_r \) was observed indicating a more complex relationship between the no-flow and flow resistances. Additional testing was accomplished over the maximum amplitude range measureable for each of the flow rates.

Analysis of these data indicated two distinctive regions over the amplitude range. These are shown in Figure 22 and 23. When flow was introduced into the waveguide and the neck pressure, which was a function of the driver output and the impedance load on the driver, was relatively low, a large increase in the resistance occurred. As the neck pressure was increased the resistance decreased in value. At a sufficiently high neck pressure, the ratio of the resistance with flow to the no-flow value became independent of any further increase in neck pressure. In the high-amplitude region, the value of the resistance was describable using Equation (5.38) with \( \varepsilon_r = 1.95 \). This was the same value obtained by Garrison (19) in his investigation of resonator arrays. In his investigation, only the high amplitude
Figure 22. Ratio of Acoustic Resistance With and Without Flow for Resonator R-1. Neck Diameter 0.75 Inch.
Figure 23. Ratio of Acoustic Resistance With and Without Flow for Resonator R-4. Neck Diameter 0.375 Inch
region was used.

Examination of Figures 22 and 23 shows the variation resulting from two neck diameters and, therefore, geometry changes. Of the two resonators, the larger diameter neck tests exhibited a much more rapid transition to the high amplitude region. Similar testing of the resonator having the largest diameter neck was not accomplished because of system limitations. Since the resistance of this resonator was relatively small, the ratio of the maximum to minimum standing-wave pressures was quite large. The magnitude of the quasi-noise caused by the flow established the lowest measurable sound pressure and the physical limits of the microphone above which it ceases to give accurate readings establishes the maximum measurable sound pressure. This range of pressures allowed a limited range of neck pressures to be used for this resonator. For the other resonators, the ratios of pressures were lower which allowed a wider range of neck pressures to be used.

The hot flow testing of the 0.750-inch neck diameter resonator indicated an identical trend. These tests were performed with an air flow temperature of 206°F and are also shown in Figure 23.

E. Resonator Response Measurements

The ability of an acoustic element to attenuate incident sound is the design requirement on which it is predicated. Therefore, the response for each of the test elements was obtained in the form of reflection and transmission factors. These measurements were made under no-flow and hot and cold flow conditions.
The no-flow theoretical response of the test element was obtained by solving Equations (5.12) and (5.16). Equations (5.34) and (5.23) were used to calculate the acoustic resistance and reactance for the linear response region. Equation (5.36) was used for the nonlinear resistance. The linear resonator reactance was used for all no-flow response calculations based on the impedance measurement results.

An incident sound pressure level was assumed for which the resonator was to be designed. At resonance the acoustic impedance of the orifice is purely resistive and the particle velocity can be calculated from the definition of impedance. If the design sound pressure is assumed to lie in the linear-response region, Equation (5.34) defines the resistance. Next, the particle velocity, \( u_0 \), is calculated from the impedance relationship

\[
\frac{u_0}{\rho_b} = \frac{p_{\text{assumed}}}{R_b} \quad (5.39)
\]

Once the particle velocity is known, the Reynolds number based on this velocity and the resonator neck diameter is calculated. A critical Reynolds number of 2100 was used in the response calculations. Thus, if the Reynolds number was greater than 2100, the acoustic resistance was recomputed using Equation (5.36).

Each of the test elements was adjusted for two different resonant frequencies. Reflection and transmission factor measurements were obtained for each configuration with both no-flow and cold flow operations. For all no-flow operations, the measured and predicted values were in good agreement, see Figures 24 through 35, pages 101 through 112.
Figure 24. No Flow Reflection Characteristics of Resonator R-1-4.30. $d_0 = 0.750$ inch, $L_B = 430$ inch
Figure 25. No Flow Transmission Characteristics of Resonator
R-1-4.30  $d_0 = 0.750$ inch, $L_B = 4.30$ inch
Figure 26. No Flow Reflection Characteristics of Resonator
R-1-2.18  \( d_o = 0.750 \) inch, \( L_B = 2.18 \) inch
Figure 27. No Flow Transmission Characteristics of Resonator
R-1-2.18 \( d_0 = 0.750 \) inch, \( L_B = 2.18 \) inch
Figure 28. No Flow Reflection Characteristics of Resonator R-3-3.96 \( d_o = 1.50 \) inch, \( L_B = 3.96 \) inch
Figure 29. No Flow Transmission Characteristics of Resonator
R-3–3.96  $d_0 = 1.50$ inch, $L_B = 3.96$ inch
Figure 30. No Flow Reflection Characteristics of Resonator
R-3-1.69 \( d_o = 1.50 \) inch, \( L_B = 1.69 \) inch
Figure 31. No Flow Transmission Characteristics of Resonator
R-3-1.69 \( d_o = 1.50 \) inch, \( L_B = 1.69 \) inch
Figure 32. No Flow Reflection Characteristics of Resonator
R-4-3.90 \( d_0 = 0.375 \) inch, \( L_B = 3.90 \) inch
Figure 33. No Flow Transmission Characteristics of Resonator
R-4-3.90 $d_o = 0.375$ inch, $L_B = 3.90$ inch
Figure 34. No Flow Reflection Characteristics of Resonator R-4-2.40 d_0 = 0.375 inch, L_B = 2.40 inch
Figure 35. No Flow Transmission Characteristics of Resonator R-4-2.40 $d_0 = 0.375$ inch, $L_B = 2.40$ inch
The changes in the transmission and reflection factors due to finite amplitudes are exhibited in Figures 36 and 37. The response curves were obtained by assuming constant neck pressures of 90 db (nonlinear region), respectively. The piston microphone was installed at the termination of the test resonator. Reflection factor measurements were obtained over the frequency range while maintaining the neck pressure as measured by the piston microphone at 90 db. Similar measurements were made over a narrow frequency range centered about the resonant frequency with a constant 110 db neck pressure. The transmission factor was computed from the impedance measurements made using the pressure-phase differential method. The reflection factor measurements were in excellent agreement with the values calculated from the simultaneous impedance measurement. A nominal variation of ± 5% based on the calculated value was obtained.

For the response measurements made with flow, a constant neck pressure was assumed. The resistance with flow and the effective length with flow were determined from the empirical relations developed from the impedance measurements. Since there is no direct means of measuring the neck pressure with the present test equipment it was not possible to maintain the desired neck pressure. Therefore, at the conclusion of the response testing, an average value of the neck pressure over a narrow frequency range centered at resonance was obtained from the test data. This new neck pressure was then used to obtain the response curves for the element.

Only limited agreement was obtained between measured and predicted values, see Figures 38 through 49. This was attributed to the variability of the neck pressure. Two restrictions inherent in the instrumentation
Figure 36. No Flow Reflection Characteristics with Finite Amplitude Incident Wave, R-1-2.18
Figure 37. No Flow Transmission Characteristics with Finite Amplitude Incident Wave, R-1-2.18
Figure 38. Reflection Characteristics with Steady Flow for Resonator R-1-4.30 and Flow Mach Number of 0.107
Figure 39. Transmission Characteristics with Steady Flow for Resonator R-1-4.30 and Flow Mach Number of 0.107
Figure 40. Reflection Characteristics with Steady Flow for Resonator R-1-2.18 and Flow Mach Number of 0.084
Figure 41. Transmission Characteristics with Steady Flow for Resonator R-1-2.18 and Flow Mach Number of 0.084
Figure 42. Reflection Characteristics with Steady Flow for Resonator R-3-3.96 and Flow Mach Number of 0.107
Figure 43. Transmission Characteristics with Steady Flow for Resonator R-3-3.96 and Flow Mach Number of 0.107
Figure 44. Reflection Characteristics with Steady Flow for Resonator R-3-1.69 and Flow Mach Number of 0.084
Figure 45. Transmission Characteristics with Steady Flow for Resonator R-3-1.69 and Flow Mach Number of 0.084
Figure 46. Reflection Characteristics with Steady Flow for Resonator R-4-3.90 and Flow Mach Number of 0.084
Figure 47. Transmission Characteristics with Steady Flow for Resonator R-4-3.90 and Flow Mach Number of 0.084
Figure 48. Reflection Characteristics with Steady Flow for Resonator R-4-2.40 and Flow Mach Number of 0.084
Figure 49. Transmission Characteristics with Steady Flow for Resonator R-4-2.40 and Flow Mach Number of 0.084
system negated the possibility of obtaining data over a sufficient range of incident pressures to allow comparison of constant-neck-pressure measurements over the entire frequency range. First, there is an upper and lower bound on the pressure amplitudes which can be measured. These are the maximum sound pressure levels above which the instrumentation system is saturated and the minimum sound pressure below which flow noise causes error. Secondly, the neck pressure cannot be measured directly, but must be calculated from other data. If the response is calculated for each frequency based on the experimentally obtained neck pressure, the agreement between measured and predicted values is greatly improved, see Figure 50.

Although the above mentioned calculation improves the match of experimental data, there was one other area of disagreement between measured and calculated response parameters for flow operations. This occurred for the 0.375-inch neck diameter resonator, R-4 (Figures 48 and 50). The measured transmission factors exceeded the predicted values by significant amounts. This was attributed to the termination not being completely anechoic. The discrepancy in the data only occurred for this resonator and only where the transmission factor approached unity. Thus, in this frequency region a large portion of the energy is transmitted beyond the test element and toward the termination where it can be reflected. Once this occurs, the assumption of a constant pressure downstream of the element is no longer valid nor is the analysis using this assumption.
Figure 50. Corrected Reflection Characteristics with Steady Flow for Resonator R-3-3.96
F. Conclusions

Finite-amplitude effects encountered in acoustic side branch resonators are dependent on the acoustic particle velocity within the resonator neck. This is a function of neck diameter and the pressure at the entrance to the side branch. The region of finite amplitude effects may be characterized by a critical Reynolds number based on the particle velocity and neck diameter. From the present work, as well as that of previous investigators, a critical Reynolds number of 2100 is indicated; however, this should not be construed as an absolute criterion since the finite amplitude effect is associated with the very complex phenomenon of turbulence.

In the region of finite-amplitude effects, the acoustic resistance of the resonator can be expressed as a linear function of the particle velocity. The constant of proportionality for this expression has been, in the past, an empirically determined expression. Based on the quantitative work of Garrison (23) and a qualitative analysis of the present work, it is concluded that this constant can be expressed as a function of the orifice coefficient of the resonator neck. It was further concluded that the variation of the effective length due to finite-amplitude waves is negligible.

The variation of the acoustic resistance with flow past the resonator was determined to be a complex function of both the flow Mach number and the pressure at the resonator neck. A region characterized by very-high-amplitude waves was detected within which the ratio of the resistances with and without flow is a linear function of the flow Mach number only. The limited examination of
heated flow operations did not reveal any variation in the results not accounted for by effect of the temperature on the wave length.

Because of the complex variation of the resistance with flow and acoustic pressure, response calculations based on an assumption of a constant pressure over the frequency range may be in large error if the assumed pressure is not within the very-high-amplitude region ($\approx 150 \text{ db}$).

The large amount of time necessary to perform accurate standing wave measurements is a pronounced drawback to this technique. In addition, the sealing problems required for heated operations add to the complexity of the problem. In investigating the variation of side branch element response, the acoustic impedance must be investigated since its' variation is the cause of the response change. Thus, the approach of using fixed microphone stations and/or a technique such as the pressure-phase differential method is deemed more appropriate by this author. In addition, it is felt that additional capabilities for operation at significantly higher gas temperatures and sound-pressure levels is a very desirable goal regardless of the measurement technique employed.
VI. EXPANSION CHAMBERS

The expansion chamber is a reactive type of acoustic filter element in wide use today. In the past, it has been designed almost solely on the basis of linear plane wave theory. Little is known about variations in its response due to finite-amplitude effects and/or to a flowing propagation medium.

A. Reflection and Transmission Factors for Sudden Area Changes

Since the expansion chamber is formed by two discontinuities in cross section connected in series, the individual discontinuities will at first be discussed separately. A common method for the analysis of discontinuities is to write the expressions for continuity of acoustic pressure and mass flow rate (1). However, since this method does not account for any flow effects which may be present, a linearized perturbation approach was decided on for the present work. The phenomena occurring at an expansion or contraction must be delt with separately since the mechanisms available for acoustic losses are considered to be quite different.

For the case of the contraction, it was assumed that no flow losses occurred, i.e., the flow was considered isentropic. This is a reasonable assumption for the flow Mach numbers used. In addition, the work of Ronneberger (20) for Mach numbers of the order of magnitude similar to those of the present investigation indicated minimal flow losses. The continuity of mass flow and the conservation of energy for the flow can be written for the contraction shown in Figure 51, as follows:
Figure 51. Schematic of Sudden Contraction

Figure 52. Schematic of Sudden Expansion
\[ \eta u_a p_a = u_b p_b \]  
(6.1)

\[ \frac{u_a^2}{2} + \frac{\gamma}{\gamma-1} \frac{p_a}{\rho_a} = \frac{u_b^2}{2} + \frac{\gamma}{\gamma-1} \frac{p_b}{\rho_b} \]  
(6.2)

where:

\[ \eta = \frac{S_a}{S_b} = \text{area ratio} \]

\[ u = \text{mean particle velocity} \]

\[ \rho = \text{density} \]

\[ p = \text{static pressure} \]

\[ \gamma = \frac{c_p}{c_v} = \text{ratio of specific heats} \]

The subscripts denote the cross section at which the equations are written. It is assumed that these sections are sufficiently removed from the discontinuity so as to be out of the region of any three-dimensional effects.

At a sudden area change part of an incident sound wave is reflected and part is transmitted. For this analysis the change in area is assumed to be terminated anechoically so that only a transmitted wave is present beyond the change. Making use of the standard isentropic relationship (6), namely, \( \rho = \frac{p}{c^2} \), the perturbations due to a superimposed acoustic wave can be written.

\[ u_a \rightarrow u_a + \frac{p_i - p_r}{\rho_a c_a} \quad u_b \rightarrow u_b + \frac{p_t}{\rho_b c_b} \]

\[ p_a \rightarrow p_a + p_i + p_r \quad p_b \rightarrow p_b + p_t \]

\[ \rho_a \rightarrow \rho_a + \frac{p_i - p_a}{c_a^2} \quad \rho_b \rightarrow \rho_b + \frac{p_t}{c_b^2} \]  
(6.3)
Applying the relationships of Equation (6.3) to Equation (6.1) and neglecting higher-order terms gives the perturbed continuity equation in linearized form

\[ \eta [p_r (1+M_a) - p_r (1-M_a)] \frac{c_b}{c_a} = p_t (1+M_b) \]  

(6.4)

Similarly, the perturbed energy equation in linearized form is given by:

\[ [p_r (1+M_a) + p_r (1-M_a)] \frac{\rho_b}{\rho_a} = p_t (1+M_b) \]  

(6.5)

which reduces to the familiar form of the continuity of pressure for zero flow, i.e., \( M_a = M_b = 0 \). Combining Equations (6.4) and (6.5), the reflection and transmission factors are obtained after simplification.

\[ R = \frac{p_r}{p_i} = \frac{(\eta \frac{c_b}{c_a} - \frac{\rho_b}{\rho_a})(1+M_a)}{(\eta \frac{c_b}{c_a} - \frac{\rho_b}{\rho_a})(1-M_a)} \]  

(6.6)

\[ T = \frac{p_t}{p_i} = \frac{2(1+M_a)\eta}{(1+M_b)(1+\frac{M_b}{M_a})\left(\frac{c_b}{c_a}\right)^2} \]  

(6.7)

A graphical illustration of the variation of these two design parameters is given in Figures 53 and 54. It is evident from these figures that the flow can cause a relatively large variation in the parameters away from the classical values obtained using only linear plane-wave theory. It should be noted that Equations (6.6) and (6.7) reduce to the classical form when the no flow case is assumed.
Figure 53. Variation of Reflection Factor with Flow
Mach Number for Sudden Contraction
Figure 54. Variation of Transmission Factor with Flow Mach Number for Sudden Contraction
The isentropic relations do not satisfactorily describe the phenomenon occurring at the sudden area expansions. Generation of ring vortices which interact with the sound field (20,32) cause losses to occur at the expansion thus negating the use of the isentropic conditions. Various models are available (33) with the two most widely used based on the assumptions of an adiabatic expansion and a constant pressure expansion (32). The assumption of an adiabatic expansion was used for this investigation as it lends itself to the concept of vortex interactions.

For an adiabatic expansion the isentropic relation relating density and pressure fluctuations is not valid. In addition entropy fluctuations occur downstream of the expansion even if they are not present upstream. This entropy wave does not generate significant pressure or velocity fluctuations (34) although it does cause density and temperature fluctuations. With these assumptions, the perturbations to be applied to the flow equations can be written (referenced to Figure 52) as follows:

\[
\begin{align*}
    & u_a \rightarrow u_a + \frac{p_i - p_r}{\rho_a c}\left(\frac{u}{a} + \frac{p}{a}\right) + \frac{p_r}{\rho_a c}\left(\frac{p}{a}\right) \\
    & u_b \rightarrow u_b + \frac{p_r}{\rho_b c}\left(\frac{p}{b}\right) \\
    & p_a \rightarrow p_a + p_i + p_r \\
    & p_b \rightarrow p_b + p_t \\
    & \rho_a \rightarrow \rho_a + \frac{p_i + p_r}{c_a} \\
    & \rho_b \rightarrow \rho_b + \frac{p_t + \rho}{c_b} \quad (6.8)
\end{align*}
\]

where:
\( \sigma / c_b^2 \) = density fluctuation due to the pressure of the entropy wave and is a function of the ratio of specific heats, and the viscosity, and conductivity of the medium.

Applying the perturbations (6.8) to the continuity equation (6.1) gives the continuity equation for the adiabatic expansion

\[
\eta [p_i(l+M_a) - p_r(l-M_a)] \frac{c_b}{c_a} = p_t(l+M_b) + M_b \sigma
\]  

(6.9)

The perturbations are next applied to the energy equation (6.2) to obtain the linearized energy equation for the expansion

\[
[p_i(l+M_a) + p_r(l-M_a)] \frac{\rho b}{\rho a} = p_t(l+M_b) - \frac{1}{\gamma - 1} \sigma
\]  

(6.10)

Because of the inclusion of the entropy wave there are now three unknowns involved in Equations (6.9) and (6.10); therefore, a third equation is necessary to produce a unique solution. This is provided by the momentum equation written across the discontinuity. First the pressure acting over the area \( S_b - S_a \) must be examined. For the quasi-stationary conditions assumed, a reasonable assumption concerning the pressure on the end surface is that it is constant over the area and is equal to the upstream static pressure. With this assumption the momentum equation can be written as follows:

\[
\rho a u_a^2 + p_a = \rho b u_b^2 + p_b
\]  

(6.11)

Applying the perturbations,

\[
p_i (\eta M_a^2 + 2\eta (M_a + 1)) + p_r (\eta M_a^2 - 2\eta M_a + 1)
\]

\[
= p_t (M_b^2 + 2M_b + 1) + M_b^2 \sigma
\]  

(6.12)
Equations (6.9), (6.10), and (6.12) are now solved to obtain the reflection and transmission factors for the adiabatic expansion. The details of this development are given in Appendix E. The variation between the results assuming adiabatic and isentropic expansions are shown in Figures 55 and 56.

One other case must be investigated in order to obtain all necessary reflection and transmission factors for analysis of a simple expansion chamber. This is the case of a sound wave propagating in the direction opposite to the flow. Powell (22,23) has shown that there is considerable difference between the cases of acoustic flow with and against the medium flow. Only the case where the sound wave encounters a contraction need be considered. It has been shown that the sound wave produced by the shed ring vortices discussed for the expansion are instigated at the edge of the expansion by the initial sound wave (20). This mechanism is not present for the case being discussed. Therefore, it was decided to assume that this contraction can be described as an isentropic process. This is in agreement with the findings of Ronneberger (12) who examined a limited number of contractions. For the isentropic process with and against the medium flow only a sign change on the Mach numbers is necessary (22). Thus, Equations (6.6) and (6.7) can be rewritten. Plots of the response functions using the flow Mach numbers as parameters are given in Figures 57 and 58.

B. Analytical Model for a Simple-Expansion Chamber

The normal approach used in analyzing the simple expansion chamber is a steady-state method in which the progress of an incident
Figure 55. Variation of Reflection Factor with Flow Mach Number for Sudden Expansion with Entropy Wave
Figure 56. Variation of Transmission Factor with Flow Mach Number for Sudden Expansion with Entropy Wave
Figure 57. Variation of Reflection Factor with Flow Mach Number for Sudden Contraction - Incident Wave Against Flow
Figure 58. Variation of Transmission Factor with Flow Mach Number for Sudden Contraction - Incident Wave Against Flow
wave into and through the element is examined. A complete description of this technique is found in Simon (11) so that it will only be described briefly in order to underscore the assumptions involved.

The reflection and transmission factors for each discontinuity are assumed known and are those developed in the preceding section for incident waves propagating in the flow direction. The only losses encountered in the transmission from the inlet to the exit end of the expansion chamber are assumed to be those due to viscous effects at the walls. It is further assumed that these losses can be described by the attenuation-constant concept described in Chapter II and that the attenuation constant is not affected by flow, i.e., Equation (2.10) still holds. All cross-sectional dimensions satisfy the restrictions for propagation of plane waves. All walls and end plates are considered rigid so that no boundary vibrations are present.

An incident plane wave is propagated down the wave guide. When the wave encounters the expansion at the inlet to the expansion chamber, part of the wave is reflected and part is transmitted. The transmitted wave travels the length of the chamber being attenuated by the viscous wall effects. At the outlet contraction this wave is split into reflected and transmitted portions. The transmitted portion propagates out the discharge and the reflected portion traverses back across the chamber. The process is repeated as the originally transmitted wave is reflected back and forth within the chamber, being reduced by attenuation and the reflection-transmission process at the end discontinuities.
By summing up each of the wavelets exiting from the chamber, the total transmitted sound wave is obtained. The total reflected wave can be obtained in the same manner. The ratios of the reflected and transmitted waves to the incident wave; i.e., the reflection and transmission factors, are now obtained. These expressions are infinite series of the form $1 + Z + Z^2 + \ldots$ which converge to $\frac{1}{1-Z}$ for $|Z|<1$.

Thus,

$$R = R_1 + \frac{R_T T_1 e^{-ikl}}{1-R_2 R_3 e^{-i2kl}} \quad (6.13)$$

and

$$T = \frac{-ikl}{\frac{T_1 T_2 e^{-ikl}}{1-R_2 R_3 e^{-i2kl}}} \quad (6.14)$$

where the subscripts denote the sequence of the discontinuities encountered.

Equations (6.13) and (6.14) were solved on a digital computer to obtain the theoretical response of the test elements. The expressions developed in the preceding sections for the individual discontinuities; i.e., Equations (6.6) and (6.7) and those developed in Appendix E, were incorporated in this calculation. The linear, or no-flow, response was obtained by letting $M_a=M_b=0$.

C. Experimental Measurements of Expansion Chamber Response

1. No-Flow Comparison to Linear Theory

Two variable length expansion chambers were constructed, see Figure 59. The chambers were fabricated from 1/4-inch wall thickness aluminum tubing having a 5-inch inside diameter and a maximum length
Figure 59. Schematic of Variable Length Expansion Chamber
of 30 inches. The variable length allowed the frequency at which
the maximum attenuation occurred to be adjusted over a wide range.
As the maximum attenuation frequency is varied so is the width of
the frequency range for which large attenuation is obtainable. In
addition to the variable length expansion chambers, a third chamber
was constructed of 1/4-inch wall thickness PVC tubing with a normal
inside diameter of approximately 2.90 inches. The end plates were
formed from PVC plate stock. The chamber length was fixed at 37 inches.

A test expansion chamber was installed between the standing wave
tube and the anechoic termination. A frequency and amplitude was
delivered by the acoustic driver and standard reflection-factor and
transmission factor measurements were made. The amplitude of the
incident wave was increased in approximately 3db steps and the tests
repeated until the maximum level obtainable with the amplifier and
acoustic driver system (~150dB) was reached. The frequency was
varied over the range for which the sound wavelength is one-half to
two times the length of the expansion chamber.

As pointed out earlier in Section B, the linear response normally
used for the design of expansion chambers is obtained by letting
\( M_a = M_b = 0 \). Very good agreement between experimental and theoretical
values of both reflection and transmission factors was obtained as
indicated in Figures 60 thru 65. No finite amplitude effects could
be found for the range of incident sound pressures investigated. This
was not unexpected as there is no resistive-type element in the
expansion chamber comparable to the neck resistance of the simple
resonator.
Figure 60. No Flow Reflection Characteristics of Expansion Chamber E-1-30
Figure 61. No Flow Transmission Characteristics of Expansion Chamber E-1-30
Figure 62. No Flow Reflection Characteristics of Expansion Chamber E-2-18
Figure 63. No Flow Transmission Characteristics of Expansion Chamber E-2-18
Figure 64. No Flow Reflection Characteristics of Expansion Chamber E-3-37
Figure 65. No Flow Transmission Characteristics of Expansion Chamber E-3-37
Another area of concern in the design of expansion chamber type elements is the high frequency region where the wavelength of the sound violates the assumptions for plane wave propagation only. For the 5-inch I.D. expansion chamber this region where three-dimensional effects may occur began at a frequency beyond the usable range of the test set-up. Therefore, an expansion chamber type element was fabricated from heavy gauge sheet metal having dimensions of 9 x 24 x 18 inches length. For this size and dimensions, the frequency above which three dimensional effects may occur as determined from Equation (2.1) was 420 Hz.

This element was tested over the frequency range 200 to 850 Hz. The linear response was calculated for this range assuming only plane wave propagation, i.e., one-dimensional waves. Comparison of the experimental data to the predicted values did not yield totally conclusive results. This was because of the fact that in order to construct an element that exhibits three-dimensional effects in the usable frequency range of the test equipment, its transmission factor was extremely low. The experimental data yielded transmission factors very close to the reflection factors of the anechoic termination up to 1000 Hz which was approximately 2.5 times the predicted transition frequency. Therefore, it was concluded that three-dimensional effects, if they existed, were negligible for the amplitude range tested.

2. Measurements with Flow Through the Element

Two flow rates, \( M=0.084 \) and \( M=0.108 \), were used for the evaluation of the response of the expansion chamber elements. Reflection and transmission factor measurements were made over the frequency
range and the results compared to the values calculated using the mathematical model previously developed.

The experimental results were in very good agreement with the predicted response for the 5 inch I.D. chambers as shown in Figures 66 through 69, pages 157 through 160. The results from the 2.90-inch I.D. chamber were not as conclusive. The measured reflection factors were in very good agreement with the predicted values; however, the measured transmission factors tended to be lower in the high attenuation frequency range, see Figures 70 and 71. This is a region of high reflection factor. Examination of Equation (6.13) indicates the dependency of the total reflection factor of the expansion chamber on the reflection factor of the expansion. The agreement between measured and predicted values lends credence to the mathematical model of the expansion. The transmission factor, in addition to being dependent upon the models of the expansion and contraction, can be strongly influenced by the viscous dissipation along the walls of the chamber.

For all the work performed, the attenuation constant given by Equation (2.10) was used to describe viscous losses. However, examination of the bore of the PVC tubing used to construct the 2.90-inch I.D. chamber revealed a rough surface texture which would cause the viscous effects along the wall to be greater than for the smooth surface finish assumed. The variation of the measured transmission factors from those predicted using the previously-developed model were thus attributed to a large increase in viscous dissipation along the chamber wall.
Figure 66. Reflection Characteristics with Steady Flow for Expansion Chamber E-1-30 and Flow Mach Number of 0.107
Figure 67. Transmission Characteristics with Steady Flow for Expansion Chamber E-1-30 and Flow Mach Number of 0.107
Figure 68. Reflection Characteristics with Steady Flow for Expansion Chamber E-2-18 and Flow Mach Number of 0.084
Figure 69. Transmission Characteristics with Steady Flow for Expansion Chamber E-2-18 and Flow Mach Number of 0.084
Figure 70. Reflection Characteristics with Steady Flow for Expansion Chamber E-3-37 and Flow Mach Number of 0.084
Figure 71. Transmission Characteristics with Steady Flow for Expansion Chamber E-3-37 and Flow Mach Number of 0.084
Two test operations were performed with heated flow and using the 2.90-inch chamber at the test element. The maximum inlet temperature attained was 180°F. Normal reflection and transmission measurements were accomplished and the results compared to the cold flow operations. No temperature effects other than the change in propagation velocity were observed. This is illustrated in Figures 70 and 71 where the frequencies of the heated-flow operation have been corrected to the cold-flow medium temperature.

Comparison of the flow and no-flow responses indicates a substantial increase in the reflection factor, while the transmission factor changes are very slight. The experimental measurements confirmed this phenomenon; see Figures 66 and 71. This is possible because of the change in the relationship of the acoustic pressure to the energy. Because of this relationship which involves the flow Mach number and the acoustic pressure, it was possible to obtain a reflection factor greater than unity and still have a balance of energy into and out of the element.

The kinetic energy of a flow with a superimposed sinusoidally excited plane sound wave is given by

\[ V_a = \frac{1}{2} \left( \rho_a + \frac{P_a}{c^2} \right) (u_a + u)^2 \]  \hspace{1cm} (6.15)

where:

- \( u_a \) = particle velocity of flow
- \( u \) = acoustic particle velocity

The potential energy is
Therefore, the energy flux density is

\[ \varepsilon = \rho_a u^2 (1+M) + \rho_a u u_a + \frac{1}{2} \rho_a u c M^2 \]

It can be shown that upon integrating over a period of the acoustic wave and multiplying by the propagation velocity, that the intensity of the acoustic wave is

\[ I_i = \frac{p_i^2}{\rho_a c^2} (1+M)^2 \]  

(6.17)

Similarly for sound propagating against the flow

\[ I_r = \frac{p_r^2}{\rho_a c^2} (1-M)^2 \]  

(6.18)

Thus, the reflection factor could exceed unity without violating energy considerations.

In order to determine if the reflection factor magnitude was greater than unity, the variation in the minimum pressures measured in the standing-wave tube was observed. For a reflection factor less than unity, the magnitude of the minimum pressures increases as one proceeds from the element toward the source. Similarly for a reflection factor greater than unity, the minimum magnitudes decrease from the element to the source. This latter case is possible only when the medium is flowing through the standing-wave tube and element, i.e., \( M > 0 \).
D. Experimental Measurements of the Acoustic Response of Combinations of Expansion Chamber and Resonator Elements

Acoustic filters are normally constructed by combining the two basic types of elements, the side branch resonator and the expansion chamber. Thus, it was decided to test a basic combination with flow to determine if the models used for the individual elements could successfully be applied.

The approach used for generating the theoretical response is a lumped-parameter approach. In this method the response of the individual elements is assumed known. The combination is assumed to be terminated anechoically. An incident wave is converted to reflected and transmitted portions at the first element. The transmitted portion is attenuated by viscous effects as it proceeds toward the second element. The form of the results are identical to those obtained for the expansion chamber in that the analysis is directly analogous. Thus, for combinations, Equations (6.13) and (6.14) are applicable using the individual element parameters. The subscripts 1 and 2 distinguish the relative position of the elements with respect to the source. And the subscript 3 designates the response of the first element to a sound wave propagated against the flow. If more than two elements are employed, the first two are combined using Equations (6.13) and (6.14). The results of this analysis are then used as the lumped model for analysis with the next element in the system. The procedure is repeated as many times as necessary to work through the complete system (7).
The 30-inch long expansion chamber and the 0.375-inch-neck-diameter resonator were chosen for this study. It was decided to join the elements using as short a connection length as possible. This was done in order to reduce the so called pass bands (7) as much as possible and to more closely approximate the manner in which combinations are normally used.

The combination was tested with two flow Mach numbers and with no medium flow. At the completion of this series of tests, the positions of the elements were interchanged. Although this change does not cause any variance in the total system response according to the linear theory, it was felt that this might not be true when flow superposition is involved.

The results of these tests are given in the form of reflection and transmission factor curves in Figures 72 through 77. In general, the data agrees reasonably well with the predicted values. The experimental values were again observed to be lower.

E. Conclusions

A linearized perturbation approach as used in this investigation is a reasonable approach for the development of the acoustic response of discontinuities in flow pipes. Based on the results of this investigation, the assumption of an adiabatic expansion and isentropic contraction for expansion chambers with low flow Mach numbers is deemed valid. Temperature effects were restricted to the change in propagation velocity and, thus, the acoustic wave length for the temperature range investigated.
Figure 72. No Flow Reflection Characteristics for a Series Combination of a Side Branch Resonator and Expansion Chamber

- Resonator, Expansion Chamber
- Expansion Chamber, Resonator
Figure 73. No Flow Transmission Characteristics for a Series Combination of a Side Branch Resonator and Expansion Chamber.

- Resonator, Expansion Chamber
- Expansion Chamber Resonator
Figure 74. Reflection Characteristics for a Series Combination of a Side Branch Resonator and Expansion Chamber with Steady Flow, $M = 0.042$

- $\circ$ - Resonator, Expansion Chamber
- $\square$ - Expansion Chamber, Resonator
Figure 75. Transmission Characteristics for a Series Combination of a Side Branch Resonator and Expansion Chamber with Steady Flow, $M = 0.042$

- ○ - Resonator, Expansion Chamber
- □ - Expansion Chamber, Resonator
Figure 76. Reflection Characteristics for a Series Combination of a Side Branch Resonator and Expansion Chamber with Steady Flow, $M = 0.107$

- Resonator, Expansion Chamber
- Expansion Chamber, Resonator
Figure 77. Transmission Characteristics for a Series Combination of a Side Branch Resonator and Expansion Chamber with Steady Flow, $M = 0.107$

- ○ - Resonator, Expansion Chamber
- □ - Expansion Chamber, Resonator
The maximum medium temperature which can be used in the present system is dictated by the seal arrangement of the standing wave tube. In addition, the standing wave measurements are very tedious and require a high degree of skill in order to be repeatable and accurate. For these reasons it would appear that a fixed microphone system should be considered for any additional work contemplated in this area of research. This would involve using the four-pole-parameter approach suggested by Baade (26). This approach appears to be more amenable to a computerized muffler design approach. Although this is a different approach than used in the present work, it is felt by the author that the results herein given can be transformed and used in the four pole parameter method.

Further investigation of three-dimensional wave phenomenon is necessary before conclusions can be reached regarding their effect on acoustic filter elements. This type of examination could be performed with the fixed microphone system suggested above.
VII. SUMMARY AND CONCLUSIONS

Basic acoustic elements were examined during this research program to investigate the effects of steady flow and finite amplitude sound waves on their acoustic response. A primary objective of this work was to provide the designer with the capability to account for the variation of the response from linear theory caused by these phenomena.

Two basic types of elements were examined; the side branch resonator and the simple expansion chamber. Different mechanisms of sound attenuation were present for the element types, each requiring appropriate analysis techniques and analytical modeling.

Finite amplitude effects were observed for no-flow operations. At the conclusion of these tests the elements were examined under conditions of flow past and flow through the element for the resonators and expansion chambers respectively. The side branch resonator experiences nonlinear response variations as a result of finite amplitudes and air flow past the resonator neck. Only flow-induced effects are encountered for the expansion chamber, at incident sound levels up to 150 dB.

Although some work examining finite amplitude effects has been reported in the past, the vast majority of these investigations have been concerned with small physical size and relatively high resonant frequencies. In addition they have primarily examined arrays of resonators; e.g., Garrison (19), as opposed to a single element. The present work extends the work to the lower frequency range (100-400 Hz resonant frequencies) and larger physical sizing (similar to
element sizing employed in automotive exhaust systems).

Finite amplitude waves impinging on the side branch resonator caused a large increase in the acoustic resistance at resonance, with no detectable change in the acoustic reactance. The resistance in the nonlinear, finite amplitude, region was dependent on the acoustic particle velocity within the neck and independent of the frequency, as compared to the linear region (small amplitude waves) where it is dependent on the frequency and independent of particle velocity.

The nonlinear region was successfully modeled as a function of particle velocity, as given by

\[ R_b = \beta_r \frac{\rho u_0}{s_o} \]

The coefficient \( \beta_r \) is an empirical proportionality constant. A qualitative comparison of the coefficient magnitudes obtained to an analogous set of coefficients obtained by Garrison (19) for high frequency arrays of resonators indicated identical trends. That is, Garrison's coefficient can be written as

\[ \beta_r = \frac{0.36}{c_f^2} \]  

(7.2)

so that, as the geometry of the resonator neck (and the flow coefficient) change, the value of \( \beta_r \) will change. It was concluded therefore that this mathematical model of the nonlinear resistance is applicable for side branch resonators in general.

Analysis and comparisons of the previous work of Garrison (19) and Phillips (30) and the present study indicate that a reasonable
approach for determining the beginning of the nonlinear region is
based on the Reynolds number of the acoustic flow through the neck
of the resonator. A Reynolds number of 2100 based on the acoustic
particle velocity and neck diameter was found to be a reasonable
criterion. The transition from the linear to nonlinear regions is
similar to the analogous flow phenomenon; because of the complexity
of turbulent flow the 2100 figure cannot be construed as an absolute
magnitude, but merely a useful guideline.

Also, as a first approximation for design purposes, the non-
linear coefficient can be assumed to be unity. This would be
the value obtained using Garrison's function and a flow coefficient
of 0.6, which is commonly used for a sharp edged orifice.

The flow past studies revealed two distinct regions of nonlinear
resistance effects. The regions could be delineated according to
the pressure at the neck of the resonator. When a flow is established
past the resonator the acoustic resistance of the element immediately
increases. For low magnitude acoustic waves the resistance becomes
three or more times the value of the no-flow resistance. As the
acoustic pressure is increased the ratio of the flow to no-flow re-
stance decreases and approaches a constant value for sufficiently high
pressures. This analysis is based on the assumption that the no-flow
acoustic resistance is defined including the finite amplitude region.
A nominal incident sound pressure level of 130 db is deemed an appro-
priate transition point to the high amplitude region. For the high
amplitude region the analogy of Garrison (19) applies; i.e., the ratio
of the flow to no-flow resistance is a linear function of the flow
Mach number only, as given by

\[
\frac{R_DV}{R_{NF}} = (1 + 1.95 M)
\]

(7.3)

The effective length of a side branch resonator is reduced by steady flow past the neck. For the single elements tested, this reduction is described as a reduction in the end length correction that accounts for oscillating gases outside of the neck. Thus the maximum reduction results in an effective length equal to the thickness of the neck; i.e., the end correction goes to zero. This is in qualitative agreement with the results of Phillips (30) for his single resonator tests. The change can be expressed as a function of Mach number with the effective length asymptotically approaching the thickness of the neck at a relatively low value of the steady flow Mach number (\(\sim 0.12-0.15\)). The variation in effective length with flow is a very complex process which appears to be dependent on geometry and/or flow turbulence.

A recommended procedure for the design of single side branch resonators is as follows:

1. Assume a nominal value for the sound pressure of the frequency component to be attenuated.
2. Calculate the particle velocity which this pressure would cause in the resonator neck at resonance, using resistance from linear theory.
3. Compute the Reynolds number based on the above particle velocity and neck diameter.
4. If the Reynolds number is less than 2100, linear theory may be used.

5. For Reynolds numbers greater than 2100, assume that the resonator will be operated in the finite amplitude region and compute the particle velocity assuming \( \beta_r = 1.0 \).

6. Using \( u_0 \) obtained in Step 5, compute the nonlinear resistance.

7. If steady flow is not present, obtain response using the reactance from linear theory and applicable resistance impedance terms.

8. If flow is present, use the flow Mach number to compute the change in the effective length as follows: for \( M > 0.15 \),

\[ l_{eff_v} = t, \text{ the thickness of the resonator neck; for } M < 0.15, \]

assume a nominal value of \( l_{eff_v} = t - 61[1-.15M] \).

9. If the assumed pressure is greater than 130 db assume the resistance is given by \( R_v = R_v^b (1+1.95M) \).

10. If the assumed pressure is less than 130 db, select a reasonable value of resistance from Figures 22-23.

11. Obtain the resonator response using the developed impedance terms; and Equations (5.12) and (5.14).

The simple expansion chamber response was not influenced by finite amplitudes for incident pressures up approximately 150 db. Flow effects were observed and the response generated using the developed analytical models were in good agreement with the measured values.

A mathematical model for a simple expansion chamber was developed by using the steady state approaches of Powell (21) and Ronneberger (20) to obtain the responses of the individual discontinuities. The
individual response parameters were then combined using a lumped parameter approach to obtain the total element response. The flow process at the expansion is assumed to be adiabatic while the process undergone at the contraction is assumed to be isentropic. It is recommended that this mathematical model be used for the design of expansion chambers in place of the normally used linear theory in that it provides a better fit to the boundary conditions at the entrance and exit of the chamber (cf. Section VI-B).

At frequencies with wavelengths greater than half the characteristic element dimensions, higher order modes are not significant, and element performance is adequately described by assuming plane waves. In practice, when higher frequency components are to be attenuated, elements of relatively small physical size or arrays of components are usually used. The reduction in physical sizing with increasing frequency is amenable to the assumption of one-dimensional acoustic flow. Furthermore, in many applications a silencer is designed to attenuate the lowest frequency of interest, and higher frequencies which violate the plane wave assumption are of secondary importance.

The effects of heating the air flow were to change the propagation velocity of the acoustic wave, which in turn causes the resonant frequency of the element to shift. Thus, for the flow temperatures in the range observed (70-200°F), measured changes in element response are adequately described by appropriate changes in wave propagation velocity.


VITA

Raymond Hershel Schaffart was born to Clarence and Vera Schaffart on June 17, 1940, in Lyons, Nebraska. He received his primary and secondary education in the public school systems of Omaha, Nebraska; Alton, Illinois; and St. Clair, Missouri. Additional educational training was obtained by Mr. Schaffart at the United States Naval Academy Preparatory School, Bainbridge, NTC, Maryland while on active duty with the United States Air Force Reserve.

He enrolled in the Mechanical Engineering curriculum of the University of Missouri School of Mines and Metallurgy, Rolla, Missouri in the fall of 1959 and received a Bachelor of Science degree in May 1963.

While working for Pratt & Whitney Aircraft, West Palm Beach, Florida, he enrolled in the Graduate School of the University of Florida in September 1963. He received a Master of Engineering degree in Mechanical Engineering in December 1965.

He has been enrolled in the Graduate School of the University of Missouri-Rolla since September 1969 and has held Graduate Teaching Assistantships in Mechanical Engineering and an NSF Research Fellowship in acoustic research during the period February 1970 to February 1972.
Appendix A

REFLECTION FACTORS IN STANDING WAVE TUBE MEASUREMENTS

Let $\bar{A}$ be an acoustic plane wave incident on the termination of the standing wave tube, see Figure A-1. The wave reflected and propagated away from the termination is expressed as $\bar{B}$. The reflection factor is defined as the complex quotient of the reflected to the incident wave. This can be expressed in exponential form as follows:

$$\frac{\bar{R}}{R} = \frac{B_1}{A_1} \, e^{-i\theta}$$

where $A_1$ and $B_1$ are the magnitudes of incident and reflected waves respectively and $\theta$ is the phase angle of $B_1$ relative to $A_1$.

Figure A-1  Schematic of Acoustic Phenomenon at a Termination
The one-dimensional solution of the wave equation for a uniformly moving medium is a wave moving with a propagation velocity consisting of the uniform velocity of the medium and the propagation velocity in free space. This velocity depends on whether the wave is traveling with the flow or against the flow. If the effects of attenuation are assumed to be independent of the medium flow and the flow is assumed uniform in the negative x-direction as shown in Figure A-1, the solution of the wave equation yields for the pressure at any x-location of the wave guide the following expression:

\[ p(x,t) = [A_1 e^{\alpha x} e^{-i(\omega-k_1 x)} + B_1 e^{-\alpha x} e^{i(\omega-k_1 x)}] e^{i\omega t} \] (A-2)

or

\[ p(x) = A_1 e^{\alpha x} e^{-i(\omega-k_2 x)} + B_1 e^{-\alpha x} e^{i(\omega-k_2 x)} \] (A-3)

where

\[ k_1 = \frac{\omega}{c_0} + u = \frac{k_0}{1+M} \]

\[ k_2 = \frac{\omega}{c_0} - u = \frac{k_0}{1-M} \]

Since \( p(x) \) is a complex quantity, the magnitude is obtained by taking the square root of the sum of squares of its real and imaginary components.

\[ |p(x)|^2 = A_1^2 e^{2\alpha x} + 2A_1 B_1 \cos(\omega-(k_1+k_2)x) + B_1^2 e^{-2\alpha x} \] (A-4)

The maximum and minimum values of \( p(x) \) occur when \( \frac{\partial |p(x)|^2}{\partial x} = 0 \). Thus from (A-4)
By order of magnitude arguments, the first term of (A-5) can be shown to be negligible compared to the second term. This then gives as the condition for the occurrence of a maximum or minimum in the wave guide,

$$\sin[\theta-(k_1+k_2)x]=0 \quad (A-6)$$

or

$$\theta-(k_1+k_2)x_{\text{max}} = \pm n\pi; \quad n=0,1,2,3... \quad (A-7)$$

$$x_{\text{min}}$$

It is obvious from inspection of Equation (A-4) that the maximum pressures occurs when \( n \) is even and the minimum pressures occurs for \( n \) off. This is expressed by the following expressions:

$$\theta-(k_1+k_2)x_{\text{max}} = \pm n\pi; \quad n=0,2,4,6... \quad (A-8)$$

$$\theta-(k_1+k_2)x_{\text{min}} = \pm n\pi; \quad n=1,3,5,7... \quad (A-9)$$

The distance between consecutive minimums can now be obtained from (A-9):

$$x_{\text{min}1} = \frac{\theta+\pi}{k_1+k_2}$$

$$x_{\text{min}2} = \frac{\theta+3\pi}{k_1+k_2}$$

Thus,

$$S = x_{\text{min}2} - x_{\text{min}1} = \frac{2\pi(1-M^2)}{2 \omega/c_0}$$
or
\[ S = \frac{\lambda}{2} (1-M^2) \] (A-10)

Using the previous development, the pressure at a reference point may be written as

\[ p_r = A_1 e^{2ax_r} + R \cos[\theta - (k_1 + k_2)x_r] + R^2 e^{2ax_r} \] (A-11)

where

- \( p_r \) = pressure at reference point
- \( x_r \) = distance from termination

Similarly, the expression for the pressure at a minimum is also obtained:

\[ p_{min} = A_1 e^{2a\min} - 2R + R^2 e^{-2a\min} \] (A-12)

Equations (A-11) and (A-12) are now solved simultaneously for the reflection factor (R):

\[ R^2 \left( \frac{p_r}{p_{min}} \right)^2 e^{-2a\min} - 2R \left( \frac{p_r}{p_{min}} \right)^2 + \phi = 0 \] (A-13)

where

\[ \phi = \theta - (k_1 + k_2)x_r = \theta - \left( \frac{2k}{1-M^2} \right)x_r \]

Since (A-13) is quadratic in R it can be solved using the familiar expression.
\[ R = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad (A-14) \]

where

\[ a = \left( \frac{p_r}{p_{\text{min}}} \right)^2 e^{-2\alpha x_{\text{min}}} e^{-2\alpha x_r} \]

\[ b = -2 \left[ \left( \frac{p_r}{p_{\text{min}}} \right)^2 \cos \phi \right] \]

\[ c = \left( \frac{p_r}{p_{\text{min}}} \right)^2 e^{2\alpha x_{\text{min}}} e^{-2\alpha x_r} \quad (A-15) \]

For the no-flow case \( (M=0) \), it can be shown by using energy considerations that the reflection factor must have magnitude less than or equal to unity. Inspection of \( (A-14) \) reveals that the negative root of the radical must be used to satisfy this condition.

If the reference pressure is chosen to be the maximum pressure adjacent to the minimum pressure used, then \( p_r = p_{\text{max}} \) and \( \phi = 0 \) or \( 2\pi \). Applying this condition to \((A-14)\) and substituting the results into \((A-14)\) yields, after simplification:

\[ R = \frac{p_{\text{max}}^e - p_{\text{min}}^e}{-p_{\text{min}}^e + p_{\text{max}}^e} e^{\alpha x_{\text{min}} - \alpha x_{\text{max}}} \quad (A-16) \]

For the case of the propagation medium in motion \( (M>0) \), the reflection factor can have a magnitude greater than unity and is obtained using the positive root of the radical in \((A-14)\)

\[ R = \frac{p_{\text{max}}^e + p_{\text{min}}^e}{p_{\text{min}}^e - p_{\text{max}}^e} e^{\alpha x_{\text{min}} + \alpha x_{\text{max}}} \quad (A-17) \]
This root is only applicable for $R > unity$.

Examination of two adjacent minimums allows detection of the conditions for $R$ greater than unity. When $R$ is greater than unity, the magnitude of adjacent minimums decreases as one travels away from the termination.
Appendix B

DERIVATION OF TRANSMISSION FACTORS

If a one-dimensional sound wave $\vec{A}_1$ is incident upon a discontinuity, a portion of the wave is reflected as a second one-dimensional wave $\vec{B}_1$ with a phase angle of $\theta$ relative to $\vec{A}_1$. In addition, a portion of the wave $\vec{A}_2$ is transmitted beyond the discontinuity with a phase angle of $\phi$ relative to $\vec{A}_1$. If the discontinuity is terminated either by an infinite pipe or an anechoic termination, $\vec{A}_2$ is the only wave present beyond the discontinuity see Figure B-1.

![Figure B-1 Schematic of Acoustic Phenomenon Through a Discontinuity](image)

If the transmission factor is defined as the ratio of the transmitted to the incident wave, it can be expressed in exponential form by the following:

$$\frac{T}{A_1} = \frac{A_2}{A_1} e^{i\phi} e^{i\omega t} \quad (B-1)$$
The pressure at the inlet side of the discontinuity can be written as follows:

\[ p_1(x_1) = [A_1 e^{\frac{\alpha x_1}{e} k_1 x_1} + B_1 e^{\frac{-\alpha x_1}{e} i(\theta - k_2 x_1)}] e^{i \omega t} \]  
(B-2)

\[ p_1(x_1) = A_1 [e^{\frac{-\alpha x_1}{e} k_1 x_1} + \text{Re} \ e^{\frac{-\alpha x_1}{e} i(\theta - k_2 x_1)}] e^{i \omega t} \]  
(B-3)

The pressure on the downstream side of the discontinuity can be written in a similar manner:

\[ p_2(x_2) = A_2 e^{\frac{-\alpha x_2}{e} i(\theta - k_1 x_2)} e^{i \omega t} \]  
(B-4)

\[ = A_2 e^{\frac{-\alpha x_2}{e} i(\phi - k_1 x_2)} e^{i \omega t} \]  
(B-5)

If \( p_1 \) and \( p_2 \) are written in exponential form

\[ p_1 = p_{1m} e^{i \psi_1}; \quad p_2 = p_{2m} e^{i \psi_2} \]  
(B-6)

then the ratio of Equations (B-6) and (B-3) can be written as

\[ \frac{A_1 e^{\frac{-\alpha x_2}{e} i(\theta - k_1 x_2)}}{A_1 [e^{\frac{-\alpha x_1}{e} k_1 x_1} + \text{Re} e^{\frac{-\alpha x_1}{e} i(\theta - k_2 x_1)}]} = \frac{p_{2m} e^{i(\psi_2 - \psi_1)}}{p_{1m}} \]  
(B-7)

Thus,

\[ \frac{p_{2m} e^{i(\psi_2 - \psi_1)}}{p_{1m}} \frac{\alpha x_1 k_1 x_1}{e^{\frac{-\alpha x_1}{e} i(\theta - k_2 x_1)}} \frac{-\alpha x_1 i(\theta - k_2 x_1)}{e^{\frac{-\alpha x_2}{e} i(\theta - k_2 x_1)}} \]  
(B-8)
Therefore, if the pressure on both sides of a discontinuity and the phase difference between them can be determined, the transmission factor can be obtained from (B-8), provided that the reflection factor has been previously determined.
Appendix C

PRESSURE-PHASE DIFFERENTIAL ANALYSIS OF A HELMHOLTZ RESONATOR

The pressure at the entrance to a resonator is given by \( P_1 \) and the pressure inside the resonator is given by \( P_2 \) as shown in Figure C-1.

![Figure C-1 Schematic of Helmholtz Resonator](image)

From the definition of acoustic impedance, the impedance of the neck is given by

\[
\overline{Z}_{\text{neck}} = \frac{\overline{P}_1 - \overline{P}_2}{s_0 u_0} = R_{\text{neck}} + iX_{\text{neck}}
\]

where \( u_0 \) = particle velocity in neck

The impedance of the volume is given by

\[
\overline{Z}_{\text{vol}} = -\frac{i\rho c^2}{\omega V} = \frac{\overline{P}_2}{s_0 u_0} = \frac{\overline{P}_2}{s_0 u_0}
\]

(C-2)
Thus, the total impedance of the resonator is obtained by combining Equation (C-1) and (C-2)

\[
\bar{Z}_{\text{tot}} = \bar{Z}_{\text{neck}} + \bar{Z}_{\text{vol}}
\]

\[
\frac{\bar{P}_1 - \bar{P}_2}{s_0 u_0} + \frac{\bar{P}_2}{s_0 u_0} = \frac{\bar{P}_1}{s_0 u_0}
\]

(C-3)

Solving Equation (C-1) for \( u_0 \) in terms of the neck impedance and the pressure differential across the neck

\[
\bar{u}_0 = \frac{\bar{P}_1 - \bar{P}_2}{s_0 \bar{Z}_{\text{neck}}}
\]

(C-4)

Upon substituting (C-4) into (C-2)

\[
\frac{\bar{P}_2}{s_0 \left( \frac{\bar{P}_1 - \bar{P}_2}{s_0 \bar{Z}_{\text{neck}}} \right)} = -i \frac{\rho c^2}{\omega v}
\]

or, upon simplification,

\[
\bar{Z}_{\text{neck}} = -i \frac{\rho c^2}{\omega v} \frac{\bar{P}_1}{\bar{P}_2} \left[ \frac{1}{\bar{P}_2} - 1 \right]
\]

(C-6)

Since \( \bar{P}_1 \) and \( \bar{P}_2 \) are vector quantities, the ratio of these quantities may be written as follows

\[
\frac{\bar{P}_1}{\bar{P}_2} = \left| \frac{\bar{P}_1}{\bar{P}_2} \right| e^{i\phi}
\]

(C-7)

where \( \phi \) = phase angle between the two vectors.
Equation (C-7) is now rewritten using standard trigonometric relations

\[
\frac{\bar{p}}{P_2} = \frac{1}{P_2} \cos \phi + i \frac{1}{P_2} \sin \phi
\]  

(C-8)

Substituting into Equation (C-6) yields the neck impedance in terms of the pressure differential and phase difference.

\[
\overline{Z}_{\text{neck}} = -i \frac{\rho c^2}{\omega v} \left[ \frac{1}{P_2} \cos \phi + i \frac{1}{P_2} \sin \phi - 1 \right]
\]  

(C-9)

The impedance of the neck can now be written in its resistive and reactive components

\[
R_{\text{neck}} = \frac{\rho c^2}{\omega v} \sin \phi
\]

\[
X_{\text{neck}} = \frac{\rho c^2}{\omega v} \left[ \frac{1}{P_2} \cos \phi - 1 \right]
\]  

(C-10)

The total impedance of the resonator may now be written as

\[
R_b = \frac{\rho c^2}{\omega v} \sin \phi
\]

\[
X_b = \frac{\rho c^2}{\omega v} \frac{1}{P_2} \cos \phi
\]  

(C-11)

Thus, once the pressure differential and phase relationship between the measured pressures are obtained, the impedance of the resonator and its components are obtained by solving Equations (C-10) and (C-11).
Appendix D
EVALUATION OF THE ANECHOIC TERMINATION

As mentioned throughout this dissertation, anechoic conditions are desired at the discharge side of a test element. This is necessary so that the reflection and transmission factors being measured arise solely from the test element.

In order to insure that the catenoidal horn used for the experimental work fulfilled the requirement of an anechoic termination, it was evaluated over the frequency range and operating conditions, namely flowing gas, to be used during element testing. The evaluation consisted of performing standing-wave analysis with only the catenoidal horn for a termination. A plot of the measured reflection factors versus frequency is given in Figure D-1. A reflection factor of 0.11 or less was deemed to be satisfactory for test operations at that frequency. It can be seen that the horn functioned satisfactorily for the frequency range 75 to 2000 Hz with minor exceptions.

The frequencies at which the reflection factor exceeded the limit of 0.11 magnitude were avoided where possible. In addition, in determining desired test frequencies, it was noted that when an element has a relatively high reflection factor, e.g., 0.75 and above, only a small percentage of the incident sound energy will be propagated beyond the element. Under these conditions therefore, even though the horn might have a relatively large reflection factor, the energy which is reflected back to, and through, the test element is quite small in comparison to the primary reflected wave.
Figure D-1 Reflection Characteristics of Anechoic Termination
Appendix E
PERTURBATION ANALYSIS FOR RESPONSE WITH FLOW

As an example of the perturbation approach to determining acoustic response parameters for discontinuities, the case of a sudden expansion shown in Figure E-1 will be examined.

The flow is assumed steady and isentropic on either side of the discontinuity. Thus the continuity equations of mass and energy can be written:

Mass:
\[ \eta u_a \rho_a = u_b \rho_b \]  \hspace{1cm} (E-1)

Energy:
\[ \frac{u_a^2}{2} + \frac{\gamma}{\gamma-1} \frac{p_a}{\rho_a} = \frac{u_b^2}{2} + \frac{\gamma}{\gamma-1} \frac{p_b}{\rho_b} \]  \hspace{1cm} (E-2)

Figure E-1 Schematic of Sudden Expansion
where

\[ \gamma = \text{ratio of specific heats} \]

\[ \eta = \frac{S_a}{S_b} = \text{area ratio} \]

With the added assumption that the pressure acting on the area \((S_b - S_a)\) is equal to the upstream pressure, \(p_a\); the conservation of momentum can be written

\[ \rho_a u_a^2 \eta + p_a = \rho_b u_b^2 + p_b \]

The acoustic perturbations are assumed to be in the form of plane waves with the incident wave propagated in the direction of the flow. At the discontinuity, an entropy wave is propagated in the direction of flow. It is assumed that the entropy wave causes a perturbation in the downstream density only, with no effect on the pressure or particle velocity. The perturbations are now written.

\[
\begin{align*}
\rho_a &\rightarrow \rho_a + \frac{p_i}{c_a} + p_r \\
\rho_b &\rightarrow \rho_b + \frac{p_i}{c_b} + p_t
\end{align*}
\]

where:

\[ \sigma = \text{perturbation to density caused by entropy wave}. \]

The perturbations for the particle velocity and density are derived from the acoustic pressure and velocity relationship and the isentropic
flow relationship respectively.

Applying the perturbations to (E-1) gives the continuity of mass with the acoustic waves present. On eliminating the steady-state flow terms, the continuity equation for the acoustic waves in the presence of steady flow is given by

\[ \frac{\rho_b}{c_a} \left[ p_{1}(1+M_a) - p_{r}(1-M_a) \right] \frac{b}{c_a} = p_{t}(1+M_b) + M_b\sigma \]  \hspace{1cm} (E-4)

Similarly, applying the perturbations, the energy Equation (E-2) yields

\[ \frac{1}{2} \left[ u_{a} + \frac{p_{1}-p_{a}}{\rho_{a}c_{a}} \right]^2 + \frac{\gamma}{\gamma-1} \left[ \frac{p_{a}+p_{1}+p_{r}}{p_{1}+p_{r}} \right] \frac{\rho_{a} + \frac{c_{a}^2}{2}}{c_{a}^2} \]

\[ = \frac{1}{2} \left[ u_{b} + \frac{p_{t}}{\rho_{b}c_{b}} \right] + \frac{\gamma}{\gamma-1} \left[ \frac{p_{t}+p_{b}}{p_{t}+\sigma} \right] \frac{\rho_{b} + \frac{c_{b}^2}{2}}{c_{b}} \]  \hspace{1cm} (E-5)

Thus, after expanding and neglecting second-order perturbation terms, the energy equation for the acoustic flow is

\[ \frac{\rho_b}{\rho_a} \left[ p_{1}(1+M_a)+p_{r}(1-M_a) \right] = p_{t}(1+M_b) - \frac{1}{\gamma-1} \sigma \]  \hspace{1cm} (E-6)

Finally, after repeating the above procedure, the momentum equation is written

\[ p_{1}(\eta M_{a}^{2}+2M_{a}\eta+1)+p_{r}(\eta M_{a}^{2}-2M_{a}\eta+1) \]

\[ = p_{t}(M_{b}^{2}+2M_{b}+1)+M_{b}^{2} \sigma \]  \hspace{1cm} (E-7)
Solving Equation (E-4) for \( \sigma \) and substituting this expression into Equation (E-6) yields

\[
\left[ p_i (1+M_a^2) + p_r (1-M_a^2) \right] \frac{\rho_b}{\rho_a} = p_t (1+M_b^2)
\]

\[
- \frac{1}{\gamma-1} \frac{\rho_b}{\rho_a} \left[ p_i (1+M_a) - p_r (1-M_a) \right] \frac{c_b}{c_a} = p_t \frac{1}{M_b}
\]

(E-8)

Thus, after combining and simplifying

\[
\frac{p_t}{p_i} \left[ (1+M_a^2)(M_b + \frac{1}{\gamma-1}) \right] - \frac{p_r}{p_t} \left[ M_b (1-M_a) \frac{\rho_b}{\rho_a} - \frac{1}{\gamma-1} \eta (1-M_a) \frac{c_b}{c_a} \right]
\]

\[
= M_b (1+M_a^2) \frac{\rho_b}{\rho_a} + \frac{1}{\gamma-1} \eta (1+M_a) \frac{c_b}{c_a}
\]

(E-9)

Similarly, substituting the expression for \( M_b \sigma \) from (E-4) into (E-7) and simplifying yields

\[
\frac{p_r}{p_i} \left[ \frac{1}{\eta (M_a^2 - 2M_a + \frac{1}{\gamma-1})} + M_b (1-M_a) \frac{c_b}{c_a} \eta \right] - \frac{p_t}{p_i} (1+M_b^2)
\]

\[
= M_b \eta (1+M_a) \frac{c_b}{c_a} - \eta (M_a^2 + 2M_a + \frac{1}{\gamma-1})
\]

(E-10)

Multiplying (E-10) by \( (M_b + \frac{1}{\gamma-1}) \) and subtracting from (E-9) yields, after simplification,

\[
\frac{p_r}{p_i} = \frac{(1+M_a) \frac{c_b}{c_a} \left[ 1 - \frac{1}{\gamma-1} (1+M_b^2 + M_a^2) \frac{c_a}{c_b} \right] - \frac{1}{\gamma-1} + M_b \left( \frac{1}{\gamma-1} + M_a^2 \right)}{(1-M_a) \frac{c_b}{c_a} \left[ 1 + \frac{1}{\gamma-1} (1+M_b^2 + M_a^2) \frac{c_a}{c_b} \right] + \frac{1}{\gamma-1} + M_b \left( \frac{1}{\gamma-1} - 2M_a^2 + M_a^2 \right)}
\]

(E-11)
Equation (E-11) can now be substituted into Equation (E-10) to obtain (after a great deal of algebra) the expression for the transmission factor for the expansion:

\[
\frac{p_t}{p_i} = \frac{2M_a M_b \eta (1-M_a^2)+2M_a^2 \eta \frac{c_a}{c_b} (M_a^2 + \frac{1}{\eta} -2)+ \frac{2}{\gamma-1} \eta \frac{c_b}{c_a} (\frac{1}{\eta} - M_a^2)}{(1+M_b) \{(1-M_a) \frac{c_b}{c_a} \frac{1}{\gamma-1} - (1+M_b) + M_b \left( M_a^2 - \frac{c_a}{c_b} \right)^2 \} + (\frac{1}{\gamma-1} + M_b) (M_a^2 - 2M_a + \frac{1}{\eta})} \]

(E-12)

As the Mach number of the incoming flow reduces to zero the expressions (E-11) and (E-12) reduce to the linear theory form; i.e.,

\[
\frac{p_t}{p_i} = \frac{\eta-1}{\eta+1} \]

(E-13)

\[
\frac{p_t}{p_i} = \frac{2\eta}{\eta+1} \]

(E-14)
Appendix F

LIST OF EQUIPMENT

1. General Radio Co., Beat-Frequency Audio Generator, Type 1304-B, Serial Number 3276.
3. General Radio Co., Graphic Level Recorder, Type 1521-B, Serial Number 2369.
5. General Radio Co., Sound-Level Calibrator, Type 1562-A, Serial Number 322.
7. General Radio Co., Ceramic Microphone, Type 1560-P5, Serial Number 8925.
8. General Radio Co., Ceramic Microphone, Type 1560-P5, Serial Number 4211.
12. Bruel and Kjaer Co., Electronic Voltmeter 2-200000 c/s, Type 2409, Serial Number 291216.
13. ADYU Electronics, Inc., Precision Phase Meter, Type 4066, Serial Number 3399L.

14. Dynakit, Mark IV 40 Watt Amplifier, Serial Number 8741024.

15. Eico, Stereo Amplifier, Model 3070W, Serial Number 70303.


17. Tektronix Dual Beam Storage Oscilloscope, Type 564, Serial Number 7764.