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A Biologically Inspired Connectionist Architecture for Directing Attention to Salient Visual Field Objects

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ABSTRACT

In an attempt to incorporate basic visual attention abilities into existing artificial vision systems, a neural model of the bidirectional interactions within and between the brain regions believed to be involved in human visual attention has been developed. This model currently gives an artificial vision system the ability to attend to "salient," or "pop-out" features and objects within the vision system's field of view. After a review of the physiology of human visual attention, a network model of the aforementioned neural interactions will be presented, followed by a demonstration of its performance.

1. INTRODUCTION

A difficulty with modern artificial vision systems regards their inability to properly attend to specific objects within the visual field. Often extensive image processing is required to extract necessary or relevant data. On the other hand, humans can perform many visual attention tasks with relatively little effort, especially when the objects being attended to are noticeably different from the surrounding information within the visual field. Therefore, it may be useful to examine the biological components that produce natural human vision, and then use this knowledge to model the areas believed to be involved in the processes associated with visual attention.

As illustrated in Figure 1, numerous bidirectional connections exist between the known areas of the human visual system. In addition to these connections, the receptive fields, or regions of input for the neurons are also known to increase in size as the information processing continues along the V1 → IT pathway [1], with the individual features becoming more complex [2]. With regard to area V4, this heightened analysis is believed to result from this area being able to process both high and low frequency inputs, as well as its ability to respond to various characteristics of the visual image, such as color, shape, and motion. For example, although cells within area V1 will respond to a single feature, the convergence of their outputs onto individual cells within area V4 will allow these later cells to become responsive to either multiple features, or extensions of a single feature. In a similar manner, a neuron in a later area such as IT will respond to even more complete objects located almost anywhere within the visual field, although unlike the responses within area V1 and area V4, the larger receptive field cells within area IT do not initially respond well to attended to stimuli [3]. In fact, area IT cells appear to only remain active for a short period of time after the presentation and matching of a stimulus, but quickly become suppressed when the same stimulus is re-attended to, indicating that they are mainly activated by a newly attended to stimulus, in

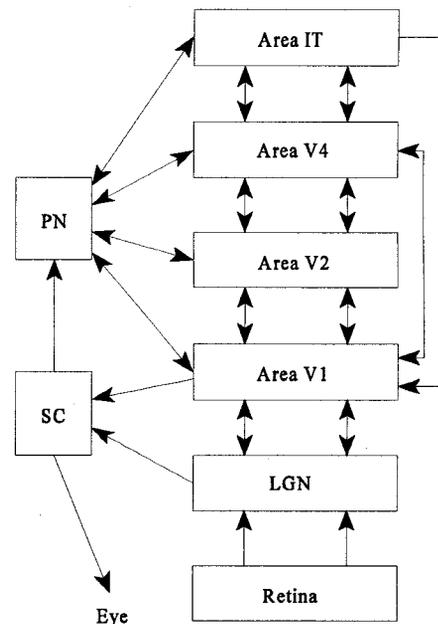Figure 1: Bidirectional Connections Between Various Visual Areas

addition to being involved in recognition. If attention is not directed to any one particular region of the visual field, the IT cells will quickly become inactive. Nonetheless, when attention is initially focused on a particular stimulus, a competition appears to occur between area IT neurons producing a single active neuron [4].

Although it is not directly known what initiates or determines the outcome of any competitive interactions, some have speculated that both bottom-up and top-down signals may determine what pathways become active [5], with cognitive factors influencing the top-down control of attention within the higher regions of the cortex [6]. But even with the involvement of bidirectional interactions within the attentional system, the spatially filtered bottom-up input provided by the retina will typically be crowded with competing stimuli, initially resulting in numerous area V4 cells becoming excited by the information contained within the visual field [7]. As such, some type of filtering of unattended stimuli is necessary to facilitate the competitive processes that appear to be occurring within area IT. To perform this filtering, some have speculated that an attention system, having the characteristic of being both anatomically separate, yet not the function of a single region, would be necessary [7]. In other words, this region would be isolated enough to carry out specific processing, while still interacting with the various layers of the visual cortex. One region that is properly positioned, and speculated to be important for performing some necessary attentional processes, is the pulvinar nucleus [8], [9].

Although little is known about the direct neural connections involved within the pulvinar, two distinct types of cells have been observed. One group consist of the inferior and lateral pulvinar cells, subsequently labeled as PI and PL. These cells are known to display receptive field characteristics [6], much like the orientation selective simple cells located in area V1. The other group of cells is located in the dorsomedial region of the pulvinar, and given the label Pdm. These particular cells are distinct in that they become enhanced only when a single item is being attended to within the visual field [10]. Selection of a target for additional processing, even if not the result of a saccadic eye movement, can also produce an amplification of the cell responses to that particular region within Pdm [6]. As previously mentioned, although little is presently known about the neural connections within the pulvinar, it has been observed that some type of internal inhibitory interactions are involved in controlling the "spotlight of attention." This hypothesis results from observations illustrating how the facilitation of inhibition will reduce the pulvinar's ability to disengage attention, while a reduction in inhibition will increase the visual system's ability to shift the current focus of attention [11].

Figure 2 illustrates one hypothesized scenario for the influence of the inhibitory characteristics observed within the pulvinar. Since bidirectional connections are known to exist from area V1 to the pulvinar [12], the direct connections the pulvinar receives from area V1 could possibly explain why some pulvinar cells, such as those within the PI and PL regions, have the characteristic of a receptive field responsive to orientation. As a result of this preference, it is likely that the cells within these regions are only receiving signals from area V1 cells displaying the same orientation response characteristic. In addition, due to the pulvinar's physical size

it is unlikely that a single PI/PL type pulvinar cell is receiving the same orientation response from every hypercolumn within area V1, although each cell is probably receiving numerous inputs from a defined section within this area. Any necessary cooperative interactions between these cells could be carried out through long range connections, as further illustrated in Figure 2. In addition to the PI/PL inputs, it can be speculated that each Pdm cell is also receiving area V1 input, except now from only a single hypercolumn, resulting in a retinotopic mapping similar to area V1. Like the hypercolumns, the formation of a retinotopic mapping will maintain the spatial aspects of the image within the pulvinar. Having each Pdm cell receive multiple orientation inputs may also explain why these cells are not usually responsive to a single orientation.
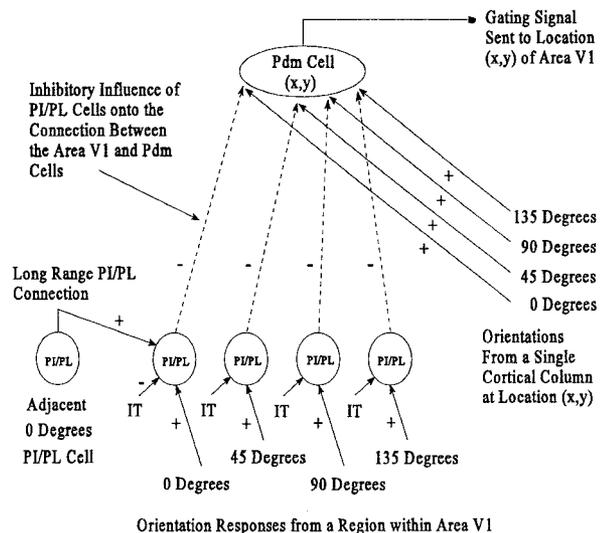

Figure 2: Pulvinar Interactions

Since the cell responses from a single hypercolumn are being sent directly to each Pdm pulvinar cell, it will be necessary to provide some type of signal to dictate how each area V1 cortical column cell will affect the individual Pdm pulvinar cells. As displayed in Figure 2, this could be carried out by providing an inhibitory signal from the PI/PL cells, although the exact interaction on each Pdm cell, or its inputs, is not directly known. Even so, the influence of any inhibitory signal may be easier to explain. For instance, consider the situation were the entire visual field contains all 90 degree orientation lines, except for an isolated 135 degree line. Due to its presence within the visual field, each 90 degree line will work to enhance its correspondingly oriented PI/PL pulvinar cell, resulting in a strong response for this cell. On the other hand, the isolated 135 degree line will provide very little input to its correspondingly oriented PI/PL pulvinar cell. As a result of the inhibitory influence of the PI/PL cells on the final response characteristic of each Pdm cell, the area V1

hypercolumn projections from cells responding to a 90 degree orientation will become inhibited, or at least have their signals attenuated before being passed on to the Pdm cells. Eventually, only the Pdm cells retinotopically located in reference to the 135 degree line will respond with an activation. This will give the appearance as though attention has concentrated around a single item within the visual field. In essence, the action of the network will be competitive, with each cell challenging for the right to draw attention to a specific region or feature within the visual field. Under the conditions when there is no top-down input, those features within the visual field that standout, or are the most "salient," will win the competition. Furthermore, the effects of any bidirectional connections back to area V1 may offer an explanation for the increased neural responses observed for those cells activated by the attended to stimulus [13].

Although this may sound somewhat unusual, this type of characteristic corresponds with studies which have determined that targets can be found easily within the visual field when they are defined by a unique color or orientation [14]. It is almost as if the most salient item will "pop-out" and draw the viewer's attention. In fact, other studies have found that the relative salience between a target and its visual distractors was more important than the absolute salience of either the target or distractor for directing and shifting attention [15]. Likewise, under the scenario in Figure 2, only the condition of a discrepancy between feature occurrences is necessary, with degrees of difference between orientation having no impact. Finally, the pulvinar network corresponds with psychological theories stressing the importance of an early parallel search involving feature inhibition strategies [16], [17].

Of course, just because the pulvinar has attended to a salient item does not insure that the cortical areas can process and recognize this item unless some type of gating is occurring. As previously mentioned, during focused attention a competition will occur between area IT neurons resulting in a single neuron remaining active, ultimately aiding the recognition of an object [4]. For the competition to be successful, it is beneficial for the object within the focus of attention to be the only item activating any area V4 neurons providing input to area IT. Since bidirectional connections exist between area V4 and the pulvinar, it can be speculated that the pulvinar cells, possibly those of the Pdm type, will make connections with retinotopically located area V4 cells. Therefore, it can be hypothesized that after attention has been focused to a single region within the pulvinar, projections made to the area V4 cells will have the effect of gating only the attended to information from area V1 to V4. This will result in a single object activating a grid of area V4 cells, allowing the competitive cellular interactions within area IT to quickly facilitate object identification. Furthermore, varying the span of the area V1 to V4 projections, or even the pulvinar gating itself, should allow for a slight amount of scale invariance to be tolerated between the various cortical areas.

2. ATTENTION NETWORK MODELING

The interactions described in the previous section, and partially illustrated in Figures 1 and 2, will now be placed into a connectionist network model that can be simulated. Initially, retina cell outputs will be provided as input to each of the area V1 cortical column cells located within a defined hypercolumn. The edge detected spatial frequency filtering of the image provided by the network of retina cells has been modeled elsewhere [18], [19]. In addition, the modeling of the bidirectional interactions between the LGN and the area V1 simple cells can also be found elsewhere [20], [21]. In essence, the area V1 network provides outputs that respond to various network feature orientations defined at each spatial location. Also, although area V4 receives the majority of its projections from area V2, this area, along with the feature and spectral processing it performs, will not be modeled. As such, area V4 will be assumed to get its input directly from area V1.

As illustrated in Figure 2, each pulvinar Pdm cell will receive inputs from the orientation simple cells within a single hypercolumn. As mentioned, these orientation inputs will be influenced by the inhibitory projections from a like oriented Pl/PL pulvinar cell. Therefore, each Pdm cell interaction will be defined by the following equation,

$$net_{p(x,y)}(t) = \sum_{k=1}^{4} (S_{(x,y)k}(t))[1 - inh_{(x,y)k}(t)] \quad (1)$$

where $net_{p(x,y)}(t)$ represents the net activation of the Pdm cell at location (x,y) within the retinotopic pulvinar grid. The term $S_{(x,y)k}(t)$ represents the kth orientation simple cell response within the area V1 hypercolumn at location (x,y). The term $inh_{(x,y)k}(t)$, defined below in equation (2), represents the output response of the inhibitory Pl/PL pulvinar cell, also at location (x,y) and with orientation k. The threshold parameter θ_{inh} has also been included to provide each Pl/PL cell with a linear activation equal to the net activation of the cell, after

$$inh_{(x,y)k}(t) = \rho_k(t)(1 - \delta_{TD})\sum_{i=1}^{x} \sum_{j=1}^{y} S_{(i,j)k}(t-1) \quad (2)$$

$$inh_{(x,y)k}(t) = 0 \quad if \quad inh_{(x,y)k}(t) < \theta_{inh}$$

surpassing a defined level. Within equation (2), $S_{(x,y)k}(t)$ once again represents the kth orientation simple cell response within the area V1 hypercolumn at location (x,y), while δ_{TD} represents a top-down signal directing the type of attention, as defined below.

$$\delta_{TD} = 1 \quad for \; external \; input, \quad otherwise \quad \delta_{TD} = 0 \quad (3)$$

Although its origin is unknown, this top-down signal could be projecting from area IT or the parietal region. Although added for completeness, δ_{TD} will equal zero during the

subsequent simulations of the bottom-up, salience driven attention processing. During a top-down search and recognition mode (not modeled here), the inhibitory contribution would be eliminated from equation (1) by way of the condition in equation (3). Also included in equation (2) is a time varying parameter, $\rho_k(t)$, defined below in equation (4).

$$\rho_k(t) = \rho_k(t-1) + 0.01 \qquad (4)$$

$$\rho_k(0) = 0 \quad \text{for all } k$$

Equation (4) is used to determine the weight, or strength of association that exist between each area V1 and PI/PL inhibitory neuron. While initially set to 0.01, the choice for a starting strength can be changed if it becomes necessary to speed up the competitive processing within an artificial vision system. Although an initially large and constant inhibitory weight will allow for a quicker recognition ability, a time varying weight is used since an initially large value may not be desirable if it results in necessary features being eliminated. This belief also corresponds with others who feel that inhibition cannot be too severe since the observer needs to be quickly made aware of objects and events within the visual field [6].

Once calculated, the inhibitory PI/PL cell signals will affect the net activation of each Pdm pulvinar cell by influencing how the hypercolumn simple cells are received. Since the network's competitive nature will result in only the salient feature characteristics activating each Pdm neuron, the output response of each Pdm neuron can be defined by the following conditions,

$$P_{(x,y)}(t) = 1 \quad \text{if} \quad net_{p(x,y)}(t) > \theta_p \qquad (5)$$

$$P_{(x,y)}(t) = 0 \quad \text{otherwise}$$

where θ_p represents the threshold of each pulvinar Pdm cell, assumed to be acting with a pure step function, or action potential response.

After comparison to the threshold, the Pdm neural signals will then be used to gate the signals projecting from area V1 to V4. Equation (6) defines the net activation of each area V4 cell, $net_{v4(x,y)}(t+1)$, at location (x,y) within the retinotopic grid.

$$net_{v4(x,y)}(t+1) = \rho_{v4} g_s \sum_{i=x-1}^{x+1} \sum_{j=y-1}^{y+1} S_{(i,j)k}(t) \qquad (6)$$

Within equation (6), $S_{(i,j)k}(t)$ is again the area V1 simple cell response to an orientation k at location (x,y). The parameter ρ_{v4} represents a bottom-up weight defining the connection strength between the area V1 and V4 cells. The term g_s represents the level of gating influence that extends from the Pdm pulvinar cells to the area V4 inputs. Although being driven to one for the present simulations, a detailed model for these projections could result in additional scale invariance beyond that already performed by the grid connections from area V1 to V4.

Like the response characteristics of the Pdm pulvinar cells, the final response of each area V4 cell, $V_{4(x,y)}(t)$, will also be defined by an internal threshold θ_{v4}, and an action potential step response, given below in equation (7).

$$V_{4(x,y)}(t) = 1 \quad \text{if} \quad net_{v4(x,y)}(t) > \theta_{v4} \qquad (7)$$

$$V_{4(x,y)}(t) = 0 \quad \text{otherwise}$$

As alluded to earlier, the network of area IT cells will interact with an external memory source that will not be directly modeled. For this reason, the present modeling and calculation of the area IT cell responses will not be explained by competitive network equations interacting with an external memory region, but will be described through processing of the area V4 inputs with a previously defined memory network. With regard to the development of artificial vision systems, numerous networks exist for carrying out any necessary memory requirements [22], [23], [24], [25], [26].

Finally, during the calculations of the area IT memory network, it will be necessary to determine a level of match between any previously stored memory and the area V4 inputs. Therefore, equation (8) will be used to determine the grid of area V4 outputs, defined here by the V_4 matrix, that will be sent to the area IT memory network. Within the equation, n_{V4} is a span parameter that allows for adjustment of the neighborhood grid and resulting V_4 matrix.

$$\overline{V}_4 = \sum_{i=x-n_{V4}}^{x+n_{V4}} \sum_{j=y-n_{V4}}^{y+n_{V4}} V_{4(i,j)}(t) \qquad (8)$$

3. ATTENTION NETWORK RESPONSE

During the following simulations the bottom-up attentional processing between areas V1, V4, IT, and the pulvinar will be simulated by repeatedly iterating equations (1) through (8). During the testing of the network, numerous artificially constructed edge detected 40 x 40 pixel images of lines, boxes, and diamonds (45 degree rotated boxes) were used to demonstrate the network's ability to locate salient features within the visual field, one of which is reproduced here. The illustrated cell responses are displayed against a black background, with black representing no activation, white full activation, and shades of gray giving intermediate intensity values in between the minimum and maximum amounts. This configuration allows for the individual signals to be compared against each other, while also being slightly more intuitive.

For instance, the image illustrated in Figure 3(a) was provided as input to the network. As observed, this image contains all 90 degree vertical lines, except for a single 135 degree line. Upon first seeing the image in Figure 3(a), the 135 degree line is clearly the most salient feature, and appears to command attention, at least initially. To test if the salient feature could be attended to and recognized, the memory network was trained to identify either a 0, 45, 90, or 135 degree line properly centered within the receptive field of an area IT cell. Network parameters were set to $x = 40$, $y = 40$, $\theta_{inh} = 0.3$, $\theta_p = 0.3$, $g_s = 1$, $\rho_{V4} = 0.3$, $\theta_{V4} = 0.3$, $n_{V4} = 3$, and $\delta_{TD} = 0$.

When the image given in Figure 3(a) was provided as input to the attentional network, the 135 degree line was quickly attended to and recognized within two iterations, as illustrated in Figure 3. For example, during the initial presentation of the image, a competition occurred between the pulvinar cells such that only those Pdm cells that initially responded to the 135 degree line remained active. These Pdm cell activations are illustrated in Figure 3(e) for both the first and second iterations (since external top-down signals were not implemented, the pulvinar competition caused the same Pdm cells to remain inhibited during each iteration). Also illustrated in Figure 3 are the cell activations for the area V1 and V4 cells for one and two iterations, with the iteration numbers shown in parentheses. Notice how each of the area V4 cells which has an object, or a portion of an object, within its receptive field is initially being activated by the area V1 simple cells. These initial activations occur since the effects of the pulvinar gating are not realized until the second iteration. This also corresponds to studies mentioned earlier which have observed how the area V4 cells will initially become excited by all the information within the visual field [7]. Since area V4 is providing area IT with too much information, the competitive interactions within this region are unable to determine a winner, resulting in no area IT cell (not illustrated) signaling the recognition of an object or feature within the visual field. Fortunately, during the second iteration the gating effects of the pulvinar Pdm cells are finally realized, as illustrated in Figure 3(f), such that only the area V1 signals corresponding to the 135 degree line are being allowed to activate the area V4 cells.

As mentioned earlier, one of the possible benefits of the increases in cortical receptive fields, as well as the pulvinar gating itself, would be the network's ability to allow for a certain amount of scale invariance of the input image with regard to the memory stored inside the area IT weight matrix. Although a pulvinar gating span was not directly implemented in the present model, the span of the signals traveling between areas V1 and V4 should allow for a slight amount of scale invariance. For instance, the original weight matrix was defined to recognize a 7 x 7 image, the exact size of the previous images which fell within the receptive field of the area IT cells. With traditional networks, recognition is often compromised when a slightly smaller or larger object is provided as input to the memory network. Likewise,

providing the memory network with an incorrectly sized input, defined by equations (8) through (10), will also normally result in a state of non-recognition. Fortunately, the neighborhood span of one, producing a 3 x 3 grid projection of signals between areas V1 and V4, should make the

a.) Original Image   d.) Simple Cells (2)

b.) Simple Cells (1)   e.) Pdm Cells (1, 2)

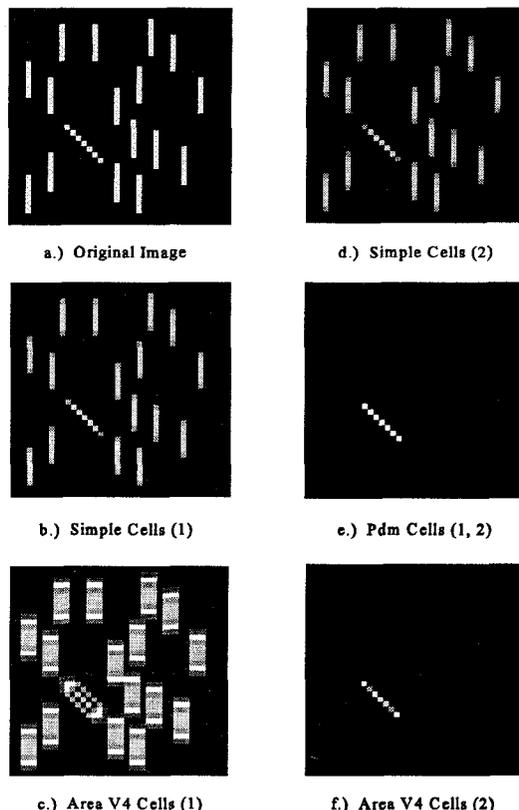c.) Area V4 Cells (1)   f.) Area V4 Cells (2)

Figure 3: Salient 135 Degree Line

necessary corrections when a series of slightly smaller 5 x 5 objects, or slightly larger 9 x 9 objects, are provided as input to the network, ultimately making it unnecessary to construct any new weight matrices. Although not illustrated in this paper, 5 x 5 and 9 x 9 objects were also subsequently identified by the network during testing.

4. CONCLUDING REMARKS

As illustrated in the previous section, modeling the competitive interactions within the pulvinar, along with the connections this structure makes with the other visual areas, has produced an attentional model that can detect salient features in a bottom-up manner. While not only producing characteristics observed in psychological studies, the benefit of such a model can be further realized when used to detect uncharacteristic defects during engineering and manufacturing applications, such as misaligned components, bent connector pins, and unknown salient defects that are uncharacteristic of items presently stored in memory.

Furthermore, by isolating single features or objects within the visual field, translation invariance is incorporated into the model, in addition to a slight amount of scale invariance resulting from the receptive field projections between the input and the memory network. Finally, the addition of an area V2 model in the future will also increase the current capabilities of the present model to include spectral aspects, as well as offering a more detailed analysis of individual features and objects.

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