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Considerations for Magnetic-Field Coupling Resulting in Radiated EMI

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Abstract: Parasitic inductance in printed circuit board geometries can worsen the EMI performance and signal integrity of high-speed digital designs. Partial-inductance theory is a powerful tool for analyzing inductance issues in signal integrity. However, partial inductances may not adequately model magnetic flux coupling to EMI antennas because the EMI antennas are typically open loops. Therefore, partial inductances may not always accurately predict radiated EMI from noise sources, unless used in a full-wave analysis such as PEEC. Partial inductances can be used, however, to estimate branch inductances, which can be used to predict EMI. This paper presents a method for decomposing loop or self inductances into branch inductances. Experimental as well as analytical investigations are used to compare branch- and partial-inductances.

I. INTRODUCTION

Equivalent circuits that effectively model the physics of EMI issues are desirable for EMI estimation at the design stage. Inductance may be decomposed into smaller pieces associated with the various conductors in a loop, the sum of which equals the total loop inductance. Partial-inductance theory has been successfully applied to analyze structures for signal integrity purposes. Magnetic-field coupling between traces and between pins in a high-density IC package, among others, have been analyzed using partial-inductance or partial-element theory [1], [2].

A method for decomposing loop inductance in a fashion that is useful for predicting EMI is presented herein. The decomposed inductance elements are called branch inductances. The branch inductance of a conductor models the magnetic flux that penetrates a conducting loop, and couples an EMI antenna. The branch inductance can then be used to model the resulting effective noise voltage that drives an EMI antenna. The branch-inductance model is presented herein, and compared with the partial-inductance theory. Two examples are investigated that demonstrate the difficulties in predicting EMI with partial inductances, and the advantage of branch inductances.

II. CONCEPTS

The inductance of a conducting loop can be decomposed into parts that sum to the total loop inductance. In general, the decomposition is not unique. Partial-inductance theory was developed as a method for analyzing signal-integrity issues [3], [4], [5]. The formulation was extended into the Partial-Element Equivalent-Circuit (PEEC) method, which may be used to yield a full-wave equivalent-circuit model [6]. Loop inductance can also be decomposed into branch inductances, which are, in general, different from partial inductances. Branch-inductances are useful for determining the effects of EMI noise sources resulting from magnetic-field coupling.

Total inductance may be defined as the ratio of the magnetic flux that penetrates a loop to the current generating the magnetic flux as

\[ L_{loop} \equiv \frac{\Psi_1}{I} = \frac{1}{I} \int_{S_{loop}} \mu \vec{H} \cdot d\vec{s} \]  

(1)

The magnetic vector-potential \( \vec{A} \) is related to the magnetic field \( \vec{H} \) by \( \vec{H} \equiv \frac{1}{\mu} \nabla \times \vec{A} \). Employing Stokes's theorem, the flux integral can be written in terms of a line integral,

\[ L_{loop} = \frac{\Psi_1}{I} = \frac{1}{I} \int_{S_{loop}} \nabla \times \vec{A} \cdot d\vec{s} = \frac{1}{I} \oint_{C} \vec{A} \cdot d\vec{l} \]  

(2)

A. Partial Inductance

Partial-inductance theory has been well documented [4], [5], [6], and is briefly reviewed here for completeness. The partial inductance of the \( i_{th} \) segment may be defined as the integral of the magnetic vector-potential along the \( i_{th} \) segment divided by the loop current \( I \) [5],

\[ L_{partial}^{i_{th}} = \frac{1}{I} \int_{i_{th}} \vec{A} \cdot d\vec{l} \]  

(3)

The magnetic vector-potential used in this definition is the total magnetic vector-potential. The partial inductance of the \( i_{th} \) segment is therefore independent of conductors orthogonal to the \( i_{th} \) segment. The independence results because the magnetic vector-potential is oriented parallel to the current density. Consequently, \( A_j \cdot d\vec{l} = 0 \) if the \( j^{th} \) and \( i^{th} \) segments are orthogonal to each other.

In addition to its generality, an advantage of Ruehli’s formulation is that the resulting equivalent circuit model incorporates the mutual interactions among elements. Partial inductances are used to decompose the voltage drop associated with conductors in a loop that results from magnetic field storage. The partial inductance can be used to find the potential difference that results along
The partial inductance of the vertical conductors may be calculated using Eq. 3 or it may be found in a reference.
The method for calculating and assigning partial inductances is very rigorous, however, some choices must be made when assigning branch inductances. Branch inductance calculations are useful for considering an EMI antenna and source geometry. The branch inductance of interest is associated with the conductor around which magnetic flux couples to the EMI antenna.

Partial inductance theory may be used to approximate branch inductance. For example, in Figure 3 the branch inductance of the right vertical wire could be computed with partial inductances, given the partial inductance of the top and bottom plates to the right of the loop are

\[ L_{partial} = \frac{\mu}{2\pi} \left[ h \ln \left( \frac{d}{a} \right) - h \ln \frac{1}{2} \left( 1 + \sqrt{1 + \frac{d^2}{h^2}} \right) \right] \]

The branch inductance of the right vertical wire in Figure 3 is

\[ L_{branch} = L_{partial} + L_{wire} \]

The example shown in Figure 3 can also be treated using image theory. The wires connecting two conducting plates can be equivalently modeled as two infinitely long wires for calculating the fields between the two plates. The solution for the magnetic-field distribution between the plates for the image problem is the same as the solution for the non-image problem. The resulting expression for the loop inductance using image theory is the same as Eq. 7. However, the development using image theory shows more intuitively that no magnetic flux wraps the plates, because the magnetic vector-potential that results from image theory is oriented completely in \( \hat{z} \) direction. Therefore, \( \mathbf{A} \cdot \mathbf{\hat{z}} \, dz \) equals zero. Dividing the region external to the signal loop by the infinite plates and the vertical wires, the branch inductance of the plates is zero, and the branch inductance of each of the wires is half of the total inductance of the conducting loop, by symmetry, i.e.,

\[ L_{branch} = 0 \]

\[ L_{wire} = \frac{\mu}{2\pi} h \ln \left( \frac{d}{a} \right) \]

\[ h, d \gg a. \]

The branch inductances of the wires and plates are, therefore, generally different from the partial inductances. However, as required by definition, the sum of all partial inductances comprising a loop equals the sum of all branch inductances comprising a loop, which is the total loop inductance,

\[ L_{total} = \sum_i L_{partial} = \sum_j L_{branch} \]
A. Stacked-Card PCB Model

Stacked-card and modules-on-backplane printed circuit-board geometries are advantageous for conserving real-estate in many designs. Unfortunately, at high frequencies, EM1 resulting from the finite impedance of the signal return may develop at the connector. This effective noise source may drive the daughter-card against the mother-board and attached cables, resulting in common-mode radiation.

Port 1 was located between the mother-board and the signal conductor in the connector. The signal conductor was terminated directly to the daughter-card. The reference planes were constructed of single-sided electro-deposited copper on an FR4 dielectric substrate. The cable extending from the mother-board was 0.085" semi-rigid coaxial cable. The cable was connected to the bottom of the mother-board and penetrated the mother-board at the signal conductor of the connector. The shield of the coaxial cable was soldered to the mother-board with a 360° connection. The center conductor of the coaxial cable was extended through the mother-board and connected to the signal conductor. The shield of the coaxial cable was soldered to the mother-board and daughter-card reference-planes. The signal-input end of the coaxial cable was connected to the network analyzer through the aluminum plate as shown in Figure 4(a), and the common-mode current on the coaxial cable was measured. Swept-frequency measurements were made between 10 MHz and 100 MHz.

An equivalent circuit for the connector region of the stacked-card model is proposed in Figure 4(b). The EMI antenna impedance \( Z_{anr} \) is shown as a capacitor, which is a low-frequency model. The inductance of the signal loop in Figure 4 is decomposed into general inductances. The values of the decomposed inductances are dependent on the method of decomposition. A more thorough treatment of the stacked-card configuration may be found in [9].

| \(|S_{z11}\)| differences for \(d = 1 \text{ cm} \& d = 2 \text{ cm}, d = 3 \text{ cm}, d = 4 \text{ cm} \& d = 5 \text{ cm}\) for the stacked-card model.

| \(\text{Frequency (MHz)}\) | \(10\) | \(20\) | \(30\) | \(40\) | \(50\) | \(60\) | \(70\) | \(80\) | \(90\) | \(100\)
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Figure 4. Stacked-card model (without traces) for investigating the common-mode current predicted by the partial- and branch-inductances. (a) Experimental model and (b) low-frequency equivalent circuit (cross-sectional view).

Figure 5. Results for the measured \(|S_{z11}\)| differences for \(d = 1 \text{ cm} \& d = 2 \text{ cm}, d = 3 \text{ cm}, d = 4 \text{ cm} \& d = 5 \text{ cm}\) for the stacked-card model.
stacked-card geometry is not easily derived. However, an experimental results shown in Figure 6(a) when the connector geometry was changed. The ratio of the common-mode current with an average difference of 1.1 dB. The valleys and peaks may be discerned in the $|S_{21}|$ plots, as well. The change in common-mode current predicted using partial inductance does not agree well with the measurements.

**B. Parallel-Plate Model**

An analytical expression for the branch inductance of the stacked-card geometry is not easily derived. However, an analytical expression was developed for the branch inductance of a vertical wire between two large plates [9]. A schematic representation of a parallel-plate configuration is shown in Figure 6(a). The plates of the model are necessarily finite, but large enough that the magnetic field distribution between the plates is approximately the same as for the infinite plate case. The plates were constructed of RT-Duroid single-sided copper-clad boards. The vertical wires were 24 AWG wire. A 0.085" semi-rigid coaxial cable was used to excite the differential-mode loop, and to return common-mode current for measurement. The coaxial cable extended 8 cm from the bottom plane, and was parallel to the planes. The wire separation $d$ varied from $d = 1$ cm, 5 cm, and $d = 10$ cm. The location of Port 1 was shifted for each separation $d$ to maintain symmetry with respect to plate edges, and limit possible artifacts resulting from the proximity of the plate edge. A low-frequency equivalent circuit-model is shown in Figure 6(b). The equivalent circuit shown in Figure 6(b) shows a current-driven noise source-mechanism that results in a potential difference between the top and bottom plates. The EMI antenna impedance $Z_{ant}$ is shown as a capacitor, which is a low-frequency model. The loop inductance in Figure 6 is decomposed into general inductances. The value of the decomposed inductances is dependent on the method of decomposition. The branch inductance of the two plates is zero as discussed in Section II-B, although the partial inductance of the two plates is finite and non-zero.

The common-mode current was measured via $|S_{21}|$ measurements using the network analyzer. The increase in common-mode current is predictable with a common-mode inductance model, such as shown in the equivalent circuit diagram of Figure 1. The partial and branch inductances for a wire in the parallel-plate geometry were calculated using Eq. 5 and Eq. 9, respectively. The increase in common-mode current predicted by the two decomposed inductance models was compared to the increase in $|S_{21}|$. The results are tabulated in Table II, and are shown graphically in Figure 7. The measured results show fair agreement with the change in common-mode current predicted using branch inductances. The valleys and peaks in Figure 7 result from small changes to the measurement system when the wire separation was changed. The ratio of the common-mode current with $d = 5$ cm to $d = 10$ cm shows the same valleys and peaks with an average difference of 1.1 dB. The valleys and peaks may be discerned in the $|S_{21}|$ plots, as well. The change in common-mode current predicted using partial inductance does not agree well with the measurements. The partial-inductance theory does not model the magnetic flux that couples the EMI antenna, because the partial inductance is calculated by integrating the magnetic vector-potential over a finite length. Consequently, the partial inductance of the vertical wires can not be used to accurately predict the level of the common-mode voltage-source for the geometry shown in Figure 6.

**Table 1**

<table>
<thead>
<tr>
<th>Results for Signal-Return Conductor in Stacked-Card Configuration</th>
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<td>$d$ (cm)</td>
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**Figure 6.** Parallel-plate model for investigating the common-mode current predicted by the partial-and branch-inductances. (a) Experimental model and (b) low-frequency equivalent circuit (cross-sectional view).
TABLE II

CALCULATED PARTIAL INDUCTANCE AND BRANCH INDUCTANCE RESULTS FOR $d = 1 \text{ cm}$, $5 \text{ cm}$, and $10 \text{ cm}$ for the parallel-plate configuration. The change in partial and branch inductance with respect to $L_{\text{partial}}(1 \text{ cm})$ and $L_{\text{branch}}(1 \text{ cm})$, respectively, is given in decibels, and contrasted to the average change in $|S_{21}|$.

| $d$ (cm) | $L_{\text{wire}}_{\text{partial}}(d)$ (nH) | $L_{\text{wire}}_{\text{partial}}(1 \text{ cm})$ (dB) | $L_{\text{wire}}_{\text{branch}}(d)$ (nH) | $L_{\text{wire}}_{\text{branch}}(1 \text{ cm})$ (dB) | $(|S_{21}|(d))$ (dB) |
|--------|-----------------|----------------|----------------|----------------|----------------|
| 1      | 12.9            | 0              | 14.7           | 0              | 0              |
| 5      | 15.4            | 1.5            | 21.1           | 3.1            | 3.0            |
| 10     | 13.8            | 1.8            | 24.0           | 4.2            | 4.1            |

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REFERENCES


