Applications of method of least squares to the adjustment of level circuits: Adjustment of elevations in Dawson and St. James Townships, Phelps County, Missouri

Clarence Edward Bardsley

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APPLICATIONS OF THE METHOD OF LEAST SQUARES
TO THE ADJUSTMENT OF LEVEL CIRCUITS.

ADJUSTMENT OF ELEVATIONS IN DAWSON
AND ST. JAMES TOWNSHIPS,
PHelps COUNTY,
MISSOURI.

BY

CLARENCE EDWARD SOLOMAN Bardsley

A
THESIS
submitted to the faculty of the
SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI
in partial fulfillment of the work required for the
DEGREE OF
CIVIL ENGINEER
Rolla, Mo.
1922.

ASSISTED BY

CHARLES EDWARD Cooke

Approved by
Professor of Civil Engineering.
APPLICATIONS OF THE METHOD OF LEAST SQUARES
TO THE ADJUSTMENT OF LEVEL CIRCUITS.
ADJUSTMENT OF ELEVATIONS IN DAWSON
AND ST. JAMES TOWNSHIPS,
PHELPS COUNTY,
MISSOURI.

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PELPS COUNTY,
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BASED ON THE LINE OF LEVELS RUN BY THE
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INTRODUCTION.

An introduction to a thesis on least squares could be drawn out to considerable length in explanation: as the purpose of this thesis has been an investigation in applications of the method of least squares to the adjustment of level circuits, it will be found in the body of the report considerable space has been devoted to the explanation of the different applications of the method, with a view that these investigations may be of use to engineers in the profession; so the introductory remarks will be brief.

It is well known among engineers and others that measurements made with apparatus of most any kind, chains, rods, scales, verniers, micrometers, etc., and no matter how careful one may be in making measurements, there will be discrepancies among the observations. If a series of benchmarks be connected by lines of levels, some of which are check lines forming with the others complete circuits, it is necessary to adjust the difference in elevation so that all the circuits will close exactly, in order that the difference of elevation between any two benchmarks will be constant when computed through two or more series of lines by the several routes. This is illustrated by several applications of the method of least squares to the adjustment of elevations in Dawson and St. James Townships, Phelps County, Missouri.

It is understood that observations are always made as carefully as possible, for the most probable results and minimum errors. Positive and negative errors of the same magnitude occur with equal frequency; they are equally probable. Errors of increasing magnitude occur with decreasing frequency. Small errors occur more frequently; are more probable that large ones. Very large errors seldom occur; they are likely to belong in the class of mistakes rather than that of accidental errors. Accidental errors are systematically modified by the circumstances of the observations. The chief circumstances affecting a set of observations are atmospheric conditions, the skill of the observer, and the precision of the instruments.
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Net Problems ... Finas.
The principle from which the term "least squares" arises is the following: In measurements of equal precision, the most probable values of observed quantities are those that render the sum of the squares of the residual errors a minimum. The errors that may be corrected, however, by this method, are only those accidental errors that are governed by no laws other than those of chance.

The method of least squares takes first rank as an arithmetical process and by its use uniformity is secured in the process of adjustment and comparison, and the most probable balancing of errors of observations may be expected.

It is believed that the method of least squares is not in general use among engineers and that approximate methods are more often substituted for the adjustment of quantities, although far superior results might be expected through the employment of the more exact method. There are those, no doubt, who although versed in mathematical science, might find it difficult to utilize this method, for lack of concentration and patience to carry this process through to completion.

It is recommended, however, whenever precision is demanded and subsequent measurements are dependent upon the results.

In the adjustment of level circuits, the several lines are weighted inversely as their lengths, this causes us to state the above principle as follows: In observations where unequal weights are applied, the most probable values of the observed quantities are those that render the sum of the weighted squares of the residual errors a minimum.

The levels in Dawson and St. James Townships were run under my supervision by Messrs. E. E. Decker, W. R. Denison, and F. R. Bolland.

In the arrangement of this thesis, and in the computations and construction of drawings, I wish to acknowledge the valuable assistance rendered by Major Charles E. Cooke, Professor of Topographic Engineering.
PLATE B.

Showing Level Lines and the directions in which run. Elevations are in black at jet points. Differences of elevation are in red and brown between points.

Distances are in green. The values on this Plate are the values as instrumentally determined and correspond to the Original Level Notes and Description.

Books. Levels were run along Prince R.R. both ways from Lobadin to Arlington, a total distance of 39 miles, in 1908. This plan is based on the 1912 adjustment of the BM at Lobadin. R.R. BASE.
This Plate is a re-arrangement of the original data on Plate B using the differences of elevation therein and applying some consistently to the circuit A, L, K, I, P, M, H, F, G, X, D to A. With the exception of line A, B, C, D, E, the entire scheme is adjusted as a whole, arrows indicate direction of levels in circuits. Juncition point elevations are in black, differences of elevation are in red and colored, closures are in blue, distances are in green. Distance is not considered when shown in blue. Lines P, Q, N, R, and O are considered lost in the adjustment scheme.

PLATE C

Arrangement of Levels in circuits for rigorous adjustment. Line A, B, C, D and E to D is adjusted between A and E on railroad line of levels. First, and the whole scheme to the north is hinged on A, B, C, D as a base.
MAP of the
northeast portion of
PHPELS CO.
showing loca-
tion of
LEVEL LINES,
BENCH MARKS,
JUNCTION POINTS,
with respect to section
lines and
physical
features, etc.,
in DAWSON TWP.
and vicinity.
This MAP also
shows the
DIRECTION in
which the
LEVELS were
built.

In following
the discussion
of the
BASE MAP
and
ADJUSTMENT
are taken up.

The SCALE of
this MAP is
1/2 in.

PLATE A.
PART I.

THE INDEPENDENT ADJUSTMENT OF A MULTIPLE LINE.

METHOD OF LEAST SQUARES APPLIED TO THE ADJUSTMENT OF THE LINE A VIA B, C, AND D TO E.

(BASED ON THE LINE OF LEVELS RUN BY THE U. S. GEOLOGICAL SURVEY IN 1908 ALONG THE FRISCO R. R.
WITH PERMANENT BENCHMARKS "A" AT ST. JAMES, AND "E" AT KNOBVIEW).
MULTIPLE LINES. By a multiple line of levels is meant a set of two or more lines connecting the same two bench marks by routes of different length. In order to find the most probable value for the difference of elevation between the terminals of a multiple line, it is necessary to weight each constituent line inversely as its length. Having weighted the several lines as thus explained, the case becomes identical with any case of weighted measurements, and hence the probable error of a single measurement of unit weight is given by the formula:

\[ r = 0.6745 \sqrt{\frac{\sum p v^2}{n-1}} \]

the probable error of any of the lines of the weight \( p \) by the formula:

\[ \frac{r}{p} = 0.6745 \sqrt{\frac{\sum p v^2}{n-1}} \]
and the probable error of the weighted arithmetic mean by the formula:

\[ r_{pa} = \frac{r_1}{\sqrt{\sum p}} = 0.6745 \sqrt{\frac{\sum pv^2}{\sum p(n-1)}} = \frac{r_1}{\sqrt{\sum p}} \]

Our problem is as follows:

Given two lines of levels, as shown in the above figure, which give the following results:

- A via B, C, to D, 6.25 mile line, +11.111 ft.
- A via E to D, 1.00 mile line, +11.049 ft.

Notes: A to E given weight of infinity.

The elevation of B.W. at A = 1087.800 ft., and the elevation of B.W. at E = 1074.005 ft.

Requires the most probable elevation of the point D, and the probable error of this result.

**Solution**

<table>
<thead>
<tr>
<th>M</th>
<th>pM</th>
<th>v</th>
<th>v²</th>
<th>pv²</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.111</td>
<td>0.160</td>
<td>+0.053</td>
<td>0.002809</td>
<td>0.00044944</td>
</tr>
<tr>
<td>11.049</td>
<td>1.000</td>
<td>-0.009</td>
<td>0.000081</td>
<td>0.00008100</td>
</tr>
</tbody>
</table>

\[ \Sigma p = 11.160 \]

\[ \Sigma pv^2 = 0.00053044 \]

n = 2.

Determination of the above quantities:

- A = 1087.800 ft.
- D = 1098.849 ft.
- E = 1074.005 ft.

+11.111 ft.
-13.795 ft.

Diff. Elev. A via B,
Diff. Elev. A to E.
C to D.

Diff. Elev. A via E to D.

D from Ozark Hwy line = 1098.911 ft.
D from Knobview B. W. = 1098.849 ft.

Observed error at D = 0.062 ft.

Residuals: In observations of unequal weight, the difference between the weighted arithmetic mean and the observed value of each observation is a residual. If the observed value is greater than the weighted arithmetic mean, the residual quantity is plus, and if less than the weighted arithmetic mean, the residual quantity is minus.

First observed diff. in Elev W = 11.111 ft.
Weighted arithmetic mean pM = 11.058
First residual error = +0.053
GENERAL PRINCIPLE: In observations of unequal precision, the most probable values of the observed quantities are those that render the sum of the weighted squares of the residual errors a minimum. It is on account of this principle that the method of least squares has been so named.

The weights \( P \) are the reciprocals of the distances for each line. Thus:

A via B, C, to D = 6.25 miles, reciprocal or wt. = 0.160.
E to D = 1.00 mile

Note: Reciprocals and squares can be very conveniently looked up in Barlow's Tables of Squares, Roots, Etc.

\( n \) = the number of observations.

CHECK: As a check on the solution so far, the summation of weighted residuals must equal zero.

\[
\begin{array}{ccc}
P & \Delta P & \Delta PV \\
0.160 & -0.053 & -0.009 \\
1.000 & -0.089 & -0.022 \\
\hline \\
\sum & & 0.000
\end{array}
\]

Then the probable error of the weighted arithmetic mean is:

\[
\sigma_p = 0.6745 \sqrt{\frac{0.00052044}{0.160 \times 1}} = 0.0144 \text{ ft.}
\]

1087.800 + 11.058 = 1098.858 ft.

And the most probable value for the elevation \( D \) is:

\[
D = 1098.858 \pm 0.0144 \text{ ft.}
\]

INTERMEDIATE POINTS. By an intermediate point is meant one lying only on a single line of levels, and hence having no influence on the general adjustment. These may occur on a line whose ends have been satisfactorily adjusted, or on a closed circuit. In either case, the required adjustment is distributed uniformly throughout the line, making the correction between any two points directly proportional to the length between those two points.

ADJUSTMENT OF INTERMEDIATE JUNCTION POINTS B & C.

Observed value of \( D \) = 1098.911 ft. (Ozark Hwy. Line)

Adjusted value of \( D \) = 1098.858 ft.

Adjustment = + 0.053 ft. lower at 0 and taper to 0 at A.
As the distance A via B, C to D = 6.25 miles, and the distance A to B = 1.00 mile, then the adjustment to point B is as follows:

**Adjustment to B = \(-\frac{1}{6.25} \times 0.053 = -0.008\) ft.**

And the most probable elevation of point B is:

\[-1074.666 - 0.008 = 1074.658\] ft.

In like manner, as the distance A to C = 4.00 miles, the adjustment to C = \(-\frac{1}{6.25} \times 0.053 = -0.034\) ft.

And the most probable elevation of point C is:

\[-1083.400 - 0.034 = 1083.366\] ft.

---

**CHECK on elevation of D from E.**

Elevation of D from E = 1098.849 ft.
Elevation of E = 1074.005 ft.

**Diff. in Elev. E to D = 24.844 ft.**

Correction for 1.00 mile E to D = -0.009 ft.

Elev. E = 1074.005 ft.

**Most probable Elev. of D = 1098.858 ft**

Observed Elev. at D = 1098.849 ft
Adjusted Elev. of D = 1098.858 ft

Adjustment = -0.009 ft. (O.K. for 1 Mile).

---

See Plate C for the adjusted values for this circuit.

---

**DIAGRAM SHOWING ADJUSTMENT OF POINTS BET. A AND D.**

From the above diagram it is easily seen how the values along the Observed (red) line are lowered proportionally to the distances from A down to the Adjusted (black) line. This diagram also serves as a check on the adjustments of the intermediate points if drawn to scale.
PART II.

THE LEAST SQUARE ADJUSTMENT OF A LEVEL NET
BY METHOD OF OBSERVATION EQUATIONS.
APPLIED TO THE ADJUSTMENT
OF THE
DAWSON-ST. JAMES TOWNSHIP LEVEL NET.
PRELPS COUNTY,
MISSOURI.

(BASED ON THE A VIA B, C, AND D TO E BASE).
THE DETERMINATION AND ADJUSTMENT OF ELEVATIONS IN
PHELPS COUNTY MISSOURI.
(Dawson Township Level Net)

Referring to the Dawson Township Level Net Diagrams indicated on pages and , the field notes show the following results:

\[
\begin{align*}
A \text{ via } L, K \text{ to } I &= -25.875 \text{ ft.} \\
I \text{ via } P \text{ to } M &= -107.201 \text{ ft.} \\
J \text{ to } H &= +73.460 \text{ ft.} \\
M \text{ to } J &= +13.695 \text{ ft.} \\
H \text{ to } F &= -40.447 \text{ ft.} \\
F \text{ to } G &= -80.066 \text{ ft.} \\
G \text{ via } X, E \text{ to } A &= +166.589 \text{ ft.} \\
I \text{ to } J &= -93.531 \text{ ft.} \\
A \text{ via } B \text{ to } H &= -45.750 \text{ ft.} \\
A \text{ via } C \text{ to } F &= -86.657 \text{ ft.} \\
M \text{ via } N, O \text{ to } G &= -33.201 \text{ ft.}
\end{align*}
\]

It is desired to find the most probable values for the elevations of the level net junction stations I, M, J, H, F, and G. Upon finding these junction elevations, it is then desired to find all intermediate and remaining elevations in this net. Referring to page 2, it will be noted that the line of levels running from A via B, C, E, to E has previously been adjusted; levels along the Frisco R. R. have also been adjusted; these lines will be considered base lines and their values will remain unchanged. The Dawson Township net will be considered to hinge on the U. S. Geological Survey Stations A (at St. James) and E (at Knobview).

SOLUTION OF PROBLEM BY METHOD OF LEAST SQUARES.

As there are but six unknown Bench Marks (B.Ms.) (I, M, J, E, F, and G), there can be but six independent unknowns in the observation equations. As the lines AI, IM, MJ, JH, HF, and FG, may evidently be selected as the independent unknowns, we may write for the most probable values
of the corresponding differences of elevation:

\[ A \text{ via } L, K, \text{ to } I = -25.875 + V_1; \]
\[ I \text{ via } P \text{ to } M = -107.201 + V_2; \]
\[ M \text{ to } J = +13.695 + V_3; \]
\[ J \text{ to } H = +73.460 + V_4; \]
\[ H \text{ to } F = -40.447 + V_5; \]
\[ F \text{ to } G = -80.066 + V_6. \]

The conditional equations involved in the several closed circuits may then be avoided by writing all the observation equations in terms of these quantities. Writing the reduced observation equations directly from the diagram on page 1C, we have, by comparison with the observed values,

\[ A \text{ via } L, K, \text{ to } I = V_1 = +0.000 \text{ (weight 0.160);} \]
\[ I \text{ via } P \text{ to } M = V_2 = 0.000 \text{ (weight 0.154);} \]
\[ M \text{ to } J = V_3 = 0.000 \text{ (weight 0.312);} \]
\[ J \text{ to } H = V_4 = 0.000 \text{ (weight 0.625);} \]
\[ H \text{ to } F = V_5 = 0.000 \text{ (weight 0.571);} \]
\[ F \text{ to } G = V_6 = 0.000 \text{ (weight 2.000);} \]
\[ G \text{ via } X, L, \text{ to } A = -V_1 - V_2 - V_3 - V_4 - V_5 - V_6 = +0.155 \text{ (weight 0.222);} \]
\[ A \text{ via } E \text{ to } H = V_1 + V_2 + V_3 + V_4 = -0.171 \text{ (weight 0.294);} \]
\[ A \text{ via } C \text{ to } F = V_1 + V_2 + V_3 + V_4 + V_5 = +0.289 \text{ (weight 0.332);} \]
\[ M \text{ via } N, O, \text{ to } G = V_3 + V_4 + V_5 + V_6 = -0.157 \text{ (weight 0.124);} \]
\[ I \text{ to } J = V_2 + V_3 = +0.025 \text{ (weight 0.625).} \]

To show how the above reduced observation equations were formed, a compilation of the values for elevations of the several level net junction points determined from the successive differences of elevation along the selected circuit, A via L, K, I, P, M, J, H, F, G, X, E, to A is shown below. The closures shown above for the several circuits are extended below, as well as the calculation of the respective weights.
Observed values for the elevations of stations along the Circuit A via L, K, I, P, M, J, H, F, G, X, L, to A, determined from successive differences of elevation:

\[ A = 1087.800 \text{ ft. U.S.S. Iron Post at St. James.} \]
\[ \downarrow \]
\[ I = 1061.925 \text{ ft.} \quad I = 1061.925 \text{ ft.} \]
\[ - 25.875 \quad - 25.875 \]
\[ M = 954.724 \text{ ft.} \quad 93.506 \text{ ft.} \]
\[ J = 968.419 \text{ ft.} \quad 1041.875 \text{ ft.} \]
\[ H = 964.490 \text{ ft.} \quad 86.557 \text{ ft.} \]
\[ A = 1087.800 \text{ ft.} \quad 0.155 \text{ ft Closure.} \]

It will be noted that in each case the respective weights are the reciprocals of the distances.
We next write the normal equations, which are arrived at as summarized in the following rule:

**RULE:** To form the normal equation for each one of the unknown quantities, multiply each observation equation by the product of the weight of that observation and the algebraic coefficient of that unknown quantity in that equation, and add the results.

Having formed the several normal equations, their solution as simultaneous equations gives the most probable values of the unknown quantities.

Then by the above rule; we have:

**NORMAL EQUATION IN $V_1$. (Solution for)**

\[
\begin{align*}
0.160V_1 &= +0.000000 \\
0.222V_1 + 0.222V_2 + 0.222V_3 + 0.222V_4 + 0.222V_5 + 0.222V_6 &= -0.034410 \\
0.294V_1 + 0.294V_2 + 0.294V_3 + 0.294V_4 &= -0.050274 \\
0.333V_1 + 0.333V_2 + 0.333V_3 + 0.333V_4 + 0.333V_5 &= +0.096333 \\
1) &= +0.22V_1 + 0.22V_2 + 0.22V_3 + 0.22V_4 + 0.22V_5 + 0.22V_6 = -0.039410 \\
0.156V_2 &= +0.000000 \\
0.222V_1 + 0.222V_2 + 0.222V_3 + 0.222V_4 + 0.222V_5 + 0.222V_6 &= -0.034410 \\
0.294V_1 + 0.294V_2 + 0.294V_3 + 0.294V_4 &= -0.050274 \\
0.333V_1 + 0.333V_2 + 0.333V_3 + 0.333V_4 + 0.333V_5 &= +0.096333 \\
+0.625V_2 + 0.625V_3 &= +0.015625 \\
2) &= +0.312V_3 &= +0.000000 \\
0.222V_1 + 0.222V_2 + 0.222V_3 + 0.222V_4 + 0.222V_5 + 0.222V_6 &= -0.034410 \\
0.294V_1 + 0.294V_2 + 0.294V_3 + 0.294V_4 &= -0.050274 \\
0.333V_1 + 0.333V_2 + 0.333V_3 + 0.333V_4 + 0.333V_5 &= +0.096333 \\
+0.124V_3 + 0.124V_4 + 0.124V_5 + 0.124V_6 &= -0.019468 \\
+0.625V_2 + 0.625V_3 &= +0.015625 \\
3) &= +0.312V_3 + 0.474V_3 + 0.312V_3 + 0.377V_4 + 0.377V_5 + 0.377V_6 = +0.007808
\end{align*}
\]
NORMAL EQUATION IN $V_4$.

$+0.625V_4 = +0.000000$

$0.222V_1 + 0.222V_2 + 0.222V_3 + 0.222V_4 + 0.222V_5 + 0.222V_6 = -0.034410$

$0.294V_1 + 0.294V_2 + 0.294V_3 + 0.294V_4 = -0.050274$

$0.333V_1 + 0.333V_2 + 0.333V_3 + 0.333V_4 + 0.333V_5 = +0.096333$

$+0.124V_1 + 0.124V_2 + 0.124V_3 + 0.124V_5 + 0.124V_6 = -0.019468$

NORMAL EQUATION IN $V_5$.

$+0.571V_5 = +0.000000$

$0.222V_1 + 0.222V_2 + 0.222V_3 + 0.222V_4 + 0.222V_5 + 0.222V_6 = -0.034410$

$0.333V_1 + 0.333V_2 + 0.333V_3 + 0.333V_4 + 0.333V_5 = +0.096333$

$+0.124V_1 + 0.124V_2 + 0.124V_3 + 0.124V_5 + 0.124V_6 = -0.019468$

NORMAL EQUATION IN $V_6$.

$+2.000V_6 = +0.000000$

$0.222V_1 + 0.222V_2 + 0.222V_3 + 0.222V_4 + 0.222V_5 + 0.222V_6 = -0.034410$

$+0.124V_1 + 0.124V_2 + 0.124V_3 + 0.124V_5 + 0.124V_6 = -0.019468$

$+0.346V_1 + 0.346V_2 + 0.346V_3 + 0.346V_4 + 0.346V_5 + 0.346V_6 = -0.053878$

Whence from the solution for the Normal Equations, we write the six following normal equations that are to be solved simultaneously for the values of $V_1, V_2, V_3, V_4, V_5, \& V_6$.

1) $+1.099V_1 + 0.849V_2 + 0.849V_3 + 0.849V_4 + 0.555V_5 + 0.222V_6 = +0.011643$

2) $+0.849V_1 + 1.630V_2 + 1.474V_3 + 0.849V_4 + 0.555V_5 + 0.222V_6 = +0.027274$

3) $+0.849V_1 + 1.474V_2 + 1.910V_3 + 0.973V_4 + 0.679V_5 + 0.346V_6 = +0.007806$

4) $+0.849V_1 + 0.849V_2 + 0.973V_3 + 1.598V_4 + 0.679V_5 + 0.346V_6 = -0.007819$

5) $+0.555V_1 + 0.555V_2 + 0.679V_3 + 0.679V_4 + 1.250V_5 + 0.346V_6 = +0.042455$

6) $+0.222V_1 + 0.222V_2 + 0.346V_3 + 0.346V_4 + 0.346V_5 + 2.346V_6 = -0.053878$

Now we are to solve the above equations simultaneously.

SOLUTION OF NORMAL EQUATIONS.
SOLUTION OF NORMAL EQUATIONS.

First Set.

ILLINIMATE $V_3$.

1) $+1.0090V_1 +0.8490V_2 +0.8490V_3 +0.8490V_4 +0.5550V_5 +0.2220V_6 = +0.011649$
2) $0.8490V_1 +0.9390V_2 +0.8490V_3 +0.4890V_4 +0.3197V_5 +0.1279V_6 = +0.015713$
3) $+0.9200V_1 -0.0900V_2 = +0.3600V_4 +0.2353V_5 +0.0941V_6 = -0.004064$

Second Set.

ILLINIMATE $V_4$.

7) $+0.5200V_1 -0.0900V_2 +0.3600V_3 +0.2353V_5 +0.0941V_6 = -0.004064$
8) $0.3000V_1 +0.4925V_2 +0.0981V_4 +0.0310V_5 -0.0450V_6 = +0.021250$
9) $0.0371V_1 -0.0087V_2 -0.0981V_4 -0.0232V_5 -0.0151V_6 = -0.001050$
10) $0.1587V_1 +0.4838V_2 = +0.0018V_5 -0.0601V_6 = +0.022300$

-12-
Summary of Second Set of Equations:

12) -0.1912V1 - 1.8973V2 + 0.1115V5 + 0.2592V6 = -0.082046
13) +0.1567V1 + 0.4838V2 + 0.0018V5 - 0.0401V6 = +0.022300
14) -0.6317V1 - 0.0067V2 + 4.4946V5 - 0.8518V6 = -0.214545
11) +0.1193V1 + 0.1193V2 + 0.5710V5 - 4.2573V6 = +0.148197

Third Set:

12) -0.1912V1 - 1.8973V2 + 0.1115V5 + 0.2592V6 = -0.082046
13) +0.1567V1 + 0.4838V2 + 0.0018V5 - 0.0401V6 = +0.022300
14) -0.6317V1 - 0.0067V2 + 4.4946V5 - 0.8518V6 = -0.214545
11) +0.1193V1 + 0.1193V2 + 0.5710V5 - 4.2573V6 = +0.148197

Fourth Set:

15) -1.3070V1 + 0.1237V2 + 0.1859V5 = -0.054836
16) +0.4821V1 + 1.1131V2 + 0.5710V5 = +0.030920
17) +0.6250V1 - 1.4711V2 + 23.3796V6 = +0.570113

Summary of Third Set of Equations:

15) -1.3070V2 + 0.1237V6 + 0.1859V6 = -0.054836
16) +0.4821V2 + 1.1131V2 + 0.5499V6 = +0.136882
17) +0.6250V2 + 0.4711V2 + 23.3796V6 = +0.470822
Summary of Fourth Set of Equations:

18) \(-2.8940V_5 - 0.5499V_6 = -0.138662\)
19) \(0.0216V_5 + 17.7766V_6 = 0.470682\)

Summary of Fifth Set of Equations:

18) \(-2.8940V_5 - 0.5499V_6 = -0.138662\)
19) \(/2.8940V_5 + 2381.7352V_6 = -63.062671\)
20) \(+2381.1853V_6 = -63.201333\)

Summary of Fifth Set of Equations:

20) \(+2381.1853V_6 = -63.201333\)
Then \(V_6 = -0.02654\)

Substitute this value for \(V_6\) in equation 18) and solve for \(V_5\):
18) \(-2.8940V_5 + 0.014594346 = -0.138662\)
Then \(-2.8940V_5 = -0.153256346\)
And \(V_5 = +0.052956\)

Substitute the values found for \(V_6\) and \(V_5\) in equation 15) and solve for \(V_2\):
15) \(-1.3070V_2 + 0.0065506572 - 0.004933786 = -0.054836\)
Then \(-1.3070V_2 = -0.0564528712\)
And \(V_2 = +0.0431927\)

Substitute the values found for \(V_6\), \(V_5\), and \(V_2\) in equation 11) and solve for \(V_1\):
11) \(+0.1193V_1 + 0.00515288911 + 0.030237876 + 0.113004666 = 0.148187\)
Then \(+0.1193V_1 = -0.00020843111\)
And \(V_1 = -0.001747\)

Substitute the values found for \(V_6\), \(V_5\), \(V_2\), and \(V_1\) in equation 7) and solve for \(V_4\):
7) \(-0.00090844 - 0.0038873437 + 0.3600V_4 + 0.8124605458 - 0.002497414 = 0.004064\)
Then \(+0.3600V_4 = -0.0092313438\)
And \(V_4 = -0.025643\)
Substitute the values found for \( V_6, V_5, V_2, V_1 \), and \( V_4 \) in equation 1) and solve for \( V_3 \):

\[
1) -0.001762723 + 0.0366706023 + 0.849 V_3 - 0.021770907 - 0.00589188 = +0.011649.
\]

Then \( +0.849 V_3 = -0.0249866723 \)

And \( V_3 = -0.02943 \)

Check these \( V \) values thru one of the equations in the first set, and we find:

\[
4) -0.001483203 + 0.036670857 - 0.02863539 - 0.040377514 + 0.035957124 - 0.00318284 = -0.007819
\]

And we find that these values will check \( 0 = 0 \) thru the fourth decimal place.

Summary of residual values.

\[
\begin{align*}
V_1 &= -0.001747 \text{ ft;} \\
V_2 &= +0.043193 \text{ ft;} \\
V_3 &= -0.029430 \text{ ft;} \\
V_4 &= -0.025642 \text{ ft;} \\
V_5 &= +0.052956 \text{ ft;} \\
V_6 &= -0.026540 \text{ ft.}
\end{align*}
\]

Now we find that the most probable values for the differences of elevation are:

\[
\begin{align*}
\text{A via L, K, to I} &= -25.875 - 0.0017 = -25.8767 \text{ ft;} \\
\text{I via P to M} &= -107.201 + 0.0432 = -107.1578 \text{ ft;} \\
\text{M to J} &= + 13.695 - 0.0294 = + 13.6656 \text{ ft;} \\
\text{J to H} &= + 73.460 - 0.0256 = + 73.4344 \text{ ft;} \\
\text{H to F} &= - 40.447 + 0.0530 = - 40.3940 \text{ ft;} \\
\text{F to G} &= - 80.066 - 0.0265 = - 80.0925 \text{ ft.}
\end{align*}
\]

Then from A via the route L, K, I, P, M, J, H, F, G, back to A, we have the most probable elevations of the unknown stations.

\[
\begin{align*}
\text{A} &= 1087.8000 \text{ ft.} & \text{M} &= 954.7655 \text{ ft.} & \text{H} &= 1041.8655 \text{ ft.} \\
& -25.8767 & & \text{+13.6656} & & \text{-40.3940} \\
\text{I} &= 1081.9233 \text{ ft.} & \text{J} &= 968.4311 \text{ ft.} & \text{F} &= 1001.4715 \text{ ft.} \\
& -197.1578 & & \text{+73.4344} & & \text{-80.0925} \\
\text{M} &= 954.7655 \text{ ft.} & \text{H} &= 1041.8655 \text{ ft.} & \text{G} &= 921.3730 \text{ ft.}
\end{align*}
\]

-15-
(O) NORMAL EQUATIONS.

<table>
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<th>$V_2$</th>
<th>$V_3$</th>
<th>$V_4$</th>
<th>$V_5$</th>
<th>$V_6$</th>
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<td>+0.849</td>
<td>+0.849</td>
<td>+0.849</td>
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<td>+0.849</td>
<td>+0.555</td>
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<td>+1.010</td>
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(Numbers below the red line are redundant numbers).

(P) Solution of Normal Equations.

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(S) Find most probable values for the differences of elevation.

A via L, K, to I = -25.879 - 0.0016 = -25.8783 ft;
J via F to M = -107.201 + 0.0440 = -107.1570 ft;
M to J = 13.665 - 0.0304 = 13.6356 ft;
J to H = 73.460 - 0.0257 = 73.4343 ft;
H to F = 40.447 + 0.0530 = 40.3940 ft;
F to G = 80.066 - 0.0285 = 80.0381 ft.

(T) Then from A via the route L, K, I, P, M, J, H, F, G, back to A, we have the most probable elevations of the unknown stations:

A = 1087.8000 ft.,  M = 954.7664 ft.,  H = 1041.8653 ft.,
I = 1061.9234 ft.,  J = 969.7410 ft.,  F = 1001.4713 ft.,
M = 1021.1570 ft.,  P = 173.4243 ft.,  G = 921.3788 ft.

-16-
(q) Solution of Normal Equations.

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<td></td>
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<td></td>
</tr>
<tr>
<td>q14</td>
<td>+0.8694</td>
<td>+0.1646</td>
<td>-0.04182</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q15</td>
<td>+2.3460</td>
<td>+0.053878</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q16</td>
<td>-0.0488</td>
<td>+0.002563</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q17</td>
<td>-0.0014</td>
<td>+0.000671</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q18</td>
<td>-0.0289</td>
<td>-0.003901</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q19</td>
<td>-0.0175</td>
<td>-0.002288</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q20</td>
<td>-0.0312</td>
<td>+0.007890</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(r) Solution of Normal Equations.

<table>
<thead>
<tr>
<th></th>
<th>V1</th>
<th>V2</th>
<th>V3</th>
<th>V4</th>
<th>V5</th>
<th>V6</th>
</tr>
</thead>
<tbody>
<tr>
<td>r1</td>
<td>+0.011545</td>
<td>+0.019082</td>
<td>-0.030011</td>
<td>-0.019114</td>
<td>+0.047943</td>
<td>-0.026514</td>
</tr>
<tr>
<td>r2</td>
<td>+0.005633</td>
<td>+0.001018</td>
<td>+0.006096</td>
<td>+0.003860</td>
<td>+0.005019</td>
<td></td>
</tr>
<tr>
<td>r3</td>
<td>-0.029134</td>
<td>-0.005089</td>
<td>-0.013124</td>
<td>-0.004243</td>
<td>+0.002962</td>
<td></td>
</tr>
<tr>
<td>r4</td>
<td>+0.021614</td>
<td>+0.003779</td>
<td>+0.002874</td>
<td>+0.002588</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r5</td>
<td>+0.025549</td>
<td>+0.025194</td>
<td>-0.003965</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Adjustment Quantities:

V fruity -0.0016 & V fruity = -0.0257

Check these V values thru one of the equations in (o), and we find:

1) -0.001614400 +0.037541587 -0.025779885 -0.021810112 +0.029938310 -0.005685108 -0.011449000 = 0

And we find that these values check 0 = 0 thru the seventh decimal place.

-17-
ADJUSTMENT OF INTERMEDIATE JUNCTION POINTS L, K, P, M, O.

Notes: (See discussion on Intermediate Points on page 13.)

Observed value of \( l = 1061.925 \) ft. (Republic Road Line)

Adjusted value of \( l = 1061.923 \) ft.

Adjustment = \( +0.002 \) ft. Lower line at \( l \) and taper to 0.00 at \( A \).

As the \( A \) via \( L \) to \( l = 6.25 \) miles, and the distance \( A \) to \( L = 0.5 \) miles; then the adjustment to point \( L \) is as follows:

\[
\text{Adjustment to } L = \frac{6.25}{6.25} \times 0.002 = 0.00016 \text{ ft.}
\]

And the most probable elevation of point \( L \) is

\[
= 1078.427 - 0.000 = 1078.427 \text{ ft.}
\]

In like manner, as the distance \( A \) to \( K = 1.75 \) miles,

the adjustment to \( K = \frac{1.75}{6.25} \times 0.002 = 0.00056 \text{ ft.}

And the most probable elevation of point \( K \) is

\[
= 1092.694 - 0.001 = 1092.693 \text{ ft.}
\]

Diagram showing adjustment of points bet. \( A \) and \( L \).

From the above diagram it is easily seen how the values along the Observed line are lowered proportional to the distance from \( A \) to the Adjusted line. The diagram also serves as a check on the adjustment of the intermediate points if drawn to scales horizontally and vertically.
Observed value of point 1 = 1061.925 ft. (Republic Road)
Adjusted value of point 1 = 1061.923
Adjustment = + 0.002 ft. Line to be lowered at 1.

Arranged value of point W
(in large circuit from summation of consecutive diffs. in elevation.) = 954.724 ft. (from the west)

Adjusted value of point W = 954.766
Adjustment = - 0.042 ft. Line to be raised at W.

From the accompanying diagram by proportionate lines in the similar triangles, we affect the adjustment to the point P as follows:
Relations stated as a proportion:
\[
\frac{6.4}{4.0} = \frac{0.042 + 0.002}{x + 0.002}
\]

Solve algebraically:
\[
x + 0.002 = \frac{0.042 + 0.002}{6.4} \\
x = \frac{(0.042 + 0.002) \times 4.0}{6.4} - 0.002
\]

And we find:
\[
x = 0.0255 \text{ ft. (Adjustment to point P.)}
\]

Then the most probable elevation of point P =
\[
= 928.552 + 0.026 = 928.578 \text{ ft.}
\]
Adjusted value of point W = 954.766

Adjustment = -0.042 ft. Line to be raised at W.

Adjusted value of point N = 875.092 ft.

Construction Line.

ADJUSTMENT OF POINT O.

In the same manner as above:

Solving: y = 0.0342 ft. The adjustment to point O.

Then the most probable elevation of point

O = 969.736 - 0.034 = 969.702 ft.
THE DETERMINATION AND ADJUSTMENT OF ELEVATIONS IN
PHELPS COUNTY MISSOURI.
(Dawson Township Level Net)

Adjustment of lines PG and NR as Parallel Duplicate Lines of Levels, and the adjustment of the line OSO as a Duplicate Line of Levels.
See sketch below and the three Plates of Dawson Township.

Adjusted values are as follows

- \( P = 928.578 \) ft.
- \( M = 954.766 \) ft.
- \( N = 875.092 \) ft.
- \( O = 969.702 \) ft.

It is required to find the most probable elevations of the points Q, R, and S.

ADJUSTMENT OF POINT Q.
Parallel Duplicate Lines of Levels: By a parallel duplicate line of levels is meant a line that is run twice over the same route with equal care and in the same direction. Levels are sometimes run in this manner when there are two instruments on the work, as was the case when the line PG was run, also the line NR.

- \( Q = 928.552 \) ft. 1st instrument.
- \( Q = 928.552 \) ft. 2nd instrument.

- Difference in elevation = \( +68.690 \) ft.
- Difference in elevation = \( +68.685 \) ft.
UNWEIGHTED MEASUREMENTS. If the difference of elevation of two stations is measured a number of times in the same manner, over the same length of line, and under such conditions that the different determinations may be regarded as of equal weight, then the arithmetic mean of the several results is the most probable value of this difference of elevation. The probable error of a single measurement is given by the formula:

\[ r_1 = 0.6745 \sqrt{\frac{\Sigma v^2}{n - 1}} \]

The probable error of the arithmetic mean of \( n \) measurements by the formula:

\[ r_a = \frac{r_1}{\sqrt{n}} = 0.6745 \sqrt{\frac{\Sigma v^2}{n(n - 1)}} \]

ADJUSTMENT OF POINT Q.

<table>
<thead>
<tr>
<th>Observed Value</th>
<th>( V )</th>
<th>( V^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>68.690 ft.</td>
<td>+0.0025</td>
<td>0.00000625</td>
</tr>
<tr>
<td>68.685 ft.</td>
<td>-0.0025</td>
<td>0.00000625</td>
</tr>
<tr>
<td>( 2 \times 68.6875 )</td>
<td>0.0000</td>
<td>0.00001250 = ( \Sigma v^2 )</td>
</tr>
<tr>
<td>( z = 68.6875 \text{ ft.} )</td>
<td>( \text{Check} )</td>
<td>( n = 2 ) ((\text{No. of Obs.}))</td>
</tr>
</tbody>
</table>

\[ r_a = 0.6745 \sqrt{\frac{0.00001250}{2(2 - 1)}} = \pm 0.0017 \text{ ft.} \]

Then the most probable value for the difference in elevation = 68.6875 ± 0.0017 ft.

And the most probable elevation of the point Q is as follows:

Adjusted value of point P = 928.578 ft.

Adjusted lift in elev. P to Q = 68.688

Most probable elev. point Q = 997.266 ft.

ADJUSTMENT OF POINT R.

<table>
<thead>
<tr>
<th>Observed elevation of point N</th>
<th>( R )</th>
<th>1st instrument</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 875.250 ) ft.</td>
<td>( R = 866.383 )</td>
<td></td>
</tr>
</tbody>
</table>

Difference in elevation = -8.867

<table>
<thead>
<tr>
<th>Observed elevation of point N</th>
<th>( N )</th>
<th>2nd instrument</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 875.250 ) ft.</td>
<td>( N = 866.293 )</td>
<td></td>
</tr>
</tbody>
</table>

Difference in elevation = -8.957
ADJUSTMENT OF POINT R.

The line N to R is a parallel duplicate line of levels.

<table>
<thead>
<tr>
<th>Observed values</th>
<th>V</th>
<th>V²</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.867 ft.</td>
<td>-0.045</td>
<td>0.002025</td>
</tr>
<tr>
<td>8.957 ft.</td>
<td>+0.045</td>
<td>0.002025</td>
</tr>
</tbody>
</table>

\[ 2z = 17.824 \]

\[ z = 8.912 \text{ ft.} \]

\[ r_a = 0.6745 \sqrt{\frac{0.004050}{2(2 - 1)}} = \pm 0.0304 \text{ ft.} \]

Then the most probable value for the difference in elevation = 8.912 ± 0.0304 ft.

And the most probable elevation of the point R is as follows:

Adjusted value of point N = 875.092 ft.

Adjusted Diff. in elev. N to R = -8.912

Most probable elev. point R = 866.180 ft.

ADJUSTMENT OF POINT S.

Duplicate Lines of Levels: By a duplicate line of levels is meant a line that is run twice over the same route with equal care, but in opposite directions.

Letting \( d \) represent the discrepancy between the result obtained from the forward line and that obtained from the reverse line, we have:

\[ v_1 = + \frac{d}{2} \]

and

\[ v_2 = - \frac{d}{2} \]

Substituting these values in the following equation, and replacing \( r \) with \( r_a \) for the case of duplicate lines, we have the probable error of a single determination (forward or reverse) by a line of the length 1:

\[ r = 0.6745 \sqrt{\frac{\Sigma v^2}{n - 1}} \]

\[ \Sigma v^2 = v_1^2 + v_2^2 = \left( + \frac{d}{2} \right)^2 + \left( - \frac{d}{2} \right)^2 = \frac{d^2}{4} \]

\[ \Sigma v^2 = \frac{d^2}{2} \]
Substituting:

$$r_L = 0.6745 \frac{d^2}{2(n - 1)} + \frac{0.6745}{1.4142136} \sqrt{\frac{d^2}{n - 1}}$$

In the case of duplicate measurements, $(n - 1) = (2 - 1) = 1$.

Then:

$$r_L = 0.4769 \sqrt{d^2} = \pm 0.4769 \, d$$

And for the error of the arithmetic mean in duplicate leveling:

$$r_A = \frac{r_L}{\sqrt{n}} = \frac{0.4769 \, d}{\sqrt{2}} = \frac{0.4769 \, d}{1.4142136} = \pm 0.3373 \, d$$

$$r_A (\text{approximately}) = \pm 1/3 \, d$$

**Adjustment of Point S.**

**Observed elevation of point 0:** 969.758 ft (From South)

**Observed elevation of point 8:** 942.516 ft (From South)

**Difference in elevation:** -27.242 ft (Going North)

**Observed elevation of point S:** 942.516 ft (From North)

**Observed elevation of point 0:** 969.743 ft (From South)

**Difference in elevation:** -27.227 ft (Going South)

**Observed Values**

27.242 ft. Then: $r_L = \pm 0.4769 \times 0.015 = \pm 0.0071$ ft.

27.227 ft. And: $r_A = \pm 0.3373 \times 0.015 = \pm 0.0051$ ft.

Then the most probable value for the difference in elevation = 27.242 - 0.0071 = 27.2495 ± 0.0051 ft.

And the most probable elevation of the point S is as follows:

Adjusted value of point 0 = 969.702 ft (On Circuit)

Adjusted Diff. in elev. O to S = -27.250 ft.

Most probable elev. point S = 942.452 ft.

**Summary of the Elevations of All of the Junction Points.**

<table>
<thead>
<tr>
<th>Point</th>
<th>Elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1047.838 ft.</td>
</tr>
<tr>
<td>B</td>
<td>1074.573 ft.</td>
</tr>
<tr>
<td>C</td>
<td>1044.565 ft.</td>
</tr>
<tr>
<td>D</td>
<td>1044.573 ft.</td>
</tr>
<tr>
<td>E</td>
<td>1044.583 ft.</td>
</tr>
<tr>
<td>F</td>
<td>1001.471 ft.</td>
</tr>
<tr>
<td>G</td>
<td>1041.673 ft.</td>
</tr>
<tr>
<td>H</td>
<td>1041.689 ft.</td>
</tr>
<tr>
<td>I</td>
<td>1061.373 ft.</td>
</tr>
<tr>
<td>J</td>
<td>1041.673 ft.</td>
</tr>
<tr>
<td>K</td>
<td>1092.673 ft.</td>
</tr>
<tr>
<td>L</td>
<td>1074.573 ft.</td>
</tr>
<tr>
<td>M</td>
<td>1092.673 ft.</td>
</tr>
</tbody>
</table>

---
THE DETERMINATION AND ADJUSTMENT OF ELEVATIONS IN
PHELPS COUNTY MISSOURI.
(Dawson Township Level Net)

ADJUSTMENT OF INTERMEDIATE TEMPORARY
BENCH MARKS BETWEEN THE POINTS
A(at St. James) AND I(at Miles
School).

Diagram to scale showing adjustment of all
points between A and I.

The adjustment of these intermediate bench marks
can be effected by dividing the adjustment correction
from the above diagram, and subtracting each correction
from the observed elevation as it appears in the Level
Notes; or, if desired, the adjustment corrections can be
computed by the following formula for this section (A to I):

FORMULA: Dist. A to Inter. Pt. Adjustment to Intermediate Point ______ x 0.0020

In this case, the adjustment corrections are to be
subtracted from the observed elevations, as stated above.

MERAMEC SPRINGS AND RED BIRD QUADRANGLES.
PHELPS COUNTY.

ST. JAMES AND DAWSON TOWNSHIPS.

The elevations in the following list are based on
a precise-level line run in 1908 by E. L. McNair from the
United States Army Engineers' bench mark at Labadie Mo.,
the accepted elevation of which is 472.572 feet.

The leveling under adjustment was run in the Fall
of 1921 by the Vocational Department of the Missouri
School of Mines and Metallurgy from the United States
Geological Survey bench marks at St. James and Knobview,
the accepted elevations of which are 1087.800 feet and
1074.005 feet respectively.

-25-
MERAMEC SPRINGS QUADRANGLE.

(Latitude 37°45'-38°; longitude 91°30'-91°45')

(St. James Township)

LINE FROM ST. JAMES NORTHWEST VIA JEFFERSON ROAD TO INTERSECTION WITH REPUBLIC ROAD, THENCE NORTH VIA REPUBLIC ROAD TO PHILPS-GASCONEAD COUNTY LINE.

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Elevation</th>
<th>Adjustment</th>
<th>Adjusted Elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>St. James, corner of Meramec and Washington Sts., about 70 ft. north of center line of Frisco R.R. track, 25 ft. from corner of Commercial Hotel, in angle between cross walk and side walk; iron post stamped &quot;1098&quot;. (See Bulletin No. 68, page 10.)</td>
<td>1087.800</td>
<td>0.000</td>
<td>1087.800 P.B.M.</td>
</tr>
</tbody>
</table>

RED BIRD QUADRANGLE.

(Latitude 38°35'-38°45'; longitude 91°30'-91°45')

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Elevation</th>
<th>Adjustment</th>
<th>Adjusted Elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>St. James, 0.5 miles north of, on Jefferson road at intersection of Parker Lane road, concrete culvert, north end wall, top of; chiseled square.</td>
<td>1078.427</td>
<td>0.000</td>
<td>1078.427</td>
</tr>
</tbody>
</table>

| K   | St. James, 1.75 miles north of, on Jefferson road, at intersection of Republic road, on south side of road, 10" black jack tree, in base of; copper nail and washer. | 1092.694 | -0.001 | 1092.693 |

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<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Elevation</th>
<th>Cor. By Fld. Br.</th>
<th>Adj. Elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>St. James, 2.25 miles north of, on Republic road, Jefferson road, 0.5 miles north of, culvert, west end, sandstone slab, top of; painted square.</td>
<td>1034.345</td>
<td>-0.001</td>
<td>1034.344</td>
</tr>
<tr>
<td>2.</td>
<td>Jefferson road, 1.0 mile north of on Republic road, T road east, at Cox's residence, 100 ft. east of, sandstone boulder in N.E. Cor. of T road; top corner broken off, painted white.</td>
<td>1024.715</td>
<td>-0.001</td>
<td>1024.714</td>
</tr>
<tr>
<td>3.</td>
<td>Jefferson road, 1.5 miles north of, Robinson creek, 0.75 miles south of, Fitzenreider's residence, 100 ft. west of, wagon gate post, in base of; copper nail and washer.</td>
<td>998.469</td>
<td>-0.001</td>
<td>998.468</td>
</tr>
<tr>
<td>4.</td>
<td>Robinson Creek, 0.25 miles south of, on Republic road, on east side of road, 10&quot; post oak tree near Kennedy's mail box, in base of; nail.</td>
<td>923.575</td>
<td>-0.001</td>
<td>923.574</td>
</tr>
<tr>
<td>5.</td>
<td>Jefferson road, 2.25 miles north of, on Republic road, Robinson Creek, west bank of, 24&quot; white oak tree, in root of; copper nail and washer.</td>
<td>917.604</td>
<td>-0.001</td>
<td>917.603</td>
</tr>
<tr>
<td></td>
<td>(Dawson Township)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Robinson Creek, 0.25 miles north of, 15 ft. west of road, sandstone boulder on St. James-Dawson Twp. Line, opposite wire fence corner, point is on south hillside, 300 ft. south of Woolchopper's cabin; painted cross on top of boulder.</td>
<td>1022.394</td>
<td>-0.001</td>
<td>1022.393</td>
</tr>
<tr>
<td>No.</td>
<td>Description</td>
<td>Elevation</td>
<td>Adj. Elev.</td>
<td></td>
</tr>
<tr>
<td>-----</td>
<td>-------------</td>
<td>-----------</td>
<td>------------</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>Robinson Creek, 0.75 miles north of, on Republic road, Township line, 0.5 miles north of, at north end of ridge, 30 ft. west of road, sandstone boulder, top of; chiseled square.</td>
<td>1039.423</td>
<td>-0.001 1039.422</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>Robinson Creek, 1.0 mile north of, on Republic road, at foot of hill, concrete culvert across road, W. end wall, top of; chiseled Sq.--</td>
<td>334.325</td>
<td>-0.002 334.323</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>Robinson Creek, 1.25 miles north of, Miles School House, 1.0 mile south of, T road west, center of roads; road elevation.</td>
<td>976.005</td>
<td>-0.002 976.003</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>Miles School House, 0.5 miles S. of, on Republic road, Marris Cemetery, 400 ft. north of, east side of road, sandstone boulder at rail fence corner, top of; chiseled square.</td>
<td>1059.077</td>
<td>-0.002 1059.075</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>St. James, 6.0 miles north of, on Republic road, Safe, 2.5 miles E. of, Miles School, 25 ft. W. of, S. W. Cor. S.E. Ctr. of the S.W. Str. Sec. 20, T. 39 N., R. 6 W., 60 ft. S. E. of, concrete well curb, top of; bronze tablet stamped</td>
<td>1061.925</td>
<td>-0.002 1061.923</td>
<td></td>
</tr>
</tbody>
</table>

---

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ADJUSTMENT OF INTERMEDIATE TEMPORARY BENCH MARKS

BETWEEN THE POINTS I (at Miles School) AND P (on Republic road one mile south of County Line.)

<table>
<thead>
<tr>
<th>Distance from I (ft.)</th>
<th>0.75</th>
<th>1.50</th>
<th>3.00</th>
<th>4.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adj. Line (ft.)</td>
<td>12</td>
<td>19</td>
<td>14</td>
<td>16</td>
</tr>
</tbody>
</table>

DIAGRAM TO SCALE SHOWING ADJUSTMENT OF ALL POINTS BETWEEN I AND P.

The adjustment of these intermediate bench marks can be effected by scaling the adjustment corrections from the above diagram, and adding each correction to the observed elevation as it appears in the level notes; or, if desired, the adjustment corrections can be computed by the following formula for this section (I to P).

FORMULA:

\[
\text{Adj. Cor. to Inter. Pt.} = \frac{0.028 \times \text{Dist. I to Inter. Pt.} - 0.002}{4.0}
\]

Add these corrections to the observed values.

---

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I (See description on page )</td>
<td>1051.925</td>
<td>-0.002</td>
<td>1051.923</td>
</tr>
</tbody>
</table>

12. Miles School, 0.75 miles north of, on Republic road, W.L. Martin's house, 150 ft. S.F. of, on top of hill, on east side of road, 12" post oak tree, in base of copper nail and washer.--- 1023.857 +0.003 1023.860

13. Miles School, 1.2 miles north of, on Republic road, Bourbeuse River, 0.75 miles south of ford, at dim cross road, southeast corner of, sandstone boulder; painted square.---------- 924.251 +0.006 924.257

---

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<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Elevation By Fld. Sk. Cor. feet</th>
<th>Adj. Elev. feet</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.</td>
<td>Miles School, 1.75 miles north of, on Republic road, Bourbeuse River, 0.2 miles south of, farm house, 600 ft. east of, on east side of road, 15” walnut tree, in base of, copper nail and washer.</td>
<td>873.567 +0.010</td>
<td>873.577</td>
</tr>
<tr>
<td>15.</td>
<td>Wash School House, 0.75 miles south of, Bourbeuse River, at ford, 100 ft. south of, east bank of river, 30” sycamore tree, in base of, copper nail and washer.</td>
<td>839.998 +0.014</td>
<td>840.012</td>
</tr>
<tr>
<td>15a.</td>
<td>Bourbeuse River, at ford, on Republic road, center of river; road elevation.</td>
<td>834.000 +0.014</td>
<td>834.014</td>
</tr>
<tr>
<td>16.</td>
<td>Bourbeuse River, 0.25 miles N. of, on Republic road, at T. road east, northeast corner of, in base of, fence post; copper nail and washer.</td>
<td>840.106 +0.016</td>
<td>840.122</td>
</tr>
<tr>
<td>17.</td>
<td>Bourbeuse River, 0.75 miles N. of, Wash School, 400 ft. S.W. of, J.W. Crossner’s house, 300 ft. S.E. of, 8 ft. west of three mail boxes, on west side of road, 14” scaly bark hickory tree, in root of; copper nail and washer.</td>
<td>947.596 +0.019</td>
<td>947.615</td>
</tr>
<tr>
<td>18.</td>
<td>Wash School, 0.5 miles north of, on Republic road, Little Bourbeuse creek, at ford, on south bank, west side of road, 10” scaly bark hickory tree, in base of; copper nail and washer.</td>
<td>835.927 +0.023</td>
<td>835.950</td>
</tr>
</tbody>
</table>

P. Wash School, 1.0 miles north of, Little Bourbeuse Creek, 0.5 Mi. north of, Harrison School, 0.5 Mi. south of, Phelps-Gasconade County Line, 1.0 mile south of, Austin E. Breuer's house, 200 ft. west of, at T. road east, west side of Republic road, 2 ft. inside of fence, 10" post oak tree, in base of; copper nail and washer. ----------- 928.552 +0.026 928.578

DIAGRAM TO SCALE SHOWING THE ADJUSTMENT OF ALL POINTS BETWEEN P AND G.

It will be remembered that the line P to G is a spur line of levels in our scheme of adjustment. The line P to 0 was run as a parallel duplicate line of levels, and the most probable values for the elevations of the points on such a line is the average of the several determinations. The above diagram shows a (red) line of levels run by a Brande instrument and a (blue) line of levels run by a Gurley instrument; these instruments were of equal refinement, equal weight was applied to to the work done by each instrument. The (green) line represents the average of the instrumental determinations. In the level net adjustment just preceding, it was found that the point P was 0.026 ft. low. As the spur line P to 0 hinges directly on the point P, then the whole line P to G will have to be raised 0.026 ft. In the diagram this adjustment is shown by the (black) line.
The adjustment is a constant of 0.026 ft. which is to be added to each of the average instrumental observed values between P and Q. This is apparent from the diagram.

FORMULA:

\[ \text{Adj. Cor. to Int. Pt. - Av. Obs Value} + 0.026. \]

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Elevation By Fl. Bk.</th>
<th>Cor.</th>
<th>Adj. Cor.</th>
<th>Elev. feet</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.</td>
<td>(See description on page 31)</td>
<td>928.562</td>
<td>+0.026</td>
<td>928.578</td>
<td></td>
</tr>
</tbody>
</table>

19. Nash School, 1.25 miles north of, Little Bourbeuse Creek, 0.75 mi. north of, Harrison School, 0.25 miles South of, Phelps-Gascoïade County Line, 0.75 miles south of, Steve Lorts' house, 70 ft. north of, T. road west (Mail road to High Gate in Maries County), northwest corner of roads, '24" oak stump in fence corner, top of; ten penny nail.---------- Brandis 962.732
   Gurley 962.726
   Average 962.729 +0.026 962.755

20. Harrison School House, on St. James-Republic road, west side of School Building, north concrete door step, top of; chiselled square.---------- Brandis 965.674
   Gurley 965.658
   Average 965.666 +0.026 965.692

21. Harrison School House, 0.25 Mi. north of, on Republic road, Phelps-Gascoïade County Line, 0.25 miles south of, private T. road west, southwest corner of, 16" blazed black oak tree; bent nail in base of. ---- Brandis 959.594
   Gurley 959.578
   Average 959.588 +0.026 959.612

-32-
<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Elevation</th>
<th>Adj. Elev.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Harrison School House, 0.50 miles north of, on Republic road, intersection</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>St. James-Republic road and High Gate-Red Bird road, 0.5 miles south of,</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Phelps-Gasconade County Line, 28 ft. south of, center of road, 30 ft. west</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>of, 5 ft. inside wire fence, in line with center of road north into</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gasconade County, dim T. road west, 50 ft. south of, iron post stamped</td>
<td>997.242</td>
<td>997.266</td>
</tr>
<tr>
<td></td>
<td>&quot;997&quot;. Brandis 997.242 Gurley 997.237</td>
<td>997.240</td>
<td>+0.026</td>
</tr>
</tbody>
</table>

This completes the description and adjustment of the line of levels from St. James northwest via Jefferson Road to intersection with Republic Road, thence north via Republic Road to Phelps-Gasconade County Line.
MERAMEC SPRINGS QUADRANGLE.
(Latitude 37°45'-38°; longitude 91°30'-91°45°)

(ST. JAMES TOWNSHIP)

LINE FROM ST. JAMES EAST VIA OZARK HIGHWAY TO PHELPS-CRAWFORD COUNTY LINE, SAID POINT ONE MILE NORTHEAST OF KNOBVIEW RAILROAD DEPOT.

ADJUSTMENT OF INTERMEDIATE BENCH MARKS ALONG THE ROUTE A VIA B AND C TO D. (A is the P.B.W. at St. James and D is the T.B.M. at the intersection of the Ozark Highway and the Phelps-Crawford County Line.)

BELOW IS DIAGRAM TO SCALE SHOWING THE ADJUSTMENT OF ALL INTERMEDIATE POINTS ALONG THE OZARK HIGHWAY BETWEEN A AT ST. JAMES AND D AT THE COUNTY LINE NEAR KNOBVVIEW.

The adjustment of these intermediate bench marks can be effected by scaling the adjustment corrections from the diagram, and subtracting each correction from the observed elevation as it appears in the level notes and description books; or, if desired, the adjustment corrections can be computed by the following formula for the section A to D:

FORMULA:

\[
\text{ADJ. OF INTER. PT.} = \frac{\text{Dist. A to Inter. Pt.}}{6.25} \times 0.053.
\]

(See Diagram on page )

Elevations on this section previously adjusted, as follows:

<table>
<thead>
<tr>
<th>Point</th>
<th>Elevation (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1087.8000</td>
</tr>
<tr>
<td>B</td>
<td>1074.6560</td>
</tr>
<tr>
<td>C</td>
<td>1083.3660</td>
</tr>
<tr>
<td>D</td>
<td>1098.8580</td>
</tr>
<tr>
<td>No.</td>
<td>Description</td>
</tr>
<tr>
<td>-----</td>
<td>-------------</td>
</tr>
<tr>
<td>A.</td>
<td>(See description on page 26)</td>
</tr>
<tr>
<td>22.</td>
<td>St. James, 0.4 miles east of, on St. James-Cuba road (Ozark Highway), Frisco R.R., 50 ft north of, turn in road to north, 500 ft. W. of, concrete culvert across road, north end wall, top of; chiseled square.</td>
</tr>
<tr>
<td>23.</td>
<td>St. James, 1.0 mile N.E. of, on Ozark Highway, T. road east (intersection of Ozark Highway and Red Bird road), 400 ft. S. of, Dr. A. Scott's house, 300 ft W. of, on W. side of road, 10&quot; elm tree, in root of; copper nail and washer.</td>
</tr>
<tr>
<td>24.</td>
<td>St. James, 1.75 miles northeast of, on Ozark Highway, concrete culvert across road, south end wall, top of; chiseled triangle.</td>
</tr>
<tr>
<td>25.</td>
<td>St. James, 2.0 miles N.E. of, on Ozark Highway, on top of hill, center of road; road elevation.</td>
</tr>
<tr>
<td>No.</td>
<td>Description</td>
</tr>
<tr>
<td>-----</td>
<td>-------------</td>
</tr>
<tr>
<td>26.</td>
<td>St. James, 2.5 miles northeast of, on Ozark Highway, Friendship School, 1.5 miles west of, concrete culvert (erected 8-13-20 by T. Hall), north end wall, top of; chiseled square.</td>
</tr>
<tr>
<td>27.</td>
<td>Friendship School, 1.25 miles west of, on Ozark Highway, at T. road N. to Hinkle P.O., Corrugated iron culvert, rock on east end of culvert, top of; chiseled triangle.</td>
</tr>
<tr>
<td>28.</td>
<td>Friendship School, 0.5 miles W. of, on Ozark Highway, Gerber's Store, 500 feet W. of, concrete culvert, north end wall, top of; chiseled triangle.</td>
</tr>
<tr>
<td>29.</td>
<td>Friendship School, on Ozark Highway, southwest corner of concrete porch of school house, top of; chiseled triangle, painted &quot;1083.3&quot;</td>
</tr>
<tr>
<td>C.</td>
<td>Friendship School, on Ozark Highway, 50 ft. S.W. of, in School Yard, 10 ft. from fence corner, 30 ft. N. of center of Ozark Highway, 30 ft. E. of center of T. road north; iron post stamped &quot;1083&quot;.</td>
</tr>
<tr>
<td>29.</td>
<td>Friendship School, 0.25 miles E. of, private T. road N. to Louis Hall's residence, 75 ft. W. of, concrete culvert, S. end wall, top of; chiseled square.</td>
</tr>
</tbody>
</table>
30. Friendship School, 0.9 miles E. of, on Ozark Highway, Phelps-Crawford County Line, 1.1 miles west of, concrete culvert, south end wall, top of; chiseled square. ------------------- 1042.297 -0.044 1042.253

31. Friendship School, 1.5 miles east of, on Ozark Highway, Phelps-Crawford County Line, 0.5 miles west of, Knobview, 0.5 miles north of across field, concrete culvert under Ozark Highway, north end wall, top of; chiseled square. 1028.412 -0.049 1028.363

D. Friendship School, 2.0 miles east of, on Ozark Highway, at cross roads on Phelps-Crawford County Line, southwest corner of roads, 14" hickory tree, in base of; copper nail and washer. ------------- 1038.911 -0.053 1038.858

This completes the description and adjustment of the line of levels from St. James east via Ozark Highway to the Phelps-Crawford County Line, said point one mile northeast of Knobview Railroad Depot.
PART III

THE LEAST SQUARE ADJUSTMENT OF A LEVEL NET
BY METHOD OF OBSERVATION EQUATIONS.

APPLIED TO THE ADJUSTMENT
OF THE
DAWSON-ST. JAMES TOWNSHIPS LEVEL NET,
PHELPS COUNTY,
MISSOURI.

(BASED ON THE LINE OF LEVELS RUN BY THE
U. S. GEOLOGICAL SURVEY IN 1908 ALONG THE FRISCO R. R.
WITH PERMANENT BENCHMARKS, "A" AT ST. JAMES, AND
"E" AT KNOBVIEV).
Showing elevations and closures when level circuits are arranged as shown in plat on page 40.

A = 1087.800 (St. James) Adjusted value.
  - 25.875
I = 1061.925 (Miles School)  C = 1083.221
  - 107.201 + 15.511
M = 954.724 (Royal)  D = 1098.732
J = 968.419 + C . 153 Closure at D.
  + 73.460
H = 1041.879 (Red Bird and Jakes Prairie Rds.)
  + 32.606
B = 1074.467 (Red Bird Rd. and Ozark Highway.)
  + 8.734
C = 1083.221 (Friendship School )  F = 1001.432
  - 82.223 - 40.447
F = 1000.998 in arr. Circ.
  - 80.066 + 0.434 Closure at F.
G = 920.932 (Jakes Prairie and Tribby Rds.)
  + 177.664
D = 1098.579 (Ozark Highway and County Line.)
  - 24.844
E = 1073.735 (Knobview Depot.)
  - 24.844
E = 1074.005 ( " " ) Adjusted value.
  - 236.270 Closure at Knobview.
I = 1061.925
  - 93.531
J = 968.394
J = 968.419 in above arranged circuit.
  - 0.025 Closure on circuit above. At J.
M = 954.724
  - 33.201
G = 921.523,
G = 920.932 in above arranged circuit.
  + 0.591 Closure on circuit above. At G.
A = 1087.800
  - 13.134
B = 1074.686
B = 1074.487 in above arranged circuit.
  + 0.179 Closure on circuit above. At B.

Note: The values given on this page are not adjusted values, but only an arrangement to facilitate the adjustment that is to be made on the following pages by the theory of Least Squares.
Observation Equations.

(A to I) \[ +v_1 = +0.000 \text{ (weight 0.160)} \]

(I to M) \[ +v_2 = +0.000 \text{ (weight 0.156)} \]

(J to H) \[ +v_3 = +0.000 \text{ (weight 0.625)} \]

(B to B) \[ +v_4 = +0.000 \text{ (weight 0.312)} \]

(S to C) \[ +v_5 = +0.000 \text{ (weight 0.284)} \]

(C to F) \[ +v_6 = +0.000 \text{ (weight 0.333)} \]

(F to G) \[ +v_7 = +0.000 \text{ (weight 0.333)} \]

(G to D) \[ +v_8 = +0.000 \text{ (weight 2.000)} \]

(H to B) \[ +v_9 = +0.000 \text{ (weight 0.222)} \]

(B to C) \[ +v_{10} = +0.434 \text{ (weight 0.571)} \]

(M to G) \[ +v_{11} = +0.153 \text{ (weight 0.445)} \]

Formation of the Normal Equations.

Normal Equations are formed in the following manner from the Observation Equations:

**RULE:** To form the normal equations for each one of the unknown quantities, multiply each observation equation by the product of the weight of that observation and the algebraic coefficient of that unknown quantity in that equation, and add the results.

Having formed the several normal equations, their solution as simple simultaneous equations gives the most probable values of the unknown quantities.

As simple simultaneous equations of the first degree, the normal equations may be solved by any of the algebraic methods of elimination; by addition or subtraction, by substitution, or by comparison. In fact, these methods are satisfactory when there are but two equations to be solved. But in larger sets, of three or more, it is possible to shorten the numerical work by taking advantage of the peculiar symmetry which all normal equations possess. It is much easier to solve a set of normal equations than a set of ordinary, simultaneous equations of the same number which do not have this symmetry.
Normal Equation in $V_1$.
\[ +0.160V_1 + 1.000V_2 + 1.000V_3 + 1.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = +0.270000 \]
\[ +1.000V_1 + 1.000V_2 + 1.000V_3 + 1.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = +0.179000 \]
1) \[ +2.160V_1 + 2.000V_2 + 2.000V_3 + 2.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = +0.448000 \]

Normal Equation in $V_2$.
\[ +0.156V_1 + 1.000V_2 + 1.000V_3 + 1.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = +0.000000 \]
\[ +1.000V_1 + 1.000V_2 + 1.000V_3 + 1.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = +0.270000 \]
\[ +1.000V_1 + 1.000V_2 + 1.000V_3 + 1.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = -0.015625 \]
2) \[ +2.000V_1 + 2.781V_2 + 2.625V_3 + 2.000V_4 + 2.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = +0.433375 \]

Normal Equation in $V_3$.
\[ +0.312V_1 + 1.000V_2 + 1.000V_3 + 1.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = +0.000000 \]
\[ +1.000V_1 + 1.000V_2 + 1.000V_3 + 1.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = +0.270000 \]
\[ +1.000V_1 + 1.000V_2 + 1.000V_3 + 1.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = -0.015625 \]
\[ -0.124V_1 + 0.124V_2 + 0.124V_3 + 0.124V_4 + 0.124V_5 + 0.124V_6 + 0.124V_7 + 0.124V_8 + 0.124V_9 = +0.073284 \]
3) \[ +2.000V_1 + 2.325V_2 + 3.061V_3 + 2.124V_4 + 2.124V_5 + 1.124V_6 + 1.124V_7 + 1.124V_8 + 1.000V_9 = +0.506859 \]

Normal Equation in $V_4$.
\[ +0.625V_1 + 1.000V_2 + 1.000V_3 + 1.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = +0.000000 \]
\[ +1.000V_1 + 1.000V_2 + 1.000V_3 + 1.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = +0.270000 \]
\[ +1.000V_1 + 1.000V_2 + 1.000V_3 + 1.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = +0.179000 \]
\[ +0.124V_1 + 0.124V_2 + 0.124V_3 + 0.124V_4 + 0.124V_5 + 0.124V_6 + 0.124V_7 + 0.124V_8 + 0.124V_9 = +0.073284 \]
4) \[ +2.000V_1 + 2.000V_2 + 2.124V_3 + 2.749V_4 + 2.124V_5 + 1.124V_6 + 1.124V_7 + 1.124V_8 + 1.000V_9 = +0.522284 \]
Normal Equation in \( V_3 \):

\[
\begin{align*}
+0.294 V_3 &= +0.000000 \\
+1.000 V_1 + 1.000 V_2 + 1.000 V_4 + 1.000 V_5 + 1.000 V_6 + 1.000 V_7 + 1.000 V_8 + 1.000 V_9 = +0.270000 \\
+0.571 V_3 + 0.571 V_4 + 0.571 V_5 + 0.571 V_6 + 0.571 V_7 + 0.571 V_8 + 0.571 V_9 = +0.247814 \\
-0.124 V_3 - 0.124 V_4 - 0.124 V_5 - 0.124 V_6 - 0.124 V_7 - 0.124 V_8 - 0.124 V_9 = -0.073284 \\
\end{align*}
\]

5) \(+2.000 V_1 + 2.000 V_2 + 2.124 V_3 + 2.124 V_4 + 2.989 V_5 + 1.695 V_6 + 1.124 V_7 + 1.000 V_9 = +0.770098\)

Normal Equation in \( V_3 \):

\[
\begin{align*}
+0.333 V_3 &= +0.000000 \\
+1.000 V_1 + 1.000 V_2 + 1.000 V_3 + 1.000 V_4 + 1.000 V_5 + 1.000 V_6 + 1.000 V_7 + 1.000 V_8 + 1.000 V_9 = +0.270000 \\
+0.571 V_3 + 0.571 V_4 + 0.571 V_5 + 0.571 V_6 + 0.571 V_7 + 0.571 V_8 + 0.571 V_9 = +0.247814 \\
-0.124 V_3 - 0.124 V_4 - 0.124 V_5 - 0.124 V_6 - 0.124 V_7 - 0.124 V_8 - 0.124 V_9 = -0.073284 \\
\end{align*}
\]

6) \(+1.000 V_1 + 1.000 V_2 + 1.124 V_3 + 1.124 V_4 + 1.695 V_5 + 2.028 V_6 + 1.124 V_7 + 1.000 V_9 = +0.591098\)

Normal Equation in \( V_3 \):

\[
\begin{align*}
+0.333 V_3 &= +0.000000 \\
+1.000 V_1 + 1.000 V_2 + 1.000 V_3 + 1.000 V_4 + 1.000 V_5 + 1.000 V_6 + 1.000 V_7 + 1.000 V_8 + 1.000 V_9 = +0.270000 \\
+0.571 V_3 + 0.571 V_4 + 0.571 V_5 + 0.571 V_6 + 0.571 V_7 + 0.571 V_8 + 0.571 V_9 = +0.247814 \\
-0.124 V_3 - 0.124 V_4 - 0.124 V_5 - 0.124 V_6 - 0.124 V_7 - 0.124 V_8 - 0.124 V_9 = -0.073284 \\
\end{align*}
\]

7) \(+1.600 V_1 + 1.000 V_2 + 1.124 V_3 + 1.124 V_4 + 1.695 V_5 + 2.473 V_6 + 1.569 V_7 + 1.445 V_9 = +0.859183\)

Normal Equation in \( V_3 \):

\[
\begin{align*}
+2.000 V_3 &= +0.000000 \\
+1.000 V_1 + 1.000 V_2 + 1.000 V_3 + 1.000 V_4 + 1.000 V_5 + 1.000 V_6 + 1.000 V_7 + 1.000 V_8 + 1.000 V_9 = +0.270000 \\
-0.124 V_3 - 0.124 V_4 - 0.124 V_5 - 0.124 V_6 - 0.124 V_7 - 0.124 V_8 - 0.124 V_9 = -0.073284 \\
\end{align*}
\]

8) \(+1.000 V_1 + 1.000 V_2 + 1.124 V_3 + 1.124 V_4 + 1.124 V_5 + 1.124 V_6 + 1.124 V_7 + 1.569 V_8 + 3.569 V_9 + 1.445 V_9 = +0.411569\)
Normal Equation in $V_9$.

\[
+1.000V_1 + 1.000V_2 + 1.000V_3 + 1.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = +0.222V_9 = +0.000000 \\
+1.000V_1 + 1.000V_2 + 1.000V_3 + 1.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = +0.270V_9 = +0.000000 \\
+1.000V_1 + 1.000V_2 + 1.000V_3 + 1.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = +0.445V_9 + 0.445V_9 + 0.445V_9 = +0.068085 \\
9) +1.000V_1 + 1.000V_2 + 1.000V_3 + 1.000V_4 + 1.000V_5 + 1.000V_6 + 1.000V_7 + 1.000V_8 + 1.000V_9 = +1.445V_9 + 1.445V_9 + 1.667V_9 = +0.338085
\]

### Normal Equations

<table>
<thead>
<tr>
<th></th>
<th>$V_1$</th>
<th>$V_2$</th>
<th>$V_3$</th>
<th>$V_4$</th>
<th>$V_5$</th>
<th>$V_6$</th>
<th>$V_7$</th>
<th>$V_8$</th>
<th>$V_9$</th>
<th>$\text{Absolute Term}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>+2.160</td>
<td>+2.000</td>
<td>+2.000</td>
<td>+2.000</td>
<td>+1.000</td>
<td>+1.000</td>
<td>+1.000</td>
<td>+1.000</td>
<td>-0.449000</td>
<td>= 0</td>
</tr>
<tr>
<td>2)</td>
<td>+2.000</td>
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- The solution involves solving the normal equations using the given data.
- Each row represents the solution for a specific term.
- The values are rounded to three decimal places.
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Check in Normal Equation (1):

-0.064015 +0.070858
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-0.043722 +0.347324
-0.449000 +0.060732
-0.032807 +0.029479 +0.035429 -0.108844 +0.046226
+0.050395 -0.054422
-0.029637

In Dawson Township, Phelps County.

\[ \begin{align*}
\Delta \text{observed} & \quad \text{correction} \\
A & \quad -25.875 \quad -0.0296 \\
I & \quad -107.201 \quad -0.0544 \\
M & \quad +13.895 \quad +0.0354 \\
J & \quad +73.460 \quad +0.0231 \\
H & \quad +32.808 \quad +0.1737 \\
B & \quad +8.734 \quad +0.0607 \\
C & \quad +32.808 \quad -8.7947 \\
F & \quad +80.066 \quad +0.0045 \\
G & \quad +177.647 \quad -0.0437 \\
D & \quad -24.844 \quad -0.0357
\end{align*} \]

Adjusted Elevations of Junction Points:

\[ \begin{align*}
\Delta & \quad \text{corrected} \\
A & \quad -25.9046 \\
I & \quad -107.2554 \\
M & \quad 13.7304 \\
J & \quad +73.4831 \\
H & \quad +32.7817 \\
B & \quad +8.7947 \\
C & \quad +80.0870 \\
F & \quad +177.6033 \\
G & \quad -24.8797 \\
D & \quad -24.8797
\end{align*} \]
PART IV.

THE LEAST SQUARE ADJUSTMENT OF A LEVEL NET
BY METHOD OF CONDITION EQUATIONS
AND
CORRELATIVES.

APPLIED TO THE ADJUSTMENT
OF THE
DAWSON-ST. JAMES TOWNSHIPS LEVEL NET,
PHELPS COUNTY,
MISSOURI.

(BASED ON THE LINE OF LEVELS RUN BY THE
U. S. GEOLOGICAL SURVEY IN 1908 ALONG THE FRISCO R. R.
WITH PERMANENT BENCHMARKS "A" AT ST. JAMES,
AND "E" AT KNOWVIEW).
Least Square Adjustment of Level Net.

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Condition Equations:

1) \( 0 = - 0.032 - d - c - b - a \)
2) \( 0 = + 0.153 + g - f - e + c \)
3) \( 0 = - 0.434 + e + h - i + b \)
4) \( 0 = + 0.157 - h + f + p - m - j \)
5) \( 0 = + 0.204 + a + i + j - l - k \)
6) \( 0 = - 0.025 + m - n + l \)

Each complete circuit furnishes the condition that the sum of its adjusted differences of elevation shall equal zero when given the proper signs as if run continuously around the circuit, clockwise or counter-clockwise.

Number of Conditions = (No. of lines) - (No. of Jnt. points) + 1.

\[ n - n' = +15 = 10 + 1 = \delta \]

The minus signs applied to the small case letters in the above condition equations result from changing the direction of the arrows (when red arrow goes in opposite direction from the black arrows in diagram on preceding page) so as to be continuous around each circuit. It is not necessary that all the circuits be traversed in the same direction in a given problem, but for clearness the work will appear more consistent if all the circuits are traversed in one direction, either clockwise or counter-clockwise.

-50-
TABLE OF CORRELATES.

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<td>0.3535</td>
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<tr>
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Product of Cs with Correlates.

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<tr>
<td>n</td>
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<tr>
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</table>

NORM AL EQUATIONS.

The normal equations may be written by inspection as follows owing to the simplicity of the condition equations.

To form the Normal Equations:

1st. Square each quantity in column 1 left hand side table of correlates and multiply by its 1/10 p and add the results = +0.7260;
2nd. Multiply each quantity in column 1 by quantities in column 2 on same line, multiply by their 1/10 p, and add results = -0.2250;
3rd. Multiply each quantity in column 1 by quantities in column 3 on same line multiply by their 1/10 p and add results = -0.8000;
4th. Multiply each quantity in column 1 by quantities in column 4 on same line, multiply by their 1/10 p, and add results = +0.0000;
5th. Multiply each quantity in column 1 by quantities in column 5 on same line, multiply by their 1/10 p, and add results = +0.0000;
6th. Multiply each quantity in column 1 by quantities in column 6 on same line, multiply by their 1/10 p, and add results = +0.0000.
This gives the first normal equation.
7th. Square each quantity in column 2 and multiply by its 1/10 p, and add results = +1.0250;
8th. Multiply each quantity in column 2 by each adjacent quantity in turn in the other columns by their 1/10 p as above and add results;
9th. Proceed as above until all terms have been multiplied, and the 8th normal equation has been completed.
### Normal Equations

<table>
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<tr>
<th></th>
<th>1</th>
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<th>5</th>
<th>6</th>
<th>(q)</th>
<th>Sum Check</th>
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Note: Sum and Check columns will not be used in the following solution of the above normal equations; but will be used later in a subsequent solution of normal equations. The (q) is the absolute term.

### (p) Solution of Normal Equations

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### Notes on The p,G,R Solution of Normal Equations

Opposite page 118 of this thesis will be found the Least Square figure adjustment of a Geodetic Quadrilateral on the folded plate. The problem on the plate was taken from the Topographic Instructions of the United States Geological Survey page 61 to page 71, for the instruction of students in my class in Geodetic Computations. On comparison with the Geological Survey publication in the description of the method for solving normal equations, it will be found that the enclosed plate goes much more into the details of the method.

As the p,G,R, solution of normal equations used here in this level net adjustment is identical with that used in the adjustment of the triangulation scheme, by referring to the plate opposite page 118, it will be found very easy to follow the process and steps in the given computation and also in other computations in this thesis where the p,G,R solution of normal equations has been employed.

-52-
(q) Solution of Normal Equations.

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(r) Solution of Normal Equations.

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Some authorities on Least Square, call "(r) Solution of Normal Equations" the, "Back Solution".
Summary of Junction Elevations in Dawson Township, Phelps County, Missouri.

A = 1087.8000 ----> 1087.8000
- 13.1646 - 25.9044
B = 1074.6354  I = 961.8956
+ 8.7948
C = 1083.4202
+ 15.4541
D = 1098.8843 ----> 1098.8843
- 24.8794 - 177.6017
E = 1074.0049  G = 921.2826
Check + 80.0613
F = 1001.3439 ----> 1001.3439
+ 82.0862 + 40.5103
C = 1083.4202  H = 1041.8542 ----> 1041.8542
Check + 32.7811 - 73.4836
B = 1074.6353  J = 968.3706
Check - 13.7303
G = 921.2826
Check

ADJUSTED ELEVATIONS.
A = 1087.8000  F = 1001.3439
B = 1074.6354  G = 921.2826
C = 1083.4302  H = 1041.8542
D = 1098.8843  J = 968.3706
E = 1074.0050

M = 954.6403.

Points K, L, N, O, P, are adjusted as intermediate points and take corrections proportionate to their distances from adjacent junction points.

ADJUSTED ELEVATIONS.
K =
L =
P =

Points Q, R, S, are at the ends of spur lines and will take the same corrections as P, N, O, respectively.

ADJUSTED ELEVATIONS.
Q =
R =
S =
Abridged Method A.

**NORMAL EQUATIONS. Cont'd.**

- **(1)**:\n  \[ \begin{array}{cccccc}
  A & B & C & D & E & F \\
  +0.7250 & -0.2250 & -0.3000 & +0.0000 & -0.1000 & +0.0000 \\
  \end{array} \]

- **(2)**:\n  \[ \begin{array}{cccccc}
  A & B & C & D & E & F \\
  +1.0250 & -0.3000 & -0.0500 & +0.0000 & +0.0000 & +0.1500 \\
  \end{array} \]

- **(3)**:\n  \[ \begin{array}{cccccc}
  A & B & C & D & E & F \\
  +1.1150 & -0.1750 & -0.3400 & +0.0000 & +0.0000 & -0.4340 \\
  \end{array} \]

- **(4)**:\n  \[ \begin{array}{cccccc}
  A & B & C & D & E & F \\
  +1.5100 & -0.1800 & -0.3210 & -0.1570 & +0.6470 \\
  \end{array} \]

- **(5)**:\n  \[ \begin{array}{cccccc}
  A & B & C & D & E & F \\
  +1.3850 & -0.1600 & +0.2040 & +0.8290 \\
  \end{array} \]

- **(6)**:\n  \[ \begin{array}{cccccc}
  A & B & C & D & E & F \\
  +1.1210 & -0.0250 & +0.6160 \\
  \end{array} \]

**Solution of Normal Equations by Abridged Method.**

- **(2)**: \( A + B - C + D + E + F \)
- **(7)**: \( -0.0686 + 0.0931 + 0.0000 + 0.0310 + 0.0000 + 0.1926 \)
- **(11)**: \( +1.1150 - 0.0750 - 0.3400 + 0.0000 - 0.0310 + 0.0000 \)
- **(19)**: \( +0.0000 - 0.0000 + 0.0000 \)
- **(20)**: \( +0.0000 - 0.0000 + 0.0000 \)
- **(21)**: \( -0.0410 - 0.0067 + 0.1683 \)
- **(VI)**: \( +1.0095 - 0.0096 + 0.9240 \)

**Normal Equation (2):**

\( A + B - C + D + E + F \) or \( x = 0.3103 \)

**Normal Equation (3):**

\( A + B - C + D + E + F \) or \( x = 0.4138 \)

**Normal Equation (4):**

\( A + B - C + D + E + F \) or \( x = 0.1379 \)

**Normal Equation (5):**

\( A + B - C + D + E + F \) or \( x = 0.0523 \)

**Normal Equation (6):**

\( A + B - C + D + E + F \) or \( x = 0.0000 \)

It must be remembered that the (Check) column includes all of the coefficients of an equation, whether written or not, so that, when the abridged form is used, as just to the left, the coefficients must be read down and to the right; i.e. \( x \) line (2); add column 2 with line (2) = +0.3030.

---

**Note:**

Throughout each fraction the numerator of each fraction used as a multiplier has its sign changed.

---

**Process:**

- **(1)**: \( x = 0.3103 \)
- **(2)**: \( x = 0.4138 \)
- **(3)**: \( x = 0.1379 \)
- **(4)**: \( x = 0.0523 \)
- **(5)**: \( x = 0.0000 \)
- **(6)**: \( x = 0.0000 \)

---

**Check Process:**

- **(1)**: \( x = 0.3103 \)
- **(2)**: \( x = 0.4138 \)
- **(3)**: \( x = 0.1379 \)
- **(4)**: \( x = 0.0523 \)
- **(5)**: \( x = 0.0000 \)

---

**Note:**

Throughout each fraction the numerator of each fraction used as a multiplier has its sign changed.
ABRIDGED METHOD A.

**Correlates**

<table>
<thead>
<tr>
<th>Column</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transverse</td>
<td>+0.0620</td>
<td>-0.1338</td>
<td>+0.4046</td>
<td>+0.0353</td>
<td>+0.0858</td>
<td>+0.0353</td>
</tr>
</tbody>
</table>

**Process**

1. **Copy (VI)(V)(IV)(III)(II)(I) Const Column.**
2. **Divide (VI) Const Column by (VI) F Column = +0.0848.**
3. **Multiply (V)(IV)(III)(II)(I) F Col. x +0.0848.**
4. **Place at head of Col. 6 Table of Correlates.**

---

The method given above is a substitute for the \((F),(q),(r)\) Solution of Normal Equations; all other steps in the process of level net adjustment are identical with those given in Part 2 of this Thesis, and will not be given again here.

Note: the ✓ (Check mark) at the ends of rows (II), (III), (IV), (V), and (VI), are written to indicate that the \(\Sigma\) of Column and Row are exactly satisfied. Unavoidable discrepancies in the last figure of the Check-term, due to remainders, should be removed by arbitrarily correcting the check-term before proceeding with the next step in the elimination; this is best done by drawing a line thru the erroneous figure and writing the correct one above it.
### ABRIDGED METHOD B.

#### NORMAL EQUATIONS.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+0.7250</td>
<td>-0.2250</td>
<td>-0.3000</td>
<td>+0.0000</td>
<td>-0.1000</td>
<td>+0.0000</td>
<td>-0.0620</td>
</tr>
<tr>
<td>2</td>
<td>-0.3000</td>
<td>+0.0000</td>
<td>-0.0500</td>
<td>+0.1500</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
</tr>
<tr>
<td>3</td>
<td>+1.1500</td>
<td>-0.1750</td>
<td>-0.3400</td>
<td>+0.0000</td>
<td>+0.1830</td>
<td>+0.0000</td>
<td>+0.0000</td>
</tr>
<tr>
<td>4</td>
<td>+1.6100</td>
<td>-0.1600</td>
<td>-0.3210</td>
<td>+0.1570</td>
<td>+0.4670</td>
<td>+0.0000</td>
<td>+0.0000</td>
</tr>
<tr>
<td>5</td>
<td>+1.3850</td>
<td>-0.1400</td>
<td>+0.2040</td>
<td>+0.0820</td>
<td>+0.0380</td>
<td>+0.0000</td>
<td>+0.0000</td>
</tr>
<tr>
<td>6</td>
<td>-0.2120</td>
<td>+0.0580</td>
<td>+0.0950</td>
<td>-0.3620</td>
<td>-0.0620</td>
<td>+0.6150</td>
<td></td>
</tr>
</tbody>
</table>

#### Solution of Normal Equations by Abridged Method.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+0.7250</td>
<td>-0.2250</td>
<td>-0.3000</td>
<td>+0.0000</td>
<td>-0.1000</td>
<td>+0.0000</td>
<td>-0.0620</td>
</tr>
<tr>
<td>2</td>
<td>+0.3103</td>
<td>+0.4138</td>
<td>+0.0000</td>
<td>+0.1379</td>
<td>+0.0855</td>
<td>+0.0824</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.0898</td>
<td>-0.9931</td>
<td>+0.0000</td>
<td>-0.0310</td>
<td>-0.0182</td>
<td>+0.0118</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>+0.9552</td>
<td>+0.3931</td>
<td>-0.0500</td>
<td>+0.1838</td>
<td>+0.6148</td>
<td>+0.0000</td>
<td>+0.0000</td>
</tr>
<tr>
<td>5</td>
<td>+0.4115</td>
<td>+0.0128</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>+0.0259</td>
<td>+1.4754</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>+0.2302</td>
<td>+0.0101</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Note.

It must be remembered that the (CheckX) column includes all of the coefficients of an equation, whether written or not, so that when the abridged form is used, the coefficients must be read down and to the right; i.e., form of check opposite line (14).

#### BACK SOLUTION.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
</tr>
<tr>
<td>2</td>
<td>+0.0101</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
</tr>
<tr>
<td>3</td>
<td>+0.0472</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
</tr>
<tr>
<td>4</td>
<td>+0.0849</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
</tr>
<tr>
<td>5</td>
<td>+0.0096</td>
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<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
</tr>
</tbody>
</table>

Copy (VI), (V), (IV), (III), (II), (I) Col. N.
ADJUSTMENT BY THE METHOD OF CORRELATES.
THEORY.

The final adjusted values of the observed quantities must satisfy the conditions of the problem in hand, and must be the most probable values, according to the Theory of Least Squares, that will satisfy all the conditions. Therefore, the sum of the weighted squares of the corrections (residuals, \( v \)) must be a minimum.

Then:

\[
U = w_a a^2 + w_b b^2 + w_c c^2 + w_d d^2 \cdots + w_n z^2 = \text{a minimum}
\] (1)

which must be satisfied simultaneously with the following conditions. Multiplying the condition equations on page 50 in succession by the factors, \(-2C_a, -2C_b, -2C_c, \) etc., respectively,

\[
\begin{align*}
+2C_a q_1 &+ 2C_a d + 2C_a c + 2C_b b + 2C_a a = 0 \\
-2C_b q_2 &- 2C_b g + 2C_b f + 2C_b e - 2C_c c = 0 \\
+2C_b q_3 &- 2C_b e - 2C_b h + 2C_b i - 2C_b b = 0 \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad Related text...
\[
\frac{dU}{dc} = 2w_0 \cdot c - 2(c_1 + c_2) = 0, \text{ dividing by 2 and solving for } c, \\
\quad c = \frac{1}{w_0} (-c_1 + c_2)
\]

\[
\frac{dU}{dz} = \ldots \ldots = 0, \text{ etc.}
\]

Where \( w_1, w_2, w_3, \text{ etc.} = \frac{1}{d} \) are the weights respectively, \( d = \) distances

\( U = \Sigma \) weighted residuals squared, desired to be a minimum,

\( a, b, c, \text{ etc.} = \) corrections for the links,

\( c_1, c_2, c_3, \text{ etc.} = \) the adjustment factors solved for in the normal equations.

\( q_1, q_2, q_3, \text{ etc.} = \) constants of observed closure respectively in the several circuits.

\[
\frac{dU}{da} = \text{ the differential of } U \text{ with respect to } a, \text{ or the first derivative.}
\]

The constant 2 comes from the differentiation of the expressions in each equation equated to 0.

Substituting these values of the \( a, b, c, \text{ etc.} \) in the condition equations on page 50, and combining the coefficients of \( c_1, c_2, c_3, \text{ etc.}, \) we obtain the Normal Equations. Which partake of the following form:

\[
[a \cdot a] c_1 + [a \cdot b] c_2 + [a \cdot c] c_3 + \ldots + q_1 = 0
\]

\[
[a \cdot b] c_1 + [b \cdot b] c_2 + [b \cdot c] c_3 + \ldots + q_2 = 0 \quad (4)
\]

\[
[a \cdot c] c_1 + [b \cdot c] c_2 + [c \cdot c] c_3 + \ldots + q_3 = 0
\]

If the foregoing substitutions and and like \( C_3 \) combined, we arrive at the normal equations as appear on the top of page 52.

In forming the normal equations, it may be found somewhat simpler to follow the rules of thumb formulated from the foregoing theory which will be found on page 51 for the formation of normal equations from the condition equations, and on pages 10 and 41, by a similar demonstration, for the formation of normal equations from observation equations.
APPLICATION OF THE THEORY.

Least Squares computation consists in rendering the value of $U$ in the following expression a minimum.

$$U = a^2 + .333b^2 + .444c^2 + d^2 + .333e^2 + 2f^2 + .222g^2 + .571h^2$$
$$+ .294i^2 + .625j^2 + .16k^2 + .825l^2 + 312m^2 + .156n^2 + .124p^2.$$

Condition Terms.

- $-2C_a [-.062 - d - c - b - a]$  
- $-2C_b [+153 + g - f - e + c]$  
- $-2C_c [-.434 + e + h - i + b]$  
- $-2C_d [-157 - h + f + p - m - j]$  
- $-2C_e [+204 + a + i + j - l - k]$  
- $-2C_f [-.025 + m - n + l]$

The coefficients of the above condition terms is taken arbitrarily as $-2$ in order to simplify the expressions for the $V$ corrections (ie. a, b, c, etc.) for the links.

To make the above expression for $U$ a minimum requires that its differentials with respect to $a$, $b$, $c$, etc. be each equated to zero, as follows:

$$2a + 2C_a - 2C_b = 0,$$
then
$$a = - C_a + C_b \ldots \ldots = 1 \text{ times} (-C_a + C_b)$$

$$.666b + 2C_a - 2C_b = 0,$$
then
$$b = - 3C_a + 3C_b \ldots \ldots = 3 \ " (-C_a + C_b)$$

$$.888c + 2C_a - 2C_b = 0,$$
then
$$c = -2.25C_a + 2.25C_b = 2.25 \ " (-C_a + C_b)$$

$$2d + 2C_a = 0,$$
then
$$d = - C_a \ldots \ldots = 1 \ " -C_a$$

$$.666e + 2C_a - 2C_b = 0,$$
then
$$e = - 3C_a + 3C_b \ldots \ldots = 3 \ " (-C_a + C_b)$$

These expressions when simplified and with the Cs transposed to the right-hand side of the equations, will then appear as extended to the left, and form the correlates as they are tabulated on page 51.
\[ 4f + 2C_2 - 2C_4 = 0, \]
then
\[ f = -0.5C_2 + 0.5C_4 \ldots \ldots = 0.5 \times (-C_2 + C_4) \]

\[ 0.444g - 2C_3 = 0, \]
then
\[ g = +4.5C_3 \ldots \ldots = 4.5 \times (-C_3) \]

\[ 1.142h - 2C_4 + 2C_6 = 0, \]
then
\[ h = +1.75C_4 - 1.75C_6 \ldots = 1.75 \times (+C_4 - C_6) \]

\[ 0.588i + 2C_5 - 2C_7 = 0, \]
then
\[ i = -3.4C_5 + 3.4C_7 \ldots = 3.4 \times (-C_5 + C_7) \]

\[ 1.250j + 2C_6 - 2C_8 = 0, \]
then
\[ j = -1.6C_6 + 1.6C_8 \ldots = 1.6 \times (-C_6 + C_8) \]

\[ 0.32k + 2C_8 = 0, \]
then
\[ k = -0.625C_8 \ldots \ldots = 0.625 \times -C_8 \]

\[ 1.250l + 2C_9 - 2C_10 = 0, \]
then
\[ l = -1.8C_9 + 1.8C_{10} \ldots = 1.8 \times (-C_9 + C_{10}) \]

\[ 0.624m + 2C_{10} - 2C_{12} = 0, \]
then
\[ m = -0.321C_{10} + 0.321C_{12} \ldots = 0.321 \times (-C_{10} + C_{12}) \]

\[ 0.312n + 2C_{12} = 0, \]
then
\[ n = +6.4C_{12} \ldots \ldots = 6.4 \times +C_{12} \]

\[ 0.248p - 2C_{14} = 0, \]
then
\[ p = +0.805C_{14} \ldots \ldots = 8.05 \times +C_{14} \]
PART V.

PROPORTIONAL ADJUSTMENT OF INTERMEDIATE BENCHMARKS
FROM CORRECTED ELEVATIONS OF ADJACENT JUNCTION POINTS.
APPLIED TO THE ADJUSTMENT
OF
INTERMEDIATE BENCHMARKS
IN THE
DAWSON-ST. JAMES TOWNSHIPS LEVEL NET,
PHELPS COUNTY,
MISSOURI.

(BASED ON THE LINE OF LEVELS RUN BY THE
U. S. GEOLOGICAL SURVEY IN 1908 ALONG THE FRISCO R. R.
WITH PERMANENT BENCHMARKS "A" AT ST. JAMES,
AND "E" AT KNOBVIEW).
Adjustment of intermediate bench marks between point A at St. James and point I at Miles School on Republic Road.

The adjustment of these intermediate bench marks can be effected by solving the adjustment corrections from the diagram below and subtracting each correction from the observed elevation as it appears in the level notes; or, if desired, the adjustment corrections can be computed by the following formula for this section (A to I).

**DERIVATION OF ADJUSTMENT FORMULA.**

**Elev. A = 1087.900**
**Orig. = 0.000**

**Elev. I = 1061.9250 Obs.**
**Elev. I = 1061.9254 Adjus.**
**Cor. = -0.0296 Cor.**

Correction C for any intermediate point at distance d, from A and between points A and I:

\[ C = \frac{0.0296 \times d}{5.25} \text{ or } = -0.004736d. \]

Graph for the adjustment of intermediate point between A and I.
Merimec Springs Quadrangle.

(Latitude 37 45'-38; longitude 91 30'-91 45')

(St. James Township)

Line from St. James, northwest via Jefferson road to the intersection of Republic road, thence north via Republic road, 9.5 miles to the Phelps-Gasconade County Line.

A. St. James, corner of Meramec and Washington Sts., about 70 ft. N. of center line of Frisco R.R. track, 25 ft. from corner of Commercial Hotel, in angle between cross walk and sidewalk; iron post stamped "1098". -------- 1087.800

B. St. James, 0.5 miles N.W. of, on Jefferson road at intersection of Parker Lane road, concrete culvert, north end wall, top of; chiseled square. -------- 1078.427

K. St. James, 1.75 miles N.W. of, on Jefferson road at intersection of Republic road, on south side of road, 10" black jack tree, in base of; copper nail and washer. 1092.694 - 0.008 1092.686

1. St. James, 2.25 miles N. of, on Republic road, Jefferson road, 0.5 miles N. of, culvert, west end of, sandstone slab, top of; painted square. -------- 1034.345

2. Jefferson road, 1.0 mile N. of, on Republic road, T road east at Cox's house, 100 ft. E. of, sandstone boulder in northeast corner of T road; top corner broken off and painted white. -------- 1024.715

3. Jefferson road, 1.5 miles north of, Robinsons Creek, 0.75 miles south of, Fitznerieder's residence, 100 feet west of, wagon gate post, in base of; copper nail and washer. 998.469 - 0.016 998.453
4 Robinson's Creek, 0.25 miles south of, on Republic road, on east side of road, 10" post oak tree, near Kennedy's mail box, in base of tree; nail. --------------- 923.575 -0.018 923.557

5 Jefferson road, 2.25 miles north of, on Republic road, Robinsons Creek, west bank of, 24" white oak tree, in root of; copper nail and washer. 917.604 -0.019 917.585

(Dawson Township)

6 Robinsons Creek, 0.25 miles north of, 15 feet west of road, sandstone boulder on St. James-Dawson Township line, opposite wire fence corner on east side of road, point is on south hill side 300 feet south of wood chopper's cabin; painted cross on top of boulder. ------------------------ 1022.394 -0.020 1022.374

7 Robinsons Creek, 0.75 miles north of, on Republic road, township line, 0.5 miles north of, at north end of ridge, 30 feet west of road, sandstone boulder, top of; chiseled square. ------------ 1039.423 -0.021 1039.402

8 Robinsons Creek, 1.0 mile north of, on Republic road, at foot of hill, concrete culvert across road, west end wall, top of; chiseled square. 934.325 -0.023 934.302

9 Robinsons Creek, 1.25 miles north of, Miles School House, 1.0 mile south of, T road west, center of roads; road elevation. ------------------------ 976.005 -0.024 975.981

10 Miles School House, 0.5 miles south of, on Republic road, Marrs Cemetery, 400 feet north of, east side of road, sandstone boulder at rail fence corner, top of; chiseled square. ------------ 1059.077 -0.025 1059.052

11 St. James, 6.0 miles N. of, on Republic Rd., Safe, 2.5 miles E. of, cross Rds. at Miles School, N.E. corner of School House, limestone wheel guard, top of; painted square. ------------------ 1061.925 -0.030 1061.895
Adjustment of point P on Republic Road 0.5 miles south of Harrison School, and the determination of intermediate points between I and P only.

\[
\text{Elev. I} = 1061.9250 \quad \text{Observed} \quad \text{Elev. P} = 928.5520 \quad \text{Observed}
\]
\[
\text{Elev. I} = 1061.8954 \quad \text{Adjusted} \quad \text{Elev. P} = 924.734 \quad \text{Adjusted}
\]

**Calculation of the Correction for Point P.**

\[
x + 0.0064 = 2.4
\]
\[
0.0330 \times 2.4 = 0.0660
\]
\[
x = \frac{0.0660 - 0.0064}{6.4}
\]
\[
x = 0.0135 - 0.0064
\]
\[
x = -0.0071 \quad \text{corr. for P.}
\]

To be applied to observed elevation.

**Correction c. for any intermediate point at distance d. from P and between points I and P.**

\[
C = \frac{0.0071}{\frac{d}{4.0}}
\]

**Derivation of Adjustment Formula.**

\[
C = \frac{0.0225 \times d}{4.0} + 0.0071
\]

Then
\[
C = (-0.05825d - 0.0071)
\]

Graph for the adjustment of intermediate points between I and P.
I. St. James, 6.0 miles north of, on Republic road, Safe, 2.5 miles east of, Miles School, 25 feet west of, S.W. Cor. S.E. Qtr. of the S.W. Qtr., Sec. 20, T 39 N., R. 6 W., 60 feet southeast of, concrete well curb, top of; bronze tablet. 1059.528 -0.030 1059.498

12 Miles School, 0.75 miles north of, on Republic road, W. L. Martin's house, 150 feet southeast of, on top of hill, on east side of road, 12" post oak tree, in base of; copper nail and washer. 1023.857 -0.025 1023.832

13 Miles School, 1.2 miles north of, on Republic road, Bourbeuse River, 0.75 miles south of ford, at dim cross road, southeast corner of; painted square on sandstone boulder. 924.251 -0.023 924.228

14 Miles School, 1.75 miles north of, on Republic road, Bourbeuse River, 0.2 miles south of, farm house, 600 feet east of, on east side of road, 15" walnut tree, in base of; copper nail and washer. 873.567 -0.020 873.547

15 Wash School House, 0.75 miles south of, Bourbeuse river ford, 100 ft. S. of, east bank of river, 30" sycamore tree, in base of; copper nail and washer. 839.998 -0.017 839.981

15a Bourbeuse River, at ford on Republic road, center of creek; road elevation. 834.000 -0.017 833.983

16 Bourbeuse river, 0.25 miles north of, on Republic road, at T road east, N. E. corner of, in base of fence post; copper nail and washer. 840.106 -0.016 840.090

17 Bourbeuse Riv., 0.75 miles N. of, Wash Sch., 400 ft. S. W. of, J. W. Crossner's house, 300 ft. S. E. of, 8 ft. W. of three mail boxes, on W. side of road, 14" scaley bark hickory tree, in root of; copper nail and washer. 947.596 -0.015 947.583

-67-
Adjustment of intermediate bench marks between the points P and Q on the Republic Road near the Phelps-Gasconade County Line.

It will be remembered that the line P to Q is a spur line of levels in our scheme of adjustment. The line P to Q was run as a parallel duplicate line of levels, and the most probable values for the elevations of the points on such a line is the average of the several determinations. The diagram below shows lines of levels run by Brandis and Gurley Instruments, as these instruments were of equal fineness, equal weight was applied to the work done by each instrument. The green line represents the average of the instrumental determinations. In the level net adjustment proceeding, it was found that the point P was 0.0071 ft. high. As the spur line P to Q hinges directly on the point P, then the whole line P to Q will have to be lowered 0.0071 ft. In the diagram the adjusted line is shown in black.

Elev. P = 928.5520 Observed
Elev. Q = 928.5449 Adjusted
-0.0071 Cor.

Correction C for any intermediate point between P and Q:
C = average observed elev minus 0.0071.

-68-
18. Wash School, 0.5 miles north of, on Republic road, Little Bourbeuse Creek, at ford, on south bank, on west side of road, 10" scaly bark hickory tree, in base of; copper nail and washer. ------- 835.927 -0.010 835.917

P. Wash School, 1.0 miles north of, Little Bourbeuse Creek, 0.5 miles north of, Harrison School, 0.5 miles south of, Phelps-Gasconade County Line, 1.0 mile south of, Austin E. Bruer's house, 200 feet west of, at T road east from Republic road, on west side of road, 2 feet inside of fence, 10" post oak tree, in base of; copper nail and washer. ------------------------- 928.552 -0.007 928.545

19. Wash School, 1.25 north of, Little Bourbeuse Creek, 0.75 miles north of, Harrison School, 0.25 miles south of, Phelps-Gasconade County Line, 0.75 miles south of, Steve Lort's house, 70 feet north of, T road west (Mail road to High Gate), northwest corner of roads, 24" oak stump in fence corner, top of; ten penny nail. ----------------------- 962.732 Brandis, 962.726 Gurley.

Average 962.729 -0.007 962.722


Average 965.666 -0.007 965.659

21. Harrison School House, 0.25 miles north of, on Republic road, Phelps-Gasconade County Line, 0.25 miles south of, private T road west, south west corner of, 16" blazed black oak tree; bent nail in base of. -- 959.594 Brandis, 959.578 Gurley.

Average 959.586 -0.007 959.579

---69--
Q. Harrison School House, 0.5 miles north of, on Republic road, intersection of St. James-Republic road and High Gate-Red Bird road, 0.5 miles south of, Phelps-Gasconade County Line, 28 feet south of, center of road, 30 feet west of, 5 feet inside wire fence, in line with center of road north into Gasconade County, dim T road west, 50 feet south of; iron post stamped "997". 

P.B. M.  
997.242 Brandis,
997.237 Gurley.

Average 997.240 -0.007 997.223
Adjustment of intermediate temporary bench marks between point I at Miles School of Republic Road and Point J two miles due east on the Red Bird Road.

DERIVATION OF ADJUSTMENT FORMULA.

Correction c. for any intermediate point at distance d. from J and between I and J:

\[
C - 0.0236 = \frac{d}{1.6}
\]

\[
C = \frac{0.0080 \cdot d}{1.6} + 0.0236
\]

\[
C = - (0.00375 \cdot d + 0.0236)
\]

\[
C = (-0.00375 \cdot d - 0.0236)
\]
Red Bird Quadrangle.

(Latitude 38° 38' 15''; longitude 91° 30' 45'')

(Dawson Township)

Line from Miles School House on Republic Road, east via road 2.0 miles, to point on Red Bird Road, two miles north of St. James-Dawson Township Line.

1. St. James, 6.0 miles north of, on Republic Road, Safe, 2.5 miles east of, cross roads at Miles School, northeast corner of school house, sandstone wheel guard, top of, painted square. --- 1061.925 -0.030 1061.895

32 Miles School, 0.25 miles east of, on north and south center line, and between sections 20 and 29, R.6 W., T.39 N., at dim T road north, center of road; road elev. 1052.867 -0.029 1054.838

33 Miles School, 0.75 miles east of, Rinehart's house, 200 feet north of, at cross roads, southwest corner of, sandstone boulder, top of, doeb of, paint. ------ 1045.151 -0.027 1045.124

34 Miles School, 1.2 miles east of, via crooked road through woods, Red Bird Road, 0.4 miles west of, on south side of road, 16" post oak tree, in root of, copper nail and washer. ------------------ 1037.012 -0.025 1036.987

J. Robinsons Branch, 2.5 miles north of, St. James-Dawson Township Line, 30 miles north of, on top of hill, Red Bird Road at T road west to Miles School, northwest corner of roads, 15" white oak tree, in root of, copper nail and washer. ---- 968.394 -0.024 968.370

-72-
Adjustment of intermediate temporary bench marks between point B at the intersection of Ozark Highway and Red Bird Road, and point H at the intersection of Jakes Prairie and Red Bird Roads.

**Derivation of Adjustment Formula.**

B = 1074.6952 Adjus.  
- 0.0308 Cor.

Elev. H = 1042.0580 Obs.  
H = 1041.8535 Adjus.  
- 0.2045 Cor.

Correction $c$ for any intermediate point at distance $d$ from B and between B and H.

$$c = \frac{0.1727}{3.4} = -0.051088 d + 0.0308$$

simplifying:

$$c = -0.051088 d + 0.0308$$

and finally

$$c = ( -0.051088 d - 0.0308 )$$

**Graph for the Adjustment of Intermediate Points Between B and H.**
Red Bird Quadrangle.
(Latitude 38°38'15"N; longitude 91°30'45"W)

(St. James Township)

Line from intersection of Ozark Highway and Red Bird Road, which is 1.0 mile northeast of St. James, thence north via Red Bird Road, to Royal P.O., and Phelps-Gasconade County Line.

35 St. James, 1.0 mile northeast of, on Ozark Highway, T road east, (intersection Ozark Highway and Red Bird Road), 400 feet south of, Dr. E. A. Scott's house, 300 feet west of, on west side of road, 10" elm tree, in root of; copper nail and washer. ------------------ 1074.666  0.031  1074.635

36 Flag Spring School, 0.25 miles east of, Britton's Store, 0.25 miles west of, on Red Bird road, turn in road to east, 30 feet south of, sandstone culvert across road, west end wall, top of; chiseled square. -------------- 1029.621  0.084  1029.537

37 Flag Spring School, 0.5 miles east of, Britton's Store, in front of, center of road at turn to north; road elevation. 1039.320  0.096  1039.224

38 Flag Spring School, 0.75 miles northeast of, St. James, 2.5 miles north of, at Y road to north, in center of Y, 10" post oak tree, in base of; copper nail and washer. ------------------ 1023.293  0.118  1023.175

-74-
39 Flag Spring School, 1.25 miles northeast of, Britton's Store, 0.75 miles north of, dim cross roads, southeast corner of, 10" post oak tree, in base of; copper nail and washer. -------- 1016.547 0.134 1016.413

40 Britton's Store, 1.25 miles north of, St. James 3.5 miles north of, on Red Bird Road, at ford across Robinson's Branch, 50 feet south of, projecting Cotton Rock Stone in bluff, on west side of road; chiseled square. -------------- 952.626 0.159 952.467 (Bench Mark obliterated by dynamite)

41 Robinson's Branch, 0.25 miles north of, on Red Bird Road, at dim cross roads, tile culvert on: west side of road, south end wall, top of; chiseled square. -------- 1025.576 0.172 1025.404 (Dawson Township)

42 Robinson's Branch, 0.5 miles north of, on Red Bird Road, on St. James-Dawson Township Line, on east side of road, opposite T.-Snow's Mail Box, #12A, gate post, in base of; copper nail and washer. -------------- 1054.905 0.185 1054.720

H! Robinson's Branch, 0.9 miles north of, on Red Bird Road, at intersection of Red Bird and Jakes Prairie roads, (Y road to northeast), F.Berries house, 300 feet south of, southeast corner of: roads, 14" post oak tree, root of; copper nail and washer. -------- 1042.947 0.205 1042.742

H. Robinson's Branch, 0.9 miles north of, on Red Bird road, at intersection of Red Bird and Jakes Prairie roads, (Y road northeast), F.Berries house, 230 feet south of, crotch in Y roads, opposite east end of wagon gate, five feet south of P.B.M. fence; iron post stamped "1042"- - 1042.058 0.205 1041.853
Adjustment of intermediate temporary bench marks between point H at intersection of Jakes Prairie and Red Bird Roads, and point J on Red Bird Road two miles north of St. James-Dawson Township Line.

**DERIVATION OF ADJUSTMENT FORMULA.**

\[
\text{Elev. } H = 1042.0580 \quad \text{Elev. } J = 968.5880
\]

\[
\text{Adj. } H = 1041.8535 \quad \text{Adj. } J = 968.3704
\]

\[
\text{Cor. } = -0.2045
\]

Correction c. for any intermediate point at distance d from H and between H and J.

\[
c = \frac{-0.2045}{0.0221} = \frac{-d}{1.6} \quad \text{and} \quad c = \frac{0.0221 \cdot d + 0.2045}{1.6}
\]

Then:

\[
c = -(0.0144375 \cdot d + 0.2045) \quad \text{and} \quad c = (-0.0144375 \cdot d - 0.2045)
\]
Adjustment of intermediate bench marks between point J on Red Bird Road two miles north of St. James–Dawson Twp. Line, and point M 500 feet north of Royal Post Office.

The adjustment of these intermediate bench marks can be effected by scaling the adjustment corrections from the large diagram below and subtracting each correction from the observed elevation as it appears in the level notes; or, if desired, the adjustment corrections can be computed by the following formula for this section, J to M.

\[
C = (0.017134d + 0.1726)
\]

DERIVATION OF ADJUSTMENT FORMULA.

\[
\begin{align*}
\text{Elev. J} &= 968.598 \text{ Observed.} \\
\text{J} &= 968.3704 \text{ Adjusted.} \\
-0.2276 \text{ cor.} \\
\text{Elev. M} &= 954.9830 \text{ Observed.} \\
\text{M} &= 954.7504 \text{ Adjusted.} \\
-0.2326 \text{ cor.}
\end{align*}
\]

Correction \( C \) for any intermediate point at distance \( d \) from M and between the points J and M.

\[
C = \frac{0.2276 - 0.1726}{0.0550} = 8.21\quad \text{and} \quad C = \frac{0.2276}{8.21} + 0.1726
\]
43 Robinson's Eranch, 1.4 miles north of, Township line, 0.9 miles north of, on Red Bird road, on top of hill, at T road west, northwest corner of, 12" white oak tree base of; copper nail and washer.------------------- 1036.078 -0.212 1035.866

44 Robinson's Eranch, 2.0 miles north of, Dawson Township Line 1.5 miles north of, on Red Bird Road, Eikman's house, 50 feet east of, on west side of road at gate, sandstone boulder, top of; chiseled square.--------- 996.226 -0.220 996.006

J. Robinson's Eranch 2.5 miles north of, Dawson St. James Township Line, 2.0 miles north of, on top of hill, at T road west to Mile School, northwest corner of roads, 15" white oak tree, root of; copper nail and washer ---- 968.598 -0.228 968.370

45 Robinson's Branch, 2.75 miles north of, on Red Bird Road, on top of hill, N.D.Putnam's house, 40" southeast of, on west side of road, five feet inside of yard fence, 12" walnut tree, root of; copper nail and washer.-------- 967.970 -0.224 967.746

46 Robinson's Branch, 3.5 miles north of, intersection of Red Bird and Jakes Prairie Road, 2.6 miles north of, ford at Clear-creek, 0.7 miles south of, on Red Bird Road, sandstone boulder at side of road, top of painted square.-------------------------- 975.138 -0.211 974.977

47 Clear Creek, 0.5 miles south of, on Red Bird Road, intersection of Red Bird Road with Jakes prairie Road, 2.8 miles north of, 18" Black Oak tree at side of road, in root of; copper nail and washer 894.860 -0.208 894.652
Adjustment of point N on Red Bird Road 0.9 miles north of Royal P.O., and point O on Jakes Prairie road 0.25 miles south of Phelps-Gascoine County Line. N and O are intermediate junction points on the line M via N and O to G.

Let \( x = \text{cor. to N} \), then:

\[
\begin{align*}
X & = -0.1726 = \frac{0.0720}{8.05} \\
x & = \frac{10.0720 \times 0.1726}{8.05} \\
x & = -0.1803 \text{ Cor. to N.}
\end{align*}
\]

Let \( Y = \text{cor. to O} \), then:

\[
\begin{align*}
Y & = -0.1726 = \frac{3.3}{8.05} \\
& = -0.2009 \text{ Cor. to O.}
\end{align*}
\]

\[
\begin{align*}
\text{Elev. M} & = 954.7240 \text{ Arranged} \\
\text{Elev. N} & = 875.0710 \text{ Arranged} \\
\text{Elev. O} & = 964.9020 \text{ Arranged} \\
\text{Elev. G} & = 921.8230 \text{ Arranged} \\
\text{Elev. G} & = 969.7360 \text{ Arranged} \\
\end{align*}
\]
Adjustment of intermediate bench marks between point M 500 feet north of Royal Post Office, and point N 1100 feet south of Red Bird Road Ford across Big Bourbeuse River.

The adjustment of these intermediate bench marks can be effected by scaling the adjustment corrections from the large diagram below and subtracting each correction from the observed elevation as it appears in the level notes; or, if desired, the adjustment corrections can be computed by the following formula for this section of the net, M to N.

**DERIVATION OF ADJUSTMENT FORMULA.**

![Diagram of adjustment formula](image)

Correction \(c\) for any intermediate point at distance \(d\) from M and between points M and N.

\[ c = \frac{-0.1726}{0.1867} \frac{d}{0.9} \quad \text{and} \quad c = \frac{0.1867}{0.9} d + 0.1726 \]

\[ c = -(0.20744 d + 0.1726) \quad \text{or} \quad c = (-0.20744 d - 0.1726). \]

![Graph for adjustment](image)
48 Royal Post Office, 1.5 miles south of, on Red Bird Road, Clear Creek, bridge over, south east end of concrete bridge, elevation on hand rail. ------- 872.690 -0.200 872.490

49 Royal Post Office, 1.2 miles south of, Clear Creek bridge, 0.3 miles north of, T road east, 100" south of, concrete culvert across road, center of; road elevation. ------------------- 924.914 -0.194 924.720

50 Royal Post Office, 1.0 mile south of, at T road northwest, 24" Black Oak tree on west side of road, in base of; copper nail and washer. ------- 983.208 -0.191 983.017

51 Royal Post Office, 0.25 miles south of, on Red Bird Road, on east side of road, 30" White Oak tree, in root of; copper nail and washer. ------- 985.210 -0.178 985.032

Mt Royal Post Office, on Red Bird Road, Steen's Store, north end of concrete porch, one foot out from Building, six inches from north end of porch; bronze P.E.M. tablet stamped "961." ------- 960.631 -0.173 960.458

M. Royal Post Office, 500 feet north of, Royal School House, 1000 feet north of, at T road west to Republic Road, southwest corner of, telephone pole, in base of; copper nail and washer. ------- 954.903 -0.173 954.730

52 Royal Post Office, 0.6 miles north of, on Red Bird Road, Huntsman's House, 200 feet north west of, on east side of road, 14" Red Oak Tree, in root of; copper nail and washer. ------- 935.969 -0.274 935.695
Adjustment of intermediate bench marks between the points N and R on the Red Bird Road near the Phelps-Gasconade County Line.

**Diagram:**

**Observed Line:**
- **Point N:** Elevation = 875.2500
- **Point R:** Elevation = 866.3300

**Adjusted Line:**
- **Point N:** Elevation = 874.8907
- **Point R:** Elevation = 865.9787

**Correction for any intermediate point between N and R, is:**

\[ C = \text{averaged observed elevation minus 0.3593 feet.} \]

**Graph for the adjustment of intermediate points between N and R.**
N. Royal Post Office, 1.25 miles north of, on Red Bird Road, at T road east to Jakes Prairie Road, 50" feet south of T road, on east side of road, 14" Red Oak Tree, in base of; copper nail and washer.-------------------------- 875.250 -0.359 874.891

53 Royal Post Office, 1.45 miles north of, on Red Bird Road, Ford across Big Bourbuese River, 125 feet southwest of, center of road, 30 feet west of, main channel of River, 80 feet south of, leaning 12" slippery Elm Tree in south bank of River, in base of; ten penny nail.------------------------ 812.023 -0.359 811.664

R. Royal Post Office, 1.75 miles north of, on Red Bird Road, Red Bird Post Office, 2.00 miles south of, intersection Red Bird - St. James and Red Bird - High Gate roads, 1.50 miles south of, Ford across Big Bourbuese River, 0.3 miles north of, large Red Barn on east side of road, 100 feet southwest of, center of road, 25 feet west of, (Primary Traverse Station#87), worm fence, 4 feet inside of, in lot; Iron Post set on Phelps Gasconade County Line, P.B.M. stamped "866."-------------------------- 866.338 -0.359 865.979

-83-
Adjustment of intermediate bench marks between point P on Republice Road 0.5 miles south of Harrison School, and point M on Red Bird Road 500 feet north of Royal Post Office.

**DERIVATION OF ADJUSTMENT FORMULA.**

Correction c. for any intermediate point at distance d. from M and between P and M.

\[ c = \frac{0.0135}{2.4} \cdot d + 0.1726 \]

and finally

\[ c = -(0.005625 \cdot d + 0.1726) \]

GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN P AND M.
Red Bird Quadrangle.
(Latitude 38°38'15"N; longitude 91°30'45"W)
(Dawson Township)

Line from point on Red Bird Road 500 feet north of Royal Post Office, thence west 2.5 miles via road to point on Republic Road 1.0 mile north of Wash School.

| M   | Royal Post Office, 500 feet north of, Royal School House, 1000 feet north of, at T road west to Republic Road, southwest corner of, telephone pole, in base of, copper nail and washer. | 954.903 | -0.173 | 954.730 |
| 57  | Royal Post Office, 0.5 miles west of, at foot of hill, at private road southwest to farm house, on north side of road, fence corner, twin white oak tree, in base of, copper nail and washer. | 863.080 | -0.176 | 862.904 |
| 56  | Bourbeuse River Ford, at mouth of Little Bourbeuse, middle of ford; road elevation. | 813.000 | -0.178 | 812.822 |
| 55  | Royal Post Office, 1.0 west of, S.T. Mitchell's house, 600 feet east of, Bourbeuse River ford, 700 feet west of, on north side of road, at corner of woods, 20" white oak tree, in root of, copper nail and washer. | 831.792 | -0.179 | 831.613 |
| 54  | Royal Post Office, 1.65 miles west of, Bourbeuse River ford, 0.75 miles west of, Republic Rd., 0.75 miles east of, Phelps-Gascoigne County Line, 1.0 mile south of, south side of road, at T road south, 12" white oak tree, in base of, copper nail and washer. | 924.692 | -0.182 | 924.510 |
P. Wash School, 1.0 mile north of, Little Bourbeuse Creek, 0.5 miles north of, Harrison School, 0.5 miles south of, Phelps-Gasconade County Line, 1.0 mile south of, Austin E. Breur's house 200 feet west of, at T road east from Republic Road, on west side of road, 2 feet inside of fence, 10" post oak tree, in base of; copper nail and washer.--------- 928.731 -0.186 928.545
Adjustment of intermediate bench marks between point E at Knobview Depot, and point D at the intersection of the Ozark Highway and the Phelps-Crawford Line.

**DERIVATION OF ADJUSTMENT FORMULA.**

<table>
<thead>
<tr>
<th>Elev. E</th>
<th>1074.805 Adjust.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cor.</td>
<td>0.000 ft.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Elev. D</th>
<th>1098.8490 Observe.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>1098.8847 Adjust.</td>
</tr>
<tr>
<td>Cor.</td>
<td>0.0357</td>
</tr>
</tbody>
</table>

Correction c. for any intermediate point at distance d. from E, and between the points E and D.

\[
C = \frac{-d}{1.0}
\]

Then:

\[
C = (+0.0357 \cdot d).
\]

**GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN E AND D.**
Adjustment of intermediate bench marks between point D at the intersection of the Ozark Highway and the Phelps-Crawford County Line, and point G at the intersection of Jakes Prairie and Tribby roads, 0.5 miles south of Hinkle P.O.

**DERIVATION OF ADJUSTMENT FORMULA.**

\[
\text{Elev. } D = 1090.849 \text{ Obs.} \\
\text{Elev. } G = 921.2814 \text{ Adjust.} \\
\text{Correction } c = 0.0357 \text{ cor.} \\
\text{Correction } C = 0.0437 \text{ d} + 0.0357 \\
C = (+0.009712 \text{ d} + 0.0357)
\]

Correction \( c \) for any intermediate point at distance \( d \) from point D and between points D and G.

**GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN D AND G.**
Red Bird Quadrangle.
(Latitude 38°38'15"; longitude 91°30'91 45"

St. James Township)

Line from Knobview Depot, east 0.5 miles via Frisco Railroad to Phelps-Crawford County Line, thence north via Phelps-Crawford County Line road, 3.5 miles to T road west, to Jakes Prairie road, thence west via T road, 1.5 miles to Jakes Prairie road at a point 0.5 miles south of Hinkle Post Office.

E. Knobview Depot, 90 feet east of, 45 feet south of center of track, 12 feet east of R.R. crossing, 2 feet north of R.R. Property line, iron post stamped "1084". --------- 1074.005 0.000 1074.005

58 Knobview Depot, 0.5 miles east of, via R.R., concrete semaphore base, southwest corner, top of; signal 949 on Phelps-Crawford County line; painted square. --------- 1081.484 0.018 1081.502

D. Friendship School, 2.0 miles east of, on Ozark Highway, at cross roads on Phelps-Crawford County Line, south-west corner of roads, 14" hickory tree, in base of; copper nail and washer. 1098.849 0.036 1098.885

59 Knobview Depot, 1.5 miles north of, on Phelps-Crawford County line road, Ozark Highway, 0.5 miles north of, Cucarolo's house, 75 feet northeast of, on west side of road, in fence line, 12" locust tree, in base of; copper nail and washer. --------- 1071.411 0.040 1071.451

(Dawson Township)

60 Knobview Depot, 2.25 miles north of, on Phelps-Crawford County line road, Ozark Highway, 1.25 miles north of, drain to northwest, 100 feet south of, on west side of road, 20" white oak tree, in base of; copper nail and washer. -------------- 1064.348 0.048 1064.396
61 Knobview Depot, 2.5 miles north of, on Phelps-Crawford County Line Road, Ozark Highway, 1.5 miles north of, on top of hill, opposite mail box on east side of road, center of road; road elevation. -------------- 1087.589 0.050 1087.639

62 Knobview Depot, 2.75 miles north of, on Phelps-Crawford County Line Road, Ozark Highway, 1.75 miles north of, farm house, 350 feet northwest of, on rock projecting out of ground in center of road, 15 feet northwest of fence corner, top of rock; painted square. -------------- 1012.794 0.053 1012.847

63 Ozark Highway, 2.25 miles north of, on Phelps-Crawford County Line Road, at dim cross roads, on top of hill, 15 feet southwest of roads, on stump; painted square. -------------- 1071.623 0.057 1071.680

64 Ozark Highway, 2.65 miles north of, on Phelps-Crawford County Line Road, Hinkle P.O., 225 miles southeast of, on top of hill, in old abandoned field, west side of road, conglomerate boulder, top of; painted square. -------------- 1040.245 0.060 1040.305

65 Ozark Highway, 3.00 miles north of, on Phelps-Crawford County Line road, right angled turn in road east, 75 feet south of, T road west to Hinkle P.O., just opposite, on east side of road, sandstone boulder, top of; painted square. -------------- 994.501 0.065 994.566

66 Phelps-Crawford County Line, 0.5 miles west of, on Tribby road, Hinkle P.O., 1.5 miles south east of, 30 feet east of drain, on south side of road, sandstone boulder, top of; painted square. 994.905 0.070 994.975
67 Phelps-Crawford County Line,
1.0 mile west of, on Tribby road,
Hinkle P.O., 1.0 mile southeast
of, at dim cross roads, 10 feet
south of center of roads, sand-
stone boulder projecting out of
ground, top of; painted square.-- 950.010 0.074 950.084

68 Phelps-Crawford County Line,
1.25 miles west of, on Tribby
road, southeast corner, S.W. Qtr.
of the S.E. Qtr. Sec. 25, T. 39 N.,
R. 6 W., at private road north,
mail box on north side of road,
center of road just opposite;
road elevation. ---------------- 949.773 0.076 949.849

G. Hinkle Post Office, 0.5 miles
south of, intersection Jakes Prairie
and Tribby roads, (T road east) (sign
board 10 miles to Cuba), east side
of Jakes Prairie road, 30 feet north
of road intersection, 8" walnut tree,
in base of; copper nail and
washer. ---------------------- 921.202 0.079 921.281
Adjustment of intermediate temporary bench marks between point A at St. James and point B at the intersection of Red Bird Road and Ozark Highway.

DERIVATION OF ADJUSTMENT FORMULA.

Elev. A = 1087.800
Cor. = 0.000

Elev. B = 1074.666
Obsrv. = 1074.6352
Adj. = 0.0308
Cor. = 0.000

Correction c. for any intermediate point at distance d. from A and between points A and B.

\[ c = \frac{0.0308}{1.0} \]
\[ C = \frac{0.0308 d}{1.0} \]
\[ C = -0.0308 \, d \]

GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN A AND B.
Adjustment of intermediate bench marks between point B at the intersection of Red Bird Road and the Ozark Highway, and point C on the Ozark Highway at Friendship School.

The adjustment of these intermediate bench marks can be effected by scaling the adjustment corrections from the large diagram below and combining each correction algebraically with the observed elevation as it appears in the level field notes; or, if desired, the adjustment corrections for the intermediate points can be computed by the following formula for this section of the level net, B to C.

**DERIVATION OF ADJUSTMENT FORMULA.**

| Cor. 0.0508 | Cor. 0.0299 |

Correction c for any intermediate point at distance d from C and between the points B and C.

\[ c = \frac{0.020233 d - 0.0299}{8.0} \]

\[ c = -(0.020233 d - 0.0299) \]

\[ C = (+0.0299 - 0.020233 d). \]
**MERAMEC SPRINGS QUADRANGLE.**

(Latitude 37°45' -38°; longitude 91°30' -91°45°)

(St. James Township)

**LINE FROM ST. JAMES EAST VIA OZARK HIGHWAY TO PHELPS-CRAWFORD COUNTY LINE, SAID POINT ONE MILE NORTHEAST OF KNOB VIEW RAILROAD DEPOT.**

A. (See description on page 64)

<table>
<thead>
<tr>
<th>Observed Elevation</th>
<th>COR.</th>
<th>Adjusted Elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1087.800</td>
<td>0.000</td>
<td>1087.800</td>
</tr>
</tbody>
</table>

22 St. James 0.4 miles east of, on St. James-Cuba road (Ozark Highway), Frisco R.R., 50 ft. north of, turn in road to north, 500 ft. west of, concrete culvert, across road, north end wall, top of; chiseled square. ------ 1065.363 -0.0124 1065.351

**RED BIRD QUADRANGLE.**

(Latitude 38°-38°15'; longitude 91°30' -91°45°)

(St. James Township)

B. St. James, 1.0 miles N.E. of, Ozark Highway, T road east (intersection of Ozark Highway and Red Bird road), 400 ft. S.of, Dr. E.A. Scott's house, 300 ft. W. of, on W. side of road, 10" elm tree, in root of; copper nail and washer. ------------------ 1074.666 -0.031 1074.635

69 St. James, 1.5 miles N.E. of, on Ozark Highway, 0.5 miles east of Red Bird road intersection, on N. side of road, 12" post oak tree, in base of; copper nail and washer. ------------------------ 1062.669 -0.021 1062.648

70 St. James, 1.75 miles northeast of, on Ozark Highway, concrete culvert across road, south end wall, top of; chiseled triangle. 1034.437 -0.015 1034.422

71 St. James, 9.2 miles N.E. of, on Ozark Highway, on top of hill, center of road; road elevation. 1054.436 -0.010
Adjustment of intermediate bench marks between point C on the Ozark Highway at Friendship School, and point D at the intersection of the Ozark Highway and the Phelps-Crawford County Line.

The adjustment of these intermediate temporary bench marks can be effected by scaling the adjustment corrections from the large diagram below and combining each correction according to its sign with the observed elevation as it appears in the level field notes; or, if desired, the adjustment corrections can be computed by the following formula for this section of the level net, C to D.

\[
C = \frac{0.0299}{0.00562} d - 2.25 \quad \text{and} \quad C = 0.0562 \cdot d - 0.0299
\]

Graph for the adjustment of intermediate points between C and D.
<table>
<thead>
<tr>
<th>Page</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>St. James, 2.5 miles northeast of, on Ozark Highway, Friendship School, 1.5 miles west of, concrete culvert (erected 8-13-20 by T. Ball), north end wall, top of; chiseled square. 1010.057</td>
</tr>
<tr>
<td>73</td>
<td>Friendship School, 1.25 miles west of, on Ozark Highway, at T. road N. to Binkle P.O., Corrugated iron culvert, rock on east end of culvert, top of; chiseled triangle. 1018.926</td>
</tr>
<tr>
<td>74</td>
<td>Friendship School, 0.5 miles W. of, on Ozark Highway, Gerber's Store, 600 feet W. of, concrete culvert, north end wall, top of; chiseled triangle. 1062.045</td>
</tr>
<tr>
<td>C.</td>
<td>Friendship School, on Ozark Highway, southwest corner of concrete porch of school house, top of; chiseled triangle, painted &quot;1083.3.&quot; 1083.400</td>
</tr>
<tr>
<td>01.</td>
<td>Friendship School, on Ozark Highway, 50 feet S.W. of, in School Yard, 10 ft. from fence corner, 30 ft. N. of center of Ozark Highway, 30 ft. E. of center of T. road north; iron post stamped &quot;1983.&quot; 1083.458</td>
</tr>
<tr>
<td>75</td>
<td>Friendship School, 0.25 miles E. of, private T. road N. to Louis Hall's residence, 75 ft. W. of, concrete culvert, S. end wall top of; chiseled square. 1037.041</td>
</tr>
<tr>
<td>76</td>
<td>Friendship School, 0.9 miles E. of, on Ozark Highway, Phelps-Crawford County Line, 1.1 miles west of, concrete culvert, south end wall, top of; chiseled Square. 1042.297</td>
</tr>
</tbody>
</table>

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D. Friendship School, 2.0 miles east of, on Ozark Highway, at cross roads on Phelps - Crawford county Line, southwest corner of roads, 14" hickory tree, in base of; copper nail and washer.------------------------ 1098.911 -0.026 1098.885

This completes the description and adjustment of the line of levels from St. James east via Ozark Highway to the Phelps-Crawford County Line, said point one mile northeast of Knowview Railroad Depot.
Adjustment of intermediate bench marks between point C on the Ozark Highway at Friendship School, and point F on the Jakes Prairie Road 1.0 mile south of Hinkle Post Office.

DERIVATION OF ADJUSTMENT FORMULA:

\[
\begin{align*}
\text{Elev. } C &= 1083.4000 \text{ Observed} \\
\text{Elev. } F &= 1001.1770 \text{ Observed} \\
\text{Elev. } C &= 1083.4299 \text{ Adjusted} \\
\text{Elev. } F &= 1001.3429 \text{ Adjusted} \\
&+ 0.0299 \text{ Cor.} \\
&+ 0.1659 \text{ Cor.}
\end{align*}
\]

Correction C for any intermediate point at distance d from C and between the points C and F.

GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN C AND F.

<table>
<thead>
<tr>
<th>Miles</th>
<th>78</th>
<th>79</th>
<th>80</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.00</td>
<td>0.50</td>
<td>0.50</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Algebraic Solution:

\[
\begin{align*}
C &= \frac{-0.0222}{0.1360} = \frac{-d}{3.0} \\
C &= \frac{-1360}{8.0} \cdot d + 0.0299 \\
C &= (+0.04533 \cdot d + 0.0299).
\end{align*}
\]
Red Bird Quadrangle.
(Latitude 38°-38° 15'; Longitude 91° 30'-91° 45')

(St. James Township)

Line from Friendship School on Ozark Highway, in center of section 11, T. 38 N., R. 6 W., north via road 3.0 miles to point on St. James-Jakes Prairie road, 1.0 mile south of Hinkle Post Office, at a T road south east.

C. Friendship School, on Ozark Highway, in center of sec. 11, T. 38 N., R. 6 W., southwest corner of concrete porch of School House, top of chiseled triangle. ------------ 1083.400 0.030 1083.430

C'. Friendship School, on Ozark Highway, in center of sec. 11, T. 38 N., R. 6 W., 50 feet southwest of, in school yard, 10 feet from fence corner, 30 feet north of center of Ozark Highway, 30 feet east of center of T road north; iron post P. B. M. stamped. --------------- 1083.458 0.030 1083.488

78 Friendship School, 0.5 miles north of, Hinkle P. 0., 3.5 miles south of, T road west, east side of road, 8" black oak tree, in root of; copper nail and washer. ------------ 1018.238 0.052 1018.290

(Dawson Township)

79 Friendship School, 1.5 miles north of, Hinkle P. 0., 2.5 miles south of, on township line between St. James and Dawson Townships, bridge on township line road over south fork of Clear Creek, northwest corner of bridge; copper nail and washer. ------ 951.418 0.098 951.516

80 Friendship School, 2.0 miles north of, Hinkle P. 0., 2.0 miles south of, east side of road, in rock ledge; chiseled triangle. ----------- 945.415 0.120 945.535
F. Friendship School, 3.0 miles north of, Hinkle P.O., 1.0 mile southwest of, on Jakes Prairie road, at T road southeast (road to Friendship Sch.) southeast corner of roads, 15" white oak tree, in base of; copper nail and washer.----- 1001.177 0.166 1001.343
Adjustment of intermediate bench marks between point H at the intersection of Red Bird and Jakes Prairie roads, and point F on Jakes Prairie Road 1.0 miles south of Hinkle P.O. and 3.0 miles north of Friendship School.

**DERIVATION OF ADJUSTMENT FORMULA.**

\[
\begin{align*}
\text{Elev. } H &= 1042.0580 \text{ Observed.} \\
\text{H} &= 1041.0535 \text{ Adjusted.} \\
- \text{ 0.2045 Cor.} \\
\text{Elev. } F &= 1001.6110 \text{ Observed.} \\
\text{F} &= 1001.3429 \text{ Adjusted.} \\
- \text{ 0.2681 Cor.}
\end{align*}
\]

Correction \( c \) for any intermediate point at distance \( d \) from \( H \) and between the points \( H \) and \( F \).

\[
\begin{align*}
C &= \frac{-0.2045}{0.0636} - \frac{d}{1.75} \\
C &= 0.036343d + 0.2045 \\
C &= -(0.036343d + 0.2045) \\
C &= (-0.036343d - 0.2045).
\end{align*}
\]
Red Bird Quadrangle.
(Latitude 38°38'15"; longitude 91°30'-91°45'
(Dawson Township)

Line from intersection of Red Bird and Jakes Prairie roads, thence northeast 2.75 miles via Jakes Prairie road to Collins School and Hinkle P.O., thence north 4.5 miles via Jakes Prairie road to Phelps-Gasconade County Line, 2.0 miles west of Jakes Prairie Post Office.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>Robinsons Branch, 0.9 miles north of, on Red Bird Road, at intersection of Red Bird and Jakes Prairie roads, (Y road Northeast), F. Berries house, 230 feet south of, in crotch in Y roads, opposite east end of wagon gate, 5 feet south of fence; iron post stamped &quot;1042&quot;.</td>
</tr>
<tr>
<td>81</td>
<td>Intersection Red Bird and Jakes Prairie roads, 0.75 miles northeast of, on Jakes Prairie road, main fork of Clear Creek, 0.5 miles southwest of, via way of road, H. Gilman's house, 100 feet east of, on east side of road, 14&quot; post oak tree, in base of; copper nail and washer.</td>
</tr>
<tr>
<td>82</td>
<td>Intersection of Red Bird and Jakes Prairie roads, 1.25 miles northeast of, on Jakes Prairie road, Hinkle P.O., 1.5 miles southwest of, Clear Creek Ford, (Main fork), 50 feet east of, at angle in road to north, southeast corner of road, in fence line, 20&quot; hickory tree, in base of; copper nail and washer.</td>
</tr>
<tr>
<td>F</td>
<td>Friendship School, 3.0 miles north of, Hinkle P.O., 1.0 mile southwest of, on Jakes Prairie road, at T road southeast (road to Friendship Sch.), southeast corner of roads, 15&quot; white oak tree, in base of; copper nail and washer.</td>
</tr>
</tbody>
</table>
Adjustment of intermediate temporary bench marks between point F on Jakes Prairie Road 1.0 Mile south of Hinkle P.O. and 3.0 miles north of Friendship School, and point G 0.5 miles south of Hinkle P.O. at the intersection of Jakes Prairie and Tribby roads.

Derivation of Formula for the Adjustment of Intermediate Points.

\[ \text{Elev. } F = 1001.6110 \text{ Observed.} \quad \text{Elev. } G = 921.5450 \text{ Observed.} \]

\[ \text{Elev. } F = 1001.3429 \text{ Adjusted.} \quad \text{Elev. } G = 921.2814 \text{ Adjusted.} \]

\[ C = 0.2681 \text{ Cor.} \quad C = 0.2686 \text{ Cor.} \]

\[ C = \frac{-0.2626}{0.0045} - d \]

\[ C = 0.0045 \cdot d + 0.2626 \]

\[ C = -(0.009 \cdot d + 0.2626) \]

\[ C = (-0.009 \cdot d - 0.2626) \]

Between F and G, a distance of 0.5 miles, the level field notes show that no intermediate points were established in this section of the net.
Adjustment of intermediate bench marks between point G 0.5 miles south of Hinkle P.O. at the intersection of Jakes Prairie and Tribby roads, and point O on Jakes Prairie Road 0.25 miles south of the Phelps-Gasconade County Line.

The adjustment of these intermediate temporary bench marks can be effected by scaling the adjustment corrections from the large diagram below and subtracting each correction from the observed elevation as it appears in the field level notes; or, if desired, the adjustment corrections can be computed by the following formula for this section of the level net, G to O.

\[ C = -\left(0.008568 \, d + 0.2229\right) \text{ or } C = \left(-0.008568 \, d - 0.2229\right) \]

Correction \( c \), for any intermediate point at distance \( d \) from \( G \) and between \( G \) and \( O \).

\[ \frac{C - 0.2229}{0.0407} = -\frac{d}{4.75} \quad \text{and} \quad C = \frac{0.0427 \, d}{4.75} + 0.2229 \]

-104-
6 Hinkle P.O., 0.5 miles south of, intersection Jake's Prairie and Tribby roads, (T road east) (sign board 10 miles to Cuba), east side Jake's Prairie road, 30 feet north of road intersection, 8" walnut tree, in base; copper nail and washer. 921.545 -0.264 921.281

83 Collins School, 800 feet north of, Hinkle Store and P.O., 40 feet southeast of, 40 feet west of center of road, in line with tangent of road to northeast; iron post P. B. M. stamped "1039!" 1038.938 -0.258 1038.680

84 Collins School, 800 feet north of, Hinkle Store and P.O., southeast corner of porch, top of floor; copper nail and washer. 1040.617 -0.258 1040.359

85 Hinkle P.O., 0.6 miles north of, on Jake's Prairie road, southwest corner sec.13, R.3W, T.39 N., 350 feet north of, on west side of road, opposite private T road east to white house, 16" black jack tree, in base of; copper nail and washer. 984.964 -0.254 984.710

86 Hinkle P.O., 1.2 miles north of, on Jake's Prairie road, at private T road east, southeast corner of road, 10" post oak tree, in base of; copper nail and washer. 1057.863 -0.249 1057.614

87 Hinkle P.O., 1.75 miles north of, Oak Grove School, 2.0 miles south of, old deserted farm house on west side of road, 125 feet southeast of, on top of hill, east side of road, 10" black jack tree, in root of; copper nail and washer. 1086.932 -0.244 1086.688
Adjustment of Permanent Bench Mark S on Jakes Prairie Road at the Phelps-Gasconade County Line. The adjustment is to be made between the points O and S with the line tied back at O.

This section, O to S, is a spur line to the level net; it was run as a duplicate line of levels, that is: the line was run over the same route twice, but in opposite directions; i.e. O to S and from S back to O. See page 2 of this thesis for a discussion on duplicate lines of levels.

**Analytic Adjustment of Point S.**

<table>
<thead>
<tr>
<th>Elev. O = 969.7580</th>
<th>Elev. O = 969.7430</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elev. O = 969.5351</td>
<td>Elev. O = 969.5351</td>
</tr>
<tr>
<td>Adjusted.</td>
<td>Adjusted.</td>
</tr>
<tr>
<td>- 0.2229 Cor.</td>
<td>- 0.2079 Cor.</td>
</tr>
</tbody>
</table>

Now for the adjustment of S.

Let X be the correction desired.

Then:

\[ X = \frac{0.2079}{0.5150} = 0.4050 \quad \text{and} \quad X = \frac{0.0150}{2} + 0.2079 \]

and:

\[ X = -(0.0075 + 0.2079) \]

and finally:

\[ X = -0.2154 \quad \text{ft., the correction to be applied to the observed elevation of S.} \]

Then: Elev. S = 942.5160 Observed.

\[ -0.2154 \quad \text{Correction.} \]

\[ S = 942.3006 \quad \text{Adjusted.} \]

**Diagram for the Graphic Adjustment of Point S.**

-106-
88 Oak Grove School, 1.5 miles south of, on Jakes Prairie road, dim T road west, 200 feet north of, farm house on east side of road, 200 ft west of, in fence corner, east side of road, 14" red oak tree, in base of; copper nail and washer. ------ 999.372 -0.238 999.134

90 Oak Grove School, on Jakes Prairie road, Dawson Township, Phelps-Gasco­nade County Line, 0.75 miles S. of, in front of School House, 125 feet west of, east side of road, 14" scaley bark hickory tree, in root of; copper nail and washer. ------ 900.143 -0.226 899.917

8. Oak Grove School, 0.75 miles north of, T road east on County Line, Jakes Prairie P.0., 2.0 miles west of, at private T road west on County Line, Sewell's house, 60 ft southwest of, Phelps-Gasco­nade County Line, 10 feet north of, north east corner of roads; iron post stamped "942". -------------------------- P. B. M. 942.515 -0.215 942.301

0. Oak Grove School, 0.5 miles north of, T road east, 50 feet south of, T road west to Red Bird road, 20 ft north of, northwest corner of roads, 10" post oak tree, in base of; copper nail and washer. ------ 969.743 -0.208 969.535

969.758 -0.223 969.535

999.134

966.842

89 Oak Grove School, 0.5 miles south of, on Jakes Prairie road, W. E. Wright's house, 200 feet north of, at T road east and private road west thru gate, top of hill, center of roads; road elevation. ------ 967.072 -0.230 966.842

89 Oak Grove School, on Jakes Prairie road, W. E. Wright's house, 200 feet north of, at T road east and private road west thru gate, top of hill, center of roads; road elevation. ------ 967.072 -0.230 966.842

Oak Grove School, in Dawson Twp., 0.5 miles north of, T road east, 50 feet south of, T road west to Red Bird road, 20 feet north of, northwest corner of roads, 10" post oak tree, in base of; copper nail and washer. ------ 969.758 -0.223 969.535

969.758 -0.223 969.535

969.535
Adjustment of intermediate temporary bench marks between point N on the Red Bird Road 0.9 miles north of Royal P.O., and point O on the Jakes Prairie Road 0.25 miles south of the Phelps-Gasconade County Line.

The adjustment of these intermediate temporary bench can be effected by scaling the adjustment corrections from the large diagram below and subtracting each correction from the observed elevation as it appears in the level field note book; or, if desired, the adjustment corrections can be computed by the following formula for this section, N to O.

\[
C = \frac{0.2023}{2.4} \cdot d + 0.2023
\]

\[
C = -(0.008583 d + 0.2023)
\]

\[
C = (-0.008583 d - 0.2023).
\]

Graph for the adjustment of intermediate points between N and O.
Red Bird Quadrangle.
(Latitude 38° 38' 15"; longitude 91° 30' 45")

(Dawson Township)

Line from point on Jakes Prairie Road 0.25 miles south of County Line, thence west two-one half miles to point on Red Bird Road 0.5 miles south of Phelps-Gasconade County Line and 1.25 miles north of Royal P.O.

0  Oak Grove School in Dawson Twp, 0.5 miles north of, T road east, 50 feet south of, T road west to Red Bird road, 20 feet north of, northwest corner of roads, 10" post oak tree, in base of; copper nail and washer. --- 969.758  -0.223  969.535

93 Oak Grove Church, 800 feet north east of, J.B. Copeland's house, 800 feet west of, in bottom of branch to north, north side of road, 24" sycamore tree, in root of; copper nail and washer. ---- 834.182  -0.219  833.963

92 Oak Grove Church, 0.6 miles west of, T road south on line between Secs. 2 and 3 produced north, 30 feet west, concrete culvert across road, north end wall, top of; painted square. --- 895.903  -0.214  895.689

91 Oak Grove Church, 1.2 miles west of, Red Bird Road, 0.65 miles east of, Spurgeon's house, 300 feet southwest of, on north side of road, twin trunk walnut tree, in root of; copper nail and washer. ---- 863.954  -0.209  863.745

N  Royal P. O., 1.25 miles north of, on Red Bird Road, Oak Grove Church, 1.85 miles west of, Phelps-Gasconade County Line, 0.5 miles south of, Red Bird Ford across Bourbeuse River, 1100 feet south of, at T road east to Jakes Prairie road, 50 feet south of T road, on east side of road, 14" red oak tree, in base of; copper nail and washer. 875.093  -0.202  874.891

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CONCLUSION.

In conclusion, a brief outline will be given covering the main classes of problems which have been considered and the methods of solution and their appropriate applications to the problem at hand.

Direct Observations of a Single Quantity.

Adjustment. Take the mean or the weighted mean of the observed quantities.

Indirect Observations.

Adjustment. Write the observation equations and from them the normal equations; the solution of the latter gives the unknown quantities themselves or the corrections to their assumed approximate values. The number of the observation equations will be the same as that of the observations; the number of the normal equations will equal that of the unknown quantities, which must always be less than that of the observations.

Conditioned Observations.

Adjustment. Write the condition equations in their general form and then in their simple form involving the corrections. From them form the normal equations, the same in number as the conditions. The solution of the normal equations gives a set of factors, called correlates, one for each condition equation, from which the desired corrections to the observed quantities are determined.

In summarizing what has been stated above, there are in general two methods of finding the most probable values of the unknown quantities in cases involving conditioned quantities.

In the first method the condition equations are avoided (or eliminated) by impressing their significance on the observation equations, which reduces the problem to the cases as considered in Parts (II) and (III).

In the second method the observation equations are eliminated by impressing their significance on the condition equations, when the solution may be effected by the method of correlates as considered in Part (IV).

The first method is the most direct in elementary problems, say with not more than three residual unknowns, but the second method by using conditioned equations instead of observation equations and completing the solution by the method of correlates in connection with the results obtained from the solution of the normal equations, greatly reduces the work of computation in the case of complicated problems with four or more unknown quantities to be solved.

The best methods of solving normal equations, as I have found them, are those given in Part (IV) pages 55, 56, and 57.
APPLICATIONS OF THE METHOD OF LEAST SQUARES
TO THE ADJUSTMENT OF LEVEL CIRCUITS.
ADJUSTMENT OF ELEVATIONS IN DAWSON
AND ST. JAMES TOWNSHIPS,
PELPS COUNTY,
MISSOURI.

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BIBLIOGRAPHY.

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Method of Least Squares, Comstock. Ginn, Boston, 1895.

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Theory of Errors and Method of Least Squares, Johnson. Wiley 1892.


Application of Least Squares to the Adjustment of Triangulation,


Adjustment of Triangulation, E.M. Douglas, School of Mines Quarterly No. 4 Vol. 14 Columbia College, New York.

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John Wiley & Sons, New York, 1912.

Of the above treatise on the subject of Least Squares, the last five mentioned were consulted as well as the book further up in the list by Wahl. The book by Leland is recommended as an exceptionally good book on the subject.
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SQUARE ADJUSTMENT

Rules for sides:
Stand outside angle; sides both read towards angles vertex; left hand side read, right hand side next and read. Consider four figures - $\pm 0.1^\circ$

Front and back angles:
Front angles $+\frac{1}{2}^\circ$
Back angles $-\frac{1}{2}^\circ$

Vertical error gotten from elimination computation.

Secant groups have signs changed in (I) stated when the diff for $\mp$ is minus, the sign remains same.
Note that the diff for $-\frac{1}{2}^\circ$ for angles over $90^\circ$ is minus.

Sides Equation

<table>
<thead>
<tr>
<th>Sides</th>
<th>Angle</th>
<th>Diff/$^\circ$</th>
<th>Sine</th>
<th>Corrected Sine</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.1^\circ$</td>
<td>83° - 3° 21' 13&quot;</td>
<td>2.39796</td>
<td>-0.03</td>
<td>-3.229756</td>
</tr>
<tr>
<td>$-0.0^\circ$</td>
<td>83° 0° 26' 47&quot;</td>
<td>3.32300</td>
<td>0.02</td>
<td>4.32300</td>
</tr>
<tr>
<td>$+0.1^\circ$</td>
<td>83° 0° 26' 47&quot;</td>
<td>3.32300</td>
<td>-0.02</td>
<td>4.32300</td>
</tr>
<tr>
<td>$+0.0^\circ$</td>
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<td>83° 0° 26' 47&quot;</td>
<td>3.32300</td>
<td>-0.02</td>
<td>4.32300</td>
</tr>
</tbody>
</table>

Whole equations divided by 100.

Equations of Condition

(f) $0 - -3.45^\circ - 0.0^\circ - 0.0^\circ + 1.4^\circ$ (from a)
(g) $0 + 1.70^\circ - 0.0^\circ - 0.0^\circ + 1.7^\circ$ (from b)
(h) $0 + 3.77^\circ - 0.0^\circ - 0.0^\circ + 3.7^\circ$ (from c)
1 Title Page
6 Contents
2 Introduction
4 Front Plates in Folder
118 Body of Thesis
1 Rear Plate

132 Pages Total.

Thesis
C. E. S. Bardsley,
Candidate for the Degree of
Civil Engineer.
GEODETIC COMPUTATION

LEAST SQUARE ADJUSTMENT

Station Side Observed Angle Cor for Angle Corrected Sa Angle

(5) Solution of Normal Equations

Equations of Condition
(a) 0 = 3.06 % - % (from a)
(b) 0 = -7.0 % - % (from b)
(c) 0 = -12.7 % - % (from c)

Table of Correlates

All figures distributed

All figures

Note: The correction for any angle is the difference between the corrections for the two sides bounding it (table of corrections)

DIRECTIONS

Computation by
R. J. Yessler
Missouri School of Mines
AND METALLURGY