12-1-1991

Illustrative examples based on the ASCE standard specification for the design of cold-formed stainless steel structural members

Shin-Hua Lin

Wei-wen Yu

University of Missouri--Rolla

Theodore V. Galambos

Follow this and additional works at: http://scholarsmine.mst.edu/ccfss-library

Part of the Structural Engineering Commons

Recommended Citation

Lin, Shin-Hua; Yu, Wei-wen; and Galambos, Theodore V., "Illustrative examples based on the ASCE standard specification for the design of cold-formed stainless steel structural members" (1991). Center for Cold-Formed Steel Structures Library. Paper 174.
http://scholarsmine.mst.edu/ccfss-library/174

This Technical Report is brought to you for free and open access by the Wei-Wen Yu Center for Cold-Formed Steel Structures at Scholars' Mine. It has been accepted for inclusion in Center for Cold-Formed Steel Structures Library by an authorized administrator of Scholars' Mine. For more information, please contact weaverjr@mst.edu.
Civil Engineering Study 91-2

Cold-Formed Steel Series

Final Report

ILLUSTRATIVE EXAMPLES BASED ON THE ASCE STANDARD SPECIFICATION FOR THE DESIGN OF COLD-FORMED STAINLESS STEEL STRUCTURAL MEMBERS

by

Shin-Hua Lin
Consultant
Engineering Design & Management Inc.
St. Louis, Missouri

Wei-Wen Yu
Project Director
University of Missouri-Rolla

Theodore V. Galambos
Consultant
University of Minnesota

A Research Project Sponsored by the Nickel Development Institute and Chromium Centre

December 1991

Department of Civil Engineering
Center for Cold-Formed Steel Structures
University of Missouri-Rolla
Rolla, Missouri
PREFACE

During the past four years, two methods were developed for the design of stainless steel structural members at the University of Missouri-Rolla with consultation of Professor T. V. Galambos at the University of Minnesota. One of the methods is based on the load and resistance factor design (LRFD) and the other is based on the allowable stress design (ASD). Both design methods are now included in the new ASCE Standard 8-90, Specification for the Design of Cold-Formed Stainless Steel Structural Members.

At the September 21, 1990 meeting of the Control Group of the ASCE Stainless Steel Cold-Formed Section Standards Committee held in Washington, D.C., the urgent need for design examples using the new ASCE Standard was discussed at length. The University of Missouri-Rolla was asked to submit a proposal for preparation of such illustrative examples beginning October 1, 1990.

During the period from October 1990 through December 1991, a total of 27 illustrative problems have been prepared as included herein. Most of the given data used for these examples are similar to those used in the 1986 edition of the AISI Cold-Formed Steel Manual except that for each problem, two examples are illustrated by using LRFD and ASD methods.

The research work reported herein was conducted in the Department of Civil Engineering at the University of Missouri-Rolla with the consulting work provided by Dr. Shin-Hua Lin and Professor T. V. Galambos. The financial assistance provided by the Nickel Development Institute and the Chromium Centre is gratefully acknowledged. Appreciation is also expressed to Dr. W. K. Armitage, Mr. J. P. Schade, Professor P. Van der Merwe and Professor G. J. Van den Berg for their technical review and suggested revisions.
# TABLE OF CONTENTS

PREFACE ........................................................................................................... ii

I. INTRODUCTION ......................................................................................... 1

II. COMPUTATION OF SECTIONAL PROPERTIES OF COLD-FORMED
    SECTIONS USING LINEAR METHOD .......................................................... 1

III. CORRELATION OF SPECIFICATION AND ILLUSTRATIVE EXAMPLES ...... 2

IV. ILLUSTRATIVE EXAMPLES ................................................................. 12

A. FLEXURAL MEMBERS

   Example 1.1 Channel w/Unstiffened Flanges (LRFD) .................. 13
   Example 1.2 Channel w/Unstiffened Flanges (ASD) ................. 29
   Example 2.1 Channel w/Stiffened Flanges (LRFD) .................... 31
   Example 2.2 Channel w/Stiffened Flanges (ASD) ...................... 42
   Example 3.1 C-Section w/Bracing (LRFD) ............................... 44
   Example 3.2 C-Section w/Bracing (ASD) ................................. 53
   Example 4.1 Z-Section w/Stiffened Flanges (LRFD) ............... 55
   Example 4.2 Z-Section w/Stiffened Flanges (ASD) .................. 64
   Example 5.1 Deep Z-Section w/Stiffened Flanges (LRFD) ........ 66
   Example 5.2 Deep Z-Section w/Stiffened Flanges (ASD) .......... 82
   Example 6.1 Hat Section (LRFD) .................................................. 84
   Example 6.2 Hat Section (ASD) .................................................. 95
   Example 7.1 Hat Section w/Intermediate Stiffener (LRFD) ...... 98
   Example 7.2 Hat Section w/Intermediate Stiffener (ASD) ...... 111
   Example 8.1 I-Section w/Unstiffened Flanges (LRFD) ............ 114
   Example 8.2 I-Section w/Unstiffened Flanges (ASD) ............... 121
   Example 9.1 Channel w/Lateral Buckling Consideration (LRFD) .. 122

iii
<table>
<thead>
<tr>
<th>Example</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.2</td>
<td>Channel w/Lateral Buckling Consideration (ASD)</td>
<td>136</td>
</tr>
<tr>
<td>10.1</td>
<td>Hat Section Using Inelastic Reserve Capacity (LRFD)</td>
<td>140</td>
</tr>
<tr>
<td>10.2</td>
<td>Hat Section Using Inelastic Reserve Capacity (ASD)</td>
<td>147</td>
</tr>
<tr>
<td>11.1</td>
<td>Deck Section (LRFD)</td>
<td>148</td>
</tr>
<tr>
<td>11.2</td>
<td>Deck Section (ASD)</td>
<td>176</td>
</tr>
<tr>
<td>12.1</td>
<td>Cylindrical Tubular Section (LRFD)</td>
<td>182</td>
</tr>
<tr>
<td>12.2</td>
<td>Cylindrical Tubular Section (ASD)</td>
<td>184</td>
</tr>
<tr>
<td>13.1</td>
<td>Flange Curling (LRFD)</td>
<td>185</td>
</tr>
<tr>
<td>13.2</td>
<td>Flange Curling (ASD)</td>
<td>190</td>
</tr>
<tr>
<td>14.1</td>
<td>Shear Lag (LRFD)</td>
<td>192</td>
</tr>
<tr>
<td>14.2</td>
<td>Shear Lag (ASD)</td>
<td>198</td>
</tr>
<tr>
<td></td>
<td><strong>B. COMPRESSION MEMBERS</strong></td>
<td></td>
</tr>
<tr>
<td>15.1</td>
<td>C-Section (LRFD)</td>
<td>199</td>
</tr>
<tr>
<td>15.2</td>
<td>C-Section (ASD)</td>
<td>208</td>
</tr>
<tr>
<td>16.1</td>
<td>C-Section w/Wide Flanges (LRFD)</td>
<td>210</td>
</tr>
<tr>
<td>16.2</td>
<td>C-Section w/Wide Flanges (ASD)</td>
<td>219</td>
</tr>
<tr>
<td>17.1</td>
<td>I-Section (LRFD)</td>
<td>221</td>
</tr>
<tr>
<td>17.2</td>
<td>I-Section (ASD)</td>
<td>228</td>
</tr>
<tr>
<td>18.1</td>
<td>I-Section with Lips (LRFD)</td>
<td>230</td>
</tr>
<tr>
<td>18.2</td>
<td>I-Section with Lips (ASD)</td>
<td>237</td>
</tr>
<tr>
<td>19.1</td>
<td>T-Section (LRFD)</td>
<td>239</td>
</tr>
<tr>
<td>19.2</td>
<td>T-Section (ASD)</td>
<td>246</td>
</tr>
<tr>
<td>20.1</td>
<td>Tubular Section - Square (LRFD)</td>
<td>248</td>
</tr>
<tr>
<td>20.2</td>
<td>Tubular Section - Square (ASD)</td>
<td>252</td>
</tr>
<tr>
<td>21.1</td>
<td>Tubular Section - Round (LRFD)</td>
<td>253</td>
</tr>
</tbody>
</table>
Example 21.2 Tubular Section - Round (ASD) .................... 256

C. BEAM-COLUMN MEMBERS

Example 22.1 C-Section (LRFD) .................................. 257
Example 22.2 C-Section (ASD) ................................. 283
Example 23.1 Tubular Section (LRFD) ........................... 290
Example 23.2 Tubular Section (ASD) ............................. 298

D. CONNECTIONS

Example 24.1 Flat Section w/Bolted Connection (LRFD) ......... 301
Example 24.2 Flat Section w/Bolted Connection (ASD) ............ 304
Example 25.1 Flat Section w/Lap Fillet Welded Connection (LRFD) 306
Example 25.2 Flat Section w/Lap Fillet Welded Connection (ASD) 308
Example 26.1 Flat Section w/Groove Welded Connection
    in Butt Joint (LRFD) ........................................ 309
Example 26.2 Flat Section w/Groove Welded Connection
    in Butt Joint (ASD) ......................................... 311
Example 27.1 Built-Up Section- Connecting Two Channels (LRFD) . 312
Example 27.2 Built-Up Section- Connecting Two Channels (ASD) .. 317
I. INTRODUCTION

This publication contains 54 examples for calculation of sectional properties, and the design of beams, compression members, beam-columns, and connections. They are prepared for the purpose of illustrating the application of various provisions of the new ASCE Standard 8-90, Specification for the Design of Cold-Formed Stainless Steel Structural Members.

II. COMPUTATION OF SECTIONAL PROPERTIES OF COLD-FORMED SECTION USING LINEAR METHOD

In the calculation of sectional properties of cold-formed stainless steel sections, the computation can be simplified by using a so-called linear method, in which the material of the section is considered to be concentrated along the centerline of the steel sheet and the area elements replaced by straight or curved "line elements." The thickness dimension, t, is introduced after the linear computations have been completed. This method has long been used for the design of cold-formed carbon steel sections. *

In the application of the linear method, the total area of the section is found from the following relation:

\[ \text{Area} = L \times t \]

where "L" is the total length of all line elements.

The moment of inertia of the section, I, is found from the following relation:

\[ I = I' \times t \]

* Cold-Formed Steel Design Manual (1986). American Iron and Steel Institute, Washington, D.C.
where "I'" is the moment of inertia of the centerline of the steel sheet.

The section modulus is computed as usual by dividing I or I' \(x\) t by the distance from the neutral axis to the extreme fiber, not to the centerline of the extreme element.

First power dimensions, such as \(x\), \(y\), and \(r\) (radius of gyration) are obtained directly by the linear method and do not involve the thickness dimension.

When the flat width of an element is reduced for design purpose, the effective design width, \(b\), is used directly to compute the total effective length, \(L_{\text{eff}}\), of the line elements, as shown in the examples.

The element into which most sections may be divided for application of the linear method consist of straight lines and circular arcs. For convenient reference, the moments of inertia and location of centroid of such elements are identified in the sketches and formulas in Fig. 1, Properties of Line Elements.

The formulas for line elements are exact, since the line as such has no thickness dimension; but in computing the properties of an actual element with a thickness dimension, the results will be approximate for the reasons given in the AISI Manual.

III. CORRELATION OF SPECIFICATION AND ILLUSTRATIVE EXAMPLES

The tables on pages 4 through 11 provide an easy cross reference between design provisions of the Specification and the illustrative examples. The first table is based on the type of design examples. The second table is based on various sections of the Specification.
\[ \theta (\text{expressed in radians}) = 0.01745 \theta (\text{expressed in degrees and decimals thereof}) \]

\[ l = (\theta_2 - \theta_1) R \]

\[ c_1 = \frac{\sin \theta_2 - \sin \theta_1}{\theta_2 - \theta_1} R, \quad c_2 = \frac{\cos \theta_1 - \cos \theta_2}{\theta_2 - \theta_1} R \]

\[ I_1 = \left[ \frac{\theta_2 - \theta_1 - \sin \theta_2 \cos \theta_2 - \sin \theta_1 \cos \theta_1}{2} \right] R^3, \quad I_4 = \left[ \frac{\sin^2 \theta_1}{2} + \frac{\sin \theta_2 \cos \theta_2 + \sin \theta_1 \cos \theta_1}{2} \right] R^3, \quad I_{14} = \left[ \frac{\sin^2 \theta_2 - \sin^2 \theta_1}{2} \right] R^3 \]

CASE I: \( \theta_1 = 0, \theta_2 = 90^\circ \)

CASE II: \( \theta_1 = 0, \theta_2 = \theta \)

\[ l = 1.57 \text{ R}, \quad c = 0.637 \text{ R} \]

\[ I_1 = I_4 = 0.149 \text{ R}^3 \]

\[ I_{12} = -0.137 \text{ R}^3 \]

\[ I_3 = I_4 = 0.783 \text{ R}^3 \]

\[ I_{14} = 0.5 \text{ R}^3 \]

Figure 1 Properties of Line Elements
CROSS REFERENCE BY EXAMPLE TO SPECIFICATION SECTION

<table>
<thead>
<tr>
<th>PROBLEM NO. *</th>
<th>TITLE OF EXAMPLE</th>
<th>USE OF SECTION NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. FLEXURAL MEMBERS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Channel w/Unstiffened Flanges</td>
<td>1.5.1,1.5.2,1.5.5,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.1.1,2.2.1,2.2.2,2.3.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.3.1.1, App. E.</td>
</tr>
<tr>
<td>2</td>
<td>Channel w/Stiffened Flanges</td>
<td>1.5.1,1.5.2,1.5.5,2.1.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.1.2,2.2.1,2.2.2,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.4,2.4.2,3.3.1.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>App. E.</td>
</tr>
<tr>
<td>3</td>
<td>C-Section w/Bracing</td>
<td>1.5.1,1.5.2,1.5.5,2.1.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.1.2,2.2.1,2.2.2,2.4,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.4.2,3.3.1.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>App. E.</td>
</tr>
<tr>
<td>4</td>
<td>Z-Section w/Stiffened Flanges</td>
<td>1.5.1,1.5.2,1.5.5,2.1.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.1.2,2.2.1,2.2.2,2.4,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.4.2,3.3.1.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>App. E.</td>
</tr>
</tbody>
</table>

* Two design examples are included for each problem. The first example uses the LRFD method and the second example uses the ASD method.
<p>| 5 | Deep Z-Section w/Stiffened Flanges | 1.5.1,1.5.2,1.5.5, 2.2.1,2.1.1,2.2.2, 2.4,2.4.2,3.3.1.1, App. E. |
| 6 | Hat Section | 1.5.1,1.5.2,1.5.5, 2.1.1,2.1.2,2.2.1, 2.2.2,3.3.1,3.3.1.1, 3.3.2,3.3.4, App. E. |
| 7 | Hat Section w/Intermediate Stiffener | 1.5.1,1.5.2,1.5.5, 2.1.1,2.1.2,2.2.1,2.2.2, 2.4,2.4.1,3.3.1,3.3.1.1, App. E. |
| 8 | I-Section w/Unstiffened Flanges | 1.5.5,2.1.1,2.2.1,2.2.2, 3.3.1.1,3.3.1.2, App. E. |
| 9 | Channel w/Lateral Buckling Consideration | 1.5.5,2.1.1,2.2.1,2.2.2, 2.3.1,3.3.1.1,3.3.1.2, 3.3.2,3.3.3,3.3.4, 3.3.5, App. E. |
| 10 | Hat Section Using Inelastic Reserve Capacity | 1.5.5,2.1.1,2.1.2,2.2.1, 2.2.2,3.3.1.1, App. E. |
| 11 | Deck Section | 1.5.5,2.1.1,2.1.2, 2.2.1,2.2.2,2.4, 2.4.2,3.3.1.1,3.3.2, 3.3.3,3.3.4, App. E. |</p>
<table>
<thead>
<tr>
<th></th>
<th>Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>Cylindrical Tubular Section</td>
<td>1.5.5, 3.6.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>, App. E</td>
</tr>
<tr>
<td>13</td>
<td>Flange Curling</td>
<td>1.5.5, 2.1.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>, 2.1.2, 2.2.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.2.1, 2.2.2,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.3.1, 3.3.1.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>App. E</td>
</tr>
<tr>
<td>14</td>
<td>Shear Lag</td>
<td>2.1.1, 2.1.2,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.2.1, 2.2.2,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.3.1.1, App. E</td>
</tr>
<tr>
<td>15</td>
<td>C-Section</td>
<td>1.5.5, 2.1.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.2.1, 2.4,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.4.2, 3.4.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.4.2, 3.4.3,</td>
</tr>
<tr>
<td>16</td>
<td>C-Section w/Wide Flanges</td>
<td>1.5.5, 2.1.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.2.1, 2.4.2,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.4.1, 3.4.2,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.4.3, App. E</td>
</tr>
<tr>
<td>17</td>
<td>I-Section</td>
<td>1.5.5, 2.2.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.2.2, 2.2.4,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.4.2, 3.4.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.4.2, 3.4.3,</td>
</tr>
<tr>
<td>18</td>
<td>I-Section w/Lips</td>
<td>1.5.5, 2.2.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.2.2, 2.2.4,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.4.2, 3.4.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.4.2, 3.4.3,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>App. E</td>
</tr>
<tr>
<td>19</td>
<td>T-Section</td>
<td>1.5.5, 2.2.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.2.2, 2.2.4,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.4.2, 3.4.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.4.2, 3.4.3,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>App. E</td>
</tr>
<tr>
<td>20</td>
<td>Tubular Section - Square</td>
<td>1.5.5, 2.1.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.2.1, 3.4.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.4.1, App. E</td>
</tr>
<tr>
<td>21</td>
<td>Tubular Section - Round</td>
<td>1.5.5, 3.4.1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.6.1, 3.6.2,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>App. E</td>
</tr>
<tr>
<td></td>
<td>C. BEAM-COLUMN MEMBERS</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>----------------------------------------------------------------------------------------</td>
<td>---</td>
</tr>
<tr>
<td>22</td>
<td>C-Section</td>
<td>1.5.5,2.1.2,2.2.1, 2.2.2,2.4,2.4.2,3.3.1.2, 3.4,3.4.1,3.4.2, 3.4.3,3.5,App. E</td>
</tr>
<tr>
<td>23</td>
<td>Tubular Section</td>
<td>1.5.5,2.1.1,2.1.2, 2.2.1,2.2.2,3.3.1.1, 3.4,3.4.1,3.5,App. E</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>D. CONNECTIONS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>Flat Section w/Bolted Connection</td>
<td>3.2,5.3,5.3.1,5.3.2, 5.3.3,5.3.4,App. E</td>
</tr>
<tr>
<td>25</td>
<td>Flat Section w/Lap Fillet Welded Connection</td>
<td>5.2,5.2.2,App. E</td>
</tr>
<tr>
<td>26</td>
<td>Flat Section w/Groove Welded Connection in Butt Joint</td>
<td>5.2,5.2.1,App. E</td>
</tr>
<tr>
<td>27</td>
<td>Built-Up Section - Connecting Two Channels</td>
<td>4.1,4.1.1,5.2.3,App. E</td>
</tr>
</tbody>
</table>
**CROSS REFERENCE BY SPECIFICATION SECTION TO DESIGN EXAMPLE**

<table>
<thead>
<tr>
<th>SECTION NO.</th>
<th>ILLUSTRATED IN PROBLEM NO. *</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>1.5.1</td>
<td>1,2,3,4,5,6,7</td>
</tr>
<tr>
<td>1.5.2</td>
<td>1,2,3,4,5,6,7</td>
</tr>
<tr>
<td>1.5.3</td>
<td></td>
</tr>
<tr>
<td>1.5.4</td>
<td></td>
</tr>
<tr>
<td>1.5.5</td>
<td>1,2,3,4,5,6,7,8,9,10,11,12,13,</td>
</tr>
<tr>
<td></td>
<td>15,16,17,18,19,20,21,22,23</td>
</tr>
<tr>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>2. Elements</td>
<td></td>
</tr>
<tr>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td>2.1.1</td>
<td>1,2,3,4,5,6,7,8,9,10,11,13,14,</td>
</tr>
<tr>
<td></td>
<td>15,16,17,20,21,22,23</td>
</tr>
<tr>
<td>2.1.2</td>
<td>2,3,4,5,6,7,10,11,14,19,20,21,</td>
</tr>
<tr>
<td></td>
<td>22,23</td>
</tr>
</tbody>
</table>

* Two design examples are included for each problem. The first example uses the LRFD method and the second example uses the ASD method.
2.2
2.2.1 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14,
       15, 16, 17, 18, 19, 20, 22, 23
2.2.2 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14,
       22, 23

2.3
2.3.1 1, 9, 13
2.3.2 1, 9, 13

2.4
2.4.1 2, 3, 4, 5, 7, 11, 15, 16, 17, 18, 19, 22
2.4.2 7
       2, 3, 4, 5, 11, 15, 16, 17, 18, 19, 22

2.5

2.6

3. Members

3.1

3.2 21

3.3
3.3.1 6, 7
3.3.1.1 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13,
       14, 23
3.3.1.2 8, 9, 22
3.3.2 6, 9, 11
3.3.3 9, 11
3.3.4 6, 9, 11
3.3.5 9

3.4 15, 16, 17, 19, 20, 22, 23
3.4.1 15,16,17,18,19,20,21,22,23
3.4.2 15,16,17,18,19,22
3.4.3 15,16,17,18,19,22
3.5 22,23
3.6 12,21
3.6.1 12,21
3.6.2 21
3.6.3
3.7

4. Structural Assemblies
4.1 27
   4.1.1 27
   4.1.2
4.2
4.3
   4.3.1
   4.3.2
   4.3.3

5. Connections and Joints
5.1
5.2 25,26
   5.2.1 26
   5.2.2 25
   5.2.3 27
5.3 24
   5.3.1 24
5.3.2  24
5.3.3  24
5.3.4  24

6. Tests

6.1

6.2

6.3

App. B Modified Ramberg-Osgood
Equation

App. E Allowable Stress Design (ASD)

All examples using ASD method
IV. ILLUSTRATIVE EXAMPLE
EXAMPLE 1.1 CHANNEL W/UNSTIFFENED FLANGES (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\phi_b M_n$, based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the service moment. Use the following two types of stainless steels: (A) Type 301, 1/4-Hard and (B) Type 409. Assume dead load to live load ratio $D/L = 1/5$ and $1.2D + 1.6L$ governs the design.

![Figure 1.1 Section for Example 1.1](image)

Given:
1. Section: 6" x 1.625" x 0.060" channel with unstiffened flanges.
2. Compression flange braced against lateral buckling.

Solution:
(A) Type 301 Stainless Steel, 1/4-Hard.
1. Calculation of the design flexural strength, $\phi_b M_n$: 

![Corner Line Element](image)
a. Properties of 90° corners:

\[ r = R + \frac{t}{2} = \frac{3}{32} + \frac{0.060}{2} = 0.124 \text{ in.} \]

Length of arc, \( u = 1.57r = 1.57 \times 0.124 = 0.195 \text{ in.} \)

Distance of c.g. from center of radius,
\[ c = 0.637r = 0.637 \times 0.124 = 0.079 \text{ in.} \]

b. Computation of \( I_x, S_e, \) and \( M_n : \)

For the first approximation, assume a compression stress of
\[ f = F_y = 50 \text{ ksi (yield strength in longitudinal compression, Table A1 of the Standard Specification) in the top fiber of the section and that the web is fully effective.} \]

Compression flange: \( k = 0.50 \) (for unstiffened compression element, see Section 2.3.1)

\[ \frac{w}{t} = \frac{1.471}{0.060} = 24.52 < 50 \text{ OK (Section 2.1.1-(1)-(iii))} \]

\[ \lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o} \quad \text{(Eq. 2.2.1-4)} \]

The initial modulus of elasticity, \( E_o \), for Type 301 stainless steel is obtained from Table A4 of the Standard, i.e., \( E_o = 27000 \text{ ksi.} \)

\[ \lambda = (1.052/\sqrt{0.50})(24.52)\sqrt{50/27000} = 1.570 > 0.673 \]

\[ \rho = \left[1-(0.22/\lambda)\right]/\lambda \quad \text{(Eq. 2.2.1-3)} \]
\[ = \left[1-(0.22/1.570)\right]/1.570 = 0.548 \]

\[ b = \rho w \quad \text{(Eq. 2.2.1-2)} \]
\[ = 0.548 \times 1.471 \]
\[ = 0.806 \text{ in.} \]
Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L Effective Length (in.)</th>
<th>y Distance from Top Fiber (in.)</th>
<th>Ly (in.²)</th>
<th>Ly² (in.³)</th>
<th>I' About Own Axis (in.³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web</td>
<td>5.692</td>
<td>3.000</td>
<td>17.076</td>
<td>51.228</td>
<td>15.368</td>
</tr>
<tr>
<td>Upper Corner</td>
<td>0.195</td>
<td>0.075</td>
<td>0.015</td>
<td>0.001</td>
<td>--</td>
</tr>
<tr>
<td>Lower Corner</td>
<td>0.195</td>
<td>5.925</td>
<td>1.155</td>
<td>6.846</td>
<td>--</td>
</tr>
<tr>
<td>Compression Flange</td>
<td>0.806</td>
<td>0.030</td>
<td>0.024</td>
<td>0.001</td>
<td>--</td>
</tr>
<tr>
<td>Tension Flange</td>
<td>1.471</td>
<td>5.970</td>
<td>8.782</td>
<td>52.428</td>
<td>--</td>
</tr>
<tr>
<td>Sum</td>
<td>8.359</td>
<td>27.052</td>
<td>110.504</td>
<td></td>
<td>15.368</td>
</tr>
</tbody>
</table>

Distance from top fiber to x-axis is

\[ y_{cg} = \frac{27.052}{8.359} = 3.236 \text{ in.} \]

Since the distance from top compression fiber to the neutral axis is greater than one half the beam depth, a compression stress of 50 ksi will govern as assumed (i.e., initial yield is in compression).

To check if web is fully effective (Section 2.2.2):

\[ f_1 = \frac{[(3.236-0.154)/3.236] \times 50}{3.236} = 47.62 \text{ ksi (compression)} \]

\[ f_2 = -\frac{[(2.764-0.154)/3.236] \times 50}{3.236} = -40.33 \text{ ksi (tension)} \]

\[ \psi = \frac{f_2}{f_1} = \frac{-40.33}{47.62} = -0.847 \]

\[ k = 4 + 2(1-\psi)^2 + 2(1-\psi) \] (Eq. 2.2.2-4)

\[ = 4 + 2[1-(-0.847)]^2 + 2[1-(-0.847)] \]

\[ = 20.296 \]

\[ h = w = 5.692 \text{ in.}, \frac{h}{t} = \frac{w}{t} = \frac{5.692}{0.060} = 94.87 \]
\( h/t = 94.87 < 200 \text{ OK (Section 2.1.2)-(1))} \)

\[ \lambda = \frac{1.052/\sqrt{20.296}}{\sqrt{47.62/27}} = 0.930 > 0.673 \]

\[ \rho = \frac{1-(0.22/0.930)}{0.930} = 0.821 \]

\( b_e = 0.821 \times 5.692 = 4.673 \text{ in.} \)

\( b_2 = \frac{b_e}{2} \)  
\( = \frac{4.673}{2} = 2.337 \text{ in.} \)  
(Eq. 2.2.2-2)

\( b_1 = \frac{b_e}{3-\Psi} \)  
\( = \frac{4.673}{3-(-0.847)} = 1.215 \text{ in.} \)  
(Eq. 2.2.2-1)

The effective widths, \( b_1 \) and \( b_2 \), of web are defined in Figure 2 of the Standard.

\( b_1 + b_2 = 1.215 + 2.337 = 3.552 \text{ in.} \)

Compression portion of the web calculated on the basis of the effective section = \( y_{cg} - 0.154 = 3.236 - 0.154 = 3.082 \text{ in.} \)

Since \( b_1 + b_2 = 3.552 \text{ in.} > 3.082 \text{ in.}, b_1 + b_2 \) shall be taken as 3.082 in.. This verifies the assumption that the web is fully effective.

\( I'_x = Ly^2 + I'_x - Ly^2_{cg} \)
\( = 110.504 + 15.368 - 8.359(3.236)^2 \)
\( = 38.339 \text{ in.}^3 \)

Actual \( I_x = I'_x t \)
\( = 38.339 \times 0.060 \)
\( = 2.300 \text{ in.}^4 \)

\( S_e = \frac{I_x}{y_{cg}} \)
\( = \frac{2.300}{3.236} \)
\( = 0.711 \text{ in.}^3 \)

\( M_n = S_e F_y \)  
(Eq. 3.3.1.1-1)
c. The design flexural strength, $\Phi_b M_n$, based on initiation of yielding is determined as follows: (Section 3.3.1.1(1))

$$\Phi_b = 0.85 \text{ (for section with unstiffened compression flanges)}$$

$$\Phi_b M_n = 0.85 \times 35.55 = 30.22 \text{ kips-in. (positive bending)}$$

2. Calculation of the effective moment of inertia for deflection determination at the service moment $M_s$:

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of $1.2D+1.6L$, the service moment can be determined as follows:

$$\Phi_b M_n = 1.2 M_{DL} + 1.6 M_{LL}$$

$$= \left[1.2 \left(\frac{M_{DL}}{M_{LL}}\right) + 1.6\right] M_{LL}$$

$$= \left[1.2 \left(\frac{1}{5}\right) + 1.6\right] M_{LL}$$

$$= 1.84 M_{LL}$$

$$M_{LL} = \frac{\Phi_b M_n}{1.84} = \frac{30.22}{1.84} = 16.42 \text{ kips-in.}$$

$$M_s = M_{DL} + M_{LL} = (1/5 + 1) M_{LL} = 1.2(16.42) = 19.70 \text{ kips-in.}$$

where

- $M_{DL} =$ Moment determined on the basis of nominal dead load
- $M_{LL} =$ Moment determined on the basis of nominal live load

The procedure is iterative: one assumes the actual compressive
stress $f$ under this service moment $M_s$. Knowing $f$, proceeds as usual to obtain $S_e$ and checks to see if $(f \times S_e)$ is equal to $M_s$ as it should. If not, reiterate until one obtains the desired level of accuracy.

a. For the first iteration, assume a stress of $f = \frac{F_y}{2} = 25$ ksi in the top and bottom fibers of the section and that the web is fully effective.

For deflection determination, the value of $E_r$, reduced modulus of elasticity determined by using Eq. (2.2.1-7), is substituted for $E_o$ in Eq. (2.2.1-4). For a compression and tension stresses of $f = 25$ ksi, the corresponding $E_{sc}$ and $E_{st}$ values for Type 301 stainless steel are obtained from Table A2 or Figure A1 of the Standard as follows:

$$E_{sc} = 25650 \text{ ksi, } \quad E_{st} = 27000 \text{ ksi}$$

$$E_r = \frac{(E_{sc} + E_{st})}{2} \quad \text{(Eq. 2.2.1-7)}$$

$$E_r = \frac{(25650 + 27000)}{2} = 26325 \text{ ksi}$$

Thus, for compression flange:

$$\lambda = \frac{(1.052/\sqrt{0.50}) (24.52) \sqrt{25/26325}}{1.124} = 1.124 > 0.673$$

$$\rho = \frac{[1-(0.22/1.124)]}{1.124} = 0.716 \quad \text{(Eq. 2.2.1-6)}$$

$$b_d = \rho w = 0.716 \times 1.471 = 1.053 \text{ in.}$$

Effective section properties about $x$-axis:

$L = 8.359 - 0.806 + 1.053 = 8.606 \text{ in.}$

$Ly = 27.052 - 0.024 + 1.053 \times 0.030 = 27.060 \text{ in}.$

$Ly^2 = 110.504 - 0.001 + 1.053 \times (0.030)^2 = 110.504 \text{ in}.$
\( I'_1 = 15.368 \text{ in.}^3 \)

\( y_{cg} = \frac{27.060}{8.606} = 3.144 \text{ in.} \) which is greater than one half beam depth. Thus, the top compression fiber controls the determination of \( S_e \).

To check if web is fully effective (Section 2.2.2-(2)):

\[ f_1 = \left(\frac{3.144-0.154}{3.144}\right) \times 25 = 23.78 \text{ ksi} \]

\[ f_2 = -\left(\frac{2.856-0.154}{3.144}\right) \times 25 = -21.49 \text{ ksi} \]

\[ \Psi = \frac{-21.49}{23.78} = -0.904 \]

\[ k = 4+2\left[1-(-0.904)\right]^2+2\left[1-(-0.904)\right] = 21.613 \]

For a compression stress of \( f = 23.78 \text{ ksi} \) and a tension stress of \( f = 21.49 \text{ ksi} \), the values of \( E_{sc} \) and \( E_{st} \) are found as follows: \( E_{sc} = 26244 \text{ ksi} \), \( E_{st} = 27000 \text{ ksi} \).

\[ E_r = \frac{E_{sc} + E_{st}}{2} = \frac{26244 + 27000}{2} = 26622 \text{ ksi} \]

\[ \Lambda = \frac{1.052/\sqrt{21.613}}{\sqrt{23.78/26622}} = 0.642 < 0.673 \]

\[ b_e = w \]

\[ = 5.692 \text{ in.} \]

\[ b_2 = \frac{5.692}{2} = 2.846 \text{ in.} \]

\[ b_1 = \frac{5.692}{3-(-0.904)} = 1.458 \text{ in.} \]

Compression portion of the web calculated on the basis of the effective section = 3.144 - 0.154 = 2.990 in..

Since \( b_1 + b_2 = 4.304 \text{ in.} > 2.990 \text{ in.} \), \( b_1 + b_2 \) shall be taken as 2.990 in.. This verifies the assumption that the web is fully effective.
\[ I'_x = 110.504 + 15.368 - 8.606(3.144)^2 \]
\[ = 40.804 \text{ in.}^3 \]

Actual \( I'_x = 40.804 \times 0.060 \)
\[ = 2.448 \text{ in.}^4 \]

\[ S_e = 2.448/3.144 = 0.779 \text{ in.}^3 \]

\[ M = f \times S_e = 25 \times 0.779 \]
\[ = 19.48 \text{ kips-in.} < M_s = 19.70 \text{ kips-in.} \]

Need to do another iteration by increasing \( f \).

b. For the second iteration, assume \( f = 25.50 \text{ ksi} \) in the top and bottom fibers of the section and that the web is fully effective.

Compression flange:

For a stress of \( f = 25.50 \text{ ksi} \), \( E_{sc} = 25375 \text{ ksi} \) and \( E_{st} = 27000 \text{ ksi} \), and \( E_r = (25375+27000)/2 = 26188 \text{ ksi} \). Thus,
\[ \lambda = \frac{(1.052/\sqrt{0.50})(24.52)\sqrt{25.50/26188}}{25.50} = 1.138 > 0.673 \]
\[ \rho = \left[1-(0.22/1.138)\right]/1.138 = 0.709 \]
\[ b_d = 0.709 \times 1.471 = 1.043 \text{ in.} \]

Effective section properties about \( x \)-axis:
\[ L = 8.359 - 0.806 + 1.043 = 8.596 \text{ in.} \]
\[ L_y = 27.052 - 0.024 + 1.043 \times 0.030 = 27.059 \text{ in.}^2 \]
\[ L_y^2 = 110.504 - 0.001 + 1.043(0.030)^2 = 110.504 \text{ in.}^3 \]
\[ I'_1 = 15.368 \text{ in.}^3 \]
\[ y_{cg} = \frac{27.059}{8.596} = 3.148 \text{ in.} \] which is greater than one half beam depth. Thus, the top compression fiber controls the determination of \( S_e \).

To check if web is fully effective:

\[
\begin{align*}
 f_1 &= \left(\frac{(3.148 - 0.154)}{3.148}\right) \times 25.50 = 24.25 \text{ ksi} \\
 f_2 &= -\left(\frac{(2.825 - 0.154)}{3.148}\right) \times 25.50 = -21.85 \text{ ksi} \\
 \Psi &= \frac{-21.85}{24.25} = -0.901 \\
 k &= 4 + 2\left[1 - (-0.901)\right]^2 + 2\left[1 - (-0.901)\right] = 21.542
\end{align*}
\]

For a compression stress of \( f = 24.25 \text{ ksi} \), \( E_{sc} = 26063 \text{ ksi} \), and for a tension stress of \( f = 21.85 \text{ ksi} \), \( E_{st} = 27000 \text{ ksi} \). Thus,

\[
E_r = \frac{(26063 + 27000)}{2} = 26532 \text{ ksi}.
\]

\[
\lambda = \frac{1.052}{\sqrt{21.542}} \times (94.87) \times \sqrt{24.25/26532} = 0.650 < 0.673
\]

\[
b_e = 5.692 \text{ in.}
\]

\[
b_2 = 5.692/2 = 2.846 \text{ in.}
\]

\[
b_1 = 5.692/\left[3 - (-0.901)\right] = 1.459 \text{ in.}
\]

Compression portion of the web calculated on the basis of the effective section = 3.148 - 0.154 = 2.994 in..

Since \( b_1 + b_2 = 4.305 \text{ in.} > 2.994 \text{ in.} \), \( b_1 + b_2 \) shall be taken as 2.994 in. This verifies the assumption that the web is fully effective.

\[
I'_{x} = 110.504 + 15.368 - 8.596(3.148)^2 = 40.686 \text{ in.}^3
\]

Actual \( I_x = 40.686 \times 0.060 \)

\[
= 2.441 \text{ in.}^4
\]

\[
S_e = 2.441/3.148 = 0.775 \text{ in.}^3
\]
\[ M = f \times S_e = 25.50 \times 0.775 \]
\[ = 19.76 \text{ kips-in.} = M_s \text{ OK} \]

Thus, use \( I_x = 2.441 \text{ in.}^4 \) for deflection determination.

(B) Type 409 Stainless Steel.

1. Calculation of the design flexural strength, \( \Phi_b M_n \):

a. Properties of 90° corners:

From Case (A) above,
\[ r = 0.124 \text{ in.}, \quad u = 0.195 \text{ in.}, \quad c = 0.079 \text{ in.} \]

b. Computation of \( I_x, S_e, \) and \( M_n \):

For the first approximation, assume a compression stress of \( f = F_y = 30 \text{ ksi} \) (yield strength in longitudinal compression, Table A1 of the Standard Specification) in the top fiber of the section and that the web is fully effective.

Compression flange: \( k = 0.50 \) (for unstiffened compression element, see Section 2.3.1)
\[ \frac{w}{t} = \frac{1.471}{0.060} = 24.52 < 50 \text{ OK (Section 2.1.1-(1)-(iii))} \]
\[ \Lambda = \frac{(1.052/\sqrt{k})(w/t)\sqrt{f/E_o}}{E_o} \quad (Eq. 2.2.1-4) \]

The initial modulus of elasticity, \( E_o \), for Type 409 stainless steel is obtained from Table A5 of the Standard, i.e., \( E_o = 27000 \text{ ksi} \).
\[ \Lambda = \frac{(1.052/\sqrt{0.50})(24.52)\sqrt{30/27000}}{27000} = 1.216 > 0.673 \]
\[ \rho = \frac{1-0.22/\Lambda}{\Lambda} \quad (Eq. 2.2.1-3) \]
\[
\frac{b}{pw} = \frac{1-(0.22/1.216)}{1.216} = 0.674 \quad \text{(Eq. 2.2.1-2)}
\]

\[
b = 0.674 \times 1.471 = 0.991 \text{ in.}
\]

Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L Effective Length (in.)</th>
<th>y Distance from Top Fiber (in.)</th>
<th>Ly (in.²)</th>
<th>Ly² (in.⁴)</th>
<th>I' About Own Axis (in.²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web</td>
<td>5.692</td>
<td>3.000</td>
<td>17.076</td>
<td>51.228</td>
<td>15.368</td>
</tr>
<tr>
<td>Upper Corner</td>
<td>0.195</td>
<td>0.075</td>
<td>0.015</td>
<td>0.001</td>
<td>--</td>
</tr>
<tr>
<td>Lower Corner</td>
<td>0.195</td>
<td>5.925</td>
<td>1.155</td>
<td>6.846</td>
<td>--</td>
</tr>
<tr>
<td>Compression Flange</td>
<td>0.991</td>
<td>0.030</td>
<td>0.030</td>
<td>0.001</td>
<td>--</td>
</tr>
<tr>
<td>Tension Flange</td>
<td>1.471</td>
<td>5.970</td>
<td>8.782</td>
<td>52.428</td>
<td>--</td>
</tr>
<tr>
<td>Sum</td>
<td>8.544</td>
<td>27.058</td>
<td>110.504</td>
<td>15.368</td>
<td></td>
</tr>
</tbody>
</table>

Distance from top fiber to x-axis is

\[
y_{cg} = \frac{27.058}{8.544} = 3.167 \text{ in.}
\]

Since the distance from top compression fiber to the neutral axis is greater than one half the beam depth, a compression stress of 30 ksi will govern as assumed (i.e., initial yield is in compression).

To check if web is fully effective (Section 2.2.2):

\[
f_1 = \left[\frac{(3.167-0.154)}{3.167}\right] \times 30 = 28.54 \text{ ksi(compression)}
\]

\[
f_2 = -\left[\frac{(2.833-0.154)}{3.167}\right] \times 30 = -25.38 \text{ ksi(tension)}
\]

\[
\Psi = \frac{f_2}{f_1} = \frac{-25.38}{28.54} = -0.889
\]
\[ k = 4 + 2(1 - \Psi)^2 + 2(1 - \Psi) \]  
\[ = 4 + 2[(1 - (-0.889))^2 + 2(1 - (-0.889))] \]  
\[ = 21.259 \]

\[ h = w = 5.692 \text{ in., } h/t = w/t = 5.692/0.060 = 94.87 \]

\[ h/t = 94.87 < 200 \text{ OK (Section 2.1.2-(1))} \]

\[ \lambda = \frac{(1.052/\sqrt{21.259})(94.87)/28.54/27000}{} = 0.704 > 0.673 \]

\[ \rho = \frac{[1 - (0.22/0.704)]/0.704}{} = 0.977 \]

\[ b_e = 0.977 \times 5.692 = 5.561 \text{ in.} \]

\[ b_2 = b_e/2 \quad \text{(Eq. 2.2.2-2)} \]

\[ = 5.561/2 = 2.781 \text{ in.} \]

\[ b_1 = b_e/(3 - \Psi) \quad \text{(Eq. 2.2.2-1)} \]

\[ = 5.561/[3 - (-0.889)] = 1.430 \text{ in.} \]

The effective widths, \( b_1 \) and \( b_2 \), are defined in Figure 2 of the Standard.

\[ b_1 + b_2 = 1.430 + 2.781 = 4.211 \text{ in.} \]

Compression portion of the web calculated on the basis of the effective section = \( y_{cg} - 0.154 = 3.167 - 0.154 = 3.013 \text{ in.} \)

Since \( b_1 + b_2 = 4.211 \text{ in.} > 3.013 \text{ in.} \), \( b_1 + b_2 \) shall be taken as 3.013 in.. This verifies the assumption that the web is fully effective.

\[ I'_x = Ly'^2 + I'_{1, cg} - Ly'^2 \]

\[ = 110.504 + 15.368 - 8.544(3.167)^2 \]

\[ = 40.177 \text{ in.}^3 \]

Actual \( I_x = I'_x t \)

\[ = 40.177 \times 0.060 \]

\[ = 2.411 \text{ in.}^4 \]
\[ S_e = \frac{I_x}{y_{cg}} \]
\[ = \frac{2.411}{3.167} \]
\[ = 0.761 \text{ in.}^3 \]

\[ M_n = S'_F \frac{e}{y} \]
\[ = 0.761 \times 30 \]
\[ = 22.83 \text{ kips-in.} \]  
(Eq. 3.3.1.1-1)

\( c. \) The design flexural strength, \( \Phi_b M_n \), based on initiation of
yielding is determined as follows: (Section 3.3.1.1(1))

\[ \Phi_b = 0.85 \] (for section with unstiffened compression flanges)

\[ \Phi_b M_n = 0.85 \times 22.83 = 19.41 \text{ kips-in.} \] (positive bending)

2. Calculation of the effective moment of inertia for deflection
determination at the service moment \( M_s \):

The unfactored loads are used to determine the section properties
for deflection determination. For a load combination of
1.2D+1.6L, the service moment can be determined as follows:

\[ \Phi_b M_n = 1.2M_{DL} + 1.6M_{LL} \]
\[ = [1.2(M_{DL}/M_{LL})+1.6]M_{LL} \]
\[ = [1.2(1/5)+1.6]M_{LL} \]
\[ = 1.84M_{LL} \]

\[ M_{LL} = \Phi_b M_n / 1.84 = 19.41 / 1.84 = 10.55 \text{ kips-in.} \]

\[ M_s = M_{DL} + M_{LL} \]
\[ = (1/5+1)M_{LL} \]
\[ = 1.2(10.55) = 12.66 \text{ kips-in.} \]

where
\[ M_{DL} = \text{Moment determined on the basis of nominal dead load} \]

\[ M_{LL} = \text{Moment determined on the basis of nominal live load} \]

The procedure is iterative: one assumes the actual compressive stress \( f \) under this service moment \( M_s \). Knowing \( f \), proceeds as usual to obtain \( S_e \) and checks to see if \( (f \times S_e) \) is equal to \( M_s \) as it should. If not, reiterate until one obtains the desired level of accuracy.

a. For the first iteration, assume a stress of \( f = F_y / 2 = 15 \text{ ksi} \) in the top and bottom fibers of the section and that the web is fully effective.

For deflection determination, the value of \( E_r \), reduced modulus of elasticity determined by Eq. (2.2.1-7), is substituted for \( E_o \) in Eq. (2.2.1-4). For a compression and tension stress of \( f = 15 \text{ ksi} \), the corresponding \( E_{sc} \) and \( E_{st} \) values for Type 409 stainless steel are obtained from Table A3 or Figure A2 of the Standard as follows:

\[ E_{sc} = 26850 \text{ ksi}, \quad E_{st} = 26930 \text{ ksi} \]
\[ E_r = (E_{sc} + E_{st})/2 \quad \text{(Eq. 2.2.1-7)} \]
\[ = (26850 + 26930)/2 = 26890 \text{ ksi} \]

Thus, for compression flange:

\[ \lambda = (1.052/\sqrt{0.50})(24.52)/\sqrt{15/26890} = 0.862 > 0.673 \]
\[ \rho = [1-(0.22/0.862)]/0.862 = 0.864 \quad \text{(Eq. 2.2.1-6)} \]
\[ b_d = \rho w \]
\[ = 0.864 \times 1.471 = 1.271 \text{ in.} \]
Effective section properties about x-axis:

\[ L = 8.544 - 0.991 + 1.271 = 8.824 \text{ in.} \]
\[ Ly = 27.058 - 0.030 + 1.271 \times 0.030 = 27.066 \text{ in.}^2 \]
\[ Ly^2 = 110.504 - 0.001 + 1.271(0.030)^2 = 110.504 \text{ in.}^3 \]
\[ I'_1 = 15.368 \text{ in.}^3 \]

\[ y_{cg} = \frac{27.066}{8.824} = 3.067 \text{ in.} \text{ which is greater than one half beam depth. Thus, the top compression fiber controls the determination of } S_e. \]

To check if web is fully effective (Section 2.2.2-(2)):

\[ f_1 = \frac{(3.067-0.154)/3.067}{15} = 14.25 \text{ ksi} \]
\[ f_2 = -\frac{(2.933-0.154)/3.067}{15} = -13.59 \text{ ksi} \]
\[ \Psi = -13.59/14.25 = -0.954 \]
\[ k = 4+2[1-(-0.954)]^3+2[1-(-0.954)] = 22.829 \]

For a compression stress of \( f = 14.25 \text{ ksi} \) and a tension stress of \( f = 13.59 \text{ ksi} \), the values of \( E_{sc} \) and \( E_{st} \) are found as follows, respectively:

\[ E_{sc} = 26890 \text{ ksi}, \ E_{st} = 26940 \text{ ksi}. \]

\[ \sigma_r = \frac{E_{sc} + E_{st}}{2} \]
\[ = \frac{26890+26940}{2} = 26920 \text{ ksi} \]

\[ \lambda = \frac{(1.052/\sqrt{22.829})(94.87)/\sqrt{14.25/26920}}{0.481 < 0.673} \]

\[ b_e = w \]
\[ = 5.692 \text{ in.} \]
\[ b_2 = 5.692/2 = 2.846 \text{ in.} \]
\[ b_1 = 5.692/[3-(-0.954)] = 1.440 \text{ in.} \]

Compression portion of the web calculated on the basis of the effective section = 3.067 - 0.154 = 2.913 \text{ in.}
Since \( b_1 + b_2 = 4.286 \text{ in.} > 2.913 \text{ in.} \), \( b_1 + b_2 \) shall be taken as 2.913 in. This verifies the assumption that the web is fully effective.

\[
I'_x = 110.504 + 15.368 - 8.824(3.067)^2
\]
\[
= 42.869 \text{ in.}^3
\]

Actual \( I_x = 42.869 \times 0.060 \\
= 2.572 \text{ in.}^4 \\
S_e = 2.572/3.067 = 0.839 \text{ in.}^3 \\
M = f \times S_e = 15 \times 0.839 \\
= 12.59 \text{ kips-in.} \approx M_s = 12.66 \text{ kips-in.} \text{ (close enough)}

Therefore, need no further iteration. Use \( I_x = 2.572 \text{ in.}^4 \) for deflection determination.
EXAMPLE 1.2 CHANNEL W/UNSTIFFENED FLANGES (ASD)

Use the data given in Example 1.1 (Figure 1.1) to determine the allowable moment, \( M_a \), by using the Allowable Stress Design (ASD) method on the basis of initiation of yielding. Also determine the effective moment of inertia for deflection determination at the allowable moment. Use Type 301 stainless steel, 1/4-Hard: \( F_y = 50 \) ksi.

Solution:

1. Calculation of the allowable moment, \( M_a \):

   The effective section properties calculated by the ASD method are the same as those determined in Example 1.1 by the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

   \[ \Omega = 1.85 \] (Safety Factor stipulated in Table E of the Standard)

   \[ M_n = 35.55 \text{ kips-in. (obtained from Example 1.1)} \]

   \[ M_a = \frac{M_n}{\Omega} \] (Eq. E-1)

   \[ = \frac{35.55}{1.85} \]

   \[ = 19.22 \text{ kips-in.} \]

2. Calculation of the effective moment of inertia for deflection determination at the allowable moment \( M_a \):

   For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 1.1 for the LRFD method, except that the computed moment \( M (= fxs_e) \) should be equal to \( M_a \).
From the results of Example 1.1, it can be seen that by assuming a compression stress of $f = F_{y}/2 = 25$ ksi, the computed $f \times S_e = 25 \times 0.779 = 19.48$ kips-in., which is close to the allowable moment, $M_a = 19.22$ kips-in. Therefore, the computed $I_x = 2.448$ in$^4$ can be used for deflection determination.
EXAMPLE 2.1 CHANNEL W/STIFFENED FLANGES (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\Phi_b M_n$, based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the service moment. Use Type 301 stainless steel, 1/4-Hard. Assume dead load to live load ratio $D/L = 1/5$ and $1.2D+1.6L$ governs the design.

Given:
1. Section: 6" x 1.625" x 0.060" channel with stiffened flanges.
2. Compression flange braced against lateral buckling.

Solution:
1. Calculation of the design flexural strength, $\Phi_b M_n$:
   a. Properties of 90° corners:
      $$r = R + t/2 = 3/32 + 0.060/2 = 0.124 \text{ in.}$$
      Length of arc, $u = 1.57r = 1.57 \times 0.124 = 0.195 \text{ in.}$
Distance of c.g. from center of radius,
\[ c = 0.637r = 0.637 \times 0.124 = 0.079 \text{ in.} \]

b. Computation of \( I_x, S_e, \) and \( M_n \):

For the first approximation, assume a compression stress of 
\[ f = F_y = 50 \text{ ksi} \] (yield strength in longitudinal compression, 
Table A1 of the Standard) in the top fibers of the section 
and that the web is fully effective.

Compression flange: (Section 2.4.2)
\[ w = 1.317 \text{ in.} \]
\[ w/t = 1.317/0.060 = 21.95 \]
\[ S = 1.28\sqrt{E_o/f} \quad \text{(Eq. 2.4-1)} \]

\( E_o \) value for Type 301 stainless steel is obtained from Table A4 
of the Standard Specification, i.e., \( E_o = 27000 \text{ ksi} \).
\[ S = 1.28\sqrt{27000/50} = 29.74 \]
\[ S/3 = 9.91 < (w/t) = 21.95 < S = 29.74 \]
\[ I_s = 399t^4\left\{\left[(w/t)/S\right]-0.33\right\}^3 \quad \text{(Eq. 2.4.2-6)} \]
\[ = 399(0.060)^4\left[(21.95/29.74)-0.33\right] \]
\[ = 0.000351 \text{ in.}^4 \]
\[ D = 0.450 \text{ in.} \]
\[ d = 0.296 \text{ in.}, \ d/t = 0.296/0.060 = 4.93 \]
\[ I_s = d^3t/12 \quad \text{(Eq. 2.4-2)} \]
\[ = (0.296)^3(0.060)/12 = 0.000130 \text{ in.}^4 \]
\[ D/w = 0.450/1.317 = 0.342, \ 0.25 < D/w = 0.342 < 0.80 \]
\[ k = [4.82-5(D/w)](I_s/I_a)^{0.43}+0.43 \leq 5.25-5(D/w) \quad \text{(Eq. 2.4.2-9)} \]
\[ n = 1/2 \]
\[ (4.82 - 5(0.342))(0.000130/0.000351)^{1/2} + 0.43 = 2.323 \]

5.25 - 5(0.342) = 3.540 > 2.323

Use \( k = 2.323 \)

Since \( I_s < I_a \), the stiffener is considered to be a simple lip.

\[ \frac{w}{t} = 21.95 < 50 \text{ OK (Section 2.1.1-(1)-(i))} \]

\[ \lambda = \left( \frac{1.052}{\sqrt{k}} \right) \left( \frac{w}{t} \right) \sqrt{E/E_0} \quad \text{(Eq. 2.2.1-4)} \]
\[ = \left( \frac{1.052}{\sqrt{2.323}} \right) (21.95) \sqrt{50/27000} = 0.652 < 0.673 \]

\[ b = w \quad \text{(Eq. 2.2.1-1)} \]
\[ = 1.317 \text{ in. (i.e. compression flange fully effective)} \]

Compress (upper) stiffener:

\[ k = 0.50 \text{ (unstiffened compression element)} \]

\[ d/t = 4.93 \]

Also conservatively assume \( f=50 \text{ ksi} \) as used in top compression fiber.

\[ \lambda = \left( \frac{1.052}{\sqrt{0.50}} \right)(4.93) \sqrt{50/27000} = 0.316 < 0.673 \]

therefore,

\[ d' = d = 0.296 \text{ in.} \]

\[ d_S = d' (I_s/I_a) \leq d' \quad \text{(Eq. 2.4.2-11)} \]
\[ = 0.296(0.000130/0.000351) \]
\[ = 0.110 \text{ in.} < 0.296 \text{ in.} \]

\[ d_S = 0.110 \text{ in. (i.e. compression stiffener is not fully effective)} \]
Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L Effective Length (in.)</th>
<th>y Distance from Top Fiber (in.)</th>
<th>Ly (in.^2)</th>
<th>Ly^2 (in.^3)</th>
<th>I' About Own Axis (in.^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web</td>
<td>5.692</td>
<td>3.000</td>
<td>17.076</td>
<td>51.228</td>
<td>15.368</td>
</tr>
<tr>
<td>Upper Corners</td>
<td>2x0.195 = 0.390</td>
<td>0.075</td>
<td>0.029</td>
<td>0.002</td>
<td>--</td>
</tr>
<tr>
<td>Lower Corners</td>
<td>2x0.195 = 0.390</td>
<td>5.925</td>
<td>2.311</td>
<td>13.691</td>
<td>--</td>
</tr>
<tr>
<td>Compression Flange</td>
<td>1.317</td>
<td>0.030</td>
<td>0.040</td>
<td>0.001</td>
<td>--</td>
</tr>
<tr>
<td>Upper Stiffener</td>
<td>0.110</td>
<td>0.209</td>
<td>0.023</td>
<td>0.005</td>
<td>--</td>
</tr>
<tr>
<td>Tension Flange</td>
<td>1.317</td>
<td>5.970</td>
<td>7.862</td>
<td>46.939</td>
<td>--</td>
</tr>
<tr>
<td>Lower Stiffener</td>
<td>0.296</td>
<td>5.698</td>
<td>1.687</td>
<td>9.610</td>
<td>0.002</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td><strong>9.512</strong></td>
<td><strong>29.028</strong></td>
<td><strong>121.476</strong></td>
<td><strong>15.370</strong></td>
<td></td>
</tr>
</tbody>
</table>

Distance from top fiber to x-axis is

\[ y_{cg} = \frac{29.028}{9.512} = 3.052 \text{ in.} \]

Since the distance from top compression fiber to the neutral axis is greater than one half the beam depth, a compression stress of 50 ksi will govern as assumed (i.e., initial yield is in compression).

To check if web is fully effective (Section 2.2.2):

\[ f_1 = \left[ \frac{(3.052-0.154)}{3.052} \times 50 \right] = 47.48 \text{ ksi(compression)} \]

\[ f_2 = -\left[ \frac{(2.948-0.154)}{3.052} \times 50 \right] = -45.77 \text{ ksi(tension)} \]

\[ \psi = \frac{f_2}{f_1} = \frac{-45.77}{47.48} = -0.964 \]

\[ k = 4 + 2(1-\psi)^2 + 2(1-\psi) \quad \text{(Eq. 2.2.2-4)} \]

\[ = 4 + 2[1-(-0.964)]^2 + 2[1-(-0.964)] \]

\[ = 23.079 \]
\[ h = w = 5.692 \text{ in.}, \quad h/t = w/t = 5.692/0.060 = 94.87 \]

\[ h/t = 94.87 < 200 \text{ OK (Section 2.1.2-(1))} \]

\[ \lambda = (1.052/\sqrt[4]{23.079})(94.87)/\sqrt[4]{47.48}/27000 = 0.871 > 0.673 \]

\[ \rho = \left[ 1-(0.22/\lambda) \right]/\lambda = \left[ 1-(0.22/0.871) \right]/0.871 = 0.858 \quad \text{(Eq. 2.2.1-3)} \]

\[ b_e = \rho w = \text{(Eq. 2.2.1-2)} \]

\[ = 0.858 \times 5.692 = 4.884 \text{ in.} \]

\[ b_2 = b_e/2 = \text{(Eq. 2.2.2-2)} \]

\[ = 4.884/2 = 2.442 \text{ in.} \]

\[ b_1 = b_e/(3-\Psi) = \text{(Eq. 2.2.2-1)} \]

\[ = 5.037/[3-(-0.964)] = 1.232 \text{ in.} \]

The effective widths, \( b_1 \) and \( b_2 \), of web are defined in Figure 2 of the Standard Specification.

\[ b_1 + b_2 = 1.232 + 2.442 = 3.674 \text{ in.} \]

Compression portion of the web calculated on the basis of the effective section = \( y_{cg} - 0.154 = 3.052 - 0.154 = 2.898 \text{ in.} \)

Since \( b_1 + b_2 = 3.674 \text{ in.} > 2.898 \text{ in.} \), \( b_1 + b_2 \) shall be taken as 2.898 in.. This verifies the assumption that the web is fully effective.

\[ I_x' = L_y^2 + I_1' - L_{y_{cg}}^2 \]

\[ = 121.476 + 15.370 - 9.512(3.052)^2 \]

\[ = 48.245 \text{ in.}^3 \]

Actual \( I_x = I_x' \times t = 48.245 \times 0.060 = 2.895 \text{ in.}^4 \)

\[ S_e = I_x/y_{cg} \]
\[
\frac{M_n}{M_{n'}} = \frac{2.895}{3.052} = 0.949 \text{ in.}^3
\]

\[
M_n = S_F e_y = 0.949 \times 50 = 47.45 \text{ kips-in.}
\]

\[\text{c. The design flexural strength, } \phi_b M_n, \text{ based on initiation of yielding is determined as follows: (Section 3.3.1.1(1))}\]

\[
\phi_b = 0.90 \text{ (for section with stiffened compression flanges)}
\]

\[
\phi_b M_n = 0.90 \times 47.45 = 42.71 \text{ kips-in.}
\]

2. Calculation of the effective moment of inertia for deflection determination at the service moment \( M_s \):

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of 1.2D+1.6L, the service moment can be determined as follows:

\[
\phi_b M_n = 1.2M_{DL} + 1.6M_{LL}
\]

\[
= [1.2(M_{DL}/M_{LL})+1.6]M_{LL}
\]

\[
= [1.2(1/5)+1.6]M_{LL}
\]

\[
= 1.84M_{LL}
\]

\[
M_{LL} = \phi_b M_n / 1.84 = 42.71 / 1.84 = 23.21 \text{ kips-in.}
\]

\[
M_s = M_{DL} + M_{LL}
\]

\[
= (1/5+1)M_{LL}
\]

\[
= 1.2(23.21) = 27.85 \text{ kips-in.}
\]

where

\[
M_{DL} = \text{Moment determined on the basis of nominal dead load}
\]

\[
M_{LL} = \text{Moment determined on the basis of nominal live load}
\]
The procedure is iterative: one assumes the actual compressive stress $f$ under this service moment $M_s$. Knowing $f$, one proceeds as usual to obtain $S_e$ and checks to see if $(f \times S_e)$ is equal to $M_s$ as it should. If not, reiterate until one obtains the desired level of accuracy. (Section 2.2.1-(2))

a. For the first iteration, assume a stress of $f = F_y/2 = 25$ ksi in the top and bottom fibers of the section and that the web is fully effective.

Compression flange:

$$S = 1.28\sqrt{27000/25} = 42.07$$

$$S/3 = 14.02 < (w/t) = 21.95 < S = 42.07$$

$$I_a = 399(0.060)^4[(21.95/42.07)-0.33]^3$$

$$= 0.000036 \text{ in.}^4$$

$$I_s/I_a = 0.000130/0.000036 = 3.61; \quad 5.25-5(D/w) = 3.540$$

$$k = (4.82-5(0.342)](3.61)^{1/2}+0.43 = 6.339 > 3.540$$

Use $k = 3.540$

For deflection determination, the reduced modulus of elasticity, $E_r$, shall be substituted for $E_o$ in Eq. (2.2.1-4). For a compression and tension stresses of $f = 25$ ksi,

$$E_r = 26325 \text{ ksi as obtained from Example 1.1.}$$

$$\lambda = (1.052/\sqrt{3.540})(21.95)\sqrt{25/26325} = 0.378 < 0.673$$

$$b_d = 1.317 \text{ in. (i.e. compression flange fully effective)}$$

Compression (upper) stiffener:
Again assume conservatively \( f = 25 \text{ ksi} \) as used in top compression fiber and the corresponding \( E_r = 26325 \text{ ksi} \).

\[
\lambda = (1.052/\sqrt{0.50})(4.93)/25/26325 = 0.226 < 0.673
\]

Therefore, \( d' = 0.296 \text{ in.} \).

Since \( I_s/I_a = 3.25 > 1.0 \), it follows that \( d_s = d' = 0.296 \text{ in.} \) (i.e. compression stiffener fully effective).

Thus, one concludes that the section is fully effective.

\[ y_{cg} = 6/2 = 3.000 \text{ in.} \] (from symmetry)

**Full section properties about x-axis:**

<table>
<thead>
<tr>
<th>Element</th>
<th>( L ) (in.)</th>
<th>( y ) Distance from Centerline of Section (in.)</th>
<th>( I'_1 ) About Own Axis (in.(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web</td>
<td>5.692</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Stiffeners</td>
<td>2 x 0.296 = 0.592</td>
<td>2.698</td>
<td>4.309</td>
</tr>
<tr>
<td>Corners</td>
<td>4 x 0.195 = 0.780</td>
<td>2.925</td>
<td>6.673</td>
</tr>
<tr>
<td>Flanges</td>
<td>2 x 1.317 = 2.634</td>
<td>2.970</td>
<td>23.234</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td><strong>9.698</strong></td>
<td><strong>34.216</strong></td>
<td><strong>15.372</strong></td>
</tr>
</tbody>
</table>

Since section is singly symmetric about x-axis and fully effective, top compression fiber may be used in computing \( S_e \).

To check if web is fully effective: (Section 2.2.2-(2))

\[
f_1 = ((3.000-0.154)/3.000) \times 25 = 23.72 \text{ ksi(compression)}
\]

\[
f_2 = -23.72 \text{ ksi(tension)}
\]
For a compression and tension stresses of \( f = 23.72 \) ksi, the corresponding \( E_{sc} \) and \( E_{st} \) values are as follows:

\[
E_{sc} = 26256 \text{ ksi}, \quad \text{and} \quad E_{st} = 27000 \text{ ksi}
\]

\[
E_r = \frac{E_{sc} + E_{st}}{2} \quad \text{(Eq. 2.2.1-7)}
\]

\[
E_r = 26628 \text{ ksi}
\]

\[
\lambda = \frac{(1.052/\sqrt{24})(94.87)\sqrt{23.72/26628}}{23.72/26628} = 0.608 < 0.673
\]

\[
b_e = w \quad \text{(Eq. 2.2.1-1)}
\]

\[
b_e = 5.692 \text{ in.}
\]

\[
b_2 = \frac{5.692}{2} = 2.846 \text{ in.}
\]

\[
b_1 = \frac{5.692}{3(-1)} = 1.423 \text{ in.}
\]

\[
b_1 + b_2 = 4.269 \text{ in.}
\]

Compression portion of the web = 3.000 - 0.154 = 2.846 in.

Since \( b_1 + b_2 = 4.269 \) in. > 2.846 in., \( b_1 + b_2 \) shall be taken as 2.846 in.. This verifies the assumption that the web is fully effective.

\[
I'_x = 34.216 + 15.372 = 49.588 \text{ in}^3
\]

Actual \( I_x = 49.588 \times 0.060 = 2.975 \text{ in}^4
\]

\[
S_e = \frac{2.975}{3.000} = 0.992 \text{ in}^3
\]

\[
M = f \times S_e = 25 \times 0.992
\]

\[
= 24.80 \text{ kips-in.} < M_s = 27.85 \text{ kips-in.}
\]

Need to do another iteration by increasing \( f \).

**b. After several trials, assume that a stress of \( f = 28.07 \) ksi in the top and bottom fibers of the section and that**
the web is fully effective.

Compression flange:

\[ S = 1.28\sqrt{27000/28.07} = 39.70 \]

\[ S/3 = 13.23 < (w/t) = 21.95 < S = 39.70 \]

\[ I_a = 399(0.060)^4[(21.95/39.70)-0.33]^3 \]

\[ = 0.000057 \text{ in.}^4 \]

\[ I_s/I_a = 0.000130/0.000057 = 2.28 \]

\[ k = [4.82-5(0.342)](2.28)^{1/2}+0.43 = 5.126 > 3.540 \]

Use \( k = 3.540 \)

For a compression and tension stresses of \( f = 28.07 \text{ ksi} \), it is found that \( E_{sc} \) and \( E_{st} \) are equal to 23950 ksi and 27000 ksi, respectively.

\[ E_r = (23950+27000)/2 = 25475 \text{ ksi} \]

\[ \lambda = (1.052/\sqrt{3.54})(21.95)\sqrt{28.07/25475} = 0.407 < 0.673 \]

\( b_d = 1.317 \text{ in.} \) (i.e. compression flange fully effective)

Compression (upper) stiffener:

\( f \) conservatively taken as for top compression fiber.

\[ \lambda = (1.052/\sqrt{0.50})(4.93)\sqrt{28.07/25475} = 0.243 < 0.673 \]

\( d'_s = 0.296 \text{ in.} \)

Since \( I_s/I_a = 2.28 > 1.0 \), it follows that \( d_s = d'_s = 0.296 \text{ in.} \) (i.e. compression stiffener fully effective).

Thus, the section is fully effective.

\( y_{cg} = 6/2 = 3.000 \text{ in.} \) (from symmetry)

40
Full section properties are the same as were found in the first iteration. Thus, as before, top compression fiber may be used in computing $S_e$.

To check if web is fully effective:

$$f_1 = \frac{[(3.000-0.154)/3.000] \times 28.07}{26.63 \text{ ksi (compression)}} = 26.63 \text{ ksi (compression)}$$

$$f_2 = -26.63 \text{ ksi (tension)}$$

$$\psi = -26.63/26.63 = -1.000$$

$$k = 24.000$$

For a compression and tension stresses of $f = 26.63$ ksi, it is found that $E_{sc}$ and $E_{st}$ are equal to 24754 ksi and 27000 ksi, respectively.

$$E_r = \frac{(24754+27000)}{2} = 25877 \text{ ksi}$$

$$\lambda = (1.052/\sqrt{24})(94.87)\sqrt{26.63/25877} = 0.654 < 0.673$$

$$b_e = w = 5.692 \text{ in.}$$

Hence, as in first iteration, $b_1 + b_2 = 2.846 \text{ in.}$ and thus the web is fully effective as assumed.

$$I_x = 2.975 \text{ in.}^4$$

$$S_e = 0.992 \text{ in.}^3$$

$$M = f \times S_e = 28.07 \times 0.992 = 27.85 \text{ kips-in.} = M_s \text{ OK}$$

Thus, use $I_x = 2.975 \text{ in.}^4$ for deflection determination.
EXAMPLE 2.2 CHANNEL W/STIFFENED FLANGES (ASD)

Use the data given in Example 2.1 (Figure 2.1) to determine the allowable moment, $M_a$, by using the Allowable Stress Design (ASD) method on the basis of initiation of yielding. Also determine the effective moment of inertia for deflection determination at the allowable moment. Use Type 301 stainless steel, 1/4-Hard: $F_y = 50$ ksi.

Solution:

1. Calculation of the allowable moment, $M_a$:

   The effective section properties calculated by the ASD method are the same as those determined in Example 2.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

   $\Omega = 1.85$ (Safety Factor stipulated in Table E of the Standard)

   $M_n = 47.45$ kips-in. (obtained from Example 2.1)

   $M_a = M_n / \Omega$  \hspace{1cm} (Eq. E-1)

   $= 47.45 / 1.85$

   $= 26.65$ kips-in.

2. Calculation of the effective moment of inertia for deflection determination at the allowable moment, $M_a$:

   For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 2.1 for the LRFD method, except that the computed moment $M (= f x S_e)$ should be equal to $M_a$. 

42
From the results of Example 2.1, it can be seen that by assuming a compression stress of \( f = 28.07 \text{ ksi} \), the computed \( S_e = 0.992 \text{ in.}^3 \) which is based on the fully effective section. If the assumed stress is equal to \( f = 26.86 \text{ ksi} \), the effective section modulus is also determined by the full section properties, i.e., \( S_e = 0.992 \text{ in.}^3 \). This will give \( fxS_e = 26.65 \text{ kips-in.} \), which is equal to \( M_a \).

Therefore, the computed \( I_x = 2.975 \text{ in.}^4 \) of the full section properties can be used for deflection determination.
EXAMPLE 3.1 C-SECTION W/BRACING (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\Phi_b M_n$, based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the service moment. Use Type 304 stainless steel, 1/4-Hard. Assume dead load to live load ratio $D/L = 1/5$ and $1.2D + 1.6L$ governs the design.

Given:
1. Section: 6" x 1.625" x 0.060" channel with stiffened flanges.
2. Compression flange braced against lateral buckling.

Solution:
1. Calculation of the design flexural strength, $\Phi_b M_n$:

   a. Properties of $90^\circ$ corners:

   \[ r = R + t/2 = 3/32 + 0.060/2 = 0.124 \text{ in.} \]
Length of arc, \( u = 1.57r = 1.57 \times 0.124 = 0.195 \) in.

Distance of c.g. from center of radius,
\( c = 0.637r = 0.637 \times 0.124 = 0.079 \) in.

b. Computation of \( I_x, S_e, \) and \( M_n: \)

For the first approximation, assume a compression stress of
\( f = F_y = 50 \) ksi (yield strength in longitudinal compression,
Table A1 of the Standard) in the top fiber of the section
and that the web is fully effective.

Compression flange:
\[
\begin{align*}
  w &= 1.317 \text{ in.} \\
  w/t &= 1.317/0.060 = 21.95 \\
  S &= 1.28 \sqrt{E_o/f} \quad \text{(Eq. 2.4-1)} \\
  S &= 1.28 \sqrt{27000/50} = 29.74 \\
  S/3 &= 9.91 < (w/t) = 21.95 < S = 29.74 \\
  I_a &= 399t^4\left\{(w/t)/S-0.33\right\}^3 \quad \text{(Eq. 2.4.2-6)} \\
  &= 399(0.060)^4\left\{(21.95/29.74)-0.33\right\}^3 \\
  &= 0.000351 \text{ in.}^4 \\
  D &= 0.600 \text{ in.} \\
  d &= 0.446 \text{ in.}, \ d/t = 0.446/0.060 = 7.43 \\
  I_s &= d^4t/12 \quad \text{(Eq. 2.4-2)} \\
  &= (0.446)^4(0.060)/12 = 0.000444 \text{ in.}^4 \\
  D/w &= 0.600/1.317 = 0.456, \ 0.25 < D/w = 0.456 < 0.80 \\
  k &= (4.82-5(D/w))(I_s/I_a)^n+0.43 \leq 5.25-5(D/w) \quad \text{(Eq. 2.4.2-9)}
\end{align*}
\]
\[ n = \frac{1}{2} \]
\[ [4.82-5(0.456)](0.000444/0.000351)^{1/2}+0.43 = 3.267 \]
\[ 5.25-5(0.456) = 2.970 < 3.267 \]

Use \( k = 2.970 \)

Since \( I_s > I_a \) and \( D/w < 0.8 \), the stiffener is not considered as a simple lip.

\[ w/t = 21.95 < 90 \text{ OK (Section 2.1.1-(1)-(i))} \]

\[ \lambda = \frac{(1.052/\sqrt{k})(w/t)\sqrt{50/27000}}{E_o} \quad \text{(Eq. 2.2.1-4)} \]
\[ = \frac{(1.052/\sqrt{2.970})(21.95)\sqrt{50/27000}}{0.577 < 0.673} \]
\[ b = w \quad \text{(Eq. 2.2.1-1)} \]
\[ = 1.317 \text{ in.} \text{ (i.e. compression flange fully effective)} \]

Compression (upper) stiffener:

\[ k = 0.50 \text{ (unstiffened compression element)} \]

\[ d/t = 7.43 \]

\( f \) can be conservatively taken equal to 50 ksi as used in the top compression fiber.

\[ \lambda = \frac{(1.052/\sqrt{0.50})(7.43)\sqrt{50/27000}}{0.476 < 0.673} \]

Therefore,

\[ d_s' = d = 0.446 \text{ in.} \]

\[ d_s = d_s'(I_s/I_a) < d_s' \quad \text{(Eq. 2.4.2-11)} \]
\[ = 0.446(0.000444/0.000351) \]
\[ = 0.564 \text{ in.} > 0.446 \text{ in.} \]

\[ d_s = 0.446 \text{ in.} \text{ (i.e. compression stiffener is fully effective)} \]

Thus, one concludes that the section is fully effective.
\[ y_{cg} = \frac{6}{2} = 3.000 \text{ in. (from symmetry)} \]

Full section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L (in.)</th>
<th>( y ) Distance from Centerline of Section (in.)</th>
<th>( Ly^2 ) (in.(^3))</th>
<th>( I'_1 ) About Own Axis (in.(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web</td>
<td>5.692</td>
<td>--</td>
<td>--</td>
<td>15.368</td>
</tr>
<tr>
<td>Stiffeners</td>
<td>2 x 0.446 = 0.892</td>
<td>2.623</td>
<td>6.137</td>
<td>0.015</td>
</tr>
<tr>
<td>Corners</td>
<td>4 x 0.195 = 0.780</td>
<td>2.925</td>
<td>6.673</td>
<td>--</td>
</tr>
<tr>
<td>Flanges</td>
<td>2 x 1.317 = 2.634</td>
<td>2.970</td>
<td>23.234</td>
<td>--</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td></td>
<td>36.044 15.383</td>
</tr>
</tbody>
</table>

Since section is singly symmetric about x-axis and fully effective, a compression stress of 50 ksi will govern as assumed. (At the bottom tension fiber a tensile stress of 50 ksi will develop simultaneously from symmetry).

To check if web is fully effective: (Section 2.2.2)

\[ f_1 = \left[ (3.000 - 0.154) / 3.000 \right] \times 50 = 47.43 \text{ ksi(compression)} \]

\[ f_2 = -47.43 \text{ ksi(tension)} \]

\[ \Psi = \frac{f_2}{f_1} = \frac{-47.43}{47.43} = -1.000 \]

\[ k = 4 + 2(1-\Psi)^2 + 2(1-\Psi) = 4 + 2[1-(-1)]^2 + 2[1-(-1)] = 24.000 \]

\[ h = w = 5.692 \text{ in.}, \frac{h}{t} = \frac{w}{t} = \frac{5.692}{0.060} = 94.87 \]

\[ \frac{h}{t} = 94.87 < 200 \text{ OK (Section 2.1.2-(1))} \]
\[ \lambda = (1.052/\sqrt{24})(94.87)/\sqrt{47.43/27000} = 0.854 > 0.673 \]
\[ \rho = \left[1-(0.22/\lambda)\right]/\lambda \quad \text{(Eq. 2.2.1-3)} \]
\[ = \left[1-(0.22/0.854)\right]/0.854 = 0.869 \]
\[ b_e = \rho e \quad \text{(Eq. 2.2.1-2)} \]
\[ = 0.869 \times 5.692 = 4.946 \text{ in.} \]
\[ b_2 = b_e/2 \quad \text{(Eq. 2.2.2-2)} \]
\[ = 4.946/2 = 2.473 \text{ in.} \]
\[ b_1 = b_e/(3-\psi) \quad \text{(Eq. 2.2.2-1)} \]
\[ = 4.946/[3-(-1)] = 1.237 \text{ in.} \]

The effective widths, \(b_1\) and \(b_2\), of web are defined in Figure 2 of the Standard.
\[ b_1 + b_2 = 1.237 + 2.473 = 3.710 \text{ in.} \]
Compression portion of the web = \(y_{cg} - 0.154 = 3.000 - 0.154 = 2.846 \text{ in.} \)

Since \(b_1 + b_2 = 3.710 \text{ in.} > 2.846 \text{ in.}\), \(b_1 + b_2\) shall be taken as 2.846 in.. This verifies the assumption that the web is fully effective.

\[ I_x' = L y^2 + I_1' \]
\[ = 36.044 + 15.383 = 51.427 \text{ in.}^3 \]
Actual \(I_x = I_x' t\)
\[ = 51.427 \times 0.060 = 3.086 \text{ in.}^4 \]
\[ S_e = I_x/y_{cg} \]
\[ = 3.086/3.000 = 1.029 \text{ in.}^3 \]
\[ M_n = S F_e y \]
\[ = 1.029 \times 50 \]
\[ = 51.45 \text{ kips-in.} \]

\[ \phi_b M' = 0.90 \text{ (for section with stiffened compression flanges)} \]
\[ \phi_b M_n' = 0.90 \times 51.45 = 46.31 \text{ kips-in.} \]

2. Calculation of the effective moment of inertia for deflection determination at the service moment \( M_s \):

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of 1.2D+1.6L, the service moment can be determined as follows:

\[ \phi_b M_n = 1.2 M_{DL} + 1.6 M_{LL} \]
\[ = [1.2(M_{DL}/M_{LL})+1.6]M_{LL} \]
\[ = [1.2(1/5)+1.6]M_{LL} \]
\[ = 1.84 M_{LL} \]
\[ M_{LL} = \phi_b M_n / 1.84 = 46.31 / 1.84 = 25.17 \text{ kips-in.} \]
\[ M_s = M_{DL} + M_{LL} \]
\[ = (1/5+1)M_{LL} \]
\[ = 1.2(25.17) = 30.20 \text{ kips-in.} \]

where

\( M_{DL} \) = Moment determined on the basis of nominal dead load

\( M_{LL} \) = Moment determined on the basis of nominal live load

The procedure is iterative: one assumes the actual compressive
stress $f$ under this service moment $M_s$. Knowing $f$, one proceeds as usual to obtain $S_e$ and checks to see if $(f \times S_e)$ is equal to $M_s$ as it should. If not, reiterate until one obtains the desired level of accuracy. (Section 2.2.1-(2))

After several iterations with beginning a stress of $f = F_y/2$, the following only gives the results of final iteration.

Assume that a stress of $f = 29.35$ ksi in the top and bottom fibers of the section and that the web is fully effective.

Compression flange:

\[
S = 1.28 \sqrt{27000/29.35} = 38.82
\]
\[
S/3 = 12.94 < w/t = 21.95 < S = 38.82
\]
\[
I_a = 399(0.060)^4((21.95/38.82)-0.33)^3
= 0.000067 \text{ in.}^4
\]
\[
I_s/I_a = 0.000444/0.000067 = 6.627
\]
\[
k = (4.82-5(0.456))(6.627)^{1/2}+0.43 = 6.969 > 2.970
\]
\[
k = 2.970
\]

For deflection determination, the reduced modulus of elasticity, $E_r$, is substituted for $E_o$ in Eq. (2.2.1-4). For a compression and tension stresses of $f=29.35$ ksi, the corresponding $E_{sc}$ and $E_{st}$ values for Type 304 stainless steel are obtained from Table A2 or Figure A1 of the Standard as follows:

\[
E_{sc} = 23089 \text{ ksi}, \quad E_{st} = 26933 \text{ ksi.}
\]
\[
E_r = (E_{sc} + E_{st})/2 \quad \text{(Eq. 2.2.1-7)}
= (23089+26933) = 25011 \text{ ksi}
\]
\[
A = (1.052/\sqrt{2.970})(21.95)\sqrt{29.35/25011} = 0.459 < 0.673
\]
\( b_d = 1.317 \text{ in. (i.e. compression flange fully effective)} \)

Compression (upper) stiffener:

\( f \) can be conservatively taken equal to 29.35 ksi as used in the compression fiber.

\[
\Lambda = \frac{(1.052/\sqrt{0.50})(7.43)}{\sqrt{29.35/250}} = 0.379 < 0.673
\]

therefore, \( d' = 0.446 \text{ in.} \)

Since \( I_s/I_a = 6.627 > 1.0 \), it follows that \( d_s = d' \) = 0.446 in. (i.e. compression stiffener fully effective).

Thus the section is fully effective.

\( y_{cg} = 6/2 = 3.000 \text{ in. (from symmetry)} \)

And since the section is singly symmetric about x-axis, top compression fiber (and also bottom tension fiber) may be used in computing \( S_e \).

To check if web is fully effective:

\[
f_1 = \frac{(3.000-0.154)/3.000}{3.000} \times 29.35 = 27.84 \text{ ksi (compression)}
\]

\( f_2 = -27.84 \text{ ksi (tension)} \)

\( \Psi = f_2/f_1 = -27.84/27.84 = -1.000 \)

\( k = 24.000 \)

For a compression and tension stresses of \( f=27.84 \text{ ksi} \), the values of \( E_{sc} \) and \( E_{st} \) are found as follows:

\( E_{sc} = 24090 \text{ ksi, } E_{st} = 27000 \text{ ksi.} \)

\( E_r = \frac{(24090+27000)}{2} \) (Eq. 2.2.1-7)

\( = 25550 \text{ ksi} \)
\[ \lambda = \frac{(1.052/\sqrt{24})(94.87)}{\sqrt{27.84/25550}} = 0.672 < 0.673 \]

\[ b_e = w = 5.692 \text{ in.} \]  
\[ b_2 = \frac{5.692}{2} = 2.846 \text{ in.} \]
\[ b_1 = \frac{5.692}{(3-(-1))} = 1.423 \text{ in.} \]
\[ b_1 + b_2 = 4.269 \text{ in.} > \text{compression portion of the web} = 2.846 \text{ in.} \]

Thus, \( b_1 + b_2 \) shall be taken as 2.846 in.. This verifies the assumption that the web is fully effective.

Full section properties are the same as that used in determination of \( \phi_b M_n \) since the section is fully effective.

\[ I_x = 3.086 \text{ in.}^4 \]
\[ S_e = 1.029 \text{ in.}^3 \]
\[ M = f x S_e = 29.35 \times 1.029 = 30.20 \text{ kips-in.} = M_s \text{ OK} \]

Thus, use \( I_x = 3.086 \text{ in.}^4 \) for deflection determination.
EXAMPLE 3.2 C-SECTION W/BRACING (ASD)

Rework Example 3.1 to determine the allowable moment, $M_a$, by using the Allowable Stress Design (ASD) method on the basis of initiation of yielding. Also determine the effective moment of inertia for deflection determination at the allowable moment.

Solution:

1. Calculation of the allowable moment, $M_a$:

   The effective section properties calculated by the ASD method are the same as those determined in Example 3.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

   $\Omega = 1.85$ (Safety Factor stipulated in Table E of the Standard)

   $M_n = 51.45 \text{ kips-in. (obtained from Example 3.1)}$

   $M_a = \frac{M_n}{\Omega}$  
   \hspace{1cm} (Eq. E-1)

   $= \frac{51.45}{1.85}$

   $= 27.81 \text{ kips-in.}$

2. Calculation of the effective moment of inertia for deflection determination at the allowable moment $M_a$:

   For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 3.1 for the LRFD method, except that the computed moment $M (= f x S_e)$ should be equal to $M_a$. 

53
From the results of Example 3.1, it is found that for a stress of $f=29.35$ ksi, the section is fully effective. Therefore, it can be seen that by assuming a stress of $f=27.03$ ksi (which is less than 29.35 ksi) the section will also be fully effective, i.e., $S_e = 1.029$ in.$^3$ Thus,

$$M = S_e \times 27.03$$

$$= 27.81 \text{ kips-in. } = M_a \quad OK$$

Therefore, the computed $I_x = 3.083$ in.$^4$ can be used for deflection determination.
EXAMPLE 4.1 Z-SECTION W/STIFFENED FLANGES (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\Phi_{b,M_n}$, based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the service moment. Use Type 301 stainless steel, 1/4-Hard. Assume dead load to live load ratio $D/L = 1/5$ and $1.2D+1.6L$ governs the design.

![Corner Line Element](image)

Figure 4.1 Section for Example 4.1

Given:

1. Section: 6" x 1.500" x 0.060" Z-section with stiffened flanges.
2. Compression flange braced against lateral buckling.

Solution:

1. Calculation of the design flexural strength, $\Phi_{b,M_n}$:
   a. Properties of 90° corners:

   $$ r = R + t/2 = 3/32 + 0.060/2 = 0.124 \text{ in.} $$

   Length of arc, $u = 1.57r = 1.57 \times 0.124 = 0.195 \text{ in.}$
Distance of c.g. from center of radius, 
\[ c = 0.637r = 0.637 \times 0.124 = 0.079 \text{ in.} \]

b. Properties of 135° corners: 
\[ r = R + t/2 = 3/32 + 0.060/2 = 0.124 \text{ in.} \]

Length of arc, \( u = (45^\circ/180^\circ)(3.14)r = 0.785r = 0.785 \times 0.124 = 0.097 \text{ in.} \)

Distance of c.g. from center of radius, 
\[ c_1 = r \sin \theta/\theta = (0.124 \times \sin 45^\circ)/0.785 = 0.112 \text{ in.} \]

c. Computation of \( I_x \), \( S_e \), and \( M_n \): 

For the first approximation, assume a compression stress 
of \( f = F_y = 50 \text{ ksi} \) (yield strength in longitudinal compression, 
Table A1 of the Standard) in the top fiber of the section 
and that the web is fully effective.

Compression flange:
\[ w = 1.346 \text{ in.} \]
\[ w/t = 1.346/0.060 = 22.43 \]
\[ S = 1.28\sqrt{E_o/f} \]  
(\text{Eq. 2.4-1})
\[ E_o = 27000 \text{ ksi} \) (Table A4 of the Standard)
\[ S = 1.28\sqrt{27000/50} = 29.74 \]
\[ S/3 = 9.91 < w/t = 22.43 < S = 29.74 \]
\[ I_a = 399t^4\left\{[(w/t)/S]-0.33\right\}^3 \]  
(\text{Eq. 2.4.2-6})
\[ = 399(0.060)^4\left\{[(22.43/29.74)-0.33]\right\}^3 \]
\[ = 0.000395 \text{ in.}^4 \]
\[ d = 0.600 \text{ in.}, d/t = 0.600/0.060 = 10 \]
\[ D = d + 0.154 \tan(\theta/2) = 0.600 + 0.154 \tan(45^\circ/2) = 0.664 \text{ in.} \]

\[ I_s = d^3 t \sin^2 \theta/12 \quad \text{(Eq. 2.4-2)} \]

\[ = (0.600)^3(0.060)\sin^2(45^\circ)/12 = 0.000540 \text{ in.}^4 \]

\[ I_s/I_a = 0.000540/0.000395 = 1.367 \]

\[ D/w = 0.664/1.346 = 0.493, \quad 0.25 < D/w = 0.493 < 0.80 \]

\[ k = [4.82 - 5(D/w)](I_s/I_a)^n + 0.43 \leq 5.25 - 5(D/w) \quad \text{(Eq. 2.4.2-9)} \]

\[ n = 1/2 \]

\[ [4.82 - 5(0.493)](1.367)^{1/2} + 0.43 = 3.183 \]

\[ 5.25 - 5(0.493) = 2.785 < 3.183 \]

\[ k = 2.785 \]

Since \( I_s > I_a \) and \( D/w < 0.8 \), the stiffener is not considered as a simple lip.

\[ w/t = 22.43 < 90 \text{ OK (Section 2.1.1-(1)-(i))} \]

\[ \lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o} \quad \text{(Eq. 2.2.1-4)} \]

\[ = (1.052/\sqrt{2.785})(22.43)\sqrt{50/27000} = 0.608 < 0.673 \]

\[ b = w \quad \text{(Eq. 2.2.1-1)} \]

\[ = 1.346 \text{ in. (i.e. compression flange fully effective)} \]

Compression (upper) stiffener:

\[ k = 0.50 \text{ (unstiffened compression element)} \]

\[ d/t = 10.00 \]

\( f \) conservatively taken equal to 50 ksi as in top compression fiber.

\[ \lambda = (1.052/\sqrt{0.50})(10.00)\sqrt{50/27000} = 0.640 < 0.673 \]

Therefore,

\[ d'_{s} = d = 0.600 \text{ in.} \]

\[ d_s = d'_{s}(I_s/I_a) \leq d'_{s} \quad \text{(Eq. 2.4.2-11)} \]

\[ = 0.600(1.367) \]
\[ d_s = 0.600 \text{ in. (i.e. compression stiffener is fully effective)} \]

Thus, one concludes that the section is fully effective.

\[ y_{cg} = \frac{6}{2} = 3.000 \text{ in. (from symmetry)} \]

Full section properties about x axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L (in.)</th>
<th>( y ) Distance from Centerline of Section (in.)</th>
<th>( I'_1 ) About Own Axis (in.(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web</td>
<td>5.692</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Stiffeners</td>
<td>2 x 0.600 = 1.200</td>
<td>2.722</td>
<td>8.891</td>
</tr>
<tr>
<td>90° Corners</td>
<td>2 x 0.195 = 0.390</td>
<td>2.925</td>
<td>3.337</td>
</tr>
<tr>
<td>135° Corners</td>
<td>2 x 0.097 = 0.194</td>
<td>2.958</td>
<td>1.697</td>
</tr>
<tr>
<td>Flanges</td>
<td>2 x 1.346 = 2.692</td>
<td>2.970</td>
<td>23.746</td>
</tr>
<tr>
<td>Sum</td>
<td>10.168</td>
<td>37.671</td>
<td>15.386</td>
</tr>
</tbody>
</table>

Since section is singly symmetric about x-axis and fully effective, a compression stress of 50 ksi will govern as assumed. (At the bottom tension fiber a tensile stress of 50 ksi will develop simultaneously from geometry).

To check if web is fully effective: (Section 2.2.2)

\[ f_1 = \left(\frac{3.000 - 0.154}{3.000}\right) \times 50 = 47.43 \text{ ksi (compression)} \]

\[ f_2 = -47.43 \text{ ksi (tension)} \]
\[ \Psi = \frac{f_2}{f_1} = \frac{-47.43}{47.43} = -1.000 \]

\[ k = 4 + 2(1-\Psi)^2 + 2(1-\Psi) \]

\[ = 4 + 2[1-(-1)]^2 + 2[1-(-1)] \]

\[ = 24.000 \]

\[ h = w = 5.692 \text{ in.}, \quad h/t = w/t = \frac{5.692}{0.060} = 94.87 \]

\[ h/t = 94.87 < 200 \text{ OK (Section 2.1.2-(1))} \]

\[ \lambda = \frac{(1.052/\sqrt{24})(94.87)\sqrt{47.43/27000}}{0.854} = 0.854 > 0.673 \]

\[ \rho = \frac{[1-(0.22/\lambda)]}{\lambda} = \frac{[1-(0.22/0.854)]}{0.854} = 0.869 \quad \text{(Eq. 2.2.1-3)} \]

\[ b_e = \rho w = 0.869 \times 5.692 = 4.946 \text{ in.} \quad \text{(Eq. 2.2.1-2)} \]

\[ b_2 = \frac{b_e}{2} = \frac{4.946}{2} = 2.473 \text{ in.} \quad \text{(Eq. 2.2.2-2)} \]

\[ b_1 = \frac{b_e}{(3-\Psi)} = \frac{4.946}{3-(-1)} = 1.237 \text{ in.} \quad \text{(Eq. 2.2.2-1)} \]

The effective widths of web, \( b_1 \) and \( b_2 \), are defined in Figure 2 of the Standard.

\[ b_1 + b_2 = 1.237 + 2.473 = 3.710 \text{ in.} \]

Compression portion of the web = \( y_{cg} - 0.154 \)

\[ = 3.000 - 0.154 \]

\[ = 2.846 \text{ in.} \]

Since \( b_1 + b_2 = 3.710 \text{ in.} > 2.846 \text{ in.} \), \( b_1 + b_2 \) shall be taken as 2.846 in. This verifies the assumption that the web is fully effective.

\[ I'_{x} = Ly^2 + I'_1 \]

\[ = 37.671 + 15.386 \]

59
Actual \( I_x = I'_x t \)
\[ = 53.057 \times 0.060 \]
\[ = 3.183 \text{ in.}^2 \]

\( S_e = \frac{I_x}{y_{cg}} \)
\[ = \frac{3.183}{3.000} \]
\[ = 1.061 \text{ in.}^2 \]

\( M_n = S_e F_y \)  \( \text{(Eq. 3.3.1.1-1)} \)
\[ = 1.061 \times 50 \]
\[ = 53.05 \text{ kips-in.} \]

**d. The design flexural strength, } \Phi_b M_n, \text{ based on initiation of yielding is determined as follows: (Section 3.3.1.1(1))**

\( \Phi_b = 0.90 \) (for section with stiffened compression flanges)

\( \Phi_b M_n = 0.90 \times 53.05 = 47.75 \text{ kips-in.} \)

**2. Calculation of the effective moment of inertia for deflection determination at the service moment } M_s:**

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of 1.2D+1.6L, the service moment can be determined as follows:

\[ \Phi_b M_n = 1.2M_{DL} + 1.6M_{LL} \]
\[ = [1.2(M_{DL}/M_{LL})+1.6]M_{LL} \]
\[ = [1.2(1/5)+1.6]M_{LL} \]
\[ = 1.84M_{LL} \]

\[ M_{LL} = \Phi_b M_n / 1.84 = 47.75 / 1.84 = 25.95 \text{ kips-in.} \]
\[ M_s = M_{DL} + M_{LL} \]
\[ = (1/5+1)M_{LL} \]
\[ = 1.2(25.95) = 31.14 \text{ kips-in.} \]

where

- \( M_{DL} \) = Moment determined on the basis of nominal dead load
- \( M_{LL} \) = Moment determined on the basis of nominal live load

The procedure is iterative: one assumes the actual compressive stress \( f \) under this service moment \( M_s \). Knowing \( f \), one proceeds as usual to obtain \( S_e \) and checks to see if \((f \times S_e)\) is equal to \( M_s \) as it should. If not, reiterate until one obtains the desired level of accuracy. (Section 2.2.1-(2))

After several trials with first iteration using \( f = F_y/2 \), the following only gives the results of final iteration.

Assume that a stress of \( f = 29.35 \text{ ksi} \) in the top and bottom fibers of the section and that the web is fully effective.

Compression flange:

\[ S = 1.28 \frac{27000}{29.35} = 38.82 \]
\[ S/3 = 12.94 < w/t = 22.43 < S = 38.82 \]
\[ I_a = 399(0.060)^4[(22.43/38.82)-0.33]^3 \]
\[ = 0.000079 \text{ in.}^4 \]
\[ I_s/I_a = 0.000540/0.000079 = 6.835 \]
\[ k = [4.82-5(0.493)](6.835)^{1/2} + 0.43 = 6.587 > 2.785 \]
\[ \text{Use } k = 2.785 \]

For a compression and tension stresses of \( f = 29.35 \text{ ksi} \), the values
of $E_{sc}$ and $E_{st}$ are found as follows:

$E_{sc} = 23089$ ksi, $E_{st} = 26933$ ksi.

$E_r = \frac{(23089 + 26933)}{2}$

= 25011 ksi

$\lambda = \frac{(1.052/\sqrt{2.785})(22.43)/\sqrt{29.35}/25011} = 0.484 < 0.673$

$b_d = 1.346$ in. (i.e. compression flange fully effective)

Compression (upper) stiffener:

$f$ can be conservatively taken equal to 29.35 ksi as used in the top compression fiber.

$\lambda = \frac{(1.052/\sqrt{0.50})(10.00)/\sqrt{29.35}/25011} = 0.510 < 0.673$

therefore, $d_s' = 0.600$ in.

Since $I_s/I_a = 6.835 > 1.0$, it follows that $d_s = d_s' = 0.600$ in. (i.e. compression stiffener fully effective).

Thus, the section is fully effective.

$y_{cg} = 6/2 = 3.000$ in. (from symmetry)

And since the section is singly symmetric about x-axis, top compression fiber may be used in computing $S_e$.

To check if web is fully effective:

$f_1 = \frac{(3.000 - 0.154)/3.000 \times 29.35} = 27.84$ ksi (compression)

$f_2 = -27.84$ ksi (tension)

$\psi = f_2/f_1 = -27.84/27.84 = -1.000$

$k = 24.000$

For a stress of $f=27.84$ ksi, the $E_r = 25550$ ksi, which is
determined in Example 3.1.

\[ \lambda = \frac{1.052}{\sqrt{24}} \sqrt{94.87 \cdot 27.84 / 25550} = 0.672 < 0.673 \]

\[ b_{e} = w \quad \text{(Eq. 2.2.1-1)} \]
\[ = 5.692 \text{ in.} \]

\[ b_2 = \frac{5.692}{2} = 2.846 \text{ in.} \]

\[ b_1 = \frac{5.692}{3-(-1)} = 1.423 \text{ in.} \]

\[ b_1 + b_2 = 4.269 \text{ in.} > \text{compression portion of the web} = 2.846 \text{ in.} \]

Thus, \( b_1 + b_2 \) shall be taken as 2.846 in.. This verifies the assumption that the web is fully effective.

Full section properties are the same as that used in the determination of \( \Phi_b M_n \) since the section is fully effective.

\[ I_x = 3.183 \text{ in.}^4 \]

\[ S_e = 1.061 \text{ in.}^3 \]

\[ M = f \times S_e = 29.35 \times 1.061 \]
\[ = 31.14 \text{ kips-in.} = M_s \text{ OK} \]

Thus, use \( I_x = 3.183 \text{ in.}^4 \) for deflection determination.
EXAMPLE 4.2 Z-SECTION W/STIFFENED FLANGES (ASD)

Use the data given in Example 4.1 (Figure 4.1) to determine the allowable moment, $M_a$, by using the Allowable Stress Design (ASD) method on the basis of initiation of yielding. Also determine the effective moment of inertia for deflection determination at the allowable moment. Use Type 301 stainless steel, 1/4-Hard: $F_y = 50$ ksi.

Solution:

1. Calculation of the allowable moment, $M_a$:

The effective section properties calculated by the ASD method are the same as those determined in Example 4.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

$\Omega = 1.85$ (Safety Factor stipulated in Table E of the Standard)

$M_n = 53.05$ kips-in. (obtained from Example 4.1)

$M_a = \frac{M_n}{\Omega}$

$= \frac{53.05}{1.85}$

$= 28.68$ kips-in.

2. Calculation of the effective moment of inertia for deflection determination at the allowable moment, $M_a$:

For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 4.1 for the LRFD method, except that the computed
M (= f x S_e) should be equal to M_e.

From the results of Example 4.1, it can be seen that by using a stress of f = 29.35 ksi, the computed \( S_e = 1.061 \) in.\(^3\) which is based on the fully effective section. If the assumed stress is equal to f = 27.03 ksi, the effective section modulus is also determined by the full section properties, i.e., \( S_e = 1.061 \) in.\(^3\). This will give \( f x S_e = 28.68 \) kips-in., which is equal to M_e.

Therefore, the computed \( I_x = 3.183 \) in\(^4\) of the full section properties is used for deflection determination.
EXAMPLE 5.1 DEEP Z-SECTION w/STIFFENED FLANGES (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, \( \Phi_b M_n \), based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the service moment. Use Type 301 stainless steel, 1/4-Hard. Assume dead load to live load ratio \( D/L = 1/5 \) and \( 1.2D+1.6L \) governs the design.

![Figure 5.1 Section for Example 5.1](image)

**Given:**
1. Section: 9.5" x 1.500" x 0.060" Z-section with stiffened flanges.
2. Compression flange braced against lateral buckling.

**Solution:**
1. Calculation of the design flexural strength, \( \Phi_b M_n \):
   a. Properties of 90° corners:
\[ r = R + t/2 = 3/32 + 0.060/2 = 0.124 \text{ in.} \]

Length of arc, \( u = 1.57r = 1.57 \times 0.124 = 0.195 \text{ in.} \)

Distance of c.g. from center of radius,
\[ c = 0.637r = 0.637 \times 0.124 = 0.079 \text{ in.} \]

b. Properties of 135° corners:
\[ r = R + t/2 = 3/32 + 0.060/2 = 0.124 \text{ in.} \]

Length of arc, \( u = (45°/180°)(3.14)r = 0.785r = 0.785 \times 0.124 \)
\[ = 0.097 \text{ in.} \]

Distance of c.g. from center of radius,
\[ c_1 = r \sin \theta / \theta = (0.124 \times \sin 45°) / 0.785 = 0.112 \text{ in.} \]

c. Computation of \( I_x, S_e, \) and \( M_n \):

For the first approximation, assume a compression stress

of \( f = F_y = 50 \text{ ksi} \) (yield strength in longitudinal compression

as given in Table A1 of the Standard) in the top fiber of the

section and that the web is fully effective.

Compression flange:

\[ w = 1.346 \text{ in.} \]

\[ w/t = 1.346/0.060 = 22.43 \]

\[ S = 1.28 \sqrt{\frac{E_o}{f}} \quad \text{(Eq. 2.4-1)} \]

\[ E_o = 27000 \text{ ksi} \] (Table A4 of the Standard)

\[ S = 1.28 \sqrt{27000/50} = 29.74 \]

\[ S/3 = 10.36 < w/t = 24.52 < S = 31.09 \]

\[ I_a = 399 \tau \left( \frac{w/t}{S} - 0.33 \right)^2 \quad \text{(Eq. 2.4.2-6)} \]

\[ = 399(0.060)^2 \left[ (22.43/29.74) - 0.33 \right]^3 \]
$d = 0.600 \text{ in.}, \frac{d}{t} = 0.600/0.060 = 10$

$D = d + 0.154\tan(\theta/2) = 0.600 + 0.154\tan(\frac{45^\circ}{2}) = 0.664 \text{ in.}$

$I_s = \frac{d^3\sin^2\theta}{12}$ (Eq. 2.4-2)

$= (0.600)^3(0.060)\sin^2(45^\circ)/12 = 0.000540 \text{ in.}^4$

$I_s/I_a = 0.000540/0.000395 = 1.367$

$D/w = 0.664/1.346 = 0.493, \ 0.25 < D/w = 0.493 < 0.80$

$k = \frac{[4.82-5(D/w)](I_s/I_a)^n+0.43}{5.25-5(D/w)}$ (Eq. 2.4.2-9)

$n = 1/2$

$\left[(4.82-5(0.493))(1.367)^{1/2}+0.43\right] = 3.183$

$5.25-5(0.493) \approx 2.785 < 3.183$

use $k = 2.995$

Since $I_s > I_a$ and $D/w < 0.8$, the stiffener is not considered as a simple lip.

$w/t = 22.43 < 90 \text{ OK (Section 2.1.1-(1)-(i))}$

$\lambda = \frac{(1.052/\sqrt{k})(w/t)\sqrt{E/E_o}}{\overline{\overline{E/E_o}}}$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{2.785})(22.43)/\sqrt{50/27000} = 0.608 < 0.673$

$b = w$ (Eq. 2.2.1-1)

$= 1.346 \text{ in. (i.e. compression flange fully effective)}$

Compression (upper) stiffener:

$k = 0.50 \text{ (unstiffened compression element)}$

$d/t = 10.00$

$f$ conservatively taken equal to 50 ksi as in top compression fiber.

$\lambda = \frac{(1.052/\sqrt{0.50})(10.00)\sqrt{50/27000} = 0.640 < 0.673}$

Therefore,
\[ d'_s = d = 0.600 \text{ in.} \]
\[ d_s = d'_s \left( \frac{I_s}{I_a} \right) \leq d'_s \]  
(Eq. 2.4.2-11)

Since \( \frac{I_s}{I_a} = 1.367 > 1.000 \)
\[ d_s = d'_s = 0.600 \text{ in.} \] (i.e. compression stiffener is fully effective)

Thus, one concludes that the section is fully effective.

\[ y_{cg} = \frac{9.5}{2} = 4.750 \text{ in.} \] (from symmetry)

It follows that a compression stress of \( f=50 \text{ ksi} \) will govern as assumed.

To check if web is fully effective (Section 2.2.2):
\[ f_1 = \left( \frac{(4.750-0.154)}{4.750} \right) \times 50 = 48.38 \text{ ksi (compression)} \]
\[ f_2 = -48.38 \text{ ksi (tension)} \]
\[ \psi = \frac{f_2}{f_1} = \frac{-48.38}{48.38} = -1.000 \]
\[ k = 4 + 2(1-\psi)^2 + 2(1-\psi) \]
\[ = 4 + 2(1-(-1))^2 + 2(1-(-1)) \]
\[ = 24.000 \]
\[ h = w = 9.192 \text{ in.}, \; h/t = w/t = 9.192/0.060 = 153.20 \]
\[ h/t = 153.20 < 200 \text{ OK (Section 2.1.2-(1))} \]
\[ \lambda = \left( \frac{1.052/\sqrt{24}}{153.20} \right) \sqrt{48.38/27000} = 1.393 > 0.673 \]
\[ \rho = \left[ 1 - \left( 0.22/\lambda \right) \right]/\lambda \]  
(Eq.2.2.1-3)
\[ = \left[ 1 - \left( 0.22/1.393 \right) \right]/1.393 = 0.604 \]
\[ b_e = \rho \cdot w \]  
(Eq. 2.2.1-2)
\[ = 0.604 \times 9.192 = 5.552 \text{ in.} \]
\[ b_2 = b_e/2 \]  
(Eq. 2.2.2-2)
\[ = 5.552/2 = 2.776 \text{ in.} \]
\[ b_1 = \frac{b_e}{(3-\psi)} \]  
\[ = 5.552 \frac{[3-(-1)]}{1.388 \text{ in.}} \]

The effective widths of web, \( b_1 \) and \( b_2 \), are defined in Figure 2 of the Standard.

\[ b_1 + b_2 = 1.388 + 2.776 = 4.164 \text{ in.} \]

Compression portion of the web = \( y_{cg} - 0.154 \)
\[ = 4.750 - 0.154 \]
\[ = 4.596 \text{ in.} \]

Since \( b_1 + b_2 = 4.164 \text{ in.} < 4.596 \text{ in.} \), it follows that the web is not fully effective. Hence \( y_{cg} = 4.750 \) as assumed.

The procedure to determine the location of the neutral axis (N.A.) based on partially effective web is iterative. We start with \( y_{cg} = 4.750 \text{ in.} \) and from Figure 2 of the Standard, scale \( b_1, b_2 \) already computed with respect to \( y_{cg} = 4.750 \text{ in.} \). Then we proceed to compute a new N.A. and hence \( b_1 + b_2 \). If \( b_1 + b_2 \) is the same as before, the solution stabilizes and the location of N.A. is calculated according to this \( (b_1 + b_2) \). If \( b_1 + b_2 \) differ than before, one reiterates in the same manner until \( b_1 + b_2 \) stabilizes.

Thus, for the first iteration, the web is divided into three segments:

\[ b_1 = 1.388 \text{ in.}, \text{ ineffective portion of web, and } b_2 = (2.776) + 4.750 - 0.154 = 7.372 \text{ in.} \]

Thus the ineffective portion of web = 9.192 - 1.388 - 7.372 = 0.432 in..

The compression flange and stiffener remain fully effective since nothing is altered in their calculations.
Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>( L ) (in.)</th>
<th>Distance from Top Fiber (in.)</th>
<th>( L_y ) (in.²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_1 )</td>
<td>1.388</td>
<td>0.154+(1.388/2)</td>
<td>0.848</td>
</tr>
<tr>
<td>( b_2 +(9.5-y_{10}^1) )</td>
<td>7.372</td>
<td>9.5-0.154-(7.372/2)</td>
<td>5.660</td>
</tr>
<tr>
<td>Compression flange</td>
<td>1.346</td>
<td>0.030</td>
<td>0.040</td>
</tr>
<tr>
<td>Compression stiffener</td>
<td>0.600</td>
<td>0.154-0.124\cos45^\circ</td>
<td>0.278</td>
</tr>
<tr>
<td>Top 90° corner</td>
<td>0.195</td>
<td>0.075</td>
<td>0.167</td>
</tr>
<tr>
<td>Top 135° corner</td>
<td>0.097</td>
<td>0.154-0.112</td>
<td>0.042</td>
</tr>
<tr>
<td>Bottom 135° corner</td>
<td>0.097</td>
<td>9.5-(0.154-0.112)</td>
<td>0.917</td>
</tr>
<tr>
<td>Bottom 90° corner</td>
<td>0.195</td>
<td>9.5-0.075</td>
<td>1.838</td>
</tr>
<tr>
<td>Bottom stiffener</td>
<td>0.600</td>
<td>9.5-0.278</td>
<td>5.533</td>
</tr>
<tr>
<td>Tension flange</td>
<td>1.346</td>
<td>9.5-(0.060/2)</td>
<td>9.470</td>
</tr>
</tbody>
</table>

\[ \text{Sum} = 13.236 \quad 64.164 \]

\[ y_{cg} = \frac{L_y}{L} = \frac{64.164}{13.236} = 4.843 \text{ in. (measured from top compression fiber)} \]

\[ f_1 = \frac{4.848-0.154}{4.848} = 48.41 \text{ ksi (compression)} \]

\[ f_2 = -\frac{9.5-4.848-0.154}{4.848} = -46.39 \text{ ksi (tension)} \]

\[ \Psi = -46.39/48.41 = -0.958 \]

\[ k = 4+2[1-(-0.958)]^3+2[1-(-0.958)] = 22.929 \]

\[ \lambda = 1.052/(\sqrt{22.929})(153.20)/\sqrt{48.41/27000} = 1.425 > 0.673 \]

\[ \rho = [1-(0.22/1.425)]/1.425 = 0.593 \]

\[ b_{1e} = 0.593 \times 9.192 = 5.451 \text{ in.} \]

\[ b_2 = 5.451/2 = 2.726 \text{ in.} \]

\[ b_1 = 5.451/(3-(-0.958)) = 1.377 \text{ in.} \]

\[ b_1+b_2 = 4.103 \text{ in.} = 4.164 \text{ in. Therefore, need to reiterate.} \]
For the second iteration:

\[ b_1 = 1.377 \text{ in.} \]

\[ b_2 + (9.5 - y_{cg}^{1}) - 0.154 = 2.726 + 9.5 - 4.848 - 0.154 = 7.224 \text{ in.} \]

Ineffective portion of web = 9.192 - 1.377 - 7.224 = 0.591 in.

Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L (in.)</th>
<th>Distance from Top Fiber (in.)</th>
<th>( Ly ) (in.²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_1 )</td>
<td>1.377</td>
<td>0.843</td>
<td>1.161</td>
</tr>
<tr>
<td>( b_2 + (9.5 - y_{cg}^{1}) - 0.154 )</td>
<td>7.224</td>
<td>5.734</td>
<td>41.422</td>
</tr>
<tr>
<td>Compression flange</td>
<td>1.346</td>
<td>0.030</td>
<td>0.040</td>
</tr>
<tr>
<td>Compression stiffener</td>
<td>0.600</td>
<td>0.278</td>
<td>0.167</td>
</tr>
<tr>
<td>Top 90° corner</td>
<td>0.195</td>
<td>0.075</td>
<td>0.015</td>
</tr>
<tr>
<td>Top 135° corner</td>
<td>0.097</td>
<td>0.042</td>
<td>0.004</td>
</tr>
<tr>
<td>Bottom 135° corner</td>
<td>0.097</td>
<td>9.458</td>
<td>0.917</td>
</tr>
<tr>
<td>Bottom 90° corner</td>
<td>0.195</td>
<td>9.425</td>
<td>1.838</td>
</tr>
<tr>
<td>Bottom stiffener</td>
<td>0.600</td>
<td>9.222</td>
<td>5.533</td>
</tr>
<tr>
<td>Tension flange</td>
<td>1.346</td>
<td>9.470</td>
<td>12.747</td>
</tr>
<tr>
<td>Sum</td>
<td>13.077</td>
<td></td>
<td>63.844</td>
</tr>
</tbody>
</table>

\[ y_{cg} = \frac{65.844}{13.077} = 4.882 \text{ in.} \text{ (measured from top compression fiber)} \]

\[ f_1 = \left(\frac{(4.882 - 0.154)}{4.882}\right) (50) = 48.42 \text{ ksi} \]

\[ f_2 = -\left(\frac{(9.5 - 4.882 - 0.154)}{4.882}\right) (50) = -45.72 \text{ ksi} \]

\[ \psi = \frac{-45.72}{48.42} = -0.944 \]

\[ k = 4 + 2 \left[1 - (-0.944)\right] + 2 \left[1 - (-0.944)\right] = 22.580 \]

\[ \lambda = \left(1.052/\sqrt{22.580}\right) (153.20)/\sqrt{48.42/27000} = 1.436 > 0.673 \]

\[ \rho = \left[1 - \left(0.22/1.436\right)\right]/1.436 = 0.590 \]
\[ b_e = 0.590 \times 9.192 = 5.423 \text{ in.} \]
\[ b_2 = \frac{5.423}{2} = 2.712 \text{ in.} \]
\[ b_1 = \frac{5.423}{3 - (-0.946)} = 1.374 \text{ in.} \]
\[ b_1 + b_2 = 4.086 \text{ in.} = 4.103 \text{ in. Therefore, need to reiterate.} \]

For the third iteration:
\[ b_1 = 1.374 \text{ in.} \]
\[ b_2 + (9.5 - y_{cg}) - 0.154 = 2.712 + 9.5 - 4.882 - 0.154 = 7.176 \text{ in.} \]
Ineffective portion of web = 9.192 - 1.374 - 7.176 = 0.642 in.

Effective section properties about x-axis:
\[ L = 13.026 \text{ in.} \]
\[ Ly = 63.736 \text{ in}^2 \]
\[ y_{cg} = \frac{63.736}{13.026} = 4.893 \text{ in.} \]
\[ f_1 = \frac{(4.893 - 0.154)/4.893}{50} = 48.43 \text{ ksi} \]
\[ f_2 = -\frac{(9.5 - 4.893 - 0.154)/4.893}{50} = -45.50 \text{ ksi} \]
\[ \psi = \frac{-45.50}{48.43} = -0.940 \]
\[ k = 4 + 2\left[1 - (-0.940)\right]^2 + 2\left[1 - (-0.940)\right] = 22.483 \]
\[ \lambda = (1.052/\sqrt{22.483})(153.20)/\sqrt{48.43/27000} = 1.440 > 0.673 \]
\[ \rho = \frac{1 - (0.22/1.440)}{1.440} = 0.588 \]
\[ b_e = 0.588 \times 9.192 = 5.405 \text{ in.} \]
\[ b_2 = 5.405/2 = 2.703 \text{ in.} \]
\[ b_1 = \frac{5.405}{3 - (-0.940)} = 1.372 \text{ in.} \]
\[ b_1 + b_2 = 4.075 \text{ in.} = 4.086 \text{ in. Therefore, need to reiterate.} \]

For the fourth iteration:
\[ b_1 = 1.372 \text{ in.} \]
\[ b_2 + (9.5 - y_{cg}) - 0.154 = 2.703 + 9.5 - 4.893 - 0.154 = 7.156 \text{ in.} \]

Ineffective portion of web = 9.192 - 1.372 - 7.156 = 0.664 in.

Effective section properties about x-axis:

\[ L = 13.004 \text{ in.} \]
\[ L_y = 63.689 \text{ in.}^2 \]
\[ y_{cg} = 63.689 / 13.004 = 4.898 \text{ in.} \]
\[ f_1 = \left(\frac{4.898 - 0.154}{4.898}\right)(50) = 48.43 \text{ ksi} \]
\[ f_2 = -\left(\frac{9.5 - 4.898 - 0.154}{4.898}\right)(50) = -45.41 \text{ ksi} \]
\[ \psi = \frac{-45.41}{48.43} = -0.938 \]
\[ k = 4 + 2\left[1 - (-0.938)^2\right] + 2\left[1 - (-0.938)^2\right] = 22.434 \]
\[ \lambda = \left(1.052 / \sqrt{22.434}\right)(153.20)^2 / 48.43 \times 27000 = 1.441 > 0.673 \]
\[ \rho = \left[1 - (0.22 / 1.441)\right] / 1.441 = 0.588 \]
\[ b_e = 0.588 \times 9.192 = 5.405 \text{ in.} \]
\[ b_2 = 5.405 / 2 = 2.703 \text{ in.} \]
\[ b_1 = 5.405 / \left(3 - (-0.938)\right) = 1.373 \text{ in.} \]
\[ b_1 + b_2 = 4.076 \text{ in.} \text{ close enough to 4.075 in.} \]

Thus, the solution stabilizes.

Hence we now compute the location of N.A. and moment of inertia using \( b_1 = 1.373 \text{ in.} \) and \( b_2 = 2.703 \text{ in.} \).
Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L (in.)</th>
<th>Top Fiber (in.)</th>
<th>Ly (in.²)</th>
<th>Ly² (in.⁴)</th>
<th>I'₁ About Own Axis (in.³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>b₁</td>
<td>1.373</td>
<td>0.841</td>
<td>1.155</td>
<td>0.971</td>
<td>0.216</td>
</tr>
<tr>
<td>b₂ + (9.5 - y_{cg}) - 0.154</td>
<td>7.151</td>
<td>5.771</td>
<td>41.268</td>
<td>238.160</td>
<td>30.473</td>
</tr>
<tr>
<td>Compression flange</td>
<td>1.346</td>
<td>0.030</td>
<td>0.040</td>
<td>0.001</td>
<td>--</td>
</tr>
<tr>
<td>Compression stiffener</td>
<td>0.600</td>
<td>0.278</td>
<td>0.167</td>
<td>0.946</td>
<td>0.009</td>
</tr>
<tr>
<td>Top 90° corner</td>
<td>0.195</td>
<td>0.075</td>
<td>0.015</td>
<td>0.001</td>
<td>--</td>
</tr>
<tr>
<td>Top 135° corner</td>
<td>0.097</td>
<td>0.042</td>
<td>0.004</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Bottom 135° corner</td>
<td>0.097</td>
<td>9.458</td>
<td>0.917</td>
<td>8.677</td>
<td>--</td>
</tr>
<tr>
<td>Bottom 90° corner</td>
<td>0.195</td>
<td>9.425</td>
<td>1.838</td>
<td>17.322</td>
<td>--</td>
</tr>
<tr>
<td>Bottom stiffener</td>
<td>0.600</td>
<td>9.222</td>
<td>5.333</td>
<td>51.027</td>
<td>0.009</td>
</tr>
<tr>
<td>Tension flange</td>
<td>1.346</td>
<td>9.470</td>
<td>12.747</td>
<td>120.710</td>
<td>--</td>
</tr>
<tr>
<td>Sum</td>
<td>13.000</td>
<td>63.684</td>
<td>436.915</td>
<td>30.707</td>
<td></td>
</tr>
</tbody>
</table>

Distance from top fiber to x-axis is

\[ y_{cg} = \frac{63.684}{13.000} = 4.899 \text{ in.} \]

Since the distance from top compression fiber to the neutral axis is greater than one half the beam depth (= 4.750 in.), a compression stress of 50 ksi will govern as assumed.

\[ I'_x = L y^2 + I' - Ly^2_{cg} \]
\[ = 436.915 + 30.707 - 13.000(4.899)^2 \]
\[ = 155.619 \text{ in.}^3 \]

Actual \[ I_x = I' t \]
\[ = 155.619 \times 0.060 \]
\[ = 9.337 \text{ in.}^4 \]

\[ S_e = \frac{I_x}{y_{cg}} \]
\[ M_n = SF_e y \]

\[ = 1.906 \times 50 \]

\[ = 95.30 \text{ kips-in.} \]

The design flexural strength, \( \Phi_b M_n \), based on initiation of yielding is determined as follows: (Section 3.3.1.1(1))

\[
\Phi_b = 0.90 \text{ (for section with stiffened compression flanges)}
\]

\[
\Phi_b M_n = 0.90 \times 95.30 = 85.77 \text{ kips-in.}
\]

2. Calculation of the effective moment of inertia for deflection determination at the service moment \( M_s \):

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of \( 1.2D + 1.6L \), the service moment can be determined as follows:

\[
\Phi_b M_n = 1.2M_{DL} + 1.6M_{LL}
\]

\[
= (1.2(M_{DL}/M_{LL})+1.6)M_{LL}
\]

\[
= [1.2(1/5)+1.6]M_{LL}
\]

\[
= 1.84M_{LL}
\]

\[
M_{LL} = \Phi_b M_n / 1.84 = 85.77/1.84 = 46.61 \text{ kips-in.}
\]

\[
M_s = M_{DL} + M_{LL}
\]

\[
= (1/5+1)M_{LL}
\]

\[
= 1.2(46.61) = 55.93 \text{ kips-in.}
\]

where

\[ M_{DL} = \text{Moment determined on the basis of nominal dead load} \]
\( M_{LL} \) = Moment determined on the basis of nominal live load

The procedure is iterative: one assumes the actual compressive stress \( f \) under this service moment \( M_s \). Knowing \( f \), one proceeds as usual to obtain \( S_e \) and checks to see if \((f \times S_e)\) is equal to \( M_s \) as it should. If not, reiterate until one obtains the desired level of accuracy. (Section 2.2.1-(2))

a. For the first iteration, assume a stress of \( f = 30 \) ksi in the top and bottom fibers of the section and that the web is fully effective.

Compression flange:
\[
S = 1.28 \sqrt{27000/30} = 38.40
\]
\[
S/3 = 12.80 < w/t = 22.43 < S = 38.40
\]
\[
I_a = 399(0.060)^4[(22.43/38.40)-0.33]^3
\]
\[
= 0.000085 \text{ in.}^4
\]
\[
I_s/I_a = 0.000540/0.000085 = 6.353
\]
\[
k = [4.82-5(0.493)](6.353)^{1/2}+0.43 = 6.366 > 2.785
\]
Use \( k = 2.785 \)

For deflection determination, the value of \( E_r \), reduced modulus of elasticity determined by using Eq. (2.2.1-7), is substituted for \( E_o \) in Eq. (2.2.1-4).

For a compression and tension stresses of \( f = 30 \) ksi, the corresponding \( E_{sc} \) and \( E_{st} \) values for Type 301 stainless steel are obtained from Table A2 or Figure A1 of the Standard as follows:
\[ E_{sc} = 22650 \text{ ksi}, \quad E_{st} = 26900 \text{ ksi} \]

\[ E_r = \frac{(E_{sc} + E_{st})}{2} \quad \text{(Eq. 2.2.1-7)} \]

\[ = \frac{(22650 + 26900)}{2} = 24775 \text{ ksi} \]

\[ \Lambda = \frac{(1.052\sqrt{2.785})(22.43)}{\sqrt{30/24775}} = 0.492 < 0.673 \]

\[ b_d = 1.346 \text{ in. (i.e. compression flange fully effective)} \]

Compression (upper) stiffener:

\[ f \text{ can be conservatively taken equal to 30 ksi as used in the top compression fiber.} \]

\[ \Lambda = \frac{(1.052\sqrt{0.50})(10.00)}{\sqrt{30/24775}} = 0.518 < 0.673 \]

therefore, \( d' = 0.600 \) in.

Since \( I_s/I_a = 6.353 > 1.0 \), it follows that \( d_s = d' = 0.600 \) in. (i.e. compression stiffener fully effective).

Thus, section is fully effective (since web was assumed fully effective).

\[ y = 9.5/2 = 4.750 \text{ in. (from symmetry)} \]

To check if web is fully effective:

\[ f_1 = \frac{[(4.750 - 0.154)/4.750](30)}{4.750} = 29.03 \text{ ksi} \]

\[ f_2 = -29.03 \text{ ksi} \]

\[ \psi = -29.03/29.03 = -1.000 \]

\[ k = 24.000 \]

For a compression and tension stresses of \( f = 29.03 \) ksi, it is found that the values of \( E_{sc} \) and \( E_{st} \) are equal to 23305 ksi and 26950 ksi, respectively.

\[ E_r = \frac{(E_{sc} + E_{st})}{2} \quad \text{(Eq. 2.2.1-7)} \]
\[
\lambda = \frac{(23305+26950)}{2} = 25130 \text{ ksi}
\]

\[
\rho = \left\lfloor \frac{1-(0.22/1.118)}{1.118} \right\rfloor = 0.718
\]

\[
b_e = 0.718 \times 9.192 = 6.600 \text{ in.}
\]

\[
b_2 = 6.600/2 = 3.300 \text{ in.}
\]

\[
b_1 = 6.600/[3-(-1)] = 1.650 \text{ in.}
\]

Compression portion of the web = \(y_{cg} - 0.154\)

\[
P = \frac{1-(0.22/1.118)}{1.118} = 0.718
\]

\[
b_1 + b_2 = 4.950 \text{ in.} > 4.596 \text{ in.}
\]

Thus \(b_1 + b_2\) shall be taken as 4.596 in. This verifies the assumption that the web is fully effective.

Full section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L \ (in.)</th>
<th>Distance from Centerline of Section \ (in.)</th>
<th>(Ly^2) \ (in.(^3))</th>
<th>(I'_L) \ (in.(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web</td>
<td>9.192</td>
<td>--</td>
<td>--</td>
<td>64.722</td>
</tr>
<tr>
<td>Stiffeners</td>
<td>2 x 0.600 = 1.200</td>
<td>4.472</td>
<td>23.999</td>
<td>0.018</td>
</tr>
<tr>
<td>90° corners</td>
<td>2 x 0.195 = 0.390</td>
<td>4.675</td>
<td>8.524</td>
<td>--</td>
</tr>
<tr>
<td>135° corners</td>
<td>2 x 0.097 = 0.194</td>
<td>4.708</td>
<td>4.300</td>
<td>--</td>
</tr>
<tr>
<td>Flanges</td>
<td>2 x 1.346 = 2.692</td>
<td>4.720</td>
<td>59.973</td>
<td>--</td>
</tr>
<tr>
<td>Sum</td>
<td>--</td>
<td>96.796</td>
<td>64.740</td>
<td>--</td>
</tr>
</tbody>
</table>

\[
I'_x = Ly^2 + I'_L
\]

\[
= 96.796 + 64.740 = 161.536 \text{ in.}^3
\]

\[
I_x = 161.536(0.060) = 9.692 \text{ in.}^4
\]

\[
S_e = I_x/y_{cg} = 9.692/4.750 = 2.040 \text{ in.}^3
\]
\[ M = f \times S_e = 30 \times 2.040 \]

\[ = 61.20 \text{ kips-in. not equal to } M_s = 55.93 \text{ kips-in.} \]

Thus, need to reiterate.

However, one sees that we need to assume a smaller stress than 30 ksi and since the section was fully effective for \( f = 30 \text{ ksi} \), it will be fully effective for \( f < 30 \text{ ksi} \).

Thus \( S_e = 2.040 \text{ in.}^3 \)

Therefore, the correct \( f \) at \( M_s = M_s / S_e = 55.93 / 2.040 \)

\[ = 27.42 \text{ ksi. and } I_x = 9.692 \text{ in.}^4 \text{ for deflection determination.} \]
Remark:

It was clearly seen that in the calculation of $\Phi_{bM_n}$, the assumption of the web being fully effective was not true. However, it would be interesting to see the percentage of error if one neglected the partial effectiveness of the web and proceeded with the assumption of a fully effective web.

To demonstrate: neglect the partial effectiveness of the web in the first approximation in the calculation of $\Phi_{bM_n}$.

Thus the whole section is fully effective. Full section properties about x-axis (from part 2):

$I_x = 9.692 \text{ in.}^4$

$s_e = 2.040 \text{ in.}^3$

$\Phi_{bM_n} = 0.90(2.040\times50) = 91.80 \text{ kips-in.}$

% error = $(91.80-85.77)/85.77 \times 100\% = 7.03\%$

Since the percentage of error is small, one could rationalize that in practical cases to get a first-hand quick answer one could assume the web being fully effective.
EXAMPLE 5.2 DEEP Z-SECTION W/STIFFENED FLANGES (ASD)

Use the data given in Example 5.1 (Figure 5.1) to determine the allowable moment, $M_a$, by using the Allowable Stress Design (ASD) method on the basis of initiation of yielding. Also determine the effective moment of inertia for deflection determination at the allowable moment. Use Type 301 stainless steel, 1/4-Hard: $F_y = 50$ ksi.

Solution:

1. Calculation of the allowable moment, $M_a$:

The effective section properties calculated by the ASD method are the same as those determined in Example 5.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

$$\Omega = 1.85 \quad \text{(Safety Factor stipulated in Table E of the Standard)}$$

$$M_n = 95.30 \text{ kips-in. (obtained from Example 5.1)}$$

$$M_a = \frac{M_n}{\Omega} \quad \text{(Eq. E-1)}$$

$$= \frac{95.30}{1.85}$$

$$= 51.51 \text{ kips-in.}$$

2. Calculation of the effective moment of inertia for deflection determination at the allowable moment, $M_a$:

For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 5.1 for the LRFD method, except that the computed moment $M (= f x S_e)$ should be equal to $M_a$. 

82
From the results of Example 5.1, it can be seen that by using a stress of \( f = 30 \text{ ksi} \), the computed \( S_e = 2.040 \text{ in.}^3 \) which is based on the fully effective section. If one assumes a smaller stress of \( f = 25.25 \text{ ksi} \), the effective section modulus will also be determined on the basis of its full cross section, i.e., \( S_e = 2.040 \text{ in.}^3 \). Therefore, \( f x S_e = 25.25 \times 2.040 = 51.51 \text{ kips-in.} \), which is equal to \( M_a \) determined above.

Therefore, the computed \( I_x = 9.692 \text{ in.}^4 \) obtained from the full section properties can be used for deflection determination.
EXAMPLE 6.1 HAT SECTION (LRFD)

(Complete Flexural Design)

By using the Load and Resistance Factor Design (LRFD) method, check the adequacy of the hat section given in Figure 6.1 for bending moment, shear, web crippling, and deflection. Use Type 316 stainless steel, 1/4-Hard. Assume dead load to live load ratio $D/L = 1/5$ and $1.2D + 1.6L$ governs the design.

![Diagram of hat section](image)

*Figure 6.1 Section for Example 6.1*

Given:

1. Section: Hat section, as shown in sketch.
2. Span length: $L = 8$ ft., with simple supports, no overhang, and 6-in. support bearing lengths.

Solution:

1. Properties of $90^\circ$ corners:

   Corner Radius, $r = R + t/2 = 3/32 + 0.060/2 = 0.124$ in.
Length of arc, \( u = 1.57r = 1.57 \times 0.124 = 0.195 \text{ in.} \)

Distance of c.g. from center of radius,

\( c = 0.637r = 0.637 \times 0.124 = 0.079 \text{ in.} \)

The moment of inertia, \( I' \), of corner about its own centroidal axis is negligible.

2. Nominal Section Strength, \( M_n \) (Section 3.3.1.1)

a. Procedure I - Based on Initiation of Yielding

Computation of \( I_x, S_e \), and \( M_n \) (first approximation)

* Assume a compressive stress of \( f = F_y = 50 \text{ ksi} \) in the top fiber of the section. (See Table A1 of the Standard for yield strength.)

* Also assume web is fully effective.

Element 4:

\[ h/t = 3.692/0.060 = 61.53 < (h/t)_{\text{max}} = 200 \text{ OK (Section 2.1.2-(1))} \]

Assumed fully effective.

Element 5:

\[ w/t = 8.692/0.060 = 144.9 < 400 \text{ OK (Section 2.1.1-(1)-(ii))} \]

\[ k = 4 \]

\[ \lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o} \]

(Eq. 2.2.1-4)

\( E_o \) is equal to 27000 ksi, which is obtained from Table A4 of the Standard.

\[ \lambda = (1.052/\sqrt{4})(144.9)\sqrt{50/27000} = 3.280 > 0.673 \]

\[ \rho = [1-(0.22/\lambda)]/\lambda \]

(Eq. 2.2.1-3)
b = \rho w

= 0.284 \times 8.692

= 2.469 \text{ in.}

Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L Effective Length (in.)</th>
<th>y Distance from Top Fiber (in.)</th>
<th>Ly (in.²)</th>
<th>Ly² (in.³)</th>
<th>I' About Own Axis (in.³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 x 0.596 = 1.192</td>
<td>3.548</td>
<td>4.229</td>
<td>15.005</td>
<td>0.035</td>
</tr>
<tr>
<td>2</td>
<td>4 x 0.195 = 0.780</td>
<td>3.925</td>
<td>3.062</td>
<td>12.016</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>2 x 2.692 = 5.384</td>
<td>3.970</td>
<td>21.375</td>
<td>84.857</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>2 x 3.692 = 7.384</td>
<td>2.000</td>
<td>14.768</td>
<td>29.536</td>
<td>8.388</td>
</tr>
<tr>
<td>5</td>
<td>2.469</td>
<td>0.030</td>
<td>0.074</td>
<td>0.002</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>2 x 0.195 = 0.390</td>
<td>0.037</td>
<td>0.029</td>
<td>0.002</td>
<td>--</td>
</tr>
<tr>
<td>Sum</td>
<td>17.599</td>
<td>43.537</td>
<td>141.418</td>
<td>8.423</td>
<td></td>
</tr>
</tbody>
</table>

The distance from the top fiber to the neutral axis is

\[ y_{cg} = \frac{L_y}{L} = \frac{43.537}{17.599} = 2.474 \text{ in.} \]

Since the distance from top compression fiber to the neutral axis, \( y_{cg} \), is greater than one half the beam depth, a compressive stress of \( F_y \) will govern as assumed.

\[ I'_{x} = Ly^2 + I'_{1} - Ly_{cg}^2 \]

\[ = 141.418 + 8.423 - 17.599(2.474)^2 \]

\[ = 42.12 \text{ in.}^3 \]

Actual \( I'_{x} = tI'_{x} \)
= (0.060)(42.12) = 2.53 in.²

Check Web

\[ f_1 = (2.320/2.474)(50) = 46.89 \text{ ksi (compression)} \]

\[ f_2 = -(1.372/2.474)(50) = -27.73 \text{ ksi (tension)} \]

\[ \Psi = f_2/f_1 = -27.73/46.89 = -0.591 \]

\[ k = 4+2(1-\Psi)^2+2(1-\Psi) \quad \text{(Eq. 2.2.2-4)} \]
\[ = 4+2[1-(-0.591)]^2+2[1-(-0.591)] \]
\[ = 15.24 \]

\[ \lambda = (1.052/\sqrt{k})(w/t)/\sqrt{f/E_o}, \quad f = f_1 \quad \text{(Eq. 2.2.1-4)} \]
\[ = (1.052/\sqrt{15.24})(61.53)/\sqrt{46.89/27000} = 0.691 > 0.673 \]

\[ \rho = [1-(0.22/\lambda)]/\lambda \quad \text{(Eq. 2.2.1-3)} \]
\[ = [1-(0.22/0.691)]/0.691 = 0.986 \]

\[ b_e = \rho w \quad \text{(Eq. 2.2.1-2)} \]
\[ = 0.986 \times 3.692 \]
\[ = 3.640 \text{ in.} \]

\[ b_2 = b_e/2 \quad \text{(Eq. 2.2.2-2)} \]
\[ = 3.640/2 = 1.820 \text{ in.} \]

\[ b_1 = b_e/(3-\Psi) \quad \text{(Eq. 2.2.2-1)} \]
\[ = \frac{3.640}{3-(-0.591)} = 1.014 \text{ in.} \]

\[ b_1 + b_2 = 1.014 + 1.820 = 2.834 \text{ in.} > 2.320 \text{ in. (compression portion of web, see sketch shown above)} \]

Therefore, web is fully effective.

\[ S_e = \frac{I_x}{y_{cg}} \]
\[ = \frac{2.53}{2.474} \]
\[ = 1.02 \text{ in.}^3 \]

\[ M_n = S_F e_y \]
\[ = (1.02)(50) \]
\[ = 51.0 \text{ kips-in.} \]

b. Procedure II - Based on Inelastic Reserve Capacity

\[ \lambda_1 = \left( \frac{1.11/y}{F_y/E_o} \right) \]
\[ = \left( \frac{1.11/50}{27000} \right) = 25.79 \]

\[ \lambda_2 = \left( \frac{1.28/y}{F_y/E_o} \right) \]
\[ = \left( \frac{1.28/50}{27000} \right) = 29.74 \]

\[ w/t = 8.692/0.06 = 144.9 \]

For \( w/t > \lambda_2 \), \( C_y = 1 \)

Maximum compressive strain = \( C_y e_y = e_y \)

Therefore, the nominal moment, \( M_n \) is the same as the \( M_n \) determined by procedure I because the compression flange will yield first.

3. Design Flexural Strength, \( \Phi_b M_n \) (Section 3.3.1)

\[ \Phi_b = 0.90 \text{ (for section with stiffened compression flanges)} \]
\[ \Phi_b M_n = 0.90 \times 51.0 = 45.9 \text{ kips-in.} \]

The factored load combination is as follows:
\[ w_u = 1.2w_{DL} + 1.6w_{LL} = 1.2(0.02) + 1.6(0.25) = 0.424 \text{ kips/ft.} \]

Maximum required flexural strength for a simply supported beam is

\[ M_u = w_u L^2/8 = 0.424(8)^2(12)/8 \]
\[ = 40.70 \text{ kips-in.} \quad \Phi_b M_n = 45.9 \text{ kips-in.} \quad \text{OK} \]

4. Strength for Shear Only (Section 3.3.2)

The required shear strength at any section shall not exceed the design shear strength \( \Phi_v V_n \):

\[ \Phi_v = 0.85 \]

\[ V_n = 4.84E_o t^2 (G_s/G_o)/h \quad (\text{Eq. 3.3.2-1}) \]

\[ v_n = V_n/(ht) \]
\[ = 4.84E_o (G_s/G_o)/(h/t)^2 \]

In the determination of the shear strength, it is necessary to select a proper value of \( G_s/G_o \) for the assumed stress from Table A12 or Figure A90 of the Standard. For the first approximation, assume a shear stress of \( v = F_y/2 = 25 \text{ ksi} \) and the corresponding value of \( G_s/G_o \) is equal to 0.888. Thus,

\[ h/t = 3.692/0.060 = 61.53 \]

\[ v_n = 4.84(27000)(0.888)/(61.53)^2 \]
\[ = 30.7 \text{ ksi} \quad \text{> assumed stress } v=25 \text{ ksi} \quad \text{NG} \]

For a second approximation, assume a stress of \( f=28.82 \text{ ksi} \) and its corresponding value of \( G_s/G_o \) is 0.836.

\[ v_n = 4.84(27000)(0.836)/(61.53)^2 \]
\[ = 28.85 \text{ ksi} = \text{assumed stress } \text{OK} \]

Therefore, the total shear strength, \( V_n \), for hat section is

\[ V_n = (2 \text{ webs})(v_n)(ht) \]
The design shear strength is determined as follows:

\[ \Phi V_n = 0.85(12.78) = 10.85 \text{ kips} \]

\[ \Phi V_n < 2(0.95F_{yv}ht) = 2(0.95 \times 42 \times 3.692 \times 0.06) = 17.68 \text{ kips OK} \]

(The shear yield strength, \( F_{yv} \), is obtained from Table A1 of the Standard.)

Maximum Required Shear Strength = Reaction

\[ V_u = \omega_u L/2 = 0.424(8)/2 = 1.70 \text{ k} \leq \Phi V_n = 10.85 \text{ k OK} \]

5. Web Crippling Strength for End Reaction (Section 3.3.4)

\[ R/t = (3/32)/0.06 = 1.563 < 6 \text{ OK} \]

\[ h/t = 3.692/0.06 = 61.53 < 200 \]

Table 2 of the Standard

Stiffened Flanges

\[ P_n = t^2C_3C_4C_6 [(331 - 0.61(h/t))(1 + 0.01(N/t))] \]

\[ C_3 = (1.33 - 0.33k)k \]

\[ k = F_{y}/33 = 50/33 = 1.515 \]

\[ C_3 = (1.33 - 0.33(1.515))(1.515) = 1.257 \]

\[ C_4 = (1.15 - 0.15R/t) \leq 1.0 \text{ but not less than 0.50} \]

\[ (1.15 - 0.15R/t) = [1.15 - 0.15(1.563)] = 0.916 \leq 1.0 \text{ OK} \]

\[ > 0.50 \text{ OK} \]
\[ C_4 = 0.916 \]
\[ C_\Theta = 0.7 + 0.3(\Theta/90)^2 \]  
(Eq. 3.3.4-20)
\[ = 0.7 + 0.3(90/90)^2 = 1.0 \]
\[ P_n = (0.06)^2(1.257)(0.916)(1.0)[331-0.61(61.53)] \]
\[ \times [1 + 0.01(6/0.06)] = 2.43 \text{k/web} \]
\[ P_n = (2 \text{webs})(2.43 \text{k/web}) = 4.86 \text{k} \]
\[ \phi_w = 0.70 \]
\[ \phi_w P_n = 0.70(4.86) = 3.40 \text{k} \]
Reaction = 1.70 k < \phi_w P_n = 3.40 k OK

6. Deflection Determination at Service Moment \( M_s \)

Find \( I_{\text{eff}} \) at \( M_s = WL^2/8 = 0.27(8)^2(12)/8 = 25.92 \text{kips-in.} \)

Computation of \( I_{\text{eff}} \), first approximation

* Assume a stress of \( f = 0.6F_y = 30 \text{ksi} \) in the top and bottom fibers of the section.

* Also assume web is fully effective.

Element 5:

For deflection determination, the value of \( E_r \), reduced modulus of elasticity determined by using Eq. (2.2.1-7), is substituted for \( E_o \) in Eq. (2.2.1-4). For a compression and tension stresses of \( f = 30 \text{ksi} \), the corresponding \( E_{sc} \) and \( E_{st} \) values for Type 316 stainless steel are obtained from Table A2 or Figure A1 of the Standard as follows:

\[ E_{sc} = 22650 \text{ksi}, \quad E_{st} = 26900 \text{ksi} \]
\[ E_r = \frac{(E_{sc} + E_{st})}{2} \quad \text{(Eq. 2.2.1-7)} \]
\[ = \frac{(22650 + 26900)}{2} = 24775 \text{ ksi} \]

Thus, for compression flange (Element 5):
\[ \lambda = \frac{(1.052/\sqrt{4})(144.9)\sqrt{30/24775}}{2} = 2.652 > 0.673 \quad \text{(Eq. 2.2.1-4)} \]
\[ \rho = \frac{[1-(0.22/2.652)]}{2.652} = 0.346 \quad \text{(Eq. 2.2.1-3)} \]
\[ b_d = \rho w \quad \text{(Eq. 2.2.1-6)} \]
\[ = 0.346(8.692) = 3.007 \text{ in.} \]

Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L (in.)</th>
<th>Distance from Top Fiber (in.)</th>
<th>Ly (in.²)</th>
<th>Ly² (in.³)</th>
<th>I' About Own Axis (in.³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 x 0.596 = 1.192</td>
<td>3.548</td>
<td>4.229</td>
<td>15.005</td>
<td>0.035</td>
</tr>
<tr>
<td>2</td>
<td>4 x 0.195 = 0.780</td>
<td>3.925</td>
<td>3.062</td>
<td>12.016</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>2 x 2.692 = 5.384</td>
<td>3.970</td>
<td>21.375</td>
<td>84.857</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>2 x 3.692 = 7.384</td>
<td>2.000</td>
<td>14.768</td>
<td>29.536</td>
<td>8.388</td>
</tr>
<tr>
<td>5</td>
<td>3.007</td>
<td>0.030</td>
<td>0.090</td>
<td>0.003</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>2 x 0.195 = 0.390</td>
<td>0.075</td>
<td>0.029</td>
<td>0.002</td>
<td>--</td>
</tr>
<tr>
<td>Sum</td>
<td>18.137</td>
<td>43.553</td>
<td>141.419</td>
<td>8.423</td>
<td></td>
</tr>
</tbody>
</table>

The distance from the top fiber to the neutral axis is
\[ y_{cg} = \frac{Ly}{L} = \frac{43.553}{18.137} = 2.401 \text{ in.} \]
\[ I'_{eff} = Ly² + I'_{1} - Ly²_{cg} \]
\[ = 141.419 + 8.423 - 18.137(2.401)^{2} \]
\[ = 45.29 \text{ in.}³ \]
\[ \text{Actual } I_{eff} = tI'_{eff} \]
\[ = (0.060)(45.29) = 2.72 \text{ in.}⁴ \]
Check Web

* Should be fully effective

\[ f_1 = \frac{2.247}{2.401}(30) = 28.08 \text{ ksi (compression)} \]

\[ f_2 = -\frac{1.445}{2.401}(30) = -18.05 \text{ ksi (tension)} \]

\[ \Psi = \frac{f_2}{f_1} = -18.05/28.08 = -0.643 \]

\[ k = 4 + 2(1-\Psi)^2 + 2(1-\Psi) \]

\[ = 4 + 2(1-(-0.643))^2 + 2(1-(-0.643)) \]

\[ = 16.16 \]

\[ \lambda = \frac{1.052/\sqrt{k}(w/t)\sqrt{f/E_r}}{f} = f_1 \text{ (Eq. 2.2.1-4)} \]

For a compression stress of \( f_1 = 28.08 \text{ ksi} \) and a tension stress of \( f_2 = 18.05 \text{ ksi} \), the values of \( E_{sc} \) and \( E_{st} \) are found as follows: \( E_{sc} = 24000 \text{ ksi} \), \( E_{st} = 27000 \text{ ksi} \).

\[ E_r = (24000+27000)/2 \text{ (Eq. 2.2.1-7)} \]

\[ = 25500 \text{ ksi} \]

\[ \lambda = (1.052/\sqrt{16.16})(61.53)\sqrt{28.08/25500} = 0.534 < 0.673 \]

\[ b = w \text{ (Eq. 2.2.1-1)} \]

\[ b_e = 3.692 \text{ in.} \]

\[ b_2 = \frac{b_e}{2} \text{ (Eq. 2.2.2-2)} \]

\[ = 3.692/2 = 1.846 \text{ in.} \]
\[ b_1 = \frac{b_e}{3-\psi} \quad \text{(Eq. 2.2.2-1)} \]

\[ = \frac{3.692}{3-(-0.643)} = 1.007 \text{ in.} \]

\[ b_1 + b_2 = 1.013 + 1.846 = 2.859 \text{ in.} > 2.216 \text{ in.} \text{ (compression portion of web, see the sketch shown above)} \]

Therefore, web is fully effective.

\[ S_{\text{eff}} = \frac{S_{\text{eff}}}{y_{\text{cg}}} = 2.72/2.401 = 1.13 \text{ in.}^3 \]

\[ M = S_{\text{eff}}(0.6F_y) \]

\[ = (1.13)(30) \]

\[ = 33.9 \text{ kips-in.} \]

To determine \( I_{\text{eff}} \) at \( M_s = 25.92 \text{ kips-in.} \), an approximation is used by extrapolating the following values:

(1) \( M = 51.00 \text{ kips-in.}, I = 2.53 \text{ in.}^4 \)

(2) \( M = 33.90 \text{ kips-in.}, I = 2.72 \text{ in.}^4 \)

(3) \( M = 25.92 \text{ kips-in.}, I = ? \)

\[ \frac{(25.92-33.9)}{(I-2.72)} = \frac{(33.9-51.0)}{(2.72-2.52)} \]

\[ -7.98 = -90.00(I-2.72) \]

\[ 0.0887 = I-2.72 \]

\[ I = 2.81 \text{ in.}^4 \]

Use \( I = 2.81 \text{ in.}^4 \) in deflection calculations.

\[ \text{Deflection} = \frac{5WL^4}{384E_I} \]
EXAMPLE 6.2  HAT SECTION (ASD)

Rework Example 6.1 by using the Allowable Stress Design (ASD) method.

Solution:

1. Calculation of the allowable moment, $M_a$:

   The effective section properties calculated by the ASD method are the same as those determined in Example 6.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

   \[ \Omega = 1.85 \] (Safety Factor stipulated in Table E of the Standard)

   \[ M_n = 51.0 \text{ kips-in.} \] (obtained from Example 6.1)

   \[ M_a = \frac{M_n}{\Omega} \]  \hspace{1cm} (Eq. E-1)

   \[ = \frac{51.0}{1.85} \]

   \[ = 27.57 \text{ kips-in.} \]

   The maximum applied moment, $M_{max} = \frac{wL^2}{8}$

   \[ M_{max} = (0.25+0.02)(8)^2(12")/8 = 25.92 \text{ kips-in.} < 27.57 \text{ kips-in.} \hspace{0.5cm} \text{OK} \]

2. Strength for Shear Only.

   The nominal shear strength at the section was calculated in Example 6.1.(4) as follows:

   \[ V_n = (2 \text{ webs})(v_n)(ht) \]

   \[ = 2(28.85)(3.692\times0.060) \]

   \[ = 12.78 \text{ kips} \]

   The allowable shear strength is determined as follows:

   \[ V_a = \frac{V_n}{\Omega} = \frac{12.78}{1.85} \]

   \[ V_a = 6.91 \text{ kips} < 2x(0.95F_{yy}ht)/1.64 = 11.35 \text{ kips}, \]
Use $V_a = 6.91 \text{kips}$

Maximum Shear Force = Reaction

$V_u = \frac{wL}{2} = 0.27(8)/2 = 1.08 \text{k} < V_a = 6.91 \text{kips} \quad \text{OK}$

3. Web Crippling Strength.

The nominal web crippling strength was determined in Example 6.1 as follows:

$P_n = (0.06)^2(1.257)(0.916)(1.0) \ 331 - 0.61(61.53)$

$x 1 + 0.01(6/0.06) = 2.43 \text{k/web}$

$P_n = (2 \text{ webs})(2.43 \text{k/web}) = 4.86 \text{k}$

$\Omega = 2.0$ (for single web)

$P_a = P_n/\Omega$

$= 4.86/2.0 = 2.43 \text{kips}$

Reaction = 1.08 kips < $P_a = 2.43 \text{kips}$ \quad \text{OK}$

4. Deflection Determination at Allowable Moment $M_a$

For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 8.1 for the LRFD method, except that the computed moment $M (= f \times S_e)$ should be equal to $M_a$.

From the results of Example 6.1, it can be seen that to determine the moment of inertia $I_{eff}$ at $M_a = 25.57 \text{kips-in.}$, an approximation can be used by extrapolating the following values:

(1) $M = 51.00 \text{kips-in.}, I = 2.53 \text{ in.}^4$

(2) $M = 33.90 \text{kips-in.}, I = 2.72 \text{ in.}^4
(3) $M = 25.57$ kips-in., $I = ?$

$\frac{(25.57-33.9)}{(I-2.72)} = \frac{(33.9-51.0)}{(2.72-2.53)}$

$I = 2.81$ in.$^4$

Use $I = 2.81$ in.$^4$ in deflection calculations.

(Deflection = $5wL^4/384EI$)
EXAMPLE 7.1 HAT SECTION w/INTERMEDIATE STIFFENER (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\phi M_n$, based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the service moment. Use Type 316 stainless steel, 1/4-Hard. Compare structural economy of this section with an almost identical section without an intermediate stiffener computed in Example 6.1.

![Figure 7.1 Section for Example 7.1](image)

**Given:**
1. Section: Hat section, as shown in sketch.
2. Dead load to live load ratio $D/L = 1/5$ and $1.2D+1.6L$ governs the design.

**Solution:**
1. Properties of 90° corners:
   - Corner Radius, $r = R + t/2 = 3/32 + 0.060/2 = 0.124$ in.
   - Length of arc, $u = 1.57r = 1.57 \times 0.124 = 0.195$ in.
Distance of c.g. from center of radius,

\[ c = 0.637r = 0.637 \times 0.124 = 0.079 \text{ in.} \]

The moment of inertia, \( I' \), of corner about its own centroidal axis is negligible.

2. Nominal Section Strength, \( M_n \) (Section 3.3.1.1)

Computation of \( I_{x'} \), \( S_e \), and \( M_n \) for the first approximation:

* Assume a compressive stress of \( f = F_y = 50 \text{ ksi} \) in the top fiber of the section. (See Table A1 of the Standard for yield strength values.)

* Also assume web is fully effective.

Element 4:

\[ h/t = 3.692/0.060 = 61.53 < 200 \text{ OK (Section 2.1.2-(1))} \]
Assumed fully effective

Element 5:

\[ E_o = 27000 \text{ ksi (Table A4 of the Standard)} \]

\[ S = 1.28\sqrt{E_o/f} \quad \text{(Eq. 2.4-1)} \]

\[ = 1.28\sqrt{27000/50} = 29.74 \]

\[ b_o/t = 8.692/0.060 = 144.9 < 400 \text{ OK (Section 2.1.1-(1)-(ii))} \]

\[ 3S = 3(29.74) = 89.22 \]

For \( b_o/t > 3S \) (Case III)

\[ I_a = t^4\left\{ \frac{[128(b_o/t)/S]}{285} \right\} \quad \text{(Eq. 2.4.1-9)} \]

\[ = (0.06)^4\left\{ \frac{[128(144.9)/29.74]}{285} \right\} = 0.004038 \text{ in.} \]
Determine full section properties of stiffener 7:

All inner radii = 3/32

\[ r = R + \frac{t}{2} = \frac{3}{32} + \frac{0.060}{2} = 0.124 \text{ in.} \]

\[ u = 1.57r = 1.57(0.124) = 0.195 \text{ in.} \]

\[ c = 0.637r = 0.637(0.124) = 0.079 \text{ in.} \]

<table>
<thead>
<tr>
<th>Element</th>
<th>L Length (in.)</th>
<th>Distance from Top Fiber (in.)</th>
<th>Ly (in.²)</th>
<th>Ly² (in.³)</th>
<th>I' About Own Axis (in.³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>2 × 0.195 = 0.390</td>
<td>0.075</td>
<td>0.0293</td>
<td>0.0022</td>
<td>--</td>
</tr>
<tr>
<td>9</td>
<td>2 × 0.350 = 0.700</td>
<td>0.329</td>
<td>0.2303</td>
<td>0.0758</td>
<td>0.0071</td>
</tr>
<tr>
<td>10</td>
<td>2 × 0.195 = 0.390</td>
<td>0.583</td>
<td>0.2274</td>
<td>0.1326</td>
<td>--</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td>1.480</td>
<td>0.4870</td>
<td>0.2106</td>
<td>0.0071</td>
</tr>
</tbody>
</table>

Distance from top fiber to the neutral axis is

\[ y_{cg} = \frac{Ly}{L} = \frac{0.4870}{1.480} = 0.329 \text{ in.} \]

Total area of section, \( Lt = (1.480)(0.060) = 0.0888 \text{ in.}^2 \)

\[ I'_s = Ly^2 + I'_{1} - Ly^2_{cg} \]

\[ = 0.2106 + 0.0071 - 1.480(0.329)^2 \]

\[ = 0.0575 \text{ in.}^3 \]

Actual \( I_s = tI'_s \)

\[ = (0.060)(0.0575) = 0.00345 \text{ in.}^4 \]
Reduced Area of Stiffener

Element 9:

Stiffened element, \( k = 4 \)

\( f = F_y = 50 \text{ ksi} \)

\( w/t = 0.350/0.060 = 5.83 < 400 \text{ OK (Section 2.1.1-(1)-(ii))} \)

\[
\lambda = \frac{(1.052/\sqrt{k})(w/t)\sqrt{f/E_o}}{(1.052/\sqrt{4})(5.83)\sqrt{50/27000}} = 0.132 < 0.673 \tag{Eq. 2.2.1-4}
\]

\( b = w = 0.350 \text{ in. (fully effective)} \) \tag{Eq. 2.2.1-1}

\[
A_s' = L_t = 0.0888 \text{ in.}^2
\]

\[
A_s = A_s'(I_s/I_a) \leq A_s'
\]

\[
= 0.0888(0.00345/0.00439) = 0.0888(0.7859) = 0.0698 \text{ in.}^2 < A_s' \text{ OK}
\]

\[
L_s = (A_s/t) = (0.0698/0.060) = 1.163 \text{ in.}
\]

Continuing with element 5:

\( k = 3\left(I_s/I_a\right)^{1/3} + 1 \leq 4 \) \tag{Eq. 2.4.1-10}

\[
= 3(0.7859)^{1/3} + 1 = 3.768 < 4 \text{ OK}
\]

\( w/t = 4.098/0.060 = 68.30 \)

\[
\lambda = \frac{(1.052/\sqrt{k})(w/t)\sqrt{f/E_o}}{(1.052/\sqrt{3.768})(68.30)\sqrt{50/27000}} = 1.593 > 0.673 \tag{Eq. 2.2.1-4}
\]

\[
\rho = (1-0.22/\lambda)/\lambda
\]

\[
= (1-0.22/1.593)/1.593 = 0.541
\] \tag{Eq. 2.2.1-3}

\( b = \rho w \) \tag{Eq. 2.2.1-2}

\[
= 0.541(4.098) = 2.217 \text{ in.}
\]
Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L Effective Length (in.)</th>
<th>Distance from Top Fiber (in.)</th>
<th>Ly (in.^2)</th>
<th>Ly^2 (in.^3)</th>
<th>I' About Own Axis (in.^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 x 0.596 = 1.192</td>
<td>3.548</td>
<td>4.229</td>
<td>15.005</td>
<td>0.035</td>
</tr>
<tr>
<td>2</td>
<td>4 x 0.195 = 0.780</td>
<td>3.925</td>
<td>3.062</td>
<td>12.016</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>2 x 2.692 = 5.384</td>
<td>3.970</td>
<td>21.375</td>
<td>84.857</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>2 x 3.692 = 7.384</td>
<td>2.000</td>
<td>14.768</td>
<td>29.536</td>
<td>8.388</td>
</tr>
<tr>
<td>5</td>
<td>2 x 2.217 = 4.434</td>
<td>0.030</td>
<td>0.133</td>
<td>0.004</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>2 x 0.195 = 0.390</td>
<td>0.075</td>
<td>0.029</td>
<td>0.002</td>
<td>--</td>
</tr>
<tr>
<td>7</td>
<td>Stiffener</td>
<td>1.163</td>
<td>0.329</td>
<td>0.383</td>
<td>0.126 0.058</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td>20.727</td>
<td>43.979</td>
<td>141.546</td>
<td>8.481</td>
</tr>
</tbody>
</table>

The distance from the top fiber to the neutral axis is

\[ y_{cg} = \frac{Ly}{L} = \frac{44.018}{21.035} = 2.093 \text{ in.} \]

Since the distance from the top compression fiber to the neutral axis is greater than one half the beam depth, a compressive stress of \( F_y \) will govern as assumed.

\[ I'_x = Ly^2 + I'_1 - Ly^2_{cg} \]
\[ = 141.546 + 8.481 - 20.727(2.122)^2 \]
\[ = 56.70 \text{ in.}^3 \]

Actual \( I_x = tI'_x \)
\[ = (0.060)(56.70) = 3.40 \text{ in.}^4 \]
Check Web

\[
f_1 = (1.968/2.122)(50) = 46.37 \text{ ksi (compression)}
\]

\[
f_2 = -(1.724/2.122)(50) = -40.62 \text{ ksi (tension)}
\]

\[
\Psi = \frac{f_2}{f_1} = \frac{-40.62}{46.37} = -0.876
\]

\[
k = 4 + 2(1-\Psi)^2 + 2(1-\Psi)
\]

\[
= 4 + 2[1-(-0.876)]^2 + 2[1-(-0.876)]
\]

\[
= 20.96
\]

\[
\lambda = \frac{(1.052/\sqrt{k})(w/t)\sqrt{f/E_o}}{f} = f_1
\]

\[
= \frac{(1.052/\sqrt{20.96})(61.53)/\sqrt{46.37}/27000}{46.37} = 0.586 < 0.673
\]

\[
b = w
\]

\[
b_e = 3.692 \text{ in.}
\]

\[
b_2 = \frac{b_e}{2}
\]

\[
= 3.692/2 = 1.846 \text{ in.}
\]

\[
b_1 = \frac{b_e}{(3-\Psi)}
\]

\[
= 3.692/(3-(-0.876)) = 0.953 \text{ in.}
\]

\[
b_1 + b_2 = 0.953 + 1.846 = 2.799 \text{ in.} > 1.939 \text{ in. (compression portion of web, see sketch shown above)}
\]

Therefore, web is fully effective.

\[
e \quad S_e = I_x/y_{cg}
\]
3. Design Flexural Strength, $\phi_bM_n$ (Section 3.3.1)

$\phi_b = 0.90$ (for section with stiffened compression flanges)

$\phi_bM_n = 0.90 \times 80.0 = 72.00$ kips-in.

4. Deflection Determination at Service Moment $M_s$

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of $1.2D+1.6L$, the service moment can be determined as follows:

$\phi_bM_n = 1.2M_{DL} + 1.6M_{LL}$

$= [1.2(M_{DL}/M_{LL})+1.6]M_{LL}$

$= [1.2(1/5)+1.6]M_{LL}$

$= 1.84M_{LL}$

$M_{LL} = \phi_bM_n/1.84 = 72.00/1.84 = 39.13$ kips-in.

$M_s = M_{DL} + M_{LL}$

$= (1/5+1)M_{LL}$

$= 1.2(39.13) = 46.96$ kips-in.

where

$M_{DL} = \text{Moment determined on the basis of nominal dead load}$

$M_{LL} = \text{Moment determined on the basis of nominal live load}$
Find $I_{eff}$ at $M_s = 46.96$ kips-in.

Computation of $I_{eff}$, first approximation

* Assume a stress of $f = 0.6F_y = 30$ ksi in the top and bottom fibers of the section.

* Web is fully effective, because it was fully effective at a higher stress gradient.

* Element 9 of the stiffener, which was fully effective at $f = 50$ ksi will also be fully effective at $f = 30$ ksi.

Element 5:

\[ S = 1.28\sqrt{E_o/f}, \quad f = 30 \]  
\[ = 1.28\sqrt{27000/30} = 38.40 \]

\[ b_o/t = 144.9 \]

\[ 3S = 3(38.40) = 115.20 \]

For $b_o/t > 3S$ (Case III)

\[ I_a = t^4\{[128(b_o/t)/S]-285\} \]  
\[ = (0.06)^4\{[128(144.9)/38.40]-285\} = 0.002566 \text{ in.}^4 \]

\[ I_s = 0.00345 \text{ in.}^4 \]

\[ k = 3(I_s/I_a)^{1/3}+1 \leq 4 \]  
\[ = 3(0.00345/0.002566)^{1/3}+1 = 4.311 > 4 \]

\[ k = 4 \]

\[ w/t = 68.30 \]

For deflection determination, the value of $E_r$, reduced modulus of elasticity determined by using Eq. (2.2.1-7), is substituted for $E_o$ in Eq. (2.2.1-4). For a compression and tension stresses of $f = 30$ ksi, the corresponding $E_{sc}$ and $E_{st}$ values for
Type 316 stainless steel are obtained from Table A2 or Figure A1 of the Standard as follows:

\[ E_{sc} = 22650 \text{ ksi}, \quad E_{st} = 26900 \text{ ksi} \]
\[ E_r = \frac{(E_{sc} + E_{st})}{2} \quad \text{(Eq. 2.2.1-7)} \]
\[ = \frac{(22650 + 26900)}{2} = 24775 \text{ ksi} \]

Thus, for compression flange (Element 5):

\[ \lambda = \frac{1.052}{\sqrt{\frac{E}{f}}} (w/t) \sqrt{\frac{f}{E_r}}, \quad f = 30 \text{ ksi} \quad \text{(Eq. 2.2.1-4)} \]
\[ = \frac{1.052}{\sqrt{\frac{1}{4}} (68.30) \sqrt{30/24775}} = 1.250 > 0.673 \]
\[ \rho = \frac{1-0.22/\lambda}{\lambda} \quad \text{(Eq. 2.2.1-3)} \]
\[ = \frac{1-0.22/1.250}{1.250} = 0.659 \]
\[ b = \rho w \quad \text{(Eq. 2.2.1-2)} \]
\[ = 0.659(4.098) = 2.701 \text{ in.} \]

Stiffener, Element 7:

\[ A_s = A'_{s} \left( \frac{I_s}{I_{a}} \right) \leq A'_{s} \quad \text{(Eq. 2.4.1-11)} \]
\[ = 0.0888(0.00345/0.002566) \]
\[ = 0.133 \text{ in.}^2 > A'_{s} \]
\[ A_s = A'_{s} = 0.0888 \text{ in.}^2 \]
\[ L_s = A_s/t = 0.0888/0.060 = 1.480 \text{ in.} \]
Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L (in.)</th>
<th>Distance from Top Fiber (in.)</th>
<th>Ly (in.²)</th>
<th>Ly² (in.⁴)</th>
<th>I' About Own Axis (in.²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 x 0.596 = 1.192</td>
<td>3.548</td>
<td>4.229</td>
<td>15.005</td>
<td>0.035</td>
</tr>
<tr>
<td>2</td>
<td>4 x 0.195 = 0.780</td>
<td>3.925</td>
<td>3.062</td>
<td>12.016</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>2 x 2.692 = 5.384</td>
<td>3.970</td>
<td>21.375</td>
<td>84.857</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>2 x 3.692 = 7.384</td>
<td>2.000</td>
<td>14.768</td>
<td>29.536</td>
<td>8.388</td>
</tr>
<tr>
<td>5</td>
<td>2 x 2.701 = 5.402</td>
<td>0.030</td>
<td>0.162</td>
<td>0.005</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>2 x 0.195 = 0.390</td>
<td>0.075</td>
<td>0.029</td>
<td>0.002</td>
<td>--</td>
</tr>
<tr>
<td>7</td>
<td>Stiffener 1.480</td>
<td>0.329</td>
<td>0.487</td>
<td>0.160</td>
<td>0.058</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td>22.012</td>
<td>44.112</td>
<td>141.581</td>
<td>8.481</td>
</tr>
</tbody>
</table>

Distance from top fiber to the neutral axis is

\[ y_{cg} = \frac{L_y}{L} = \frac{44.112}{22.012} = 2.004 \text{ in.} \]

\[ I'_{eff} = L_y^2 + I'_1 - L_y^2_{cg} \]
\[ = 141.581 + 8.481 - 22.012(2.004)^2 \]
\[ = 61.66 \text{ in.}^3 \]

Actual \( I_{eff} = tI'_{eff} \)
\[ = (0.060)(61.66) = 3.70 \text{ in.}^4 \]

\[ S_{eff} = \frac{I_{eff}}{y_{cg}} = \frac{3.70}{2.004} = 1.85 \text{ in.}^2 \]

\[ M = S_{eff}(0.6F_y) \]
\[ = (1.85)(30) \]
\[ = 55.5 \text{ kips-in.} > M_s = 46.96 \text{ kips-in.} \text{ NG} \]

Computation of \( I_{eff} \): second approximation by extrapolating the following data to obtain the stress value

(1) \( M = 80.00 \text{ kips-in.}, f = F_y = 50 \text{ ksi} \)
(2) \( M = 55.50 \text{ kips-in.}, f = 0.6F = 30 \text{ ksi} \)

(3) \( M = 46.96 \text{ kips-in.}, f = ? \)

\[
\frac{f-30}{30-50} = \frac{46.96-55.5}{55.5-80.0}
\]

\( f = 23.03 \text{ ksi} \)

* Compressive stress of \( f = 23.03 \text{ ksi} \) in the top fiber of section

* Web is fully effective

* Element 9 of stiffener is fully effective

Element 5:

\[
S = 1.28\sqrt{\frac{E_o}{f}}, \quad f = 23.03 \text{ ksi} \quad \text{(Eq. 2.4-1)}
\]

\[
= 1.28\sqrt{\frac{27000}{23.03}} = 43.83
\]

\[
b_o/t = 144.9
\]

\[
3S = 3(43.83) = 131.49
\]

For \( b_o/t > 3S \) (Case III)

\[
I_a = t^4\left\{\frac{128(b_o/t)}{S}\right\} - 285 \quad \text{(Eq. 2.4.1-9)}
\]

\[
= (0.06)^4\left\{\frac{128(144.9)}{43.83}\right\} - 285 = 0.00179 \text{ in.}^4
\]

\[
I_s = 0.00345 \text{ in.}^4
\]

\[
k = 3(I_s/I_a)^{1/3} + 1 \leq 4 \quad \text{(Eq. 2.4.1-10)}
\]

Since \( I_s/I_a > 1, k = 4 \)

\[
w/t = 68.30
\]

\[
\lambda = (1.052/\sqrt{k})(w/t)\sqrt{E/E_r}, \quad f = 23.03 \text{ ksi} \quad \text{(Eq. 2.2.1-4)}
\]

For a compression and tension stresses of \( f= 23.03 \text{ ksi} \), it is found that the values of \( E_{sc} \) and \( E_{st} \) are equal to 26390 ksi and 27000 ksi, respectively.

\[
E_r = \frac{(26390+27000)}{2} \quad \text{(Eq. 2.2.1-7)}
\]

\[
= 26695 \text{ ksi}
\]

\[
\lambda = (1.052/\sqrt{4})(68.30)\sqrt{23.03/26695} = 1.055 > 0.673
\]
\[ \rho = (1-0.22/\lambda)/\lambda \]  
\[ = (1-0.22/1.055)/1.055 = 0.750 \]  
\[ b = \rho w \]  
\[ = 0.750(4.098) = 3.074 \text{ in.} \]

Stiffener, Element 7:
\[ A_s = A'_s (I_s/I_a) \leq A'_s \]  
\[ \text{Since } I_s/I_a > 1 \]  
\[ A_s = A'_s = 0.0888 \text{ in.}^2 \]  
\[ L_s = A_s/t = 0.0888/0.060 = 1.480 \text{ in.} \]

Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>Effective Length (in.)</th>
<th>Distance from Top Fiber (in.)</th>
<th>Ly (in.²)</th>
<th>Ly² (in.⁴)</th>
<th>I' About Own Axis (in.³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 x 0.596 = 1.192</td>
<td>3.548</td>
<td>4.229</td>
<td>15.005</td>
<td>0.035</td>
</tr>
<tr>
<td>2</td>
<td>4 x 0.195 = 0.780</td>
<td>3.925</td>
<td>3.062</td>
<td>12.016</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>2 x 2.692 = 5.384</td>
<td>3.970</td>
<td>21.375</td>
<td>84.857</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>2 x 3.692 = 7.384</td>
<td>2.000</td>
<td>14.768</td>
<td>29.536</td>
<td>8.388</td>
</tr>
<tr>
<td>5</td>
<td>2 x 3.074 = 6.148</td>
<td>0.030</td>
<td>0.184</td>
<td>0.006</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>2 x 0.195 = 0.390</td>
<td>0.075</td>
<td>0.029</td>
<td>0.002</td>
<td>--</td>
</tr>
<tr>
<td>7</td>
<td>Stiffener 1.480</td>
<td>0.329</td>
<td>0.487</td>
<td>0.160</td>
<td>0.058</td>
</tr>
<tr>
<td>Sum</td>
<td>22.758</td>
<td>44.134</td>
<td>141.582</td>
<td>8.481</td>
<td></td>
</tr>
</tbody>
</table>

Distance from top fiber to the neutral axis is
\[ y_{cg} = Ly/L = 44.134/22.758 = 1.939 \text{ in.} \]
\[ I'_{eff} = Ly^2 + I'_{1} - Ly_{cg}^2 \]
\[ = 141.582 + 8.481 - 22.758(1.939)^2 \]
Actual \( I_{\text{eff}} \) = \( tI_{\text{eff}}' \)
\[ = (0.060)(64.50) = 3.87 \text{ in.}^4 \]
\( S_{\text{eff}} \) = \( \frac{I_{\text{eff}}}{y_{cg}} \)
\[ = \frac{3.87}{1.939} = 2.00 \text{ in.}^3 \]
\( M \) = (2.00)(23.03) = 46.06 kips-in. close to \( M_s \) OK

Use \( I = 3.87 \text{ in.}^4 \) in deflection calculations

5. Comparison of sections with and without intermediate stiffeners.

<table>
<thead>
<tr>
<th>Hat Section</th>
<th>Total Area (in.(^2))</th>
<th>Design Flexural Strength (kips-in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without Stiffener</td>
<td>1.43</td>
<td>45.90</td>
</tr>
<tr>
<td>With Stiffener</td>
<td>1.49</td>
<td>72.00</td>
</tr>
</tbody>
</table>

Increase in weight = \( \frac{(1.49-1.43)}{1.43} \times 100\% = 4.2\% \)

Increase in moment capacity = \( \frac{(72.00-45.90)}{45.90} \times 100\% = 56.9\% \)
EXAMPLE 7.2 HAT SECTION W/INTERMEDIATE STIFFENER (ASD)

Rework Example 7.1 by using the Allowable Stress Design (ASD) method.

Solution:

1. Calculation of the allowable moment, $M_a$:

   The effective section properties calculated by the ASD method are the same as those determined in Example 7.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

   $\Omega = 1.85$ (Safety Factor stipulated in Table E of the Standard)

   $M_n = 80.0$ kips-in. (obtained from Example 7.1)

   $M_a = M_n / \Omega$  

   \[ = 80.0 / 1.85 \]

   \[ = 43.24 \text{ kips-in.} \]

2. Deflection Determination at Allowable Moment $M_a$

   For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 7.1 for the LRFD method, except that the computed moment $M (= fxS_e)$ should be equal to $M_a$.

   Computation of $I_{\text{eff}}$: assume that

   * A stress of $f = 20.50$ ksi in the top and bottom fibers of section

   * Web is fully effective

   * Element 9 of stiffener is fully effective
Element 5:

\[ S = 1.28\sqrt{\frac{E_t}{f}}, \quad f = 20.50 \text{ ksi} \]  
\[ = 1.28\sqrt{27000/20.50} = 46.45 \]  

\[ b_o/t = 144.9 \]

\[ 3S = 3(46.45) = 139.35 \]

For \( b_o/t > 3S \) (Case III)

\[ I_a = t^4\left\{\frac{128(b_o/t)/S}{285}\right\} \]  
\[ = (0.06)^4\left\{\frac{128(144.9)/46.45}{285}\right\} = 0.00148 \text{ in.}^4 \]

\[ I_s = 0.00345 \text{ in.}^4 \]

\[ k = 3\left(\frac{I_s}{I_a}\right)^{1/3} + 1 \leq 4 \]  
\[ \text{Since } I_s/I_a > 1, \quad k = 4 \]

\[ \frac{w}{t} = 68.30 \]

\[ \lambda = \frac{(1.052/\sqrt{k})(w/t)\sqrt{f/E_t}}{f}, \quad f = 20.50 \text{ ksi} \]  
\[ = (1.052/\sqrt{4})(68.30)\sqrt{20.50/26950} = 0.991 > 0.673 \]

\[ \rho = (1-0.22/\lambda)/\lambda \]  
\[ = (1-0.22/0.991)/0.991 = 0.785 \]

\[ b = \rho w \]  
\[ = 0.785(4.098) = 3.217 \text{ in.} \]

Stiffener, Element 7:

\[ A_s = A_s'(I_s/I_a) \leq A_s' \]  
\[ \text{Since } I_s/I_a > 1 \]
\[ A_s = A'_s = 0.0888 \text{ in.}^2 \]

\[ L_s = \frac{A_s}{t} = \frac{0.0888}{0.060} = 1.480 \text{ in.} \]

**Effective section properties about x-axis:**

<table>
<thead>
<tr>
<th>Element</th>
<th>L (Effective Length) (in.)</th>
<th>Distance from Top Fiber (in.)</th>
<th>( y )</th>
<th>( L_y ) (in.)</th>
<th>( L_y^2 ) (in.(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 x 0.596 = 1.192</td>
<td>3.548</td>
<td>4.229</td>
<td>15.005</td>
<td>0.035</td>
</tr>
<tr>
<td>2</td>
<td>4 x 0.195 = 0.780</td>
<td>3.925</td>
<td>3.062</td>
<td>12.016</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>2 x 2.692 = 5.384</td>
<td>3.970</td>
<td>21.375</td>
<td>84.857</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>2 x 3.692 = 7.384</td>
<td>2.000</td>
<td>14.768</td>
<td>29.536</td>
<td>8.388</td>
</tr>
<tr>
<td>5</td>
<td>2 x 3.217 = 6.434</td>
<td>0.030</td>
<td>0.193</td>
<td>0.006</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>2 x 0.195 = 0.390</td>
<td>0.075</td>
<td>0.029</td>
<td>0.002</td>
<td>--</td>
</tr>
<tr>
<td>7</td>
<td>Stiffener 1.480</td>
<td>0.329</td>
<td>0.487</td>
<td>0.160</td>
<td>0.058</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td></td>
<td>23.044</td>
<td>44.143</td>
<td>141.582</td>
<td>8.481</td>
</tr>
</tbody>
</table>

Distance from top fiber to the neutral axis is

\[ y_{cg} = \frac{L_y}{L} = \frac{44.143}{23.044} = 1.916 \text{ in.} \]

\[ I'_{eff} = L_y^2 + I'_1 - L_y^2_{cg} = 141.582 + 8.481 - 23.044(1.916)^2 = 65.47 \text{ in.}^3 \]

Actual \( I_{eff} = tI'_{eff} = (0.060)(65.47) = 3.93 \text{ in.}^4 \)

\[ S_{eff} = \frac{I_{eff}}{y_{cg}} = 3.93/1.916 = 2.05 \text{ in.}^3 \]

\[ M = (2.05)(20.50) = 42.03 \text{ kips-in. close to } M_a \text{ OK} \]

Use \( I = 3.93 \text{ in.}^4 \) in deflection calculations
EXAMPLE 8.1 I-SECTION W/UNSTIFFENED FLANGES (LRFD)

By using the Load and Resistance Factor Design (LRFD) criteria, determine the design flexural strength of an I-section (Fig. 8.1) used as a simply supported beam. Assume that the span length is 8 ft. with laterally braced at both ends and midspan and that the beam carries uniform load. Use Type 301, 1/4-Hard, stainless steel.

![Diagram of I-section](image)

Figure 8.1 Section for Example 8.1

Solution:

1. Nominal section strength (Section 3.3.1.1).
   a. Procedure I - based on initiation of yielding

   For this I-section, the elastic section modulus of the effective section, $S_e$, based on initiation of yielding can be obtained from Example 1.1 for a channel section. Therefore,

   $$ S_e = 2 \times 0.711 = 1.422 \text{ in}^3 $$
\[ M_n = S F_y \] 
\[ = 1.422 \times 50 = 71.10 \text{ kips-in.} \] 

b. Procedure II - based on inelastic reserve capacity

Since the member is subjected to lateral buckling, therefore this provision does not apply in this example. Then,

\[ (M_n)_1 = 71.10 \text{ kips-in.} \]
\[ \Phi_b = 0.85 \]
\[ \Phi_b (M_n)_1 = 0.85 \times 71.10 = 60.44 \text{ kips-in.} \]

2. Lateral buckling strength (Section 3.3.1.2).

The following equations used for computing the sectional properties for I-section without lips are adopted from the Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.

Basic parameters used for calculating the section properties of an I-section without lips:

\[ r = R + t/2 = 3/32 + 0.060/2 = 0.124 \text{ in.} \]

From the sketch \( a = 5.692 \text{ in.}, b = 1.471 \text{ in.}, c = 1.471 \text{ in.}, \)
\( A' = 6.0 \text{ in.}, B' = 1.625 \text{ in.}, C' = 1.625 \text{ in.}, \)
\( a = 1.00 \) (For I-section)

\[ \bar{a} = A'-(t/2+at/2) = 6.0-(0.060/2+0.060/2) = 5.94 \text{ in.} \]
\[ \bar{b} = B'- t/2 = 1.625-0.06/2 = 1.595 \text{ in.} \]
\[ \bar{c} = a(C'-t/2) = 1.625-0.06/2 = 1.595 \text{ in.} \]
\[ u = 1.57r = 1.57 \times 0.124 = 0.195 \text{ in.} \]
\[ x = a/2 = 2.97 \text{ in.} \]
a. Area:
\[ A = t[2a+2b+2u+a(2c+2u)] = t[2a+2b+2c+4u] \]
\[ = 0.06(2 \times 5.692 + 2 \times 1.471 + 2 \times 1.471 + 4 \times 0.195) \]
\[ = 1.083 \text{ in.}^2 \]

b. Moment of inertia about x-axis:
\[ I_x = 2t\{a(a/2+r)^2+0.0833a^3+0.358r^2+u[0.363r+t/2]^2+0.149r^3\} - A(x)^2 \]
\[ = 2 \times 0.06[5.692(5.692/2+0.124)^2+0.0833(5.692)^3 \]
\[ +0.358(0.124)^2+1.471(5.692+2 \times 0.124)^2 \]
\[ +0.195(5.692+1.637 \times 0.124)^2+0.149(0.124)^3] - 1.083(2.97)^2 \]
\[ = 5.357 \text{ in.}^4 \]

c. Moment of inertia about y-axis:
\[ I_y = 2t\{b(b/2+r+t/2)^2+0.0833b^3+u(0.363r+t/2)^2+0.149r^3\} \]
\[ +a[c(c/2+r+t/2)^2+0.0833b^3+u(0.363r+t/2)^2+0.149r^3]\}
\[ = 2t\{b(b/2+r+t/2)^2+c(c/2+r+t/2)^2+2x0.0833b^3+2u(0.363r+t/2)^2 \]
\[ +2x0.149r^3\} \]
\[ = 2 \times 0.06[1.471(1.531/2+0.124)^2+1.471(1.531/2+0.124)^2 \]
\[ +2x0.0833x(1.471)^3+2x0.195(0.363x0.124+0.06/2)^2 \]
\[ +2x0.149x(0.124)^3\} \]
\[ = 0.343 \text{ in.}^4 \]

Therefore,
\[ S_f = I_x/y_{cg} = 5.357/3.0 = 1.786 \text{ in}^3 \]
\[ C_b = 1.75+1.05(M_1/M_2)+0.3(M_1/M_2)^2 \]
\[ = 1.75+1.05(0/M_{\text{max}})+0.3(0/M_{\text{max}})^2 \]
\[ \begin{align*}
I_{yc} &= \frac{I_y}{2} = \frac{0.343}{2} = 0.172 \text{ in.}^4 \\
M_c &= n^2E_o C_b \left( \frac{E_t}{E_o} \right) dI_{yc}/L^2 \quad \text{(Eq. 3.3.1.2-2)} \\
M_n &= \frac{S_c (M_c/S_f)}{S_f} \quad \text{(Eq. 3.3.1.2-1)}
\end{align*} \]

In the determination of the lateral buckling stress, it is necessary to select a proper ratio of \( E_t/E_o \) from Table A10 or Figure A7 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of \( f=32 \text{ ksi} \). From Table A10, the corresponding value of \( E_t/E_o \) is found to be equal to 0.42. Thus,

\[ f_1 = 116.95 \times 0.42 \]

\[ = 49.12 \text{ ksi} > \text{assumed stress } f=32 \text{ ksi} \]

Because the computed stress is larger than the assumed value, the further successive approximation is needed.

Assume \( f=38.5 \text{ ksi} \), and

\[ \frac{E_t}{E_o} = 0.33 \]

\[ f_1 = 116.95 \times 0.33 \]

\[ = 38.49 \text{ ksi} = \text{assumed stress } f=38.5 \text{ ksi } \text{OK} \]

Therefore,

\[ f = \frac{M_c}{S_f} = 38.47 \text{ ksi} \]

Properties of 90° corners:

\[ r = R + t/2 = 3/32 + 0.060/2 = 0.124 \text{ in.} \]
Length of arc, \( u = 1.57r = 1.57 \times 0.124 = 0.195 \) in.

Distance of c.g. from center of radius,
\( c = 0.637r = 0.637 \times 0.124 = 0.079 \) in.

Determination of elastic section modulus of the effective section calculated at a stress of \( f = 38.47 \) ksi in the extreme compression fiber (assume the webs are fully effective):

Compression flange: \( k = 0.50 \) (unstiffened compression element)

\[
\frac{w}{t} = 1.471/0.06 = 24.52 < 50 \text{ OK (Section 2.1.1-(1)-(iii))}
\]

\[
\lambda = \frac{(1.052)\sqrt{\frac{k}{(w/t)}}\sqrt{f/E}}{\sqrt{E}} \quad \text{(Eq. 2.2.1-4)}
\]

\[
= (1.052/0.50)(24.52)/(38.47/27000) = 1.377 > 0.673
\]

\[
\rho = (1-0.22/\lambda)/\lambda \quad \text{(Eq. 2.2.1-3)}
\]

\[
= (1-0.22/1.377)/1.377 = 0.610
\]

\[
b = \rho w \quad \text{(Eq. 2.2.1-2)}
\]

\[
= 0.610 \times 1.471 = 0.897 \text{ in.}
\]

Effective section properties about x axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L Effective Length (in.)</th>
<th>Distance from Top Fiber (in.)</th>
<th>Ly (in.²)</th>
<th>Ly² (in.⁴)</th>
<th>I' About Own Axis (in.⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Webs</td>
<td>11.384</td>
<td>3.000</td>
<td>34.152</td>
<td>102.456</td>
<td>30.736</td>
</tr>
<tr>
<td>Upper Corners</td>
<td>0.390</td>
<td>0.075</td>
<td>0.029</td>
<td>0.002</td>
<td>--</td>
</tr>
<tr>
<td>Lower Corners</td>
<td>0.390</td>
<td>5.925</td>
<td>2.310</td>
<td>13.691</td>
<td>--</td>
</tr>
<tr>
<td>Compression Flanges</td>
<td>1.794</td>
<td>0.030</td>
<td>0.054</td>
<td>0.002</td>
<td>--</td>
</tr>
<tr>
<td>Tension Flanges</td>
<td>2.942</td>
<td>5.970</td>
<td>17.564</td>
<td>104.856</td>
<td>--</td>
</tr>
<tr>
<td>Sum</td>
<td>16.900</td>
<td>54.109</td>
<td>221.007</td>
<td>30.736</td>
<td></td>
</tr>
</tbody>
</table>

Distance from top fiber to x-axis is

118
\[ y_{cg} = \frac{54.109}{16.90} = 3.202 \text{ in.} \]

Since the distance of top compression fiber from neutral axis is greater than one half the beam depth, a compression stress of \( f = 38.47 \text{ ksi} \) will govern.

To check if webs are fully effective (Section 2.2.2):

\[ f_1 = \frac{[(3.202-0.154)/3.202] \times 38.47}{36.62 \text{ ksi (compression)}} \]

\[ f_2 = -\frac{[(2.798-0.154)/3.202] \times 38.47}{31.77 \text{ ksi (tension)}} \]

\[ \psi = \frac{f_2}{f_1} = \frac{-31.77/36.62}{-0.868} = 36.62 \text{ ksi (compression)} \]

\[ f_2/f_1 = -0.868 \]

\[ k = 4+2(1-\psi)^3+2(1-\psi) \]

\[ = 4+2\left[1-(-0.868)^3\right]+2\left[1-(-0.868)\right] \]

\[ = 20.772 \]

\[ h = w = 5.692 \text{ in., } h/t = w/t = 5.692/0.06 = 94.87 \]

\[ h/t = 54.47 < 200 \text{ OK (Section 2.1.2-(1))} \]

\[ \lambda = (1.052/\sqrt{20.772})(54.47)/\sqrt{36.62/27000} = 0.463 < 0.673 \]

\[ b_e = w \] (Eq. 2.2.1-1)

\[ = 5.692 \text{ in.} \]

\[ b_2 = b_e/2 \] (Eq. 2.2.2-2)

\[ = 5.692/2 = 2.846 \text{ in.} \]

\[ b_1 = b_e/(3-\psi) \] (Eq. 2.2.2-1)

\[ = 5.692/[3-(-0.868)] = 1.472 \text{ in.} \]

Compression portion of the web calculated on the basis of the effective section = \( y_{cg} = 0.154 = 3.202 - 0.154 = 3.048 \text{ in.} \)

Since \( b_1+b_2 = 4.318 \text{ in.} > 3.048 \text{ in.} \), \( b_1+b_2 \) shall be taken as 3.048 in.. This verifies the assumption that the webs are
fully effective.

\[ I'_x = L_y^2 I'_1 - L_y^2_{cg} \]

\[ = 221.007 + 30.736 - 16.90(3.202)^2 \]

\[ = 78.471 \text{ in.}^3 \]

Actual \( I_x = I'_x t \)

\[ = 78.471 \times 0.06 \]

\[ = 4.708 \text{ in.}^4 \]

\[ S_c = I_x/y_{cg} \]

\[ = 4.708/3.202 \]

\[ = 1.470 \text{ in.}^3 \]

Therefore,

\[ (M_n)_2 = S_c f = 1.470 \times 38.47 \]

\[ = 56.55 \text{ kips-in.} \]

\[ \Phi_b = 0.85 \]

\[ \Phi_b (M_n)_2 = 0.85 \times 56.55 \]

\[ = 48.07 \text{ kips-in.} < \Phi_b (M_n)_1 = 60.44 \text{ kips-in.} \]

Therefore, \( \Phi_b M_n = 48.07 \text{ kips-in.} \) (i.e., lateral buckling strength controls).
EXAMPLE 8.2 I-SECTION W/UNSTIFFENED FLANGES (ASD)

By using the Allowable Stress Design (ASD) method, rework Example 8.1 to determine the allowable bending strength of the I-section (Fig. 8.1).

Solution:

1. Nominal section strength

\[ M_n = S_e F_y \]
\[ = 1.422 \times 50 = 71.10 \text{ kips-in.} \text{ (see Example 8.1)} \]

\[ (M_n)_1 = 71.10 \text{ kips-in.} \]

Allowable bending strength

\[ \Omega = 1.85 \]

\[ (M_a)_1 = 71.10/1.85 = 38.43 \text{ kips-in.} \]

2. Lateral buckling strength

\[ M_n = S_c (M_c / S_f) \]
\[ = S_c f \]
\[ f = M_c / S_f = 38.47 \text{ ksi} \]
\[ S_c = 1.470 \text{ in}^3 \]

(For detailed calculations, see Example 8.1)

\[ (M_n)_2 = 1.470 \times 38.47 = 56.55 \text{ kips-in.} \]

Allowable lateral buckling strength

\[ \Omega = 1.85 \]

\[ (M_a)_2 = 56.55/1.85 = 30.57 \text{ kips-in.} \]

Therefore, \( M_a = 30.57 \text{ kips-in.} \) (i.e., lateral buckling controls)
EXAMPLE 9.1 CHANNEL W/LATERAL BUCKLING CONSIDERATION (LRFD)

Complete Flexural Design, Unstiffened Compression Flange

By using the LRFD criteria, check the adequacy of a channel section (Fig. 9.1) to be used as a flexural member and to support a nominal live load of 200 lb/ft. and a nominal dead load of 40 lb/ft. Assume that the beam is continuous over three 10 ft. spans with 6 in. and 3 1/2 in. bearing lengths at interior and exterior supports, respectively. Also assume that, for each span, the compression flange is braced at the center and a quarter point of span, and \( K_x = K_y = 1.0 \). Use Type 304, 1/4-Hard, stainless steel.

![Figure 9.1 Section for Example 9.1](image)

Solution:

1. Nominal section strength, \( M_n \) (Section 3.3.1.1).
   a. Procedure I - based on initiation of yielding
Properties of 90° corners:
\[ r = R + t/2 = 3/16 + 0.135/2 = 0.255 \text{ in.} \]
Length of arc, \( u = 1.57r = 1.57 \times 0.255 = 0.40 \text{ in.} \)
Distance of c.g. from center of radius,
\[ c = 0.637r = 0.637 \times 0.255 = 0.162 \text{ in.} \]

Computation of \( I_x, S_e, \) and \( M_n \):
For the first approximation, assume a compression stress of
\[ f = F_y = 50 \text{ ksi (yield strength in longitudinal compression, Table A1 of the Standard Specification) in the top fiber of the section and that the web is fully effective.} \]

Compression flange: \( k = 0.50 \) (for unstiffened compression element, see Section 2.3.1 of the Standard)
\[ w/t = 1.177/0.135 = 8.72 < 50 \text{ OK (Section 2.1.1-(1)-(iii))} \]
\[ \Lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o} \quad \text{(Eq. 2.2.1-4)} \]
The initial modulus of elasticity, \( E_o \), for Type 304 stainless steel is obtained from Table A4 of the Standard, i.e., \( E_o = 27000 \text{ ksi.} \)
\[ \Lambda = (1.052/\sqrt{0.50})(8.72)\sqrt{50/27000} = 0.558 < 0.673 \quad \text{(Eq. 2.2.1-1)} \]
\[ b = w = 1.177 \text{ in.} \]
Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L Effective Length (in.)</th>
<th>y Distance from Top Fiber (in.)</th>
<th>Ly (in.²)</th>
<th>Ly² (in.⁴)</th>
<th>I' About Own Axis (in.⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web</td>
<td>6.354</td>
<td>3.500</td>
<td>22.239</td>
<td>77.837</td>
<td>21.378</td>
</tr>
<tr>
<td>Upper Corner</td>
<td>0.400</td>
<td>0.161</td>
<td>0.064</td>
<td>0.010</td>
<td>--</td>
</tr>
<tr>
<td>Lower Corner</td>
<td>0.400</td>
<td>6.839</td>
<td>2.736</td>
<td>18.709</td>
<td>--</td>
</tr>
<tr>
<td>Compression Flange</td>
<td>1.177</td>
<td>0.068</td>
<td>0.080</td>
<td>0.005</td>
<td>--</td>
</tr>
<tr>
<td>Tension Flange</td>
<td>1.177</td>
<td>6.933</td>
<td>8.160</td>
<td>56.574</td>
<td>--</td>
</tr>
<tr>
<td>Sum</td>
<td>9.508</td>
<td>33.279</td>
<td>153.135</td>
<td>21.378</td>
<td></td>
</tr>
</tbody>
</table>

Distance from top fiber to x-axis is
\[ y_{cg} = \frac{33.279}{9.508} = 3.500 \text{ in.} \]

Since the distance from top compression fiber to the neutral axis is equal to one half the beam depth, a compression stress of 50 ksi will govern as assumed (i.e., initial yield is in compression).

To check if web is fully effective (Section 2.2.2):
\[ f_1 = \frac{[(3.500-0.323)/3.500]x50 = 45.39 \text{ ksi (compression)}}{4+2(1-\psi)^2+2(1-\psi)} \]
\[ f_2 = -[(3.500-0.323)/3.500]x50 = -45.39 \text{ ksi (tension)}}{4+2[1-(-1.00)]^2+2[1-(-1.00)]} \]
\[\psi = f_2/f_1 = -45.39/45.39 = -1.00 \]
\[ k = 4+2(1-\psi)^2+2(1-\psi) \]
\[ h = w = 6.354 \text{ in.}, \ h/t = w/t = 6.354/0.135 = 47.07 \]
\[ h/t = 47.07 < 200 \text{ OK (Section 2.1.2-(1))} \]

\[ \lambda = (1.052/\sqrt{24.0})(47.07)/\sqrt{45.39/27000} = 0.414 < 0.673 \]

\[ b_e = 6.354 \text{ in.} \]

\[ b_2 = b_e/2 \quad \text{(Eq. 2.2.2-2)} \]

\[ = 6.354/2 = 3.177 \text{ in.} \]

\[ b_1 = b_e/(3-\Psi) \quad \text{(Eq. 2.2.2-1)} \]

\[ = 6.354/[3-(1.0)] = 1.589 \text{ in.} \]

The effective widths, \( b_1 \) and \( b_2 \), of web are defined in Figure 2 of the Standard.

\[ b_1 + b_2 = 1.589 + 3.177 = 4.766 \text{ in.} \]

Compression portion of the web calculated on the basis of the effective section \( = y_{cg} - 0.154 = 3.50 - 0.323 = 3.177 \text{ in.} \)

Since \( b_1 + b_2 = 4.766 \text{ in.} > 3.177 \text{ in.} \), \( b_1 + b_2 \) shall be taken as 3.177 in.. This verifies the assumption that the web is fully effective.

\[ I'_x = I'_1 + I'_{cg} - Ly^2 \]

\[ = 153.135 + 21.378 - 9.508(3.50)^2 \]

\[ = 58.04 \text{ in.}^3 \]

Actual \( I_x = I'_x t \)

\[ = 58.04 \times 0.135 \]

\[ = 7.835 \text{ in.}^4 \]

\[ S_e = I_x/y_{cg} \]

\[ = 7.835/3.50 \]

\[ = 2.239 \text{ in.}^3 \]

\[ (M_n)_{1} = S_e F_y \quad \text{(Eq. 3.3.1.1-1)} \]

\[ = 2.239 \times 50 \]
b. Procedure II - based on inelastic reserve capacity

   For unstiffened compression element, \( C_y = 1 \).

   Maximum compressive strain = \( C_y e_y = e_y \).

   Therefore, the nominal ultimate moment, \( M_n \), is the same as the \( (M_n)_1 \) determined by Procedure I because the compression flange will yield first.

---

2. Lateral buckling strength, \( M_n \) (Section 3.3.1.2).

   The following equations used for computing the sectional properties for channel with no lips are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.

   a. Basic parameters used for calculating the sectional properties:

   (For a Channel with no lips)

   \( r = R + t / 2 = 3 / 16 + 0.135 / 2 = 0.255 \) in.

   From the sketch, \( A' = 7.0 \) in., \( B' = 1.50 \) in.

   \( a = 0.0 \) (For sections with no lips)

   \[ \begin{align*}
   a &= A' - (2r + t) \\
   &= 7.0 - (2 	imes 0.255 + 0.135) = 6.355 \text{ in.}
   \end{align*} \]

   \[ \begin{align*}
   A &= A' - t = 7 - 0.135 = 6.865 \text{ in.}
   \end{align*} \]

   \[ \begin{align*}
   b &= B' - [r + t / 2 + a(r + t / 2)] = 1.5 - (0.255 + 0.135 / 2) = 1.177 \text{ in.}
   \end{align*} \]

   \[ \begin{align*}
   b &= B' - (t / 2 + a t / 2) = 1.5 - 0.135 / 2 = 1.433 \text{ in.}
   \end{align*} \]

   \[ \begin{align*}
   u &= 1.57r = 1.57 	imes 0.255 = 0.40 \text{ in.}
   \end{align*} \]

   b. Area:

   \[ A = t[a + 2b + 2u] \]
\[ I_x = 2t \left[ 0.0417a^3 + b(a/2+r)^2 + u(a/2+0.637r)^2 + 0.149r^3 \right] \]
\[ = 2 \times 0.135 \left[ 0.0417(6.355)^3 + 1.177(6.355/2+0.255)^2 + 0.4(6.355/2+0.637\times0.255)^2 + 0.149(0.255)^3 \right] \]
\[ = 7.839 \text{ in.}^4 \]

\[ \bar{x} = \frac{(2t/A) [b(b/2+r)+u(0.363r)]}{I_x} \]
\[ = \frac{(2 \times 0.135/1.284) [1.177(1.177/2+0.255)+0.4(0.363\times0.255)]}{7.839} \]
\[ = 0.217 \text{ in.} \]

\[ I_y = 2t \left[ b(b/2+r)^2 + 0.0833b^3 + 0.356r^3 \right] - A(\bar{x})^2 \]
\[ = 2 \times 0.135 \left[ 1.177(1.177/2+0.255)^2 + 0.0833(1.177)^3 + 0.356(0.255)^3 \right] \]
\[ - 1.284(0.217)^2 \]
\[ = 0.204 \text{ in.}^4 \]

\[ m = \frac{5t}{(12I_x)} \left[ 3\beta(\Delta)^2 \right] \]
\[ = \left[ 1.433 \times 0.135/(12 \times 7.839) \right] \left[ 3 \times 1.433 \times (6.865)^2 \right] \]
\[ = 0.417 \text{ in.} \]
g. Distance between centroid and shear center:
\[ x_0 = -(\bar{x} + m) = -(0.217 + 0.417) = -0.634 \text{ in.} \]

h. St. Venant torsion constant:
\[ J = \frac{t^3}{3}(a+2b+2u) \]
\[ = \left[ \frac{(0.135)^3}{3} \right] (6.355 + 2 \times 1.177 + 2 \times 0.4) \]
\[ = 0.0078 \text{ in.}^4 \]

i. Warping Constant:
\[ C_w = \left( \frac{t^2}{12} \right) \left( \frac{3a^2 + 2a^3}{6a + 5a} \right) \]
\[ = \frac{0.135(6.865)^2(1.433)^3}{12} \times \frac{3 \times 1.433 + 2 \times 6.865}{6 \times 1.433 + 6.865} \]
\[ = 1.819 \text{ in.}^6 \]

j. Radii of gyration:
\[ r_x = \sqrt{\left( \frac{I_x}{A} \right)} = \sqrt{\left( \frac{7.839}{1.284} \right)} = 2.47 \text{ in.} \]
\[ r_y = \sqrt{\left( \frac{I_y}{A} \right)} = \sqrt{\left( \frac{0.204}{1.284} \right)} = 0.40 \text{ in.} \]
\[ r_o^2 = r_x^2 + r_y^2 + x_0^2 \]
\[ = (2.47)^2 + (0.40)^2 + (-0.634)^2 \]
\[ = 6.662 \text{ in.}^2 \]
\[ r_o = 2.581 \text{ in.} \]

Therefore, for determining the lateral buckling stress:
\[ M_n = S_c \left( \frac{M_c}{S_f} \right) \quad \text{Eq. (3.3.1.2-1)} \]

where \( M_c \) is the critical moment calculated in accordance with
Eq. (3.3.1.2-4) of the Standard.
\[ S_f = \frac{I_x}{y_{cg}} = 7.836/3.5 = 2.239 \text{ in}^3 \]
\[ C_b = 1.75 + 1.05 \left( \frac{M_1}{M_2} \right) + 0.3 \left( \frac{M_1}{M_2} \right)^2 \]

\[ = 1.75 + 1.05 \left( \frac{-0.0063}{0.10} \right) + 0.3 \left( \frac{-0.0063}{0.10} \right)^2 = 1.685 < 2.3 \]

where \( M_1 \) and \( M_2 \) are determined from the moment diagram at the interior support.

\[ M_c = C_b \frac{r_o A}{\sqrt{\sigma_{ey} \sigma_t}} \quad \text{(Eq. 3.3.1.2-4)} \]

where

\[ \sigma_{ey} = \left( \frac{n^2 E_o}{(K_y L_y/r_y)^2} \right) \left( \frac{E_t}{E_o} \right) \quad \text{(Eq. 3.4.3-3)} \]

\[ \sigma_t = \left[ \frac{1}{(A r_o^2)} \left( \frac{\sigma_o J + n^2 E_o C_w}{(K_t L_t)^2} \right) \left( \frac{E_t}{E_o} \right) \right] \quad \text{(Eq. 3.4.2-1)} \]

Therefore,

\[ \sigma_{ey} = \left( \frac{(n^2 x 27000)}{(1.0 x 2.5 x 12 / 0.40)^4} \right) \left( \frac{E_t}{E_o} \right) \]

\[ = 47.14 \left( \frac{E_t}{E_o} \right) \]

\[ \sigma_t = \left[ \frac{1}{(1.284 x 6.662)} \left[ \left( \frac{10500 x 0.0078 + n^2 x 27000 x 1.819}{(1.0 x 2.5 x 12)^2} \right) x \left( \frac{E_t}{E_o} \right) \right] \right] \]

\[ = 72.54 \left( \frac{E_t}{E_o} \right) \]

\[ M_n = S_c \frac{M_c}{S_f} \quad \text{Eq. (3.3.1.2-1)} \]

\[ = S_c f \]

where,

\[ f = \frac{M_c}{S_f} \]

\[ = (1/2.239)(1.685 x 2.581 x 1.284 x 47.14 x 72.54) \left( \frac{E_t}{E_o} \right) \]

\[ = 145.84 \left( \frac{E_t}{E_o} \right) \text{ksi} \]

In the determination of the lateral buckling stress, it is necessary to select a proper ratio of \( E_t / E_o \) from Table A10 or Figure A7 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of \( f=32 \text{ ksi} \).

From Table A10, the corresponding value of \( E_t / E_o \) is found to be equal to 0.42. Thus,
\[ f_1 = 145.84 \times 0.42 \]
\[ = 61.25 \text{ ksi} > \text{assumed stress } f = 32 \text{ ksi} \]

Because the computed stress is larger than the assumed value, the further successive approximation is needed. After several trials, assume \( f = 42.12 \text{ ksi} \), and

\[ \frac{E_t}{E_o} = 0.2888 \]
\[ f_1 = 145.84 \times 0.2888 \]
\[ = 42.12 \text{ ksi} = \text{assumed stress } f = 42.12 \text{ ksi} \text{ OK} \]

Therefore,

\[ f = \frac{M_C}{S_f} = 34.50 \text{ ksi} \]

It is noted that from the calculation of Part 1(a), the section is fully effective for \( f = F_y = 50 \text{ ksi} \). Therefore, for the lateral buckling stress of \( f = 42.12 \text{ ksi} \), the section will also be fully effective.

Thus,

\[ (M_n)_2 = S_c f = 2.239 \times 42.12 = 94.30 \text{ kips-in.} \]

3. Design flexural strength, \( \Phi_b M_n \)

Based on the above calculations, the lateral buckling stress \( (M_n)_2 \) is less than the nominal section strength \( (M_n)_1 \). Therefore, lateral buckling governs the design.

\[ M_n = 94.30 \text{ kips-in.} \]
\[ \Phi_b = 0.85 \]
\[ \Phi_b M_n = 0.85 \times 94.30 = 80.16 \text{ kips-in.} \]

This value can be used for both positive and negative bending.

\[ w_u = 1.2 w_{DL} + 1.6 w_{LL} = 1.2(0.04) + 1.6(0.20) = 0.368 \text{ kips/ft.} \]
For a continuous beam over three equal spans, the maximum bending moment is negative and occurs over the interior supports. It is given by

\[ M_u = 0.100w_u L^2 = 0.100(0.368)(10)^2(12) \]

\[ = 44.16 \text{ kips-in.} < \phi_b M_n = 80.16 \text{ kips-in. OK} \]

4. Strength for Shear Only (Section 3.3.2)

The required shear strength at any section shall not exceed the design shear strength \( \phi_v V_n \):

\[ \phi_v = 0.85 \]

\[ V_n = 4.84E_0 t^2(G_s/G_o)/h \]  \hspace{1cm} (Eq. 3.3.2-1)

\[ v_n = V_n/(ht) \hspace{0.5cm} (\text{in terms of design shear stress}) \]

\[ = 4.84E_0(G_s/G_o)/(h/t)^2 \]

In the determination of the shear strength, it is necessary to select a proper value of \( G_s/G_o \) for the assumed stress from Table A12 or Figure A9 of the Standard. For the first approximation, assume a shear stress of \( v=F_y/2=25 \text{ ksi} \) and the corresponding value of \( G_s/G_o \) is equal to 0.888. Thus,

\[ h/t = 6.354/0.135 = 47.07 \]

\[ v_n = 4.84(27000)(0.888)/(47.07)^2 \]

\[ = 52.38 \text{ ksi} > \text{ assumed stress } v=25 \text{ ksi NG} \]

For a second approximation, assume a stress of \( f=38.30 \text{ ksi} \) and its corresponding value of \( G_s/G_o \) is 0.648.

\[ v_n = 4.84(27000)(0.648)/(47.07)^2 \]

\[ = 38.24 \text{ ksi} \neq \text{ assumed stress (close enough) OK} \]

Therefore, the total shear strength, \( V_n \), for hat section is

\[ V_n = (2 \text{ webs})(v_n)(ht) \]
= 2(38.24)(6.354x0.135)
= 32.80 kips

The design shear strength is determined as follows:

\[ \phi_v V_n = 0.85(32.80) = 27.88 \text{ kips} \]

\[ \phi_v V_n < 2(0.95F_{yv}) = 2(0.95\times42\times6.354\times0.135) = 34.23 \text{ kips OK} \]

(The shear yield strength, \( F_{yv} \), is obtained from Table A1 of the Standard.)

The maximum required shear strength is given by

\[ V_u = 0.600w_u L \]
\[ = (0.600)(0.368)(10) = 2.21 \text{ kips} < \phi_v V_n = 27.88 \text{ kips OK} \]

5. Strength for combined bending and shear (Section 3.3.3).

At the interior supports there is a combination of web bending and web shear:

\[ \phi_b M_n = 80.16 \text{ kips-in.} \quad M_u = 44.16 \text{ kips-in.} \]

\[ \phi_v V_n = 27.88 \text{ kips} \quad V_u = 2.21 \text{ kips} \]

For unreinforced webs

\[ \left( \frac{M_u}{\phi_b M_n} \right)^2 + \left( \frac{V_u}{\phi_v V_n} \right)^2 \leq 1.0 \quad (\text{Eq. 3.3.3-1}) \]

\[ \left( \frac{44.16}{80.16} \right)^2 + \left( \frac{2.21}{27.88} \right)^2 = 0.31 < 1.0 \text{ OK} \]

6. Web crippling strength (Section 3.3.4).

\[ R/t = (3/16)/0.135 = 1.389 < 6 \text{ OK} \]

\[ h/t = 6.354/0.135 = 47.07 < 200 \text{ OK} \]

\[ N/t = 3.0/0.135 = 22.22 < 210 \text{ OK (at end support)} \]

\[ N/t = 6.0/0.135 = 44.44 < 210 \text{ OK (at interior support)} \]

Table 2 of the Standard applies:

For end reactions: \( \text{(Eq. 3.3.4-2)} \)
For interior reactions: (Eq. 3.3.4-4)

\[ k = \frac{F_y}{33} = \frac{50}{33} = 1.515 \]  
(Eq. 3.3.4-21)

\[ C_1 = (1.22 - 0.22k)k \]
\[ = [1.22 - 0.22(1.515)](1.515) = 1.343 \]  
(Eq. 3.3.4-10)

\[ C_2 = (1.06 - 0.06R/t) \]
\[ = [1.06 - 0.06(1.389)] = 0.977 < 1.0 \text{ OK} \]  
(Eq. 3.3.4-11)

\[ C_3 = (1.33 - 0.33k)k \]
\[ = [1.33 - 0.33(1.515)](1.515) = 1.258 \]  
(Eq. 3.3.4-12)

\[ C_4 = (1.15 - 0.15R/t) \leq 1.0 \text{ but not less than 0.50} \]  
(Eq. 3.3.4-13)

\[ 1.15 - 0.15R/t = 1.15 - 0.15(1.389) = 0.942 \leq 1.0 \text{ OK} \]  
\[ > 0.50 \text{ OK} \]

\[ C_4 = 0.942 \]

\[ C_\theta = 0.7 + 0.3(\theta/90)^2 \]  
(Eq. 3.3.4-20)

\[ = 0.7 + 0.3(90/90)^2 = 1.0 \]

For end reaction:

\[ P_n = t^2C_3C_4C_\theta[217 - 0.28(h/t)][1 + 0.01(N/t)] \]
\[ = (0.135)^2(1.258)(0.942)(1.0)[217 - 0.28(47.07)] \]
\[ \times [1 + 0.01(22.22)] = 5.38 \text{ kips} \]

\[ \Phi_w = 0.70 \]

\[ \Phi_w P_n = 0.70(5.38) = 3.77 \text{ kips} \]

End reaction is given by

\[ R = 0.400wL \]
\[ = (0.400)(0.368)(10) = 1.47 \text{ kips} < \Phi_w P_n = 3.77 \text{ kips} \text{ OK} \]

For interior reaction:

\[ P_n = t^2C_1C_2C_\theta[538 - 0.74(h/t)][1 + 0.007(N/t)] \]  
(Eq. 3.3.4-4)

\[ = (0.135)^2(1.343)(0.977)(1.0)[538 - 0.74(47.07)] \]
\[ \times [1 + 0.007(44.44)] = 15.79 \text{ kips} \]
\[ \Phi_w = 0.70 \]
\[ \Phi_{wn} = 0.70(15.79) = 11.05 \text{ kips} \]

Interior reaction is given by
\[ R = 1.10wL \]
\[ = (1.10)(0.368)(10) = 4.05 \text{ kips} < \Phi_{wn} = 11.05 \text{ kips} \text{ OK} \]

7. Combined bending and web crippling strength (Section 3.3.5).
At the interior supports there is a combination of web bending and web crippling:
\[ \Phi_{bn} = 80.16 \text{ kips-in.} \quad M_u = 44.16 \text{ kips-in.} \]
\[ \Phi_{wn} = 11.05 \text{ kips} \quad R = 4.05 \text{ kips} \]
For shapes having single unreinforced webs:
\[ 1.07(R/\Phi_{wn})+(M_u/\Phi_{bn}) \leq 1.42 \quad (\text{Eq. 3.3.5-1}) \]
\[ 1.07(4.05/11.05)+(44.16/80.16) = 0.943 < 1.42 \text{ OK} \]

8. Deflection due to service live load.
From the result of sectional properties calculated in item (1) of this example, the section is fully effective at \( F_y = 50 \text{ ksi} \).
\[ S_x = S_e = 2.239 \text{ in.}^2 \]
Therefore, for any stress \( f \) which is less than \( F_y = 50 \text{ ksi} \), the section will be fully effective, i.e.,
\[ I_x = 7.835 \text{ in.}^4 \]
This value can be used for deflection determination.

The maximum deflection occurs at a distance of 0.446L from the exterior supports. It is given by
\[ \Delta = 0.0069wL^4/(E_oI_x) \]

134
Thus, the live load deflection is calculated as follows:

\[ \Delta = 0.0069(0.20)(10)^4(12)^3/(27000 \times 7.835) \]

\[ = 0.113 \text{ in.} \]

The live load deflection is limited to 1/240 of the span, i.e.,

\[ L/240 = 10 \times 12/240 = 0.5 \text{ in.} > 0.113 \text{ in. OK} \]

From the above calculations, it can be concluded that the section is adequate.
EXAMPLE 9.2 CHANNEL W/LATERAL BUCKLING CONSIDERATION (ASD)

By using the ASD method, rework Example 9.1 for the same given data.

Solution:

1. Nominal section strength, $M_n$

   For detailed calculations see Example 9.1.

   \[
   (M_n)_1 = \frac{SF_y}{e} \\
   = 2.239 \times 50 \\
   = 111.95 \text{ kips-in.}
   \]

2. Lateral buckling strength, $M_n$

   For detailed calculations see Example 9.1.

   \[
   (M_n)_2 = \frac{S_f}{c} \\
   = 2.239 \times 42.12 \\
   = 94.30 \text{ kips-in.}
   \]

3. Allowable bending strength, $M_a$

   $M_n = 94.30 \text{ kips-in. (based on lateral buckling strength)}$

   \[
   \Omega = 1.85 \\
   M_a = \frac{94.30}{1.85} = 50.97 \text{ kips-in.}
   \]

   This value can be used for both positive and negative bending.

   \[
   w = w_{DL} + w_{LL} = 0.04 + 0.20 = 0.24 \text{ kips/ft.}
   \]

   For a continuous beam over three equal spans, the maximum bending moment is negative and occurs over the interior supports. It is given by

   \[
   M = 0.100wL^2 = 0.100(0.24)(10)^2(12) \\
   = 28.80 \text{ kips-in.} < M_a = 50.97 \text{ kips-in. OK}
   \]
4. Strength for Shear Only

The required shear strength at any section shall not exceed the allowable shear strength \( V_n \):

\[
V_n = (2 \text{ webs})(v_n)(ht)
\]

\[
= 2(38.24)(6.354 	imes 0.135) \quad (\text{See Example 9.1 for } v_n)
\]

\[
= 32.80 \text{ kips}
\]

The allowable shear strength is determined as follows:

\[\Omega = 1.85\]

\[V_a = 32.80/1.85 = 17.73 \text{ kips}\]

\(V_a\) shall be less than the allowable shear yielding strength, i.e.,

\[V_a < 2(F_{yv}ht)/1.64 = 43.94 \text{ kips} \quad \text{OK}\]

(The safety factor used for shear yielding is 1.64, and the shear yield strength, \( F_{yv} \), is obtained from Table A1 of the Standard.)

The maximum required shear strength is given by

\[V = 0.600wL\]

\[
= (0.600)(0.24)(10) = 1.44 \text{ kips} < V_a = 17.73 \text{ kips} \quad \text{OK}
\]

5. Strength for combined bending and shear

At the interior supports there is a combination of web bending and web shear:

\[M_a = 50.97 \text{ kips-in.} \quad M = 28.80 \text{ kips-in.}\]

\[V_a = 17.73 \text{ kips} \quad V = 1.44 \text{ kips}\]

For unreinforced webs

\[(M/M_a)^2 + (V/V_a)^2 \leq 1.0\]

\[
(28.80/50.97)^2 + (1.44/17.73)^2 = 0.326 < 1.0 \quad \text{OK}
\]

137
6. Web crippling strength

See Example 9.1 for detailed calculations.

For end reaction:

\[ P_n = t^2 C_3 C_4 C_6 (217 - 0.28(h/t)) [1 + 0.01(N/t)] \]

\[ = (0.135)^2(1.258)(0.942)(1.0)[217 - 0.28(47.07)] \]

\[ x[1 + 0.01(22.22)] = 5.38 \text{ kips} \]

\[ \Omega = 2.00 \]

\[ P_a = 5.38/2.0 = 2.69 \text{ kips} \]

End reaction is given by

\[ R = 0.400wL \]

\[ = (0.400)(0.240)(10) = 0.96 \text{ kips} < P_a = 2.69 \text{ kips OK} \]

For interior reaction:

\[ P_n = t^2 C_1 C_2 C_6 [538 - 0.74(h/t)] [1 + 0.007(N/t)] \]

\[ = (0.135)^2(1.343)(0.977)(1.0)[538 - 0.74(47.07)] \]

\[ x[1 + 0.007(44.44)] = 15.79 \text{ kips} \]

\[ \Omega = 2.00 \]

\[ P_a = 15.79/2.0 = 7.90 \text{ kips} \]

Interior reaction is given by

\[ R = 1.10wL \]

\[ = (1.10)(0.240)(10) = 2.64 \text{ kips} < P_a = 7.90 \text{ kips OK} \]

7. Combined bending and web crippling strength

At the interior supports there is a combination of web bending and web crippling:

\[ M_a = 50.97 \text{ kips-in.} \quad M = 28.80 \text{ kips-in.} \]

\[ P_a = 7.90 \text{ kips} \quad R = 2.64 \text{ kips} \]

For shapes having single unreinforced webs:
1.07(R/P) + (M/M_a) \leq 1.42

1.07(2.64/7.90) + (28.80/50.97) = 0.923 < 1.42 OK

8. Deflection due to service live load.

From the result of sectional properties calculated in item (1) of Example 9.1, the section is fully effective at F_y = 50 ksi. Therefore, for a stress f = 42.12 ksi which is less than F_y = 50 ksi, the section will be fully effective, i.e.,

I_x = 7.835 in.\textsuperscript{4}

This value can be used for deflection determination.

The maximum deflection occurs at a distance of 0.446L from the exterior supports. It is given by

\[ \Delta = \frac{0.0069wL^4}{(E_o I_x)} \]

Thus, the live load deflection is calculated as follows:

\[ \Delta = \frac{0.0069(0.20)(10)^4(12)^3}{(27000 \times 7.835)} = 0.113 \text{ in.} \]

The live load deflection is limited to 1/240 of the span, i.e.,

L/240 = 10 \times 12 / 240 = 0.5 \text{ in.} > 0.113 \text{ in.} \text{ OK}

From the above calculations, it can be concluded that the section is adequate.
EXAMPLE 10.1 HAT SECTION USING INELASTIC RESERVE CAPACITY (LRFD)

(Inelastic Reserve Capacity)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\Phi_b M_n$. Use Type 301 stainless steel, annealed.

Figure 10.1 Section for Example 10.1

Given:
1. Section: Hat section, as shown in sketch.
2. Top flange continuously supported.
3. Span = 8 ft., simply supported.

Solution:
1. Properties of 90° corners:
   Corner Radius, $r = R + t/2 = 3/16 + 0.135/2 = 0.255$ in.
   Length of arc, $u = 1.57r = 1.57 \times 0.255 = 0.400$ in.
   Distance of c.g. from center of radius,
   $c = 0.637r = 0.637 \times 0.255 = 0.162$ in.
I' of corner about its own centroidal axis = 0.149r³
= 0.149(0.255)³ = 0.003 in.³. This is negligible.

2. Nominal Section Strength (Section 3.3.1.1)

a. Procedure I - Based on Initiation of Yielding

Computation of Iₓ, Sₑ, and Mₑ for the first approximation:

* Assume a compressive stress of f = Fᵧc = 28 ksi (yield strength in longitudinal compression, see Table A1 of the Standard) in the top fiber of the section.

* Assume web is fully effective.

Element 3:

h/t = 2.354/0.135 = 17.44 < 200 OK (Section 2.1.2-(1))
Assumed fully effective

Element 5:

w/t = 3.854/0.135 = 28.55 < 400 OK (Section 2.1.1-(1)-(ii))

k = 4

\[ \lambda = (1.052/\sqrt{k})\sqrt{f/E_o} \]  
\[ E_o = 27000 \text{ ksi is obtained from Table A4 of the Standard.} \]

\[ \lambda = (1.052/\sqrt{4})(28.55)\sqrt{28/28000} = 0.475 < 0.673 \]  
\[ b = w \]  
\[ = 3.854 \text{ in. (Fully effective)} \]
Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L Effective Length (in.)</th>
<th>Distance from Top Fiber (in.)</th>
<th>Ly (in.²)</th>
<th>Ly² (in.³)</th>
<th>I' About Own Axis (in.³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 x 1.347 = 2.694</td>
<td>2.933</td>
<td>7.902</td>
<td>23.175</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>2 x 0.400 = 0.800</td>
<td>2.839</td>
<td>2.271</td>
<td>6.448</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>2 x 2.354 = 4.708</td>
<td>1.500</td>
<td>7.062</td>
<td>10.593</td>
<td>2.174</td>
</tr>
<tr>
<td>4</td>
<td>2 x 0.400 = 0.800</td>
<td>0.161</td>
<td>0.129</td>
<td>0.021</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>3.854</td>
<td>0.068</td>
<td>0.262</td>
<td>0.018</td>
<td>--</td>
</tr>
<tr>
<td>Sum</td>
<td>12.856</td>
<td>17.626</td>
<td>40.255</td>
<td>2.174</td>
<td>--</td>
</tr>
</tbody>
</table>

The distance from the top fiber to the neutral axis is

\[ y_{cg} = \frac{L_y}{L} = \frac{17.626}{12.856} = 1.371 \text{ in.} \]

\[(3.000 - y_{cg})/y_{cg} = (3.0 - 1.371)/1.371 = 1.188 \]

\[1.188xF_{yc} = 33.264 \text{ ksi} > F_{yt} = 30 \text{ ksi} \text{ NG} \]

\(F_{yt}\) is the yield strength in longitudinal tension, see Table A1 of the Standard.

Since the computed stress in tension flange is larger than the specified yield strength, \(F_{yt} = 30 \text{ ksi}\), the compression stress of \(F_{yc}\) will not govern as assumed. The actual compressive stress will be less than \(F_{yc}\) and so the flange will still be fully effective. The tension flange will yield first. Section properties will not change.

Therefore,

\[ I'_{x} = Ly' + I'_{1} - Ly'_{cg} \]
\[ = 40.255 + 2.174 - 12.856(1.371)^2 \]
\[ = 18.26 \text{ in.}^3 \]
Actual $I_x = tI'_x$

$= (0.135)(18.26) = 2.47$ in.$^4$

Check Web

Assume a stress of $f = 30$ ksi at the bottom of tension fiber.

$f_1 = (1.048/1.629)(30) = 19.30$ ksi (compression)

$f_2 = -(1.306/1.629)(30) = -24.05$ ksi (tension)

$\Psi = \frac{f_2}{f_1} = -24.05/19.30 = -1.246$

$k = 4 + 2(1-\Psi)^2 + 2(1-\Psi)$

$= 4 + 2[1-(-1.246)]^2 + 2[1-(-1.246)]$

$= 31.15$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}, \hspace{1em} f = f_1$

For annealed Type 301 stainless steel, $E_o$ value is equal to 28000 ksi, which is given in Table A4 of the Standard.

$\lambda = (1.052/\sqrt{31.15})(17.44)/\sqrt{19.30}/28000 = 0.086 < 0.673$

$b = w$  \hspace{1em} (Eq. 2.2.1-1)

$b_e = 2.354$ in.

$b_2 = b_e/2$  \hspace{1em} (Eq. 2.2.2-2)

$= 2.354/2 = 1.177$ in.
\[ b_1 = \frac{b_e}{(3-\psi)} \]  
\[ = \frac{2.354}{(3-(-1.246))} = 0.554 \text{ in.} \]

\[ b_1 + b_2 = 0.554 + 1.177 = 1.731 \text{ in.} > 1.048 \text{ in. (compression portion of web, see the sketch shown above)} \]

Therefore, web is fully effective.

\[ S_e = \frac{I_x}{(d-y_{cg})} = \frac{2.47}{(3-1.371)} = 1.516 \text{ in.}^3 \]

\[ M_n = S_e F_n e_y (\text{Eq. 3.3.1.1-1}) \]
\[ = (1.516)(30) \]
\[ = 45.48 \text{ kips-in.} \]

**b. Procedure II - Based on Inelastic Reserve Capacity**

\[ \lambda_1 = \frac{1.11/\sqrt{F_{yc}/E_o}}{35.10} \]  
\[ = 1.11/\sqrt{28/28000} = 35.10 \]

\[ \lambda_2 = \frac{1.28/\sqrt{F_{yc}/E_o}}{40.48} \]  
\[ = 1.28/\sqrt{28/28000} = 40.48 \]

\[ w/t = 28.55 \]

For \( w/t < \lambda_1 = 35.10 \)

\[ C_y = 3.0 \]

Compute location of \( y \) on strain diagram, the summation of longitudinal forces should be zero.

Refer to equations from Reck, Pekoz, and Winter, "Inelastic Strength of Cold-Formed Steel Beams," Journal of the Structural Division, November 1975, ASCE.

Distance from neutral axis to the outer compression fiber, \( y_c \):

\[ t = 0.135 \text{ in.} \]
\[ b_t = 2(1.670) = 3.340 \text{ in.} \]
\[ b_c = 4.500 \text{ in.} \]
\[ d = 3.000 \text{ in.} \]
\[ y_c = (1/4)(b_t - b_c + 2d) \]
\[ = (1/4)[3.340 - 4.500 + 2(3.000)] = 1.210 \text{ in.} \]

\[ y_p = y_c / c \]
\[ = 1.21 / 3.0 = 0.403 \text{ in.} \]

\[ y_t = d - y_c \]
\[ = 3.000 - 1.210 = 1.790 \text{ in.} \]

\[ y_{cp} = y_c - y_p \]
\[ = 1.210 - 0.403 = 0.807 \text{ in.} \]

\[ y_{tp} = y_t - y_p \]
\[ = 1.790 - 0.403 = 1.387 \text{ in.} \]

**Summing moments of stresses in component plates:**

\[ M_n = F_y t \left\{ b_c y_c + 2y_{cp} \left[ y_p + (y_{cp}/2) \right] + (4/3)y_p^2 \right\} + 2y_{tp} \left[ y_p + (y_{tp}/2) \right] + b_t y_t \}

\[ M_n = 28(0.135) \left[ 4.500(1.210) + 2(0.807)(0.403 + (0.807/2)) \right] \]

145
\[(4/3)(0.403)^2+2(1.387)(0.403)+(1.387/2)+3.340(1.790)\]

\[M_n = 60.42 \text{ kips-in.}\]

\[M_n \text{ shall not exceed } 1.25S F_y = 1.25(45.48) = 56.85 \text{ kips-in.}\]

Therefore,

\[M_n = 1.25S F_y = 56.85 \text{ kips-in.}\]

The inelastic reserve capacity is used in this example because the following conditions are met: (Section 3.3.1.1(2))

1) Member is not subject to twisting, lateral, torsional, or torsional-flexural buckling.

2) The effect of cold-forming is not included in determining the yield point, \(F_y\).

3) The ratio of depth of the compressed portion of the web to its thickness does not exceed \(\lambda_1\),

\[(1.210-0.323)/0.135 = 6.57 < \lambda_1 = 35.10 \text{ OK}\]

4) The shear force does not exceed \(0.35F_y\) times the web area, \(h \times t\).

This still needs to be checked for a complete design.

5) The angle between any web and the vertical does not exceed 30°.

3. Design Flexural Strength, \(\phi_b M_n\)

\[\phi_b = 0.90 \text{ (for section with stiffened compression flanges)}\]

\[\phi_b M_n = 0.90 \times 56.85 = 51.17 \text{ kips-in.}\]
EXAMPLE 10.2 HAT SECTION USING INELASTIC RESERVE CAPACITY (ASD)

Rework Example 10.1 by using the Allowable Stress Design (ASD) method.

Solution:

Calculation of the allowable moment, $M_a$:

The effective section properties calculated by the ASD method are the same as those determined in Example 10.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

$$M_a = M_n / \Omega$$  \hspace{1cm} (Eq. E-1)

$\Omega = 1.85$ (Safety Factor stipulated in Table E of the Standard)

The nominal section strength based on inelastic reserve capacity is as follows:

$M_n = 56.85$ kips-in. (obtained from Example 10.1)

$$M_a = M_n / \Omega$$

$$M_a = 56.85 / 1.85$$

$$M_a = 30.73$$ kips-in.
EXAMPLE 11.1 DECK SECTION (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\phi_{M_n}$, based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the service moment. Compute the factored uniform load, $w_u'$, as controlled either by bending or deflection. Use Type 201 stainless steel, 1/4-Hard. Assume dead load to live load ratio $D/L = 1/5$ and $1.2D + 1.6L$ governs the design.

Given:

1. Section: Deck section, as shown in sketch.
2. Deck is continuous over three 10'-0" spans.
3. Deflection due to service live load is to be limited to 1/240 of the span.
Corner Properties:

\[ \theta = 75.96^\circ \]
\[ R = 1/8" \]
\[ r = 0.155" \]
\[ a = r \sin(90^\circ - 75.96^\circ) \]
\[ = 0.155" \sin 14.04^\circ \]
\[ = 0.0376" \]
\[ b = t/2 + r - a \]
\[ = 0.060" / 2 + 0.155" - 0.0376" \]
\[ = 0.147" \]

\[ b' = b - t/2 \]
\[ = 0.147" - 0.060" / 2 \]
\[ = 0.117" \]

\[ b'/b" = \cos(90^\circ - 75.96^\circ) \]
\[ b" = b' / \cos 14.04^\circ \]
\[ = 0.117" / \cos 14.04^\circ \]
\[ = 0.121" \]
Solution:

1. Full Section Properties:

Elements 2 and 6:

Corner Radius, \( r = R + \frac{t}{2} = \frac{1}{8} + \frac{0.060}{2} = 0.155 \text{ in.} \)

Angle, \( \theta = 75.96^\circ = 1.326 \text{ rad} \)

Length of arc, \( u = \theta r = 1.326(0.155) = 0.206 \text{ in.} \)

Distance of c.g. from center of radius,
\[ c_1 = r \sin \theta / \theta = 0.155(\sin 1.326)/1.326 = 0.113 \text{ in.} \]

The moment of inertia, \( I'_1 \), of arc element about its own centroidal axis is negligible.

Element 3:

\( l = 3.819 \text{ in.} \)

\( \theta = 14.04^\circ \)
\[
\cos \theta = 0.9701
\]

\[
I'_{1} = \frac{(\cos^3 \theta)}{12} = \frac{(0.9701)^2(3.819)^3}{12} = 4.368 \text{ in}^3
\]

Element 7:

\[
\begin{align*}
L &= 1.000 \text{ in.} \\
\theta &= 14.04^\circ \\
\cos \theta &= 0.9701
\end{align*}
\]

\[
I'_{1} = \frac{(\cos^3 \theta)}{12} = \frac{(0.9701)^2(1)^3}{12} = 0.0784 \text{ in}^3
\]

Distance from top fiber to the centroid of full section is

\[
y = 4 - 0.147 - \frac{1.000}{2} \cos 14.04^\circ = 3.368 \text{ in.}
\]

2. Section Modulus for Load Determination - Based on Initiation of Yielding

Since the effective design width of flat compressive elements is a function of stress, iteration is required.

Computation of \( I_x, S_e, \) and \( M_n \) for the first approximation:

* Assume a compressive stress of \( f = F_y = 50 \text{ ksi} \) in the top fiber of the section. (See Table A1 of the Standard for \( F_y \) value.)

* Assume web is fully effective.

Element 3:

\[
h/t = \frac{3.819}{0.060} = 63.65 < 200 \text{ OK (Section 2.1.2-(1))}
\]

Assumed fully effective

Element 4:

\[
w/t = \frac{2.000}{0.060} = 33.33 < 400 \text{ OK (Section 2.1.1-(1)-(ii))}
\]

\[
k = 4
\]
\[ \Lambda = \frac{(1.052/\sqrt{k})(w/t)\sqrt{f/E_o}}{f} = F_y \quad \text{(Eq. 2.2.1-4)} \]

From Table A4 of the Standard, \( E_o \) value is equal to 27000 ksi in longitudinal compression for Type 201, 1/4-Hard, stainless steel.

\[ \Lambda = \frac{(1.052/\sqrt{4})(33.33)\sqrt{50/27000}}{0.754} = 0.754 > 0.673 \]

\[ \rho = \frac{(1-0.22/\Lambda)}{\Lambda} = \frac{(1-0.22/0.754)}{0.754} = 0.939 \quad \text{(Eq. 2.2.1-3)} \]

\[ b = \rho w \quad \text{(Eq. 2.2.1-2)} \]

\[ = 0.939 \times 2.000 \]

\[ = 1.878 \text{ in.} \]

Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L Effective Length (in.)</th>
<th>y Distance from Top Fiber (in.)</th>
<th>( Ly ) (in.²)</th>
<th>( Ly^2 ) (in.³)</th>
<th>( I' ) About Own Axis (in.⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>3.970</td>
<td>3.970</td>
<td>15.761</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>5 x 0.206 = 1.030</td>
<td>3.928</td>
<td>4.046</td>
<td>15.892</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>4 x 3.819 = 15.276</td>
<td>2.000</td>
<td>30.552</td>
<td>61.104</td>
<td>17.472</td>
</tr>
<tr>
<td>4</td>
<td>2 x 1.878 = 3.756</td>
<td>0.030</td>
<td>0.113</td>
<td>0.003</td>
<td>--</td>
</tr>
<tr>
<td>5 &amp; 8</td>
<td>2 x 2.000 = 4.000</td>
<td>3.970</td>
<td>15.880</td>
<td>63.044</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>4 x 0.206 = 0.824</td>
<td>0.072</td>
<td>0.059</td>
<td>0.004</td>
<td>--</td>
</tr>
<tr>
<td>7</td>
<td>1.000</td>
<td>3.368</td>
<td>3.368</td>
<td>11.343</td>
<td>0.078</td>
</tr>
<tr>
<td>Sum</td>
<td>26.886</td>
<td>57.988</td>
<td>167.151</td>
<td>17.550</td>
<td></td>
</tr>
</tbody>
</table>

The distance from the top fiber to the neutral axis is

\[ \gamma_{cg} = \frac{Ly}{L} = \frac{57.988}{26.886} = 2.157 \text{ in.} \]

Since the distance from the top compression fiber to the neutral axis is greater than one half of the deck depth,
a compressive stress of $F_y$ will govern as assumed.

$$I'_x = Ly^2 + I'_1 - Ly_{cg}$$

$$= 167.151 + 17.550 - 26.886(2.157)^2$$

$$= 59.61 \text{ in.}^3$$

Actual $I_x = tI'_x$

$$= (0.060)(59.61) = 3.58 \text{ in.}^4$$

Check Web

$$f_1 = (2.010/2.157)(50) = 46.59 \text{ ksi(compression)}$$

$$f_2 = -(1.696/2.157)(50) = -39.31 \text{ ksi(tension)}$$

$$\psi = f_2/f_1 = -39.31/46.59 = -0.844$$

$$k = 4+2(1-\psi)^3+2(1-\psi)$$

(Eq. 2.2.2-4)

$$= 4+2[-(-0.844)]^3+2[1-(-0.844)]$$

$$= 20.23$$

$$\lambda = (1.052/\sqrt{k})(w/t)/\sqrt{f/E_o}, \ f = f_1$$

(Eq. 2.2.1-4)

$$= (1.052/\sqrt{20.23})(63.65)/\sqrt{46.59/27000} = 0.618 < 0.673$$

$$b = w$$

(Eq. 2.2.1-1)

$$b_e = 3.819 \text{ in.}$$

$$b_2 = b_e/2$$

(Eq. 2.2.2-2)
\[ \frac{3.819}{2} = 1.910 \text{ in.} \]

\[ b_1 = \frac{b_e}{(3-\Psi)} \quad \text{(Eq. 2.2.2-1)} \]

\[ = \frac{3.819}{[3-(-0.844)]} = 0.993 \text{ in.} \]

\[ b_1 + b_2 = 0.993 + 1.910 = 2.903 \text{ in.} > 2.002 \text{ in. (compression portion of web, see the sketch shown above.)} \]

Therefore, web is fully effective.

\[ S_e = \frac{I_x}{y_{cg}} \]

\[ = \frac{3.58}{2.157} \]

\[ = 1.66 \text{ in.}^3 \]

\[ M_n = S_e F_y \quad \text{(Eq. 3.3.1.1-1)} \]

\[ = (1.66)(50) \]

\[ = 83.0 \text{ kips-in.} \]

\[ \phi_b = 0.90 \text{ (for section with stiffened compression flanges)} \]

\[ \phi_b M_n = 0.90 \times 83.0 = 74.7 \text{ kips-in.} \]

3. Moment of Inertia for Deflection Determination - Positive Bending

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of 1.2D+1.6L, the service moment can be determined as follows:

\[ \phi_b M_n = 1.2 M_{DL} + 1.6 M_{LL} \]

\[ = [1.2(M_{DL}/M_{LL})+1.6]M_{LL} \]

\[ = [1.2(1/5)+1.6]M_{LL} \]

\[ = 1.84 M_{LL} \]

\[ M_{LL} = \phi_b M_n / 1.84 = 74.70/1.84 = 40.60 \text{ kips-in.} \]

\[ M_s = M_{DL} + M_{LL} \]
\[ M = (1/5+1)M_{LL} \]
\[ = 1.2(40.60) = 48.72 \text{ kips-in.} \]

where

- \( M_{DL} \) = Moment determined on the basis of nominal dead load
- \( M_{LL} \) = Moment determined on the basis of nominal live load

Computation of \( I_{eff} \) for the first approximation:

* Assume a stress of \( f = 28.66 \text{ ksi} \) in the top and bottom fibers of the section.

* Since the web was fully effective at a higher stress gradient, it will be fully effective at this stress level.

Element 4:

\[ w/t = 33.33 \]
\[ k = 4 \]

For deflection determination, the value of \( E_r \), reduced modulus of elasticity determined by using Eq. (2.2.1-7), is substituted for \( E_o \) in Eq. (2.2.1-4). For a compression and tension stresses of \( f = 28.66 \text{ ksi} \), the corresponding \( E_{sc} \) and \( E_{st} \) values for Type 201 stainless steel are obtained from Table A2 or Figure A1 of the Standard as follows:

\[ E_{sc} = 23550 \text{ ksi}, \quad E_{st} = 26970 \text{ ksi} \]

\[ E_r = \left( E_{sc} + E_{st} \right)/2 \quad \text{(Eq. 2.2.1-7)} \]

\[ = (23550 + 26970)/2 = 25260 \text{ ksi} \]

Thus, for compression flange (Element 4):

\[ A = (1.052/\sqrt{k})(w/t)\sqrt{f/E_r} \]

\[ = (1.052/\sqrt{4})(33.33)\sqrt{28.66/25260} = 0.591 < 0.673 \]
\[ b_d = w \]  
\[ = 2.000 \text{ in. (Fully effective)} \]

Note: All elements are fully effective.

Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>( L ) (in.)</th>
<th>( y ) Distance from Top Fiber (in.)</th>
<th>( L_y ) (in.(^2))</th>
<th>( L_y^2 ) (in.(^3))</th>
<th>( I'_{\text{eff}} ) About Own Axis (in.(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>3.970</td>
<td>3.970</td>
<td>15.761</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>5 x 0.206 = 1.030</td>
<td>3.928</td>
<td>4.046</td>
<td>15.892</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>4 x 3.819 = 15.276</td>
<td>2.000</td>
<td>30.552</td>
<td>61.104</td>
<td>17.472</td>
</tr>
<tr>
<td>4</td>
<td>2 x 2.000 = 4.000</td>
<td>0.030</td>
<td>0.120</td>
<td>0.004</td>
<td>--</td>
</tr>
<tr>
<td>5 &amp; 8</td>
<td>2 x 2.000 = 4.000</td>
<td>3.970</td>
<td>15.880</td>
<td>63.044</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>4 x 0.206 = 0.824</td>
<td>0.072</td>
<td>0.059</td>
<td>0.004</td>
<td>--</td>
</tr>
<tr>
<td>7</td>
<td>1.000</td>
<td>3.368</td>
<td>3.368</td>
<td>11.343</td>
<td>0.078</td>
</tr>
<tr>
<td>Sum</td>
<td>27.130</td>
<td>57.995</td>
<td>167.152</td>
<td>17.550</td>
<td></td>
</tr>
</tbody>
</table>

The distance from the top fiber to the neutral axis is

\[ y_{cg} = \frac{L_y}{L} = \frac{57.995}{27.130} = 2.138 \text{ in.} \]

Since the distance from the top compression fiber to the neutral axis is greater than one half the deck depth, the compressive stress of 28.66 ksi will govern as assumed.

\[ I'_{\text{eff}} = L_y^2 + I'_1 - L_y^2_{cg} \]
\[ = 167.152 + 17.550 - 27.130(2.138)^2 \]
\[ = 60.69 \text{ in.}^3 \]

Actual \( I'_{\text{eff}} = tI'_{\text{eff}} \)
\[ = (0.060)(60.69) = 3.64 \text{ in.}^4 \]
\[ S_{\text{eff}} = \frac{I_{\text{eff}}}{y_{\text{cg}}} = \frac{3.64}{2.138} = 1.70 \text{ in.}^3 \]
\[ M = S_{\text{eff}}(28.66) = (1.70)(28.66) = 48.72 \text{ kips-in.} = M_s \text{ OK} \]

Thus, use \( I_{\text{eff}} = 3.64 \text{ in.}^4 \) for deflection calculations.

4. Section Modulus for Load Determination - Negative Bending (Based on Initiation of Yielding)

Following a similar procedure as in positive bending.

Computation of \( I_x', S_e \) and \( M_n \) for the first approximation:

* Assume a compressive stress of \( f = F_y = 50 \text{ ksi} \) in the bottom fiber of the section.

* Assume web is fully effective.

Element 3:

\[ \frac{h}{t} = 3.819/0.060 = 63.65 < 200 \text{ OK (Section 2.1.2-(1))} \]

Assumed fully effective

Element 1:

\[ \frac{w}{t} = 1.000/0.060 = 16.67 < 50 \text{ OK (Section 2.1.1-(1)-(iii))} \]

\[ k = 0.50 \]

\[ \lambda = (1.052/\sqrt{0.50})(16.67)/\sqrt{50/27000} = 1.067 > 0.673 \] (Eq. 2.2.1-3)

\[ \rho = (1-0.22/\lambda)/\lambda \]

\[ = (1-0.22/1.067)/1.067 = 0.744 \] (Eq. 2.2.1-2)

\[ b = \rho w \]

\[ = 0.744 \times 1.000 \]

\[ = 0.744 \text{ in.} \]
Element 5:
Same as element 4 in positive bending case.

\[ b = 1.878 \text{ in.} \]

Element 8:

\[ \frac{w}{t} = \frac{2.000}{0.060} = 33.33 < 50 \text{ OK (Section 2.1.1-(1)-(iii))} \]

\[ S = 1.28\sqrt{\frac{E_o}{f}} \quad \text{(Eq. 2.4-1)} \]

\[ = 1.28\sqrt{27000/50} = 29.74 \]

For \( w/t > S \)

\[ I_a = t\left\{\left[115\left(\frac{w}{t}\right)/S\right]+5\right\} \quad \text{(Eq. 2.4.2-13)} \]

\[ = (0.060)\left\{\left[115(33.33)/29.74\right]+5\right\} \]

\[ = 0.00174 \text{ in.}^4 \]

\[ I_s = d^3\sin^2\theta/12 \]

\[ = (1.000)^3(0.060)(\sin75.96^\circ)^2/12 = 0.00471 \text{ in.}^4 \]

\[ D = 1.000+0.185\tan(75.96^\circ/2) = 1.144 \text{ in.} \]

\[ D/w = 1.144/2.000 = 0.572 \]

For \( 0.25 < D/w < 0.80 \)

\[ k = \left[4.82-5(D/w)\right](I_s/I_a)^{1/3}+0.43 \leq 5.25-5(D/w) \quad \text{(Eq. 2.4.2-9)} \]

\[ \left[4.82-5(0.572)\right](0.00471/0.00174)^{1/3}+0.43 = 3.162 \]

\[ 5.25-5(0.572) = 2.390 < 3.162 \]

\[ k = 2.390 \]

\[ \lambda = (1.052/\sqrt{2.390})(33.33)/\sqrt{50/27000} = 0.976 > 0.673 \]

\[ \rho = \frac{(1-0.22/\lambda)}{\lambda} \quad \text{(Eq. 2.2.1-3)} \]

\[ = (1-0.22/0.976)/0.976 = 0.794 \]

\[ b = \rho w \quad \text{(Eq. 2.2.1-2)} \]

\[ = 0.794(2.000) = 1.588 \text{ in.} \]
Element 7:

\[ I_s = 0.00471 \text{ in.}^4 \] (calculated previously)

\[ I_a = 0.00174 \text{ in.}^4 \] (calculated previously)

\[ d = 1.000 \text{ in.} \]

Assume a maximum stress in element, \( f = F_y = 50 \text{ ksi} \), although it will be actually less.

\[ k = 0.50 \]

\[ w/t = 1.000/0.060 = 16.67 < 50 \text{ OK (Section 2.1.1-(1)-(iii))} \]

\[ \lambda = (1.052/\sqrt{k})(w/t)^{\sqrt{E/E_0}} \]  \hspace{1cm} (Eq. 2.2.1-4)

\[ = (1.052/\sqrt{0.50})(16.67)^{\sqrt{50/27000}} = 1.067 > 0.673 \]

\[ \rho = (1-0.22/\lambda)/\lambda \]  \hspace{1cm} (Eq. 2.2.1-3)

\[ = (1-0.22/1.067)/1.067 = 0.744 \]

\[ b = \rho w \]  \hspace{1cm} (Eq. 2.2.1-2)

\[ = 0.744(1.000) = 0.744 \text{ in.} \]

\[ d' = 0.744 \text{ in.} \]

\[ d_s = d'(I_s/I_a) \leq d' \]  \hspace{1cm} (Eq. 2.4.2-11)

Since \( I_s/I_a > 1 \)

\[ d_s = d' = 0.744 \text{ in.} \]

\[ I'_1 = (d_s)^3\sin^2\theta/12 = (0.744)^3(\sin75.96^\circ)^2/12 = 0.032 \text{ in.}^3 \]

The distance from top fiber to the centroid of the reduced section is

\[ y = 4-0.147-(0.744/2)\cos14.04^\circ = 3.492 \text{ in.} \]
Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L Effective Length (in.)</th>
<th>y Distance from Top Fiber (in.)</th>
<th>Ly (in.²)</th>
<th>Ly² (in.⁴)</th>
<th>I'₁ About Own Axis (in.⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.744</td>
<td>3.970</td>
<td>2.954</td>
<td>11.726</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>5 x 0.206 = 1.030</td>
<td>3.928</td>
<td>4.046</td>
<td>15.892</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>4 x 3.819 = 15.276</td>
<td>2.000</td>
<td>30.552</td>
<td>61.104</td>
<td>17.472</td>
</tr>
<tr>
<td>4</td>
<td>2 x 2.000 = 4.000</td>
<td>0.030</td>
<td>0.120</td>
<td>0.004</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>1.878</td>
<td>3.970</td>
<td>7.456</td>
<td>29.599</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>4 x 0.206 = 0.824</td>
<td>0.072</td>
<td>0.059</td>
<td>0.004</td>
<td>--</td>
</tr>
<tr>
<td>7</td>
<td>0.744</td>
<td>3.492</td>
<td>2.598</td>
<td>9.072</td>
<td>0.020</td>
</tr>
<tr>
<td>8</td>
<td>1.588</td>
<td>3.970</td>
<td>6.304</td>
<td>25.028</td>
<td>--</td>
</tr>
<tr>
<td>Sum</td>
<td>26.084</td>
<td>54.089</td>
<td>152.429</td>
<td>17.504</td>
<td></td>
</tr>
</tbody>
</table>

The distance from top fiber to the neutral axis is (see sketch below)

\[ y_{cg} = \frac{Ly}{L} = \frac{54.089}{26.084} = 2.074 \text{ in.} \]

The corresponding tension stress can be computed as follows:

\[ \frac{y_{cg}}{(4.00-y_{cg})} = \frac{2.074}{(4.00-2.074)} = 1.077 \]

\[ 1.077 \times F_y c = 1.077 \times 50 = 53.85 \text{ ksi} < F_{yt} = 75 \text{ ksi OK} \]

Because the distance of the top fiber from the neutral axis is greater than one half the deck depth, and also because the computed tension stress is less than the specified value, the compressive stress of \( f=F_y \) will govern as assumed.

\[ f_t = \frac{(2.074/1.926)(50)}{160} \]

\[ = 53.85 \text{ ksi} < F_{yt} \text{ OK} \]
Check Web:

\[ f_1 = \left( \frac{1.779}{1.926} \right) (50) = 46.18 \text{ ksi (compression)} \]

\[ f_2 = -\left( \frac{1.927}{1.926} \right) (50) = -50.03 \text{ ksi (tension)} \]

\[ \Psi = \frac{f_2}{f_1} = -50.03/46.18 = -1.083 \]

\[ k = 4 + 2(1-\Psi)^2 + 2(1-\Psi) = 26.24 \] (Eq. 2.2.2-4)

\[ \lambda = \left( \frac{1.052/\sqrt{k}}{w/t} \right) \sqrt{f/E_o} , \; f = f_1 \] (Eq. 2.2.1-4)

\[ \lambda = \left( \frac{1.052/\sqrt{26.24}}{63.65} \right) \sqrt{46.18/27000} = 0.541 < 0.673 \]

\[ b = w \] (Eq. 2.2.1-1)

\[ b_e = 3.819 \text{ in.} \]

\[ b_2 = b_e/2 \] (Eq. 2.2.2-2)

\[ b_2 = 3.819/2 = 1.910 \text{ in.} \]

\[ b_1 = b_e/\left(3-\Psi\right) \] (Eq. 2.2.2-1)

\[ b_1 = 3.819/\left(3-(-1.083)\right) = 0.935 \text{ in.} \]

\[ b_1 + b_2 = 0.935 + 1.910 = 2.845 \text{ in.} > 1.763 \text{ in. (compression portion of web, see sketch shown above)} \]

Therefore, web is fully effective.
Check Element 7:

Assume the maximum stress in element, \( f = 46.18 \text{ ksi} \)

\( k = 0.50 \)

\( w/t = 16.67 \)

\[
\lambda = \frac{(1.052/\sqrt{k})(w/t)\sqrt{f/E_o}}{(1.052/\sqrt{0.50})(16.67)\sqrt{46.18/27000}} = 1.026 > 0.673
\]  
(Eq. 2.2.1-4)

\[
\rho = \frac{(1-0.22/\lambda)}{\lambda} = \frac{(1-0.22/1.026)/1.026}{1.026} = 0.766
\]  
(Eq. 2.2.1-3)

\[
b = \rho w = 0.766(1.000) = 0.766 \text{ in.}
\]  
(Eq. 2.2.1-2)

\[
d' = 0.766 \text{ in.}
\]

\[
d_s = d'(I_s/I_a) \leq d'
\]  
(Eq. 2.4.2-11)

Since \( I_s/I_a > 1 \)

\[
d_s = d' = 0.766 \text{ in.}
\]

\[
I_1' = (d_s)^3\sin^2\theta/12 = (0.766)^3(\sin75.96^\circ)^2/12 = 0.035 \text{ in.}^3
\]

The distance from top fiber to the centroid of the reduced section is

\[
y = 4-0.147-(0.766/2)\cos14.04^\circ = 3.481 \text{ in.}
\]

Determine section properties, but only the properties of

element 7 have changed

\[
\Delta L = 0.766-0.744 = 0.022 \text{ in.}
\]

\[
\Delta L_y = (0.766)(3.481)-2.598 = 0.068 \text{ in.}^2
\]

\[
\Delta L_y^2 = 0.766(3.481)^2-9.072 = 0.210 \text{ in.}^3
\]

\[
\Delta I_1' = 0.035-0.032 = 0.003 \text{ in.}^3
\]

Therefore,

\[
L = 26.084+0.022 = 26.106 \text{ in.}
\]

\[
L_y = 54.089+0.068 = 54.097 \text{ in.}^2
\]
The distance from top fiber to the neutral axis is:

\[ y_{cg} = \frac{L_y}{L} = \frac{54.097}{26.106} = 2.072 \text{ in.} \]

\[ f_t = (2.072/1.928)(50) = 53.73 \text{ ksi} < F_{yt} = 75 \text{ ksi OK} \]

\[ I_x' = L_y^2 + I_1 - L_y c_g \]

\[ = 152.639 + 17.507 - 26.106(2.072)^2 \]

\[ = 58.07 \text{ in.}^3 \]

Actual \( I_x = t I_x' \)

\[ = (0.060)(58.07) = 3.48 \text{ in.}^4 \]

\[ S_e = \frac{I_x}{(4.00 - y_{cg})} \]

\[ = \frac{3.48}{(4.00 - 2.072)} \]

\[ = 1.80 \text{ in.}^3 \]

\[ M_n = S_e F_y \]

\[ = (1.80)(50) \]

\[ = 90.0 \text{ kips-in.} \]

\[ \Phi_b = 0.85 \text{ (for section with unstiffened compression flanges)} \]

\[ \Phi_b M_n = 0.85 \times 90.0 = 76.5 \text{ kips-in.} \]

5. Moment of Inertia for Deflection Determination - Negative Bending

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of 1.2D + 1.6L, the service moment can be determined as follows:

\[ \Phi_b M_n = 1.2 M_{DL} + 1.6 M_{LL} \]
\[ M_{LL} = \phi_{bn} \frac{M_n}{1.84} = 76.50/1.84 = 41.58 \text{ kips-in.} \]

\[ M_s = M_{DL} + M_{LL} = (1/5+1)M_{LL} = 1.2(41.58) = 49.90 \text{ kips-in.} \]

Computation of \( I_{\text{eff}} \) for the first approximation:

* Assume a stress of \( f = 27 \text{ ksi} \) in the top and bottom fibers of the section.

* Since the web was fully effective at a higher stress gradient, it will be fully effective at this stress level.

Element 1:

\[ \frac{w}{t} = 16.67 \]

\[ k = 0.50 \]

\[ \lambda = (1.052/\sqrt{k})(\frac{w}{t})\sqrt{f/E_r} \]

For a compression and tension stresses of \( f=27 \text{ ksi} \), the values of \( E_{sc} \) and \( E_{st} \) are equal to 24550 ksi and 27000 ksi, respectively.

\[ E_r = \frac{(24550+27000)}{2} \quad \text{(Eq. 2.2.1-7)} \]

\[ = 25775 \text{ ksi} \]

\[ \lambda = (1.052/\sqrt{0.50})(16.67)\sqrt{27/25775} = 0.803 > 0.673 \]

\[ \rho = \frac{(1-0.22/\lambda)}{\lambda} \quad \text{(Eq. 2.2.1-3)} \]

\[ = (1-0.22/0.803)/0.803 = 0.904 \]

\[ b = \rho w \quad \text{(Eq. 2.2.1-2)} \]

\[ = 0.904 \times 1.000 \]

164
Element 5:

\( \frac{w}{t} = 33.33 \)

\( k = 4 \)

\( \lambda = \frac{1.052/\sqrt{k}}{\left(\frac{w}{t}\right)} \sqrt{E/E_{\tau}} \)

\[ = \frac{1.052/\sqrt{4}}{33.33/27/\sqrt{25775}} = 0.567 < 0.673 \]

\( b_d = w \quad \text{(Eq. 2.2.1-5)} \)

\[ = 2.000 \text{ (Fully effective)} \]

Element 8:

\( \frac{w}{t} = 33.33 \)

\( S = 1.28 \frac{E_0}{f} \quad \text{(Eq. 2.4-1)} \)

\[ = 1.28 \times 27000/27 = 40.48 \]

For \( S/3 < \frac{w}{t} < S \),

\( I_a = t^399 \left\{ \left[ \frac{(w/t)}{S} \right]-0.33 \right\}^3 \quad \text{(Eq. 2.4.2-6)} \)

\[ = (0.060)^3(399) \left[ \left( \frac{33.33}{40.48} \right)-0.33 \right]^3 \]

\[ = 0.000621 \text{ in.}^4 \]

\( I_s = 0.00471 \text{ in.}^4 \) (calculated previously)

\( I_s/I_a = 0.00471/0.000621 = 7.58 > 1 \)

\( D = 1.144 \text{ in.} \) (calculated previously)

\( D/w = 0.572 \) (calculated previously)

For \( 0.25 < D/w < 0.80 \)

\( k = [4.82-5(D/w)](I_s/I_a)^{1/2}+0.43 \leq 5.25-5(D/w) \quad \text{(Eq. 2.4.2-9)} \)

Since \( I_s/I_a > 1 \)

\( k = 5.25-5(D/w) = 5.25-5(0.572) = 2.390 \)

\( \lambda = \frac{1.052/\sqrt{k}}{\left(\frac{w}{t}\right)} \sqrt{E/E_{\tau}} \)
\[ E_r = 25775 \text{ ksi for a stress of } f=27 \text{ ksi.} \]

\[ \lambda = (1.052/\sqrt{2.390})(33.33)/27/25775 = 0.734 > 0.673 \]

\[ \rho = (1-0.22/\lambda)/\lambda \quad \text{(Eq. 2.2.1-3)} \]
\[ = (1-0.22/0.734)/0.734 = 0.954 \]

\[ b = \rho w \quad \text{(Eq. 2.2.1-2)} \]
\[ = 0.954(2.000) = 1.908 \text{ in.} \]

Element 7:

\[ I_s/I_a > 1 \]

\[ d = 1.000 \text{ in.} \]

Assume the maximum stress in element, \( f = 27 \text{ ksi,} \) although it will be actually less.

\[ k = 0.50 \]

\[ w/t = 16.67 \]

\[ \lambda = (1.052/\sqrt{l})(w/t)\sqrt{f/E_r} \]
\[ = (1.052/\sqrt{0.50})(16.67)/27/25775 = 0.803 > 0.673 \]

\[ \rho = (1-0.22/\lambda)/\lambda \quad \text{(Eq. 2.2.1-3)} \]
\[ = (1-0.22/0.803)/0.803 = 0.904 \]

\[ b = \rho w \quad \text{(Eq. 2.2.1-2)} \]
\[ = 0.904(1.000) = 0.904 \text{ in.} \]

\[ d_s' = 0.904 \text{ in.} \]

\[ d_s = d_s'(I_s/I_a) \leq d_s' \quad \text{(Eq. 2.4.2-11)} \]

Since \( I_s/I_a > 1 \)

\[ d_s = d_s' = 0.904 \text{ in.} \]

\[ I_1' = (d_s')^2\sin^2\theta/12 = (0.904)^2\sin^2(75.96^\circ)/12 = 0.058 \text{ in.}^2 \]

The distance from top fiber to the centroid of the reduced section is

\[ y = 4-0.147-(0.904/2)\cos14.04^\circ = 3.415 \text{ in.} \]
Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L (in.)</th>
<th>Distance from Top Fiber (in.)</th>
<th>Ly (in.²)</th>
<th>Ly² (in.⁴)</th>
<th>I'₁ About Own Axis (in.⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.904</td>
<td>3.970</td>
<td>3.589</td>
<td>14.248</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>5 x 0.206 = 1.030</td>
<td>3.928</td>
<td>4.046</td>
<td>15.892</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>4 x 3.819 = 15.276</td>
<td>2.000</td>
<td>30.552</td>
<td>61.104</td>
<td>17.472</td>
</tr>
<tr>
<td>4</td>
<td>2 x 2.000 = 4.000</td>
<td>0.030</td>
<td>0.120</td>
<td>0.004</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>2.000</td>
<td>3.970</td>
<td>7.940</td>
<td>31.522</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>4 x 0.206 = 0.824</td>
<td>0.072</td>
<td>0.059</td>
<td>0.004</td>
<td>--</td>
</tr>
<tr>
<td>7</td>
<td>0.904</td>
<td>3.415</td>
<td>3.087</td>
<td>10.542</td>
<td>0.058</td>
</tr>
<tr>
<td>8</td>
<td>1.908</td>
<td>3.970</td>
<td>7.575</td>
<td>30.072</td>
<td>--</td>
</tr>
<tr>
<td>Sum</td>
<td>26.846</td>
<td>56.968</td>
<td>163.388</td>
<td>17.530</td>
<td></td>
</tr>
</tbody>
</table>

The distance from top fiber to the neutral axis is

\[ y_{cg} = \frac{L_y}{L} = \frac{56.968}{26.846} = 2.122 \text{ in.} \]

\[ I'_{\text{eff}} = L_y^2 + I'₁ - L_y^2_{cg} \]

\[ = 163.388 + 17.530 - 26.846(2.122)^2 \]

\[ = 60.03 \text{ in.}³ \]

Actual \( I'_{\text{eff}} = \frac{t}{I'_{\text{eff}}} \)

\[ = \frac{0.060}{(60.03)} = 3.60 \text{ in.}⁴ \]

\[ S'_{\text{eff}} = \frac{I'_{\text{eff}}}{(d-y_{cg})} = \frac{3.60/(4-2.122)} = 1.92 \text{ in.}³ \]

\[ M = (1.92)(27) = 51.84 \text{ ksi} > M_s = 49.90 \text{ ksi} \text{ N.G.} \]

Computation of \( I'_{\text{eff}} \) for the second approximation:

* Assume a stress of \( f=25.85 \text{ ksi} \) in the top and bottom fibers of the section.
Element 1:

\[ \frac{w}{t} = 16.67 \]
\[ k = 0.50 \]
\[ \lambda = \frac{1.052}{\sqrt{k}} \left( \frac{w}{t} \right) \sqrt{\frac{f}{E_r}} \]

For a compression and tension stresses of \( f = 25.85 \) ksi, the values of \( E_{sc} \) and \( E_{st} \) are equal to 25180 ksi and 27000 ksi, respectively.

\[ E_r = \frac{(25180 + 27000)}{2} = 26090 \text{ ksi} \]
\[ \lambda = \frac{1.052}{\sqrt{0.50}} (16.67) \sqrt{25.85/26090} = 0.781 > 0.673 \]

\[ \rho = \frac{(1 - 0.22/\lambda)}{\lambda} \]
\[ = \frac{(1 - 0.22/0.781)}{0.781} = 0.920 \]

\[ b = \rho w \]
\[ = 0.920 \times 1.000 \]
\[ = 0.917 \text{ in.} \]

Element 5:

Fully effective at \( f = 27 \) ksi

It will also be fully effective at \( f = 25.85 \) ksi

\[ b = 2.000 \text{ in.} \]

Element 8:

\[ \frac{w}{t} = 33.33 \]
\[ S = 1.28 \sqrt{\frac{E_o}{f}} \]
\[ = 1.28 \sqrt{27000/25.85} = 41.37 \]

For \( S/3 < \frac{w}{t} < S \),

\[ I_s/I_a > 1 \] by observation from the first approximation

\[ D/w = 0.572 \]
Since $I_s/I_a > 1$

\[ k = 2.390 \]

\[ \lambda = (1.052/\sqrt{k})(w/t)\sqrt{E/E_r} \]

\[ = (1.052/\sqrt{2.390})(33.33)\sqrt{25.85/26090} = 0.714 > 0.673 \]

\[ \rho = (1-0.22/\lambda)/\lambda \] (Eq. 2.2.1-3)

\[ = (1-0.22/0.714)/0.714 = 0.969 \]

\[ b = \rho w \] (Eq. 2.2.1-2)

\[ = 0.969 \times 2.000 \]

\[ = 1.938 \text{ in.} \]

Element 7:

$I_s/I_a > 1$

\[ d = 1.000 \text{ in.} \]

Assume a maximum stress in element, $f = 25.85 \text{ ksi}$, although it will be actually less.

\[ k = 0.50 \]

\[ w/t = 16.67 \]

\[ \lambda = (1.052/\sqrt{k})(w/t)\sqrt{E/E_r} \]

\[ = (1.052/\sqrt{0.50})(16.67)\sqrt{25.85/26090} = 0.781 > 0.673 \]

\[ \rho = (1-0.22/\lambda)/\lambda \] (Eq. 2.2.1-3)

\[ = (1-0.22/0.781)/0.781 = 0.920 \]

\[ b = \rho w \] (Eq. 2.2.1-2)

\[ = 0.920(1.000) = 0.920 \text{ in.} \]

\[ d'_s = 0.920 \text{ in.} \]

Since $I_s/I_a > 1$

\[ d_s = d'_s = 0.920 \text{ in.} \]

\[ I'_1 = (d_s)^3\sin^2\theta/12 = (0.920)^3(\sin75.96^\circ)^2/12 = 0.061 \text{ in.}^3 \]
The distance from top fiber to the centroid of the reduced section is

\[ y = 4 - 0.147 - \frac{0.920}{2} \cos 14.04^\circ = 3.407 \text{ in.} \]

Effective section properties about \( x \)-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>( L ) (in.)</th>
<th>( y ) Distance from Top Fiber (in.)</th>
<th>( L_y ) (in.(^2))</th>
<th>( L_y^2 ) (in.(^3))</th>
<th>( I' ) About Own Axis (in.(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.920</td>
<td>3.970</td>
<td>3.652</td>
<td>14.500</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>5 \times 0.206 = 1.030</td>
<td>3.928</td>
<td>4.046</td>
<td>15.892</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>4 \times 3.819 = 15.276</td>
<td>2.000</td>
<td>30.552</td>
<td>61.104</td>
<td>17.472</td>
</tr>
<tr>
<td>4</td>
<td>2 \times 2.000 = 4.000</td>
<td>0.030</td>
<td>0.120</td>
<td>0.004</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>2.000</td>
<td>3.970</td>
<td>7.940</td>
<td>31.522</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>4 \times 0.206 = 0.824</td>
<td>0.072</td>
<td>0.059</td>
<td>0.004</td>
<td>--</td>
</tr>
<tr>
<td>7</td>
<td>0.920</td>
<td>3.407</td>
<td>3.134</td>
<td>10.679</td>
<td>0.061</td>
</tr>
<tr>
<td>8</td>
<td>1.938</td>
<td>3.970</td>
<td>7.694</td>
<td>30.545</td>
<td>--</td>
</tr>
<tr>
<td>Sum</td>
<td>26.908</td>
<td>57.197</td>
<td>164.250</td>
<td>17.533</td>
<td></td>
</tr>
</tbody>
</table>

The distance from top fiber to the neutral axis is

\[ y_{cg} = \frac{L_y}{L} = \frac{57.197}{26.908} = 2.126 \text{ in.} \]

\[ I'_{\text{eff}} = L_y^2 + I'_{\text{1}} - L_y^2_{cg} \]

\[ = 164.250 + 17.533 - 26.908(2.126)^2 \]

\[ = 60.16 \text{ in.}^3 \]

Actual \( I'_{\text{eff}} = tI'_{\text{eff}} \)

\[ = (0.060)(60.16) = 3.61 \text{ in.}^4 \]

\[ S_{\text{eff}} = \frac{I'_{\text{eff}}}{(d - y_{cg})} = \frac{3.61}{(4 - 2.126)} = 1.93 \text{ in.}^3 \]

\[ M = (1.93)(25.85) = 49.90 \text{ kips-in.} = M_s \text{ OK} \]

6. Summary
Positive Bending: $\phi_b M_n = 74.7 \text{ kips-in.}$

$I_{\text{eff}} = 3.64 \text{ in.}^4$

Negative Bending: $\phi_b M_n = 76.5 \text{ kips-in.}$

$I_{\text{eff}} = 3.62 \text{ in.}^4$

7. Compute Factored Uniform Load

For a continuous deck over three equal spans, the maximum bending moment is negative and occurs over the interior supports. It is given by:

$$M_u = 0.100 w_u L^2$$

Therefore, the maximum factored uniform load is

$$w_u = M_u / 0.100 L^2 = 76.5 / 0.100 (10' \times 12")^2 = 0.0531 \text{ kips/in.}$$

$$w_u = 0.638 \text{ kips/ft}$$

The maximum deflection occurs at a distance of 0.446L from the exterior supports. It is given by:

$$\Delta = 0.0069 w LL^4 / E_o I$$

This deflection is limited to $\Delta = L / 240$ for live load. Therefore, the maximum live load which will satisfy the deflection requirement is

$$w_{LL} = E_o I / [240(0.0069)L^3] = 27000(3.64) / [240(0.0069)(10 \times 12)^3]$$

$$w_{LL} = 0.0343 \text{ kips/in.}$$

$$w_{LL} = 0.412 \text{ kips/ft}$$

$$w_u = 1.2 w_{DL} + 1.6 w_{LL}$$

$$= [1.2(w_{DL}/w_{LL}) + 1.6] w_{LL}$$

$$= [1.2(1/5) + 1.6] w_{LL}$$

$$= 1.84 w_{LL} = 1.84(0.412) = 0.742 \text{ kips/ft} > 0.638 \text{ kips/ft}$$

Therefore, design flexural strength governs.
Factored Uniform Load = 0.638 kips/ft.

8. Check Shear Strength (Section 3.3.2)

The required shear strength at any section shall not exceed the design shear strength \( \phi_v V_n \):

\[
\phi_v = 0.85 \\
V_n = 4.84E_o t^3 (G_s/G_o)/h \\
\nu_n = V_n/(ht) \\
= 4.84E_o (G_s/G_o)/(h/t)^2
\]

(Eq. 3.3.2-1)

In the determination of the shear strength, it is necessary to select a proper value of \( G_s/G_o \) for the assumed stress from Table A12 or Figure A9 of the Standard. For the first approximation, assume a shear stress of \( \nu=27 \) ksi and the corresponding value of \( G_s/G_o \) is equal to 0.863. Thus,

\[
h/t = 3.819/0.060 = 63.65 < 200 \text{ (Section 2.1.2 (1))} \\
\nu_n = 4.84(27000)(0.863)/(63.65)^2 \\
= 27.82 \text{ ksi > assumed stress } f=27 \text{ ksi NG}
\]

For a second approximation, assume a stress of \( f=27.59 \) ksi and its corresponding value of \( G_s/G_o \) is 0.855.

\[
\nu_n = 4.84(27000)(0.855)/(63.65)^2 \\
= 27.58 \text{ ksi = assumed stress OK}
\]

Therefore, the total shear strength, \( V_n \), for hat section is

\[
V_n = 4(\nu_n)(ht) \text{ (a total of 4 webs)} \\
= 4(27.58)(3.819\times0.060) \\
= 25.28 \text{ kips}
\]
The design shear strength is determined as follows:

\[ \phi_{V_n} V_n = 0.85(25.28) = 21.49 \text{ kips} \]

\[ \phi_{V_n} V_n < 4(0.95F_{yy}) = 4(0.95 \times 42 \times 3.819 \times 0.06) = 36.57 \text{ kips OK} \]

(The shear yield strength, \( F_{yy} \), is obtained from Table A1 of the Standard.)

The maximum required shear strength is given by

\[ V_u = 0.600w_u L \]

\[ = (0.600)(0.638)(10) = 3.83 \text{ kips} < (\phi_{V_n} V_n) = 21.49 \text{ kips} \text{ OK} \]

9. Check Strength for Combined Bending and Shear (Section 3.3.3)

At the interior supports, there is a combination of web bending and web shear:

\[ \phi_{b_n} M = 76.5 \text{ kips-in.} \quad M_u = 0.100w_u L^2 \]

\[ \phi_{V_n} V_n = 21.49 \text{ kips} \quad V_u = 0.600w_u L \]

For unreinforced webs

\[ \left( \frac{M_u}{\phi_{b_n} M} \right)^2 + \left( \frac{V_u}{\phi_{V_n} V_n} \right)^2 \leq 1.0 \quad \text{(Eq. 3.3.3-1)} \]

Solve for \( w_u \):

\[ \left\{ \frac{0.100w_u (10 \times 12)^2}{76.5} \right\}^2 + \left\{ \frac{0.600w_u (10 \times 12)}{21.49} \right\}^2 = 1.0 \]

\[ 354.33w_u^2 + 16.16w_u^2 = 1.0 \]

\[ 370.49w_u^2 = 1.0 \]

\[ w_u = 0.0520 \text{ kips/in.} \]

\[ = 0.624 \text{ kips/ft.} \]

Factored Uniform Load = 0.627 kip/ft is determined for the case of combined bending and shear.

10. Check Web Crippling Strength (Section 3.3.4)
h = 3.819 in.
t = 0.060 in.
h/t = 3.819/0.06 = 63.65 < 200 OK
R = 1/8 in.
R/t = 0.125/0.06 = 2.083 < 7 OK
Let N = 6 in.
N/t = 6/0.06 = 100 < 210 OK
N/h = 6/3.819 = 1.57 < 3.5 OK
Table 2 of the Standard is used to check the web crippling requirements. For end reactions, use Eq. (3.3.4-2). For interior reaction, use Eq. (3.3.4-4).
k = \frac{F_y}{33} = \frac{50}{33} = 1.515 \quad \text{(Eq. 3.3.4-21)}
C_1 = (1.22-0.22k)k \quad \text{(Eq. 3.3.4-10)}
\quad = [1.22-0.22(1.515)](1.515) = 1.343
C_2 = (1.06-0.06R/t) \quad \text{(Eq. 3.3.4-11)}
\quad = [1.06-0.06(2.083)] = 0.935 < 1.0 OK
C_3 = (1.33-0.33k)k \quad \text{(Eq. 3.3.4-12)}
\quad = [1.33-0.33(1.515)](1.515) = 1.258
C_4 = (1.15-0.15R/t) \leq 1.0 \text{ but not less than 0.50} \quad \text{(Eq. 3.3.4-13)}
\quad (1.15-0.15R/t) = [1.15-0.15(2.083)] = 0.838 \leq 1.0 OK
\quad > 0.50 OK
C_4 = 0.838
\Theta = 75.96^\circ
C_{\Theta} = 0.7+0.3(\Theta/90)^2 \quad \text{(Eq. 3.3.4-20)}
\quad = 0.7+0.3(75.96/90)^2 = 0.914
a) For end reaction:
P_n = tC_3C_4C_9[217-0.28(h/t)][1+0.01(N/t)] \quad \text{(Eq. 3.3.4-2)}
\[
= (0.06)^2(1.258)(0.838)(0.914)[217-0.28(63.65)]
\]
\[
x[1+0.01(100)] = 1.38 \text{ kips/web}
\]

Total \( P_n \) for section:

\[
P_n = (4 \text{ webs})(1.38 \text{ k/web}) = 5.52 \text{ kips}
\]

\[
\phi_w = 0.70
\]

\[
\phi_w P_n = 0.70(5.52) = 3.86 \text{ kips}
\]

End reaction is given by

\[
R = 0.400w_u L
\]
\[
= (0.400)(0.627)(10) = 2.51 \text{ kips} < \phi_w P_n = 3.86 \text{ kips OK}
\]

b) For interior reaction:

\[
P_n = t^2C_1 C_2 C_3 [538-0.74(h/t)][1+0.007(N/t)]
\]
\[
= (0.06)^2(1.343)(0.935)(0.914)[538-0.74(63.65)]
\]
\[
x[1+0.007(100)] = 3.45 \text{ kips/web}
\]

Total \( P_n \) for section:

\[
P_n = (4 \text{ webs})(3.45 \text{ k/web}) = 13.80 \text{ kips}
\]

\[
\phi_w = 0.70
\]

\[
\phi_w P_n = 0.70(13.80) = 9.66 \text{ kips}
\]

Interior reaction is given by

\[
R = 1.10w_u L
\]
\[
= (1.10)(0.627)(10) = 6.90 \text{ kips} < \phi_w P_n = 9.66 \text{ kips OK}
\]
EXAMPLE 11.2 DECK SECTION (ASD)

Rework Example 11.1 by using the Allowable Stress Design (ASD) method to determine the allowable bending moment based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the allowable moment. Compute the allowable uniform load as controlled either by bending or deflection.

Solution:

1. Element Properties:

   See section properties calculated in Example 11.1.

2. Section Modulus for Load Determination - Positive Bending
   (Based on Initiation of Yielding)

   The effective section properties calculated by the ASD method are the same as those determined in Example 11.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

   \[ M_a = \frac{M_n}{\Omega} \]  

   \( M_n = 83.0 \text{ kips-in.} \)  

   \( \Omega = 1.85 \)  

   \( M_a = \frac{83.0}{1.85} = 44.87 \text{ kips-in.} \)

3. Moment of Inertia for Deflection Determination - Positive Bending

   For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 11.1 for the LRFD method, except that the computed moment \( M (= f x S_e) \) should be equal to \( M_a \).

   From the results of Example 11.1.(3), it was found that for a compression stress of \( f=28.66 \text{ ksi} \), the section is fully effective.
Then, for an assumed stress of \( f = 26.39 \text{ ksi} \) (less than \( f = 28.66 \text{ ksi} \)), the section will also be fully effective, i.e.,

\[
S_e = \frac{I_x}{y_{cg}} = \frac{3.64}{2.138} = 1.70 \text{ in.}^3
\]

\[
M = fS_e = 26.39 \times 1.70 = 44.87 \text{ kips-in.} = M_a \text{ OK}
\]

Therefore, use \( I_{eff} = 3.64 \text{ in.}^4 \) for deflection calculation.

4. Section Modulus for Load Determination - Negative Bending

(Based on Initiation of Yielding)

The effective section properties calculated by the ASD method are the same as those determined in Example 11.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

\[
M_a = \frac{M_n}{\Omega} \quad (E-1)
\]

\[
M_n = 90.0 \text{ kips-in.}
\]

\[
\Omega = 1.85
\]

\[
M_a = 90.0 / 1.85 = 48.65 \text{ kips-in.}
\]

5. Moment of Inertia for Deflection Determination - Negative Bending

For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 11.1 for the LRFD method, except that the computed moment \( M (= fxS_e) \) should be equal to \( M_a \).

For an assumed stress of \( f = 25.20 \text{ ksi} \), it is found that the section modulus is likely to be the same as calculated in Example 11.1.(5), i.e.,

\[
S_e = \frac{I_x}{y_{cg}} = \frac{3.61}{(4.0-2.126)} = 1.93 \text{ in.}^3
\]

\[
M = fS_e = 25.20 \times 1.93 = 48.65 \text{ kips-in.} = M_a \text{ OK}
\]

Therefore, use \( I_{eff} = 3.61 \text{ in.}^4 \) for deflection calculation.
6. Summary

Positive Bending: $M_a = 44.87$ kips-in.

\[ I_{\text{eff}} = 3.64 \text{ in.}^4 \]

Negative Bending: $M_a = 48.65$ kips-in.

\[ I_{\text{eff}} = 3.61 \text{ in.}^4 \]

7. Compute Allowable Uniform Load

For a continuous deck over three equal spans, the maximum bending moment is negative and occurs over the interior supports. It is given by:

\[ M_a = 0.100wL^2 \]

Therefore, the maximum factored uniform load is

\[ w = \frac{M_a}{0.100L^2} = \frac{44.87}{0.100(10' \times 12''/1)^2} = 0.0312 \text{ kip/in.} \]

\[ w = 0.374 \text{ kip/ft} \]

The maximum deflection occurs at a distance of $0.446L$ from the exterior supports. It is given by:

\[ \Delta = 0.0069wL'/E_I \]

This deflection is limited to $\Delta = L/240$ for live load. Therefore, the maximum live load which will satisfy the deflection requirement is

\[ w_{\text{LL}} = \frac{E_I}{[240(0.0069)L^2]} = \frac{27000(3.64)}{[240(0.0069)(10\times12)^2]} \]

\[ w_{\text{LL}} = 0.0343 \text{ kip/in.} \]

\[ w_{\text{LL}} = 0.412 \text{ kip/ft} \]

Therefore, allowable bending strength governs.

Allowable Uniform Load = 0.374 kip/ft.

8. Check Shear Strength
The required shear strength at any section shall not exceed the allowable shear strength $V_a$:

$$\Omega \ = \ 1.85 \ (\text{for single web})$$

$$v = 4.84(27000)(0.855)/(63.65)^2$$

$$= 27.58 \text{ ksi (from Example 11.1.(8))}$$

Therefore, the total shear strength, $V_n$, for hat section is

$$V_n = 4(v)(ht) \ (\text{a total of 4 webs})$$

$$= 4(27.58)(3.819 \times 0.060)$$

$$= 25.28 \text{ kips}$$

The allowable shear strength is determined as follows:

$$V_a = V_n/\Omega = 25.28/1.85 = 13.66 \text{ kips}$$

$$< 4(F_{ht})/1.64 = 4(42 \times 3.819 \times 0.06)/1.64 = 23.47 \text{ kips OK}$$

The maximum required shear strength is given by

$$V = 0.600wL$$

$$= (0.600)(0.374)(10) = 2.24 \text{ kips < 13.66 kips OK}$$

Check Strength for Combined Bending and Shear

At the interior supports, there is a combination of web bending and web shear:

$$M_a = 48.65 \text{ kips-in.} \quad M = 0.100wL^2$$

$$V_a = 13.66 \text{ kips} \quad V = 0.600wL$$

For unreinforced webs

$$(M/M_a)^2 + (V/V_a)^2 \leq 1.0$$

Solve for $w$:

$$[0.100w(10 \times 12)^2 /48.65]^2 + [0.600w(10 \times 12) /13.66]^2 = 1.0$$
\[876.11w^2 + 27.78w^2 = 1.0\]
\[903.89w^2 = 1.0\]
\[w = 0.0333 \text{ kip/in.} = 0.399 \text{ kip/ft.}\]

Allowable Uniform Load = 0.399 kip/ft is determined for the case of combined bending and shear.

10. Check Web Crippling Strength

The nominal web crippling strengths are calculated in Example 11.1.(10) as follows:

a) For end reaction:

\[P_n = t^2C_3C_4C_0[217-0.28(h/t)][1+0.01(N/t)]\]  \hspace{1cm} (Eq. 3.3.4-2)
\[= (0.06)^2(1.258)(0.838)(0.914)[217-0.28(63.65)]\]
\[\times[1+0.01(100)] = 1.38 \text{ kips/web}\]

Total \(P_n\) for section:

\[P_n = (4 \text{ webs})(1.38 \text{ k/web}) = 5.52 \text{ kips}\]

\[\Omega = 2.0 \text{ (for single web)}\]

\[P_a = P_n/\Omega = 5.52/2.0 = 2.76 \text{ kips}\]

End reaction is given by

\[R = 0.400wL\]
\[= (0.400)(0.374)(10) = 1.50 \text{ kips} < P_a = 2.76 \text{ kips} \text{ OK}\]

b) For interior reaction:

\[P_n = t^2C_1C_2C_0[538-0.74(h/t)][1+0.007(N/t)]\]  \hspace{1cm} (Eq. 3.3.4-4)
\[= (0.06)^2(1.343)(0.935)(0.914)[538-0.74(63.65)]\]
\[\times[1+0.007(100)] = 3.45 \text{ kips/web}\]

Total \(P_n\) for section:

\[P_n = (4 \text{ webs})(3.45 \text{ k/web}) = 13.80 \text{ kips}\]
\[ \Omega = 2.0 \]

\[ P_a = \frac{P_n}{\Omega} = \frac{13.8}{2.0} = 6.90 \text{ kips} \]

Interior reaction is given by

\[ R = 1.10wL \]

\[ = (1.10)(0.374)(10) = 4.11 \text{ kips} < P_a = 6.90 \text{ kips OK} \]
EXAMPLE 12.1 CYLINDRICAL TUBULAR SECTION (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, \( \phi_b M_n \), for the section shown in Figure 12.1. Use Type 301, 1/4-Hard stainless steel.

![Figure 12.1 Section for Example 12.1](image)

Solution:

Ratio of outside diameter to wall thickness,

\[
D/t = \frac{8.000}{0.125} = 64.00
\]

\[
0.881E_o / F_y = 0.881 \left( \frac{27000}{50} \right) = 475.7
\]

Since \( D/t < 0.881E_o / F_y \), the ASCE Specification can be used.

The design requirement for cylindrical tubular members is based on Section 3.6.1 of the Standard.

Because \( 0.112E_o / F_y = 0.112 \left( \frac{27000}{50} \right) = 60.48 \) and

\[
0.112E_o / F_y < D/t < 0.881E_o / F_y,
\]

\[
M_n = K_c F_y S_f
\]

(Eq. 3.6.1-2)

where

\[
S_f = \pi \left[ (\text{O.D.})^4 - (\text{I.D.})^4 \right] / 32(\text{O.D.})
\]

\[
= \pi \left[ (8)^4 - (7.75)^4 \right] / 32(8)
\]

\[
= 5.995 \text{ in.}^3
\]
\[
K_c = (1-C)(E_o/F_y)/[(8.93-\lambda_c)(D/t)] + 5.882C/(8.93-\lambda_c) \quad (\text{Eq. 3.6.1-3})
\]

\[
C = F_{\text{pr}}/F_y
\]
\[
\lambda_c = 3.048C
\]

From Table A17 of the Standard, the ratio of \( F_{\text{pr}}/F_y \) is equal to 0.5 in longitudinal compression for Type 301, 1/4-Hard stainless steel.

Therefore,
\[
K_c = (1-0.5)(27000/50)/[(8.93-3.048x0.5)(64.0)] \\
+ (5.882x0.5)/(8.93-3.048x0.5) \\
= 0.967
\]
\[
M_n = 0.967(50)(5.995) \\
= 289.86 \text{ kips-in.}
\]
\[
\Phi_b = 0.90
\]
\[
\Phi_b M_n = 0.90 \times 289.86 = 260.90 \text{ kips-in.}
\]
EXAMPLE 12.2 CYLINDRICAL TUBULAR SECTION (ASD)

Rework Example 12.1 by using the Allowable Stress Design (ASD) method.

Solution:

Calculation of the allowable moment, \( M_a \):

The effective section properties calculated by the ASD method are the same as those determined in Example 12.1 for the LRFD method.

Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

\[ \Omega = 1.85 \text{ (Safety Factor stipulated in Table E of the Standard)} \]

\[ M_n = 289.86 \text{ kips-in. (obtained from Example 12.1)} \]

\[ M_a = \frac{M_n}{\Omega} \quad \text{(Eq. E-1)} \]

\[ = \frac{289.86}{1.85} \]

\[ = 156.7 \text{ kips-in.} \]

\[ M_{\text{max}} = 25.92 \text{ kips-in.} < 27.57 \text{ kips-in.} \text{ OK} \]
EXAMPLE 13.1 FLANGE CURLING (LRFD)

By using the LRFD criteria, determine the amount of curling for the com-
pression flange of the channel section used in Example 1.1.

\[ S_e = 0.711 \text{ in.}^3 \]

\[ M_n = S_e F_y \quad \text{(Eq. 3.3.1.1-1)} \]

\[ = 0.711 \times 50 = 35.55 \text{ kips-in.} \]

\[ \Phi_b = 0.85 \]
\[ \Phi_B M_n = 0.85 \times 35.55 = 30.22 \text{ kips-in.} \]

2. Determination of the average stress in compression flange, \( f_{av} \), at the service moment \( M_s \):

\[ \Phi_B M_n = 1.2M_{DL} + 1.6M_{LL} \]
\[ = (1.2(M_{DL}/M_{LL}) + 1.6)M_{LL} \]
\[ = (1.2(1/5) + 1.6)M_{LL} \]
\[ = 1.84M_{LL} \]
\[ M_{LL} = \Phi_B M_n / 1.84 = 30.22/1.84 = 16.42 \text{ kips-in.} \]
\[ M_s = M_{DL} + M_{LL} \]
\[ = (1/5 + 1)M_{LL} \]
\[ = 1.2(16.42) = 19.70 \text{ kips-in.} \]

where

- \( M_{DL} \) = Moment determined on the basis of nominal dead load
- \( M_{LL} \) = Moment determined on the basis of nominal live load

The procedure is iterative: one assumes the actual compressive stress \( f \) under this service moment \( M_s \). Knowing \( f \), proceeds as usual to obtain \( S_e \) and checks to see if \((f \times S_e)\) is equal to \( M_s \) as it should. If not, re-iterate until one obtains the desired level of accuracy.

For the first approximation, assume a compression stress of \( f = 25 \text{ ksi} \) in the top fiber of the section and that the web is fully effective.

Compression flange: \( k = 0.50 \) (for unstiffened compression element, see Section 2.3.1)

\[ w/t = 1.471/0.060 = 24.52 < 50 \text{ OK (Section 2.1.1-(1)-(iii))} \]
\[ \lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o} \]  
(Eq. 2.2.1-4)

The initial modulus of elasticity, \( E_o \), for Type 301 stainless steel is obtained from Table A4 of the Standard, i.e., \( E_o = 27000 \) ksi.

\[ \lambda = (1.052/\sqrt{0.50})(24.52)\sqrt{25/27000} = 1.110 > 0.673 \]

\[ \rho = \frac{[1-(0.22/\lambda)]/\lambda}{1.110} = 0.722 \]  
(Eq. 2.2.1-3)

\[ b = \rho w \]  
(Eq. 2.2.1-2)

\[ = 0.722 \times 1.471 \]

\[ = 1.062 \text{ in.} \]

Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L Effective Length (in.)</th>
<th>y Distance from Top Fiber (in.)</th>
<th>( L_y )</th>
<th>( L_y^2 )</th>
<th>I' About Own Axis (in.³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web</td>
<td>5.692</td>
<td>3.000</td>
<td>17.076</td>
<td>51.228</td>
<td>15.368</td>
</tr>
<tr>
<td>Upper Corner</td>
<td>0.195</td>
<td>0.075</td>
<td>0.015</td>
<td>0.001</td>
<td>--</td>
</tr>
<tr>
<td>Lower Corner</td>
<td>0.195</td>
<td>5.925</td>
<td>1.155</td>
<td>6.846</td>
<td>--</td>
</tr>
<tr>
<td>Compression Flange</td>
<td>1.062</td>
<td>0.030</td>
<td>0.032</td>
<td>0.001</td>
<td>--</td>
</tr>
<tr>
<td>Tension Flange</td>
<td>1.471</td>
<td>5.970</td>
<td>8.782</td>
<td>52.428</td>
<td>--</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td><strong>8.615</strong></td>
<td><strong>27.060</strong></td>
<td><strong>110.504</strong></td>
<td><strong>15.368</strong></td>
<td></td>
</tr>
</tbody>
</table>

Distance from top fiber to x-axis is

\[ y_{cg} = \frac{27.060}{8.615} = 3.141 \text{ in.} \]

Since the distance from top compression fiber to the neutral
axis is greater than one half the beam depth, a compression stress of 25 ksi will govern as assumed (i.e., initial yield is in compression).

To check if web is fully effective (Section 2.2.2):

\[ f_1 = \left[\frac{(3.141 - 0.154)}{3.141}\right] \times 25 = 23.77 \text{ ksi (compression)} \]

\[ f_2 = -\left[\frac{(2.859 - 0.154)}{3.141}\right] \times 25 = -21.53 \text{ ksi (tension)} \]

\[ \psi = \frac{f_2}{f_1} = \frac{-21.53}{23.77} = -0.906 \]

\[ k = 4 + 2(1 - \psi)^3 + 2(1 - \psi) \quad \text{(Eq. 2.2.2-4)} \]

\[ = 4 + 2(1 - (-0.906))^3 + 2(1 - (-0.906)) \]

\[ = 21.660 \]

\[ h = w = 5.692 \text{ in., } h/t = w/t = \frac{5.692}{0.060} = 94.87 \]

\[ h/t = 94.87 < 200 \text{ OK (Section 2.1.2-(1))} \]

\[ \lambda = \left(\frac{1.052}{\sqrt{21.66}}\right)\left(\frac{94.87}{\sqrt{23.77/27000}}\right) = 0.636 > 0.673 \]

\[ b_2 = \frac{b_e}{2} \quad \text{(Eq. 2.2.2-2)} \]

\[ = 5.692/2 = 2.846 \text{ in.} \]

\[ b_1 = \frac{b_e}{(3 - \psi)} \quad \text{(Eq. 2.2.2-1)} \]

\[ = 5.692/\left(3 - (-0.906)\right) = 1.457 \text{ in.} \]

The effective widths, \( b_1 \) and \( b_2 \), of web are defined in Figure 2 of the Standard.

\[ b_1 + b_2 = 1.457 + 2.846 = 4.303 \text{ in.} \]

Compression portion of the web calculated on the basis of the effective section = \( y_{cg} - 0.154 = 3.141 - 0.154 = 2.987 \text{ in.} \)

Since \( b_1 + b_2 = 4.303 \text{ in.} > 2.987 \text{ in.} \), \( b_1 + b_2 \) shall be taken as 2.987 in. This verifies the assumption that the web is fully effective.
\[ I' = L_y^2 + I'_t - L_y^2 \]
\[ = 110.504 + 15.368 - 8.615(3.141)^2 \]
\[ = 40.877 \text{ in.}^3 \]

Actual \( I_x = I'_t x \)
\[ = 40.877 \times 0.060 \]
\[ = 2.453 \text{ in.}^4 \]

\[ S_y = \frac{I_x}{y_{cg}} \]
\[ = \frac{2.453}{3.141} \]
\[ = 0.781 \text{ in.}^3 \]

\[ M_n = S_f e_y \]
\[ = 0.781 \times 25 \]
\[ = 19.53 \text{ kips-in.} = M = 19.70 \text{ kips-in. (close enough)} \]

Therefore,
\[ f_{av} = f \left( \frac{b}{w} \right) = 25.0 \times \left( \frac{1.062}{1.471} \right) = 18.05 \text{ ksi} \]

3. Determination of the curling of the compression flange, \( c_f \).
\[ w_f = 1.625 - 0.06 = 1.565 \text{ in.} \]
\[ w_f = \sqrt{0.061tdE/f_{av} \sqrt{(100c_f/d)}} \quad \text{(Eq. 2.1.1-1)} \]
\[ 1.565 = \sqrt{0.061 \times (0.06) \times (6) \times (27000) / 18.05 \sqrt{100c_f / 6}} \]
\[ = 5.731 \sqrt{16.67c_f} \]
\[ \sqrt{16.67c_f} = 1.565 / 5.731 \]
\[ 16.67c_f = (1.565 / 5.731)^4 \]
\[ c_f = (1.565 / 5.731)^4 / 16.67 = 0.00033 \text{ in.} \]
EXAMPLE 13.2 FLANGE CURLING (ASD)

Rework Example 13.1 by using the ASD method.

Solution:

1. Determination of the allowable bending moment, $M_a$:

The nominal bending strength, $M_n$, is obtained from Example 13.1 as follows:

$$M_n = S_F e_y = 0.711 \times 50 = 35.55 \text{ kips-in.}$$

Therefore, the allowable moment:

$$\Omega = 1.85$$

$$M_a = 35.55/1.85 = 19.22 \text{ kips-in.}$$

2. Determination of the average stress in compression flange, $f_{av}$, at the allowable moment $M_a$:

Assume that a compression stress of $f=25$ ksi in the top fiber of the section and that the web is fully effective. Therefore, from the calculation of Example 13.1.(2):

$$M = S_e f = 0.781 \times 25$$

$$= 19.53 \text{ kips-in.} = M_a = 19.22 \text{ kips-in. (close enough)}$$

Therefore,

$$f_{av} = f (b/w) = 25.0 \times (1.062/1.471) = 18.05 \text{ ksi}$$

3. Determination of the curling of the compression flange, $c_f$:

$$w_f = 1.625 - 0.06 = 1.565 \text{ in.}$$

$$t = 0.06$$

$$d = 6$$

$$1.565 = \sqrt{0.061(0.06)(6)(27000)/18.05 \cdot \sqrt{100c_f/6}}$$

$$= 5.731 \sqrt{16.67c_f}$$
\[ \sqrt{16.67c_f} = \frac{1.565}{5.731} \]
\[ 16.67c_f = \left(\frac{1.565}{5.731}\right)^4 \]
\[ c_f = \frac{(1.565/5.731)^4}{16.67} = 0.00033 \text{ in.} \]
EXAMPLE 14.1 SHEAR LAG (LRFD)

For the tubular section shown in Fig. 14.1, determine the design flexural strength, $\phi_b M_n$, if the member is to be used as a simply supported beam and to carry a concentrated load at midspan. Assume that the span length is 3 ft. and the section material is Type 316, 1/4-Hard, stainless steel.

Solution:

1. Determination of the nominal moment, $M_n$, based on initiation of yielding (Section 3.3.1.1).

Properties of $90^\circ$ corners:

$$r = R + t/2 = 3/16 + 0.135/2 = 0.255 \text{ in.}$$

Length of arc, $u = 1.57r = 1.57 \times 0.255 = 0.400 \text{ in.}$

Distance of c.g. from center of radius,
c = 0.637r = 0.637 x 0.255 = 0.162 in.

Computation of I_x:
For the first approximation, assume a compression stress of 
f = F_y = 50 ksi in the compression flange, and that the webs are fully effective.

Compression flange: k = 4.00 (stiffened compression element supported by a web on each longitudinal edge)
w/t = 7.354/0.135 = 54.47 < 400 OK (Section 2.1.1-(1)-(ii))

\[ \lambda = \frac{1.052/\sqrt{k} \cdot (w/t) \cdot \sqrt{E/\sigma_0}}{\beta} \]  \hspace{1cm} (Eq. 2.2.1-4)
\[ = \frac{1.052/\sqrt{4.00} \cdot (54.47) \cdot \sqrt{29,000}}{5} = 1.233 > 0.673 \]

\[ \rho = \frac{1-0.22/\lambda}{\lambda} \]  \hspace{1cm} (Eq. 2.2.1-3)
\[ = \frac{1-0.22/1.233}{1.233} = 0.666 \]

\[ b = \rho w \]  \hspace{1cm} (Eq. 2.2.1-2)
\[ = 0.666 \times 7.354 \]
\[ = 4.898 \text{ in.} \]
Effective section properties about x axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L Effective Length (in.)</th>
<th>y Distance from Top Fiber (in.)</th>
<th>Ly (in.²)</th>
<th>Ly² (in.³)</th>
<th>I' About Own Axis (in.³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Webs</td>
<td>14.708</td>
<td>4.000</td>
<td>58.832</td>
<td>235.328</td>
<td>66.286</td>
</tr>
<tr>
<td>Upper Corners</td>
<td>0.800</td>
<td>0.161</td>
<td>0.129</td>
<td>0.021</td>
<td>--</td>
</tr>
<tr>
<td>Lower Corners</td>
<td>0.800</td>
<td>7.839</td>
<td>6.271</td>
<td>49.160</td>
<td>--</td>
</tr>
<tr>
<td>Compression Flange</td>
<td>4.898</td>
<td>0.068</td>
<td>0.333</td>
<td>0.023</td>
<td>--</td>
</tr>
<tr>
<td>Tension Flange</td>
<td>7.354</td>
<td>7.933</td>
<td>58.339</td>
<td>462.806</td>
<td>--</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td>28.560</td>
<td>123.904</td>
<td>747.338</td>
<td>66.286</td>
<td></td>
</tr>
</tbody>
</table>

Distance from top fiber to x-axis is

\[ y_{cg} = \frac{123.904}{28.560} = 4.338 \text{ in.} \]

Since the distance of top compression fiber from neutral axis is greater than one half the beam depth, a compression stress of 50 ksi will govern as assumed (i.e., initial yielding is in compression).

To check if webs are fully effective (Section 2.2.2):

\[ f_1 = \frac{[(4.338-0.323)/4.338] \times 50}{4.338} = 46.28 \text{ ksi (compression)} \]

\[ f_2 = -\frac{[(3.662-0.323)/4.338] \times 50}{4.338} = -38.49 \text{ ksi (tension)} \]

\[ \psi = \frac{f_2}{f_1} = \frac{-38.49}{46.28} = -0.832 \]

\[ k = 4+2(1-\psi)^3+2(1-\psi) \]

\[ = 4+2[1-(-0.832)]^3+2[1-(-0.832)] \]

\[ = 19.961 \]

\[ h = w = 7.354 \text{ in.}, \ h/t = w/t = 7.354/0.135 = 54.47 \]

\[ h/t = 54.47 < 200 \text{ OK (Section 2.1.2-(1))} \]
\[ \lambda = \frac{1.052/\sqrt{19.961}}{54.47/\sqrt{46.28/27000}} = 0.531 < 0.673 \]

\[ b_e = \frac{w}{7.354 \text{ in.}} \quad \text{(Eq. 2.2.1-1)} \]

\[ b_2 = \frac{w}{2} = \frac{7.354}{2} \approx 3.677 \text{ in.} \quad \text{(Eq. 2.2.2-2)} \]

\[ b_1 = \frac{w}{3-\psi} \quad \text{(Eq. 2.2.2-1)} \]

\[ \approx \frac{7.354}{3-(-0.843)} = 1.914 \text{ in.} \]

Compression portion of the web calculated on the basis of the effective section = \( y_{cg} - 0.323 = 4.338 - 0.323 = 4.015 \text{ in.} \)

Since \( b_1 + b_2 = 5.591 \text{ in.} > 4.015 \text{ in.} \), \( b_1 + b_2 \) shall be taken as 4.015 in.. This verifies the assumption that the webs are fully effective.

\[ I'_x = Ly^2 + I'_1 - Ly_{cg}^2 \]

\[ = 747.338 + 66.286 - 28.560(4.338)^2 \]

\[ = 276.175 \text{ in.}^3 \]

Actual \( I_x = I'_x t \)

\[ = 276.175 \times 0.135 \]

\[ = 37.284 \text{ in.}^4 \]

\[ S_e = \frac{I_x}{y_{cg}} \]

\[ = \frac{37.284}{4.338} \]

\[ = 8.595 \text{ in.}^3 \]

\[ M_n = S_F e_Y = 8.595 \times 50 \]

\[ = 429.75 \text{ kips-in.} \]

2. Determination of the nominal moment, \( M_n \), based on shear lag consideration (Section 2.1.1(3)).
\[ w_f = (8-2\times0.135)/2 = 3.865 \text{ in.} \]
\[ L/w_f = 3\times12/3.865 = 9.314 < 30 \]

Because the \( L/w_f \) ratio is less than 30, and the member carries a concentrated load, consideration for shear lag is needed.

From Table 1 of the Standard:

\( L/w_f = 10 \), effective design width/actual width = 0.73
\( L/w_f = 8 \), effective design width/actual width = 0.67
\( L/w_f = 9.314 \), effective design width/actual width = ?

\[
\frac{10-9.314}{9.314-8} = \frac{0.73-x}{x-0.67}
\]

\[
0.686(x-0.67) = 1.314(0.73-x)
\]

\[ x = 0.709 \]

Therefore, the effective design widths of compression and tension flanges between webs are

\[ 0.709(8-2\times0.135) = 5.481 \text{ in.} \]
\[ b = 5.481-2R = 5.481-2(3/16) = 5.106 \text{ in.} \]

Because of symmetry and assume webs are fully effective,
\[ y_{cg} = 4.000 \text{ in.} \]

Effective section properties about \( x \)-axis:

\[
L = 28.560-4.898-7.354+5.106x2 = 26.520 \text{ in.} \]
\[
Ly^2 = 747.338-0.023-462.806+5.106(0.068)^2+5.106(7.933)^2
\]
\[ = 605.866 \text{ in.}^3 \]
\[ I'_{1} = 66.286 \text{ in.}^3 \]

To check if webs are fully effective:
\[ f_1 = \left(\frac{(4.000 - 0.323)}{4.000}\right) \times 50 = 45.96 \text{ ksi} \]

\[ f_2 = -45.96 \text{ ksi} \]

\[ \psi = \frac{-45.96}{45.96} = -1.000 \]

\[ k = 4 + 2(1 - (-1.000))^2 + 2(1 - (-1.000)) = 24.000 \]

\[ \lambda = \left(\frac{1.052(\sqrt{24.0} \times 54.47)}{27000}\right) = 0.483 < 0.673 \]

\[ b_e = 7.354 \text{ in.} \]

\[ b_2 = \frac{7.354}{2} = 3.677 \text{ in.} \]

\[ b_1 = \frac{7.354}{3 - (-1.000)} = 1.839 \text{ in.} \]

Compression portion of the web calculated on the basis of the effective section = 4.000 - 0.323 = 3.677 in.

Since \( b_1 + b_2 = 5.516 \text{ in.} > 3.677 \text{ in.} \), \( b_1 + b_2 \) shall be taken as 3.677 in.. This verifies the assumption that the webs are fully effective.

\[ I'_x = 605.866 + 66.286 - 26.520(4.000)^2 = 247.832 \text{ in.}^3 \]

Actual \( I'_x = 247.832 \times 0.135 = 33.457 \text{ in.}^4 \]

\[ S_e = \frac{33.457}{4.000} = 8.364 \text{ in.}^3 \]

\[ M_n = 8.364 \times 50 = 418.20 \text{ kips-in.} < 429.75 \text{ kips-in.} \text{ (initial yielding)} \]

3. Determination of the design flexural strength, \( \Phi_b M_n \).

\[ M_n = 418.20 \text{ kips-in.} \]

\[ \Phi_b = 0.90 \]

\[ \Phi_b M_n = 0.90 \times 418.20 = 376.38 \text{ kips-in.} \]
EXAMPLE 14.2 SHEAR LAG (ASD)

Rework Example 14.1 to determine the allowable bending moment for the tubular section.

Solution:

1. Determination of the nominal moment, \( M_n \), based on initiation of yielding

\[
M_n = S_F Y = 8.595 \times 50 = 429.75 \text{ kips-in. (from Example 14.1)}
\]

2. Determination of the nominal moment, \( M_n \), based on shear lag consideration.

\[
w_f = \frac{8 - 2 \times 0.135}{2} = 3.865 \text{ in.}
\]

\[
L/w_f = \frac{3 \times 12}{3.865} = 9.314 < 30
\]

Because the \( L/w_f \) ratio is less than 30, and the member carries a concentrated load, consideration for shear lag is needed.

\[
S_e = \frac{33.457}{4.000} = 8.364 \text{ in.}^3 \text{ (from Example 14.1)}
\]

\[
M_n = 8.364 \times 50 = 418.20 \text{ kips-in.} < 429.75 \text{ kips-in. (initial yielding)}
\]

3. Determination of the allowable bending strength, \( M_a \).

\[
M_n = 418.20 \text{ kips-in.}
\]

\[
\Omega = 1.85
\]

\[
M_a = \frac{418.20}{1.85} = 226.05 \text{ kips-in.}
\]
EXAMPLE 15.1 C-SECTION (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design axial strength for C-section as shown in Figure 15.1. Use Type 304 stainless steel, 1/4-Hard.

Figure 15.1 Section for Example 15.1

Given:
1. Section: 3.5" x 2.0" x 0.105" channel with stiffened flanges.
2. $K_x L_x = K_y L_y = K_c L_c = 6$ ft.

Solution:
The following equations used for computing the sectional properties for channel with lips are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.
1. Basic parameters used for calculating the section properties:

\[ r = R + t/2 = 3/16 + 0.105/2 = 0.240 \text{ in.} \]

From the sketch \( a = 2.914 \text{ in.}, b = 1.414 \text{ in.}, c = 0.607 \text{ in.}, \)
\( A' = 3.5 \text{ in.}, B' = 2.0 \text{ in.}, C' = 0.9 \text{ in.}, \)
\( a = 1.00 \) (Since the section has lips)

\[ \bar{a} = A' - t = 3.5 - 0.105 = 3.395 \text{ in.} \]
\[ \bar{b} = B' - (t/2 + at/2) = B' - t = 2 - 0.105 = 1.895 \text{ in.} \]
\[ \bar{c} = \bar{a}(C' - t/2) = C' - t/2 = 0.9 - 0.105/2 = 0.848 \text{ in.} \]
\[ u = 1.57r = 1.57 \times 0.240 = 0.377 \text{ in.} \]

2. Area:

\[ A = t[a+2b+2\bar{c}+\alpha(2c+2u)] = t[a+2b+2c+4u] \]
\[ = 0.105[2.914+2\times1.414+2\times0.607+4\times0.377] \]
\[ = 0.889 \text{ in.}^2 \]

3. Moment of inertia about x-axis:

\[ I_x = 2t[0.0417a^2+b(a/2+r)^2+u(a/2+0.637r)^2+0.149r^3 \]
\[ +\alpha[0.0833c^2+(c/4)(a-c)^2+u(a/2+0.637r)^2+0.149r^3] \]
\[ = 2t[0.0417a^2+b(a/2+r)^2+2u(a/2+0.637r)^2+0.298r^3 \]
\[ +0.0833c^2+(c/4)(a-c)^2] \]
\[ = 2\times0.105[0.0417(2.914)^2+1.414(2.914/2+0.240)^2 \]
\[ +2\times0.377(2.914/2+0.637\times0.240)^2+0.298(0.240)^3 \]
\[ +0.0833(0.607)^2+(0.607/4)(2.914-0.607)^2] \]
\[ = 1.657 \text{ in.}^4 \]
4. Distance from centroid of section to centerline of web:

\[ x = \frac{(2t/A)\{b(b/2+r)+u(0.363r)+a[u(b+1.637r)+c(b+2r)]\}} \]
\[ = \left[ \frac{(2\times0.105)}{0.889}\right] \left[ 1.414(1.414/2+0.240)+0.377(0.363\times0.240) +0.377(1.414+1.637\times0.240)+0.607(1.414+2\times0.240) \right] \]
\[ = 0.757 \text{ in.} \]

5. Moment of inertia about y-axis:

\[ I_y = 2t\{b(b/2+r)^2+0.0833b^3+0.356r^3+a(c(b+2r)^2) \]
\[ +u(b+1.637r)^2+0.149r^3 \} - A(x)^2 \]
\[ = 2\times0.105\{1.414(1.414/2+0.240)^2+0.0833(1.414)^3 \]
\[ +0.356(0.240)^3+0.607(1.414+2\times0.240)^2 \]
\[ +0.377(1.414+1.637\times0.240)^2+0.149(0.240)^3 \} - 0.889(0.757)^2 \]
\[ = 0.524 \text{ in.}^4 \]

6. Distance from shear center to centerline of web:

\[ m = \frac{(bt/12I_x)}{6c(a)^2+3b(a)^2-8(c)^2} \]
\[ = \left[ \frac{(1.895\times0.105)/(12\times1.657)}{6\times0.848(3.395)^2 \}
\[ +3\times1.895(3.395)^2-8(0.848)^3 \] \]
\[ = 1.194 \text{ in.} \]

7. Distance from centroid to shear center:

\[ x_o = -(x+m) = -(0.757+1.194) \]
\[ = -1.951 \text{ in.} \]

8. St. Venant torsion constant:

\[ J = \frac{(t^3/3)[a+2b+2u+a(2c+2u)]}{\}
\[ = \left[ (0.105)^3/3 \right] \left[ 2.914+2\times1.414+4\times0.377+2\times0.607 \right] \]
9. Warping Constant:

\[ C_w = \left( t^2/A \right) \left[ (\bar{A}\bar{a})^2 / t (2\bar{a} + 4\bar{c}) + (m/3t) (m\bar{a})^2 \right] + \frac{1}{6} \left[ (2\bar{c} + 3\bar{a}) - (2\bar{a} + 4\bar{c}) + (m\bar{a})^2 \right] \left( \frac{8}{6} \frac{2\bar{c} + 3\bar{a}}{4\bar{a} + 3\bar{c}} \right) \]

\[ = 0.003266 \text{ in.}^4 \]

10. Radii of gyration:

\[ r_x = \sqrt{I_x/A} = \sqrt{1.657/0.889} = 1.365 \text{ in.} \]

\[ r_y = \sqrt{I_y/A} = \sqrt{0.524/0.889} = 0.768 \text{ in.} \]

\[ (K_{L_y})/r_y = (6x12)/0.768 = 93.75 < 200 \]

\[ r_o^2 = r_x^2 + r_y^2 + x_o^2 = (1.365)^2 + (0.768)^2 + (-1.951)^2 \]

\[ = 6.259 \text{ in.}^2 \]

11. Torsional-flexural constant:

\[ \beta = 1 - \left( r_o / r_o^2 \right) \]

\[ = 1 - (-1.951)^2 / 6.259 \]

\[ = 0.75 \]
12. Determination of \( F_n \): (Section 3.4 of the Standard)

For this singly symmetric section (x-axis is the axis of symmetry), \( F_n \) shall be taken as the smaller of either (Eq. 3.4.1-1) or (Eq. 3.4.3-1):

a. For Flexural Buckling:

\[
(F_n)_1 = \left( \frac{n^2 E_t}{(K y L y / r_y)^2} \right)
\]

(Eq. 3.4.1-1)

In the determination of the flexural buckling stress, it is necessary to select a proper value of \( E_t \) from Table A13 or Figure A11 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of \( f=20 \) ksi.

From Table A13, the corresponding value of \( E_t \) is found to be equal to 27000 ksi. Thus,

\[
(F_n)_1 = \left( \frac{n^2 \times 27000}{(93.75)^2} \right)
= 30.32 \text{ ksi} > \text{assumed stress } f=20 \text{ ksi}
\]

Because the computed stress is larger than the assumed value, the further successive approximation is needed.

Assume \( f=22.7 \) ksi, and

\[
E_t = 20250 \text{ ksi}
\]

\[
(F_n)_1 = \left( \frac{n^2 \times 20250}{(93.75)^2} \right)
= 22.74 \text{ ksi} = \text{assumed stress } f=22.7 \text{ ksi} \text{ OK}
\]

Alternatively, the tangent modulus \( E_t \) can be determined by using the Modified Ramberg-Osgood equation as given in Appendix B of the Standard as follows:

\[
E_t = \frac{E_o y_f}{[F_y + 0.002 n E_o (f/F_y)^{n-1}]} \quad \text{(Eq. B-2)}
\]
From Table B in the Standard, the coefficient n is equal to 4.58 for Type 304, 1/4-Hard stainless steel. Thus, for an assumed compression stress of \( f = 23.1 \text{ ksi} \),

\[
E_t = \frac{(27000 \times 50)}{\left[50 + 0.002 \times 4.58 \times 27000 \times (23.1/50)^{3.58}\right]}
\]

\[= 20584 \text{ ksi}\]

Therefore,

\[
(F_n)_1 = \frac{(n^2 \times 20584)}{(93.75)^2}
\]

\[= 23.11 \text{ ksi} = \text{assumed stress } f=23.1 \text{ ksi OK}\]

It is found that for this example, the flexural buckling stress determined by using Eq. B-2 is approximately 2% larger than that by using the tabulated value.

b. For Torsional-Flexural Buckling:

\[
(F_n)_2 = \left(\frac{1}{2\beta}\right) \left[\left(\sigma_{ex} + \sigma_t\right) - \sqrt{\left(\sigma_{ex} + \sigma_t\right)^2 - 4\beta \sigma_{ex} \sigma_t}\right]
\]

(Ref. 3.4.3-1)

where

\[
\sigma_{ex} = \frac{(n^2E_o)/(K_L/L_x)}{r_L}(E_t/E_o)
\]

(Ref. 3.4.3-3)

\[
\sigma_t = \left[1/(Ar_o^2)\right]G_o J^+ \left[(n^2E_oC_w)/(K_tL_t)^2\right](E_t/E_o)
\]

(Ref. 3.4.2-1)

\[G_o = 10500 \text{ ksi (Table A4 of the Standard)}\]

Similar to the determination of flexural buckling stress, the plasticity reduction factor of \( E_t/E_o \) depends on the assumed stress value. For the first approximation, assume a buckling stress of \( f=20 \text{ ksi} \). The value of \( E_t/E_o \) is found to be equal to 1.0, which is obtained from Table A10 or Figure A7 of the Standard. Thus,

\[
\sigma_{ex} = \left[(n^2 \times 27000)/(6 \times 12/1.365)^2\right] x(1.0)
\]

\[= 95.78 \text{ ksi}\]

\[
\sigma_t = \left[1/(0.889 \times 6.259)\right] \left[10500 \times 0.003266 + n^2 \times 27000 \times 2.05/(6 \times 12)^2\right] x(1.0)
\]

\[= 25.10 \text{ ksi}\]
Therefore,

\[
(F_{n2}) = \frac{1}{2\beta} \left[ \left( \frac{\sigma_{ex}}{\sigma_{t}} + \frac{\sigma_t}{\sigma_{ex}} \right)^2 - \frac{4\beta}{\sigma_{ex}} \sigma_{t} \right] \\
= \left[ \frac{1}{2(\chi 0.392)} \right] \left[ (95.78+25.10)^2 - 4 \times 0.392 \times 95.78 \times 25.10 \right] \\
= 21.37 \text{ ksi} > \text{assumed value } f = 20 \text{ ksi}
\]

For the second approximation, assume a stress of \( f = 20.46 \text{ ksi} \), and \( \frac{E_t}{E_o} = 0.957 \).

\[
(F_{n2}) = 20.46 \text{ ksi} = \text{assumed value OK}
\]

The plasticity reduction factor \( \frac{E_t}{E_o} \) can be alternatively determined by using the Ramberg-Osgood equation given in the Appendix B of the Standard as follows:

\[
\frac{E_t}{E_o} = \frac{F_y}{F_y + 0.002nE_o(f/F_y)^{n-1}} \quad \text{(Eq. B-5)}
\]

From Table B in the Standard, the coefficient \( n \) is equal to 4.58 for Type 304, 1/4-Hard stainless steel. Thus, for an assumed compression stress of \( f = 18.6 \text{ ksi} \),

\[
E_t/E_o = \frac{50}{50 + 0.002 \times 4.58 \times 27000 \times (18.6/50)^{4.58}} \\
= 0.875
\]

Therefore,

\[
(F_{n2}) = 21.37 \times 0.875 = 18.7 \text{ ksi} = \text{assumed value OK}
\]

(The lateral buckling stress determined by using Eq. B-5 is approximately 8.6% less than that computed by using Table A10.)

Then, \( F_n \) should be the smaller of \( (F_{n1}) \) and \( (F_{n2}) \).

\[
F_n = 20.46 \text{ ksi} \quad \text{(based on tabulated } E_t/E_o \text{ value)}
\]

13. Determination of \( A_e \):
Flanges:

d = 0.607 in.

\[ I_s = \frac{d^2 t}{12} = \frac{(0.607)^2(0.105)}{12} = 0.001957 \text{ in.}^3 \]

D = 0.9 in.

w = 1.414 in.

\[ D/w = \frac{0.9}{1.414} \approx 0.636 < 0.80 \]

\[ S = 1.28 \sqrt{\frac{E_o}{t}}, \quad f = F_n \quad \text{(Eq. 2.4-1)} \]

The initial modulus of elasticity, \( E_o \), for Type 301 stainless steel is obtained from Table A4 of the Standard, i.e., \( E_o = 27000 \text{ ksi} \).

\[ S = \frac{1.28 \sqrt{27000}}{0.105} = 46.50 \]

\[ w/t = \frac{1.414}{0.105} = 13.47 < S/3 = 15.50 \quad \text{(Eq. 2.4.2-1)} \]

\[ I_a = 0 \text{ (no edge stiffener needed)} \quad \text{(Eq. 2.4.2-2)} \]

\[ b = w \quad \text{(Eq. 2.4.2-3)} \]

\[ b = 1.414 \text{ in. (flanges fully effective)} \]

\[ w/t = 13.47 < 90 \text{ (Section 2.1.1-(1)-(i))} \]

Web:

w = 2.914 in., k = 4.00

\[ \Lambda = \frac{(1.052/\sqrt{4})(w/t)\sqrt{f/E_o}}{20.46} = \frac{(1.052/\sqrt{4})(2.914/0.105)\sqrt{20.46/27000}}{20.46} = 0.402 < 0.673 \]

\[ b = w \quad \text{(Eq. 2.2.1-1)} \]

\[ b = 2.914 \text{ in. (web fully effective)} \]

\[ w/t = \frac{2.914}{0.105} = 27.75 < 400 \text{ (Section 2.1.1-(1)-(ii))} \]

Lips:

\[ d = 0.607 \text{ in.} \]

k = 0.50 (unstiffened compression element)
\[ d_s = d'_s \]  
\[ \lambda = \frac{(1.052/\sqrt{0.50})(0.607/0.105)}{20.46/27000} \]
\[ = 0.237 < 0.673 \]
\[ d'_s = d = 0.607 \text{ in.}, \quad d_s = 0.607 \text{ in.} \]
\[ \frac{d}{t} = 5.78 < 50 \text{ (Section 2.1.1-(1)-(iii))} \]

Since flanges, web, and lips are fully effective, the effective area is the same as the full section area, i.e.,

\[ A_e = A = 0.889 \text{ in.}^2 \]

14. Determination of \( \phi_c P_n \): (Section 3.4 of the Standard)

\[ P_n = A F_e n \]  
\[ = 0.889 \times 20.46 \]
\[ = 18.19 \text{ kips} \]

\[ \phi_c = 0.85 \]

\[ \phi_c P_n = 0.85 \times 18.19 \]
\[ = 15.46 \text{ kips} \]
EXAMPLE 15.2 C-SECTION (ASD)

Determine the allowable axial load for C-section used in Example 15.1.

Solution:

1. Basic parameters used for calculating the section properties:
   See Example 15.1 for section properties of C-section.

2. Determination of $F_n$
   The following is the result obtained from Example 15.1.

   a. For Flexural Buckling:

      $\left( F_n \right)_1 = \left( \frac{n^2E_t}{(K_y/L_y/r_y)^2} \right)$

      $\left( F_n \right)_1 = \left( \frac{n^2\times 20250}{(93.75)^2} \right)$ (Eq. 3.4.1-1)

      $= 22.74 \text{ ksi}$ (Eq. 3.4.1-1)

   b. For Torsional-Flexural Buckling:

      $\left( F_n \right)_2 = \left( 1/2\beta \right) \left[ (\sigma_{ex} + \sigma_t) - \sqrt{(\sigma_{ex} + \sigma_t)^2 - 4\beta\sigma_{ex}\sigma_t} \right]$ (Eq. 3.4.3-1)

      where

      $\sigma_{ex} = \left( \frac{n^2E_o}{(K_x/L_x/r_x)^2} \right)$ (Eq. 3.4.3-3)

      $\sigma_t = \left[ 1/(Ar_o^2) \right] \left[ G_o \times + \left( \frac{n^2E_o C_w}{(K_t/L_t)^2} \right) \right]$ (Eq. 3.4.2-1)

      $G_o = 10500 \text{ ksi}$ (Table A4 of the Standard)

      $\left( F_n \right)_2 = \left( 1/2\beta \right) \left[ (\sigma_{ex} + \sigma_t) - \sqrt{(\sigma_{ex} + \sigma_t)^2 - 4\beta\sigma_{ex}\sigma_t} \right]$ (Eq. 3.4.3-1)

      $= \left[ 1/(2\times 0.392) \right] \left( 95.78+25.10 \right)$

      $-\sqrt{(95.78+25.10)^2-4\times 0.392\times 95.78\times 25.10} \times (0.957)$

      $= 20.46 \text{ ksi}$ (controls) (Eq. 3.4.3-3)

      Then, $F_n$ should be the smaller of $(F_n)_1$ and $(F_n)_2$.

      $F_n = 20.46 \text{ ksi}$

3. Determination of $A_e$:

   The effective area is the same as the full section area, i.e.,

   $A_e = A = 0.889 \text{ in.}^2$
4. Determination of $P_a$:

\[ P_n = A_F e_n \]

\[ = 0.889 \times 20.46 \]

\[ = 18.19 \text{ kips} \]

\[ \Omega = 2.15 \]

\[ P_a = \frac{P_n}{\Omega} = \frac{18.19}{2.15} \]

\[ = 8.46 \text{ kips} \]
EXAMPLE 16.1 C-SECTION w/WIDE FLANGE (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design axial strength for C-section as shown in Figure 16.1. Use Type 304 stainless steel, 1/4-Hard.

Figure 16.1 Section for Example 16.1

Given:
1. Section: 3.5" x 3.5" x 0.105" channel with stiffened flanges.
2. \( K_{x,x} = K_{y,y} = K_{t,t} = 6 \) ft.

Solution:
The following equations used for computing the sectional properties for channel with lips are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.
1. Basic parameters used for calculating the section properties:

\[ r = \frac{R+t}{2} = \frac{3}{16}+0.105/2 = 0.240 \text{ in.} \]

From the sketch \( a = 2.914 \text{ in.}, \ b = 2.914 \text{ in.}, \ c = 0.607 \text{ in.}, \ A' = 3.5 \text{ in.}, \ B' = 3.5 \text{ in.}, \ C' = 0.9 \text{ in.}, \)

\( a = 1.00 \) (for section has lips)

\[ a = A'-t = 3.5-0.105 = 3.395 \text{ in.} \]

\[ b = B'-t = 3.5-0.105 = 3.395 \text{ in.} \]

\[ c = C'-t/2 = 0.9-0.105/2 = 0.848 \text{ in.} \]

\[ u = 1.57r = 1.57 \times 0.240 = 0.377 \text{ in.} \]

2. Area:

\[ A = t(a+2b+2c+4u) \]

\[ = 0.105[2.914+2x2.914+2x0.607+4x0.377] \]

\[ = 1.204 \text{ in.}^2 \]

3. Moment of inertia about x-axis:

\[ I_x = 2t[0.0417a^2+b(a/2+r)^2+2u(a/2+0.637r)^2+0.298r^3 \]

\[ +0.0833c^3+(c/4)(a-c)^2] \]

\[ = 2x0.105[0.0417(2.914)^2+2.914(2.914/2+0.240)^2 \]

\[ +2x0.377(2.914/2+0.637x0.240)^2+0.298(0.240)^3 \]

\[ +0.0833(0.607)^3+(0.607/4)(2.914-0.607)^2] \]

\[ = 2.564 \text{ in.}^4 \]

4. Distance from centroid of section to centerline of web:

\[ \bar{x} = \frac{(2t/A)[b(b/2+r)+u(0.363r)+u(b+1.637r)+c(b+2r)]}{211} \]
\[ \frac{(2x0.105)/1.204}{1.204} \left[ 2.914(2.914/2+0.240)+0.377(0.363x0.240) \\
+0.377(2.914+1.637x0.240)+0.607(2.914+2x0.240) \right] \]
\[ = 1.445 \text{ in.} \]

5. Moment of inertia about y-axis:
\[ I_y = 2x0.105(2.914(2.914/2+0.240)^2+0.0833(2.914)^3 \]
\[ +0.505(0.240)^3+0.607(2.914+2x0.240)^2 \]
\[ +0.377(2.914+1.637x0.240)^2 \]
\[ = 2.017 \text{ in.}^2 \]

6. Distance from shear center to centerline of web:
\[ m = \frac{(bt/12I_x)\left[6c(\ddot{a})^2+3b(\ddot{a})^2-8(\ddot{c})^2\right]}{[(3.395x0.105)/(12x2.564)](6x0.848(3.395)^2 \]
\[ +3x3.395(3.395)^2-8(0.848)^3]} \]
\[ = 1.983 \text{ in.} \]

7. Distance from centroid to shear center:
\[ x_o = -(\ddot{x}+m) = -(1.445+1.983) \]
\[ = -3.428 \text{ in.} \]

8. St. Venant torsion constant:
\[ J = (t^3/3)[a+2b+2c+4u] \]
\[ = [(0.105)^3/3][2.914+2x2.914+2x0.607+4x0.377] \]
\[ = 0.004424 \text{ in.}^4 \]
9. Warping Constant:

\[
C_w = \frac{t^2}{A} \left( \frac{xA(\bar{a})^2}{t} \right) \left( \frac{(\bar{b})^2}{3-m^2} + m^2 - m\bar{b} + (A/2t) \left( \frac{(m)^2}{2} (\bar{a})^3 \right) \right.
\]

\[
+ (\bar{b})^2 (c)^2 (2c+3\bar{a}) - (I_x^m/t)(2\bar{a}+4c) + \left[ m(c)^2/3 \right] \left( 8(\bar{b})^2 (c) 
\right.
\]

\[
+ 2m(2c(c-\bar{a})+\bar{b}(2\bar{c}-3\bar{a})) + (\bar{b})^2 (a)^2/6 \left[ (3c+b)(4c+a)-6(c)^2 
\right.
\]

\[
-m(a)^4/4 \right) \}
\]

\[
= \left[ \frac{(0.105)^2}{1.204} \right] \left[ \left( \frac{1.445 \times 1.204 \times (3.395)^2}{0.105} \right) \left( \frac{(3.395)^2}{3} \right) 
\right.
\]

\[
+ (1.983)^2-1.983 \times 3.395 + 1.204/(3 \times 0.105) \left( \frac{(1.983)^2 (3.395)^3}{3} \right) 
\]

\[
+ (3.395)^2 (0.848)^2 (2x0.848+3x3.395)) 
\]

\[
- \left[ 2.564x(1.983)^2/0.105 \left( 2x3.395+4x0.848 \right) + 1.983(0.848)^2/3 \right] \left[ 8(3.395)^3(0.848) 
\right.
\]

\[
+ 2x1.983(2x0.848(0.848-3.395)+3.395(2x0.848-3x3.395)) 
\]

\[
+ \left( (3.395)^2 (3.395)^2 / 6 \right) \left[ (3x0.848+3.395)(4x0.848+3.395) 
\right.
\]

\[
- 6(0.848)^2 \right] \left[ (1.983)^2 (3.395)^4/4 \right] \}
\]

\[= 7.572 \text{ in.}^4 \]

10. Radii of gyration:

\[
r_x = \sqrt{I_x/A} = \sqrt{2.564/1.204} = 1.459 \text{ in.}
\]

\[
r_y = \sqrt{I_y/A} = \sqrt{2.017/1.204} = 1.294 \text{ in.}
\]

\[
(K_y L_y)/r_y = (6 \times 12)/1.294 = 55.64 < 200
\]

\[
r_o^2 = r_x^2 + r_y^2 + r_z^2 = (1.459)^2 + (1.294)^2 + (-3.428)^2
\]

\[= 15.554 \text{ in.}^2 \]

11. Torsional-flexural constant:

\[
\beta = 1 - (x_o / r_o)^2 \quad \text{(Eq. 3.4.3-4)}
\]

\[= 1 - (-3.428)^2 / 15.554 \]

\[= 0.244 \]

213
12. Determination of $F_n$ (Section 3.4 of the Standard)

For this singly symmetric section ($x$-axis is the axis of symmetry), $F_n$ shall be taken as the smaller of either (Eq. 3.4.1-1) or (Eq. 3.4.3-1):

a. For Flexural Buckling:

\[
(F_n)_1 = \left( \frac{n^2E_t}{(K_Ly/r_y)^2} \right)
\]

(Eq. 3.4.1-1)

In the determination of the flexural buckling stress, it is necessary to select a proper value of $E_t$ from Table A13 or Figure A11 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of $f=32.0$ ksi. From Table A13, the corresponding value of $E_t$ is found to be equal to $11300$ ksi. Thus,

\[
(F_n)_1 = \left( \frac{n^2 \times 11300}{(55.64)^2} \right)
\]

$= 36.02$ ksi $> \text{assumed stress } f=32.0$ ksi

Because the computed stress is larger than the assumed value, further successive approximations are needed. For the second approximation, assume $f=33.77$ ksi, and

\[
E_t = 10600 \text{ ksi}
\]

\[
(F_n)_1 = \left( \frac{n^2 \times 10600}{(55.64)^2} \right)
\]

$= 33.79$ ksi $= \text{assumed stress } f=33.77$ ksi \ OK

b. For Torsional-Flexural Buckling:

\[
(F_n)_2 = \frac{1}{2\beta} \left[ (\sigma_{ex} + \sigma_t)^2 - (\sigma_{ex} + \sigma_t)^2 - 4\beta \sigma_{ex} \sigma_t \right]
\]

(Eq. 3.4.3-1)

where

\[
\sigma_{ex} = \left( \frac{n^2E_o}{(K_l x/r_x)^2} \right) (E_t/E_o)
\]

(Eq. 3.4.3-3)

\[
\sigma_t = \left[ \frac{1}{(Ar_o^2)} \right] [G_o J + (n^2E_o C_w)/(K_t L_t^2)] (E_t/E_o)
\]

(Eq. 3.4.2-1)

\[
G_o = 10500 \text{ ksi (Table A4 of the Standard)}
\]
Similar to the determination of flexural buckling stress, the plasticity reduction factor of $E_t/E_o$ used for determining the torsional-flexural buckling stress depends on the assumed stress value. For the first approximation, assume a buckling stress of $f=20$ ksi. The value of $E_t/E_o$ is found to be equal to 1.0, which is obtained from Table A10 or Figure A7 of the Standard. Thus,

$$\sigma_{ex} = \frac{(n^2x27000)/(6x12/1.459)^2}{1.0} = 109.42 \text{ ksi}$$

$$\sigma_t = \frac{[1/(1.204x15.554)][1.0500x0.004424 + n^2x27000x7.572/(6x12)^2]}{(1.0) x(1.0)} = 23.27 \text{ ksi}$$

Therefore,

$$\left(F_n\right)_2 = \frac{(1/2\beta)[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}]}{(1/(2x0.244))[109.42+23.27] -\sqrt{(109.42+23.27)^2-4x0.244x109.42x23.27}} = 19.92 \text{ ksi}$$

Because the computed stress $(F_n)_2$ is less than the assumed value of $f=20$ ksi, the second approximation will be assumed that a stress of $f=19.92$ ksi and $E_t/E_o = 1.0$. Thus,

$$\left(F_n\right)_2 = 19.92 \text{ ksi} \quad \text{OK}$$

$F_n$ should be the smaller of $(F_n)_1$ and $(F_n)_2$. Thus,

$$F_n = 19.92 \text{ ksi}$$

13. Determination of $A_e$:

Flanges:

$$d = 0.607 \text{ in.}$$

$$I_s = d^3t/12 = (0.607)^3(0.105)/12$$
\[ D = 0.9 \text{ in.} \\
w = 2.914 \text{ in. (for flange)} \]
\[ D/w = 0.9/2.914 = 0.309 < 0.80 \]
\[ S = 1.28\sqrt{E_0/f}, \quad f = F_n \quad \text{(Eq. 2.4-1)} \]

The initial modulus of elasticity, \( E_0 \), for Type 304 stainless steel is obtained from Table A4 of the Standard, i.e., \( E_0 = 27000 \text{ ksi} \)
\[ S = 1.28\sqrt{27000/19.92} = 47.12, \quad S/3 = 15.71 \]
\[ w/t = 2.914/0.105 = 27.75 \]
\[ S/3 < w/t < S \]

\[ I_a = 399t\left\{\left[(w/t)/S\right]-0.33\right\}^3 \quad \text{(Eq. 2.4.2-6)} \]
\[ = 399(0.105)^3\left[(27.75/47.12)-0.33\right] \]
\[ = 0.000842 \text{ in.}^4 < I_s = 0.001957 \text{ in.}^4 \]

\[ C_1 = 2 - (I_s/I_a) \geq 1.0 \quad \text{(Eq. 2.4.2-8)} \]
\[ = 2 - (0.001957/0.000842) = -0.32 < 1.0 \]

\[ C_2 = 1.0 \quad \text{(Eq. 2.4.2-7)} \]

\[ I_s/I_a = (0.001957/0.000842) = 2.32 > 1.0 \]

\[ C_2 = 1.0 \]

\[ 0.25 < D/w = 0.309 < 0.8 \]

\[ k = [4.82-5(D/w)](I_s/I_a)^n + 0.43 \leq 5.25-5(D/w) \quad \text{(Eq. 2.4.2-9)} \]
\[ n = 1/2 \]
\[ [4.82-5(0.309)](0.001957/0.000842)^{1/2} + 0.43 = 5.414 \]

\[ 5.25-5(0.309) = 3.705 < 5.414 \]

\[ k = 3.705 \]

\[ \Lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_0}, \quad f' = F_n \quad \text{(Eq. 2.2.1-4)} \]
\[ = (1.052/\sqrt{3.705})(27.75)\sqrt{19.92/27000} = 0.412 < 0.673 \]

216
\[ b = w \quad \text{(Eq. 2.2.1-1)} \]

\[ = 2.914 \text{ in. (flanges fully effective)} \]

\[ \frac{w}{t} = 27.75 < 90 \quad \text{(Section 2.1.1-(1)-(i))} \]

**Web:**

\[ w = 2.914 \text{ in., } k = 4.00 \]

\[ A = \frac{(1.052/\sqrt{4})(2.914/0.105)\sqrt{19.92}/27000}{27000} \]

\[ = 0.397 < 0.673 \]

\[ b = w = 2.914 \text{ in. (web fully effective)} \]

\[ \frac{w}{t} = 2.914/0.105 = 27.75 < 400 \quad \text{(Section 2.1.1-(1)-(ii))} \]

**Lips:**

\[ d = 0.607 \text{ in.} \]

\[ k = 0.50 \quad \text{(unstiffened compression element)} \]

\[ A = \frac{(1.052/\sqrt{0.50})(0.607/0.105)\sqrt{19.92}/27000}{27000} \]

\[ = 0.234 < 0.673 \]

\[ d' = d = 0.607 \text{ in.} \]

\[ d_s = d' (\frac{I_s}{I_a}) \leq d'_s \quad \text{(Eq. 2.4.2-11)} \]

\[ = 0.607(2.32) = 1.408 > d'_s = 0.607 \text{ in.} \]

\[ d_s = 0.607 \text{ in. (Lip fully effective in computing the overall effective area)} \]

\[ \frac{d}{t} = 5.78 \]

Since flanges, web, and lips are fully effective, the effective area is the same as the full section area, i.e.,

\[ A_e = A = 1.204 \text{ in.}^2 \]

14. Determination of \( \phi P_n^c : \) (Section 3.4 of the Standard)

\[ P_n = A F_n \quad \text{(Eq. 3.4-1)} \]
\[ \phi_c = 0.85 \]
\[ \phi_c P_n = 0.85 \times 23.98 = 20.38 \text{ kips} \]
EXAMPLE 16.2 C-SECTION w/WIDE FLANGE (ASD)

Determine the allowable axial load for C-section used in Example 16.1.

Solution:

1. Basic parameters used for calculating the section properties:
   See Example 16.1 for section properties of C-section.

2. Determination of $F_n$

   The following results are obtained from Example 16.1.

   a. For Flexural Buckling:

   $$(F_n)_1 = \left(\frac{n^2E_t}{(KL/y/r_y)^2}\right)$$ (Eq. 3.4.1-1)

   $$(F_n)_1 = \left(\frac{n^2 \times 10600}{(55.64)^2}\right)$$

   = 33.79 ksi

   b. For Torsional-Flexural Buckling:

   $$(F_n)_2 = \frac{1}{2}\left(\sigma_{ex}^2 + \sigma_t^2\right) - \sqrt{\frac{\sigma_{ex}^2 + \sigma_t^2 - 4\beta\sigma_{ex}\sigma_t}{\sigma_{ex}\sigma_t}}$$ (Eq. 3.4.3-1)

   where

   $$\sigma_{ex} = \left[\frac{n^2E_o}{(KL_y/y_y)^2}\right]\left(\frac{E_t}{E_o}\right)$$ (Eq. 3.4.3-3)

   $$\sigma_t = \frac{1}{(Ar^2)}\left[Go J + \left(\frac{n^2E_o C_w}{(KL_t)^2}\right)\left(E_t/E_o\right)\right]$$ (Eq. 3.4.2-1)

   $G_o = 10500$ ksi (Table A4 of the Standard)

   $$(F_n)_2 = \frac{1}{2}\left(\sigma_{ex}^2 + \sigma_t^2\right) - \sqrt{\frac{\sigma_{ex}^2 + \sigma_t^2 - 4\beta\sigma_{ex}\sigma_t}{\sigma_{ex}\sigma_t}}$$ (Eq. 3.4.3-1)

   $$= \left[\frac{1}{(2 \times 0.244)}\left(\frac{109.42 + 23.27}{109.42 + 23.27}ight)^2 - 4 \times 0.244 \times 109.42 \times 23.27\right]$$

   = 19.92 ksi (control)

   Then, $F_n$ should be the smaller of $(F_n)_1$ and $(F_n)_2$.

   $F_n = 19.92$ ksi

3. Determination of $A_e$:

   The effective area is the same as the full section area, i.e.,

   $$A_e = A = 1.204 \text{ in.}^2 \text{ (from Example 16.1)}$$
4. Determination of $P_a$:

\[
\begin{align*}
  P_n &= A F_e n \\ 
  &= 1.204 \times 19.92 \\ 
  &= 23.98 \text{ kips} \\

  \Omega &= 2.15 \\

  P_a &= \frac{P_n}{\Omega} = \frac{23.98}{2.15} \\
  &= 11.15 \text{ kips}
\end{align*}
\]
EXAMPLE 17.1 I-SECTION (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design axial strength for the I-section as shown in Figure 17.1. Use Type 409 stainless steel.

Figure 17.1 Section for Example 17.1

Given:

1. Section: 6.0" x 3.0" x 0.135" I-section with no lips.

2. $K_{Lx} = 14$ ft., $K_{Ly} = 7.0$ ft.

Solution:

The following equations used for computing the sectional properties for I-section with no lips are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.
1. Basic parameters used for calculating the sectional properties:

\[ r = R + t/2 = 3/16 + 0.135/2 = 0.255 \text{ in.} \]

From the sketch, \( A' = 6.0 \text{ in.}, \quad B' = C' = 1.5 \text{ in.} \)

\( \alpha = 1.00 \) (For I-section)

\[ a = A' - [r + t/2 + (r + t/2)] = 6.0 - (0.255 + 0.135/2 + 0.255 + 0.135/2) = 5.355 \text{ in.} \]

\[ \bar{a} = A' - (t/2 + \alpha t/2) = 6.0 - 0.135 = 5.865 \text{ in.} \]

\[ b = c = B' - (r + t/2) = 1.5 - (0.255 + 0.135/2) = 1.178 \text{ in.} \]

\[ \bar{b} = \bar{c} = B' - t/2 = 1.5 - 0.135/2 = 1.433 \text{ in.} \]

\[ u = 1.57r = 1.57 \times 0.255 = 0.40 \text{ in.} \]

2. Area:

\[ A = t \left[ 2a + 2b + 2u + \alpha(2c + 2u) \right] \]

\[ = 0.135(2 \times 5.355 + 2 \times 1.178 + 2 \times 0.40 + 2 \times 1.178 + 2 \times 0.4) \]

\[ = 2.298 \text{ in.}^2 \]

3. Moment of inertia about y-axis:

\[ I_y = 2t \left[ b(b/2 + r + t/2)^2 + 0.0833b^3 + u(0.363r + t/2)^2 + 0.149r^2 \right. \]

\[ \left. + \alpha[c(c/2 + r + t/2)^2 + 0.0833b^3 + u(0.363r + t/2)^2 + 0.149r^2] \right] \]

\[ = 2 	imes 0.135 \times 2 \left[ 1.178(1.178/2 + 0.255 + 0.135/2)^2 + 0.0833(1.178)^3 \right. \]

\[ \left. + 0.4(0.363 \times 0.255 + 0.135/2)^2 + 0.149(0.255)^3 \right] \]

\[ = 0.609 \text{ in.}^4 \]
4. Distance between centroid and flange centerline:

\[ \bar{y} = \frac{\bar{a}}{2} = \frac{5.865}{2} = 2.933 \text{ in.} \]

5. Moment of inertia about x-axis:

\[
I_x = 2t \left[ 0.358r^2 + a(a/2+r)^2 + 0.0833a^2 + a(a+1.637r)^2 + 0.149r^2 + 1.178(a+2r)^2 \right] \]
\[ - A(y)^2 \]
\[ = 2 \times 0.135 \left[ 0.358(0.255)^2 + 5.355(5.355/2+0.255)^2 + 0.0833(5.355)^3 + 0.4(5.355+1.637\times0.255)^2 + 0.149(0.255)^2 + 1.178(5.355+2\times0.255)^2 \right] \]
\[ - 2.298(2.933)^2 \]
\[ = 10.66 \text{ in.}^4 \]

6. Distance between shear center and flange centerline:

\[ m = \frac{\bar{a}}{2} = 2.933 \text{ in.} \]

7. Distance between centroid and shear center:

\[ y_o = -(\bar{y} - m) = 0 \]

8. St. Venant torsion constant:

\[ J = \frac{(2t^3/3)[a+b+u+c(u+c)]}{2(0.135)^3/3} (5.355+1.178+0.4+0.4+1.178) \]
\[ = 0.0140 \text{ in.}^4 \]

9. Warping Constant:

\[ C_w = \frac{(t\bar{a}^2/12) \times 8(\bar{b})^3(\bar{c})^3}{[(\bar{b})^3 + (\bar{c})^3]} \]
\[ = \frac{(t\bar{a}^2/12) \times 4\bar{b}^3}{(0.135(5.865)^3/12) \times 4(1.433)^3} \]
\[ = 4.55 \text{ in.}^4 \]

223
10. Radii of gyration:

\[ r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{10.66}{2.298}} = 2.154 \text{ in.} \]

\[ r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{0.609}{2.298}} = 0.515 \text{ in.} \]

\[
\frac{(K_x L_x)}{r_x} = \frac{(14 \times 12)}{2.514} = 66.83 < 200
\]

\[
\frac{(K_y L_y)}{r_y} = \frac{(7 \times 12)}{0.515} = 163.1 < 200 \quad \text{(control)}
\]

\[
r_o^2 = r_x^2 + r_y^2 + y_o^2 = (2.154)^2 + (0.515)^2 + 0
\]

\[ = 4.905 \text{ in.}^2 \]

11. Determination of \( F_n \): (Section 3.4 of the Standard)

For this doubly symmetric section (x-axis is the major axis), \( F_n \) shall be taken as the smaller of either

(Eq. 3.4.1-1) or (Eq. 3.4.2-1):

a. For Flexural Buckling:

\[
(F_n)_1 = \frac{n^2 E_t}{(K_y L_y/r_y)^2} \quad \text{(Eq. 3.4.1-1)}
\]

In the determination of the flexural buckling stress, it is necessary to select a proper value of \( E_t \) from Table A14 or Figure A12 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of \( f=8 \) ksi. From Table A14, the corresponding value of \( E_t \) is found to be equal to 27000 ksi. Thus,

\[
(F_n)_1 = \frac{(n^2 \times 27000)}{(163.1)^2}
\]

\[ = 10.02 \text{ ksi} > \text{assumed stress } f=8 \text{ ksi} \]

Because the computed stress is larger than the assumed value, the further successive approximation is needed. After several trials, assume \( f=10.0 \) ksi, and

\[ E_t = 26900 \text{ ksi} \]

\[
(F_n)_1 = \frac{(n^2 \times 26900)}{(163.1)^2}
\]
Alternatively, the tangent modulus \( E_t \) can be determined by using the Modified Ramberg-Osgood equation as given in Appendix B of the Standard as follows:

\[
E_t = \frac{E_o F_y}{[F_y + 0.002nE_o(f/F_y)^{n-1}]} \quad \text{(Eq. B-2)}
\]

From Table B in the Standard, the coefficient \( n \) is equal to 9.7 for Type 409 stainless steel in longitudinal compression.

Thus, for an assumed compression stress of \( f = 10.0 \) ksi,

\[
(F_y = 30 \text{ ksi}, \ E_o = 27000 \text{ ksi})
\]

\[
E_t = \frac{(27000 \times 30)}{[30 + 0.002 \times 9.7 \times 27000 \times (10.0/30)^{9.7}]} = 26966 \text{ ksi}
\]

Therefore,

\[
(F_n)_1 = \frac{(n^2 \times 26966)}{(163.1)^2} = 10.0 \text{ ksi} \quad \text{assumed stress } f=10.0 \text{ ksi OK}
\]

It is found that for this example, the flexural buckling stress determined by using Eq. B-2 is practically the same as that determined by using the tabulated value.

b. For Torsional Buckling:

\[
(F_n)_2 = \left[1/(A_r^2)\right][G_o J + (n^2E_o C_w)/(K_t L_t)^2](E_t/E_o) \quad \text{(Eq. 3.4.2-1)}
\]

\( G_o = 10500 \text{ ksi (Table A4 of the Standard)} \)

Similar to the determination of flexural buckling stress, the plasticity reduction factor of \( E_t/E_o \) depends on the assumed stress value. For the first approximation, assume a buckling stress of \( f = 8 \) ksi. The value of \( E_t/E_o \) is found to be equal to 1.0, which is obtained from Table A11 or Figure A8.
of the Standard. Thus,

\[
(F_n)_2 = \left[1/(2.298x4.905)\right] \left[10500x0.014 + \pi^2x27000x4.555/(7x12)^2\right]x(1.0)
\]

= 28.3 ksi > 8 ksi NG

After several trials, assume a stress of \(f=19.65\) ksi, and

\[\frac{E_t}{E_o} = 0.694.\]

\[
(F_n)_2 = \left[1/(2.298x4.905)\right] \left[10500x0.014 + \pi^2x27000x4.555/(7x12)^2\right]x(0.694)
\]

= 19.64 = assumed value OK

The plasticity reduction factor \(\frac{E_t}{E_o}\) can be alternatively
determined by using the Ramberg-Osgood equation given in the
Appendix B of the Standard as follows:

\[
E_t/E_o = F_y/F_y + 0.002nE_o(f/F_y)^{n-1}
\]

(Eq. B-5)

From Table B in the Standard, the coefficient \(n\) is equal to 9.7
for Type 409 stainless steel. Thus, for an assumed compression
stress of \(f = 19.65\) ksi,

\[
\frac{E_t}{E_o} = 30/30 + 0.002x9.7x27000x(19.65/30)^{9.7}
\]

= 0.694

Therefore,

\[
(F_n)_2 = 28.3x(0.694) = 19.65 \text{ ksi = assumed value OK}
\]

The lateral buckling stress determined by using Eq. B-5 is
practically the same as that computed by using Table A11.

Then, \(F_n\) should be the smaller of \((F_n)_1\) and \((F_n)_2\).

\[F_n = 10.0 \text{ ksi}\]

12. Determination of \(A_s\):

Unstiffened Compression Flanges: \((k=0.5)\)
\[ \frac{w}{t} = \frac{1.178}{0.135} = 8.73 < 50 \quad (\text{Eq. 2.4.2-1}) \]

\[ \lambda = \frac{1.052}{\sqrt{k}}(\frac{w}{t})\sqrt{\frac{f}{E_o}}, \quad f = F_n \quad (\text{Eq. 2.2.1-4}) \]

\[ = \frac{1.052}{\sqrt{0.5}}(8.73)\sqrt{10.0/27000} \]

\[ = 0.25 < 0.673 \]

\[ b = w \quad (\text{Eq. 2.4.2-3}) \]

\[ = 1.178 \text{ in. (flanges fully effective)} \]

Web: (Sec. 2.2.2-(2))

\[ w = 5.355 \text{ in.}, \]

\[ \frac{w}{t} = 5.355/0.135 = 39.67 \]

\[ k = 4.0 \]

\[ \lambda = \frac{1.052}{\sqrt{k}}(\frac{w}{t})\sqrt{\frac{f}{E_o}}, \quad f = F_n \quad (\text{Eq. 2.2.1-4}) \]

\[ = \frac{1.052}{\sqrt{4}}(39.67)\sqrt{10.0/27000} \]

\[ = 0.40 < 0.673 \]

\[ b = w \quad (\text{Eq. 2.2.1-1}) \]

\[ = 5.355 \text{ in. (web fully effective)} \]

Since flanges and webs are fully effective, the effective area is the same as the full section area, i.e.,

\[ A_e = A = 2.298 \text{ in.}^2 \]

13. Determination of \( \Phi_c P_n \): (Section 3.4 of the Standard)

\[ P_n = A \frac{F_n}{e_n} \quad (\text{Eq. 3.4-1}) \]

\[ = 2.298 \times 10.0 \]

\[ = 22.98 \text{ kips} \]

\[ \Phi_c = 0.85 \]

The design axial strength is

\[ \Phi_c P_n = 0.85 \times 22.98 \]

\[ = 19.53 \text{ kips} \]
EXAMPLE 17.2 I-SECTION (ASD)

Determine the allowable axial load for the I-section used in Example 17.1.

Solution:

1. Basic parameters used for calculating the sectional properties:
   See Example 17.1 for calculation of sectional properties of the I-section.

2. Determination of $F_n$
   The following results are obtained from Example 17.1.
   a. For Flexural Buckling:
      
      \[
      (F_n)_1 = \frac{n^2E_t}{K_{L_y}/r_y^2} \quad \text{(Eq. 3.4.1-1)}
      \]
      
      \[
      (F_n)_1 = \frac{n^2 \times 26900}{(163.1)^2}
      \]
      
      \[
      = 10.0 \text{ ksi (see Example 17.1 for } E_t) \]

   b. For Torsional Buckling:
      
      \[
      (F_n)_2 = \left[\frac{1}{(Ae)^2}\right]\left[\frac{G_J + (n^2E_tC_y)}{K_{L_t}}\right]E_t \quad \text{(Eq. 3.4.2-1)}
      \]
      
      \[
      = \left[\frac{1}{(2.298)^2}\right]\left[105000 \times 0.014 + n^2 \times 27000 \times 4.555 \times (7 \times 12)^2\right] \times 0.694
      \]
      
      \[
      = 19.65 \text{ ksi}
      \]

   Then, $F_n$ should be the smaller of $(F_n)_1$ and $(F_n)_2$.

   \[
   F_n = 10.0 \text{ ksi}
   \]

3. Determination of $A_e$:
   The effective area is the same as the full section area, i.e.,
   \[
   A_e = A = 2.298 \text{ in}^2 \quad \text{(See Example 17.1)}
   \]

4. Determination of $P_n$:
   \[
   P_n = A_e F_n \quad \text{(Eq. 3.4-1)}
   \]
   \[
   = 2.298 \times 10.0
   \]

228
\[ \Omega = 2.15 \]

The allowable axial load is

\[ P_a = \frac{P_n}{\Omega} = \frac{22.98}{2.15} \]

\[ = 10.69 \text{ kips} \]
EXAMPLE 18.1 I-SECTION W/LIPS (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design axial strength for the I-section as shown in Figure 18.1. Use Type 409 stainless steel.

Given:
1. Section: 6.0" x 5.0" x 0.135" I-section with lips.
2. \( K_x = K_y = 1.0 \), \( L_x = 12.0 \) ft. and \( L_y = 6.0 \) ft.

Solution:

The following equations used for computing the sectional properties for I-section with lips are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.
1. Basic parameters used for calculating the sectional properties:

(For a channel with lips)

\[ r = R + \frac{t}{2} = \frac{3}{16} + 0.135/2 = 0.255 \text{ in.} \]

From the sketch, \( A' = 6.0 \text{ in.}, \quad B' = 2.5 \text{ in.}, \quad C' = 0.82 \text{ in.} \)

\( a = 1.00 \) (For sections with lips)

\[ a = A' - (2r + t) = 6.0 - (2 \times 0.255 + 0.135) = 5.355 \text{ in.} \]

\[ \bar{a} = A' - t = 6 - 0.135 = 5.865 \text{ in.} \]

\[ b = B' - [r + t/2 + a(r + t/2)] = 2.5 - (2 \times 0.255 + 0.135) = 1.855 \text{ in.} \]

\[ \bar{b} = B' - (t/2 + at/2) = 2.5 - 0.135 = 2.365 \text{ in.} \]

\[ c = a[C' - (r + t/2)] = 0.82 - (0.255 + 0.135/2) = 0.498 \text{ in.} \]

\[ \bar{c} = a(C' - t/2) = 0.82 - 0.135/2 = 0.753 \text{ in.} \]

\[ u = 1.57r = 1.57 \times 0.255 = 0.40 \text{ in.} \]

2. Area: (lipped I-section)

\[ A = 2xt[a + b + \alpha(a + 2u + a(2c + 2u))] \]

\[ = 2 \times 0.135 \left[ 5.355 + 2 \times 1.855 + 2 \times 0.40 + 2 \times 0.498 + 2 \times 0.4 \right] \]

\[ = 3.148 \text{ in.}^2 \]

3. Moment of inertia about x-axis: (lipped I-section)

\[ I_x = 2xt \left\{ 0.0417a^2 + b(a/2 + r)^2 + u(a/2 + 0.637r)^2 + 0.149r^2 \right\} \]

\[ + \alpha \left( 0.0833c^2 + (c/4)(a - c)^2 + u(a/2 + 0.637r)^2 + 0.149r^2 \right\} \]

\[ = 2 \times 2 \times 0.135 \left[ 0.0417(5.355)^2 + 1.855(5.355/2 + 0.255)^2 + 0.4(5.355/2 + 0.637 \times 0.255)^2 + 0.4(5.355/2 + 0.637 \times 0.255)^2 \right] \]

\[ + 0.4(5.355/2 + 0.637 \times 0.255)^2 + 0.4(5.355/2 + 0.637 \times 0.255)^2 \]

\[ + (0.498/4)(5.355 - 0.498)^2 \]

\[ + (5.355 - 0.498)^2 + 0.4(5.355/2 + 0.637 \times 0.255)^2 \]

\[ + (0.498/4)(5.355 - 0.498)^2 + 0.4(5.355/2 + 0.637 \times 0.255)^2 \]

\[ = 231 \]
4. Distance between centroid and web centerline for a lipped channel:

\[
\bar{x} = \frac{2t}{A} \left\{ b \left( \frac{b}{2} + r \right) + u \left[ u(b+1.637r) + c(b+2r) \right] \right\} \\
= \frac{2 \times 0.135}{1.574} \left[ 1.855 \left( 1.855/2 + 0.255 \right) + 0.4 \left( 0.363 \times 0.255 \right) + 0.4 \left( 1.855 + 1.637 \times 0.255 \right) + 0.498 \left( 1.855 + 2 \times 0.255 \right) \right] \\
= 0.741 \text{ in.}
\]

5. Moment of inertia about y-axis:

For a channel with lips

\[
I_y = 2t \left[ b \left( \frac{b}{2} + r \right)^2 + 0.0833b^3 + 0.356r^2 + \alpha \left[ c(b+2r)^2 \right. \right. \\
\left. \left. + u(b+1.637r)^2 + 0.149r^2 \right] \right] - A(x)^2 \\
= 2 \times 0.135 \left[ 1.855 \left( 1.855/2 + 0.255 \right)^2 + 0.0833(1.855)^3 + 0.356(0.255)^3 \right. \\
\left. + 0.498(1.855+2\times0.255)^2 + 0.4(1.855+1.637\times0.255)^2 + 0.149(0.255 \right] \\
- 1.574(0.741)^2 \\
= 1.292 \text{ in.}^4
\]

For lipped I-section

\[
I_y = 2 \left[ I_y' + A(\bar{x}+t/2)^2 \right] \\
= 2 \left[ 1.292 + 1.574(0.741+0.135/2)^2 \right] = 4.642 \text{ in.}^4
\]

6. Distance between shear center and y-axis: (lipped I-section)

\[ m = 0 \]

7. Distance between centroid and shear center: (lipped I-section)

\[ x_o = 0 \]
8. St. Venant torsion constant: (lipped I-section)

\[ J = \frac{2x(0.135)^3}{3} \left[ 5.355 + 2x1.855 + 2x0.4 + 2x0.498 + 2x0.4 \right] \]

\[ = 0.0191 \text{ in.}^4 \]

9. Warping Constant: (lipped I-section)

\[ C_w = \frac{tB^2}{3} \left[ (a)^2 + 3(a)^2 c + 6(a)(c)^2 + 4(c)^2 \right] \]

\[ = \frac{0.135(2.365)^2}{3} \left[ 5.865^2 + 2(5.865)(0.753) + 4(0.753)^2 \right] \]

\[ = 45.49 \text{ in.}^6 \]

10. Radii of gyration: (lipped I-section)

\[ r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{17.15}{3.148}} = 2.334 \text{ in.} \]

\[ r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{4.642}{3.148}} = 1.214 \text{ in.} \]

\[ \left( K_{x_x} \right) / r_x = \frac{12x12}{2.334} = 61.70 < 200 \text{ (control)} \]

\[ \left( K_{y_y} \right) / r_y = \frac{6x12}{1.214} = 59.3 < 200 \]

\[ r_o^2 = r_x^2 + r_y^2 + x^2 = (2.334)^2 + (1.214)^2 + 0 \]

\[ = 6.921 \text{ in.}^2 \]

11. Determination of \( F_n \): (Section 3.4 of the Standard)

For this doubly symmetric section (x-axis is the major axis), \( F_n \) shall be taken as the smaller of either

(Eq. 3.4.1-1) or (Eq. 3.4.2-1):

a. For Flexural Buckling:

\[ (F_n')_1 = \frac{n^2E_t}{(K_{x_x} / r_x)^3} \quad \text{(Eq. 3.4.1-1)} \]

or the above equation can be written as follows:

\[ (F_n')_1 = \left[ \frac{n^2E_o}{(K_{x_x} / r_x)^3} \right] (E_t/E_o) \]

233
In the determination of the flexural buckling stress, it is necessary to select a proper value of \( \frac{E_t}{E_o} \) from Table A11 or Figure A8 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of \( f = 20 \) ksi. From Table A11, the corresponding value of \( \frac{E_t}{E_o} \) is found to be equal to 0.66. Thus,

\[
(F_n)_1 = \left( \frac{n^2 \times 27000}{(61.7)^2} \right) (0.66)
\
= 46.2 \text{ ksi} \quad \text{assumed stress } f = 20 \text{ ksi}
\]

Because the computed stress is larger than the assumed value, the further successive approximation is needed. After several trials, assume \( f = 23.50 \) ksi, and \( \frac{E_t}{E_o} = 0.336 \).

\[
(F_n)_1 = \left( \frac{n^2 \times 27000}{(61.7)^2} \right) \times 0.336
\
= 23.52 \text{ ksi} \quad \text{assumed stress } f = 23.50 \text{ ksi } \text{OK}
\]

b. For Torsional Buckling:

\[
(F_n)_2 = \left[ \frac{1}{(A)^{1/2}} \right] \left[ G_o \left( \frac{J + (n^2 E_o C_w)}{K_L L_t} \right)^2 \frac{E_t}{E_o} \right] \quad \text{(Eq. 3.4.2-1)}
\]

\[ G_o = 10500 \text{ ksi} \quad \text{(Table A4 of the Standard)} \]

Similar to the determination of flexural buckling stress, the plasticity reduction factor of \( \frac{E_t}{E_o} \) depends on the assumed stress value. For the first approximation, assume a buckling stress of \( f = 24 \) ksi. The value of \( \frac{E_t}{E_o} \) is found to be equal to 0.29, which is obtained from Table A11 or Figure A8 of the Standard. Thus,

\[
(F_n)_2 = \left[ \frac{1}{(3.148 \times 6.921)} \right] \left[ 10500 \times 0.0191 + n^2 \times 27000 \times 45.49 / (6 \times 12)^2 \right] \times (0.29)
\
= 33.79 \text{ ksi} > 24 \text{ ksi } \text{NG}
\]

After several trials, assume a stress of \( f = 25.43 \) ksi, and \( \frac{E_t}{E_o} = 0.219 \).
\[
(F_n)_2 = \left[1/(3.148 \times 6.921)\right] \left[10500 \times 0.0191 + \pi^2 \times 27000 \times 45.49/(6 \times 12)^2\right] \times 0.219
\]
\[
= 25.46 \approx \text{assumed value } = f = 25.43 \text{ ksi OK}
\]

Then, \( F_n \) should be the smaller of \((F_n)_1\) and \((F_n)_2\).

\( F_n = 23.52 \text{ ksi (based on flexural buckling)} \)

12. Determination of \( A_e \):

Flanges:

\[
d = 0.498 \text{ in.}
\]

\[
I_s = d^2 t/12 = (0.498)^2 (0.135)/12
\]

\[
= 0.001389 \text{ in.}^4
\]

\[
D = 0.82 \text{ in.}
\]

\[
w = 1.855 \text{ in.}
\]

\[
D/w = 0.82/1.855 = 0.442 < 0.80
\]

\[
S = 1.28 \sqrt{E_o/f}, \quad f = F_n \quad \text{ (Eq. 2.4-1)}
\]

The initial modulus of elasticity, \( E_o \), for Type 409 stainless steel is obtained from Table A5 of the Standard, i.e., \( E_o = 27000 \text{ ksi} \).

\[
S = 1.28 \sqrt{27000/23.52} = 43.37
\]

\[
w/t = 1.855/0.135 = 13.74 < S/3 = 14.46 \quad \text{ (Eq. 2.4.2-1)}
\]

\[
I_a = 0 \quad \text{(no edge stiffener needed)} \quad \text{ (Eq. 2.4.2-2)}
\]

\[
b = w \quad \text{ (Eq. 2.4.2-3)}
\]

\[
= 1.855 \text{ in. (flanges fully effective)}
\]

\[
w/t = 13.74 < 90 \quad \text{(Section 2.1.1-(1)-(i))}
\]

Web:

\[
w = 5.355 \text{ in.}, \quad k = 4.00
\]

\[
\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}, \quad f = F_n \quad \text{ (Eq. 2.2.1-4)}
\]

\[
= (1.052/\sqrt{4})(5.355/0.135)\sqrt{23.52/27000}
\]
\[ b = w \quad \text{(Eq. 2.2.1-1)} \]
\[ = 5.355 \text{ in. (web fully effective)} \]
\[ \frac{w}{t} = \frac{5.355}{0.135} = 39.67 < 400 \quad \text{(Section 2.1.1-(1)-(ii))} \]

Lips:
\[ d = 0.498 \text{ in.} \]
\[ k = 0.50 \quad \text{(unstiffened compression element)} \]
\[ d_s = d' \quad \text{(Eq. 2.4.2-4)} \]
\[ \lambda = \frac{1.052}{\sqrt{0.50}}(0.498/0.135)\sqrt{23.52/27000} \]
\[ = 0.162 < 0.673 \]
\[ d'_s = d = 0.498 \text{ in.}, \quad d_s = 0.498 \text{ in.} \]
\[ d/t = 3.69 < 50 \quad \text{(Section 2.1.1-(1)-(iii))} \]

Since flanges, web, and lips are fully effective, the effective area is the same as the full section area, i.e.,
\[ A_{e} = A = 3.148 \text{ in.}^2 \]

14. Determination of \( \Phi_c P_n \): (Section 3.4 of the Standard)
\[ P_n = A F_{e} \quad \text{(Eq. 3.4-1)} \]
\[ = 3.148 \times 23.52 \]
\[ = 74.04 \text{ kips} \]
\[ \Phi_c = 0.85 \]

The design axial strength is
\[ \Phi_c P_n = 0.85 \times 74.04 \]
\[ = 62.93 \text{ kips} \]
EXAMPLE 18.2 I-SECTION W/LIPS (ASD)

Determine the allowable axial load for the I-section used in Example 18.1.

Solution:

1. Basic parameters used for calculating the sectional properties:

See Example 18.1 for calculation of sectional properties of the I-section.

2. Determination of $F_n$

The following results are obtained from Example 18.1.

a. For Flexural Buckling:

$$(F_n)_1 = \left( \frac{n^2 E_o}{K_L x^2 x} \right) \left( \frac{E_t}{E_o} \right)$$

$$= \left( \frac{n^2 \times 7000}{61.7} \right) \left( \frac{0.336}{E_t/E_o} \right)$$

$$= 23.52 \text{ ksi}$$

b. For Torsional Buckling:

$$F_n = \frac{C}{\sqrt{\frac{3.148 \times 6.921 \times [10500 \times 0.0191 + n^2 \times 27000 \times 45.49/(6 \times 12)^2]} \times 0.219}}$$

$$= 25.46 \leq \text{assumed value OK}$$

Then, $F_n$ should be the smaller of $(F_n)_1$ and $(F_n)_2$.

$$F_n = 23.52 \text{ ksi}$$

3. Determination of $A_e$:

The effective area is the same as the full section area, i.e.,

$$A_e = A = 3.148 \text{ in}^2$$

4. Determination of $P_a$:

$$P_n = A_e F_n$$ (Eq. 3.4-1)
\[ \Omega = 2.15 \]

The allowable axial load is

\[ P_a = \frac{P_n}{\Omega} = \frac{74.04}{2.15} \]

\[ = 34.44 \text{ kips} \]
EXAMPLE 19.1 T-SECTION (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design axial strength for the T-section as shown in Figure 19.1. Use Type 304, 1/4-Hard stainless steel.

Figure 19.1 Section for Example 19.1

Given:
1. Section: as shown.
2. \( K_L x \times x = K_L y \times y = 8.0 \text{ ft.} \)

Solution:
The following equations used for computing the sectional properties for T-section are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.
1. Basic parameters used for calculating the sectional properties:

\[ r = R + t/2 = \frac{3}{16} + 0.135/2 = 0.255 \text{ in.} \]

From the sketch, \[ A' = 3.0 \text{ in.}, \quad B' = 2.0 \text{ in.} \]

\[ a = 0.00 \text{ (For T-section)} \]

\[ a = A' - \left[ r + t/2 + a(r + t/2) \right] = 3.0 - (0.255 + 0.135/2) = 2.678 \text{ in.} \]

\[ b = B' -(r + t/2) = 2.0 - (0.255 + 0.135/2) = 1.678 \text{ in.} \]

\[ u = 1.57r = 1.57 \times 0.255 = 0.40 \text{ in.} \]

2. Area:

\[ A = t(2a+2b+2u) = 0.135(2 \times 2.678 + 2 \times 1.678 + 2 \times 0.40) = 1.284 \text{ in}^2 \]

3. Moment of inertia about x-axis:

\[ I_x = 2t \left[ b(b/2+r+t/2)^2 + 0.0833b^3 + u(0.363r+t/2)^2 \right. \right. \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad + 0.149r^3 \right] \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad = 2 \times 0.135 \left[ 1.678(1.678/2 + 0.255 + 0.135/2)^2 + 0.0833(1.678)^3 \right. \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad + 0.4(0.363 \times 0.255 + 0.135/2)^2 + 0.149(0.255)^3 \right] \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad = 0.721 \text{ in}^4 \]

4. Distance between centroid and flange centerline:

\[ \bar{x} = (2t/A) \left[ u(0.363r) + a(a/2+r) \right] \]

\[ = (2 \times 0.135/1.284) \left[ 0.4(0.363 \times 0.255) + 2.678(2.678/2 + 0.255) \right] \]

\[ = 0.905 \text{ in.} \]
5. Moment of inertia about y-axis:

\[ I_y = 2t \left[ 0.358r^3 + a(a/2 + r)^2 + 0.0833a^3 \right] - A(x^2) \]
\[ = 2 \times 0.135 \times 0.358(0.255)^2 + 2.678(2.678/2 + 0.255)^2 + 0.0833(2.678)^3 - 1.284(0.905)^2 \]
\[ = 1.219 \text{ in.}^4 \]

6. Distance between shear center and flange centerline:

\[ m = \bar{a} \left\{ 1 - \frac{(\bar{b})^2}{(\bar{b})^2 + (\bar{c})^2} \right\} \]
\[ = 2.933 \left\{ 1 - \frac{(1.678)^2}{(1.678)^2 + 0} \right\} = 0 \]

7. Distance between centroid and shear center:

\[ x_0 = -(\bar{x} - m) = -0.905 \text{ in.} \]

8. St. Venant torsion constant:

\[ J = \frac{2xt^3}{3}[a+b+c] \]
\[ = \left[ 2x(0.135)^3/3 \right] [2.678 + 1.678 + 0.4] \]
\[ = 0.0078 \text{ in.}^4 \]

9. Warping Constant:

\[ C_w = 0 \]

10. Radii of gyration:

\[ r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{0.721}{1.284}} = 0.749 \text{ in.} \]
\[ r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{1.219}{1.284}} = 0.974 \text{ in.} \]
\[ \frac{(K_x L_x)}{r_x} = \frac{(8 \times 12)}{0.749} = 128.17 < 200 \text{ (control)} \]
\[ \frac{(K_y L_y)}{r_y} = \frac{(8 \times 12)}{0.974} = 98.56 < 200 \]
\[ r_o^2 = r_x^2 + r_y^2 + x_0^2 \]
\[\begin{align*}
&= (0.749)^2 + (0.974)^2 + (0.905)^2 \\
&= 2.329 \text{ in.}^2
\end{align*}\]

11. Torsional-flexural constant:

\[\beta = 1 - \left(\frac{x_o}{r_o}\right)^2\]

\[= 1 - (0.905)^2 / 2.329\]

\[= 0.648\]

12. Determination of \( F_n \): (Section 3.4 of the Standard)

For this singly symmetric section (x-axis is the axis of symmetric), \( F_n \) shall be taken as the smaller of either (Eq. 3.4.1-1) or (Eq. 3.4.3-1):

a. For Flexural Buckling:

\[\left( F_n \right)_1 = \left( \frac{n^2 E_t}{(K L_x / r_x)^2} \right)\]

In the determination of the flexural buckling stress, it is necessary to select a proper value of \( E_t \) from Table A13 or Figure A11 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of \( f=20 \) ksi.

From Table A13, the corresponding value of \( E_t \) is found to be equal to 27000 ksi. Thus,

\[\left( F_n \right)_1 = \left( \frac{n^2 \times 27000}{(128.17)^2} \right)\]

\[= 16.2 \text{ ksi} < \text{assumed stress } f=20 \text{ ksi}\]

Because the computed stress is less than the assumed value, flexural buckling is in the elastic region and therefore, no further approximation is needed. Thus,

\[E_t = 27000 \text{ ksi}\]

\[\left( F_n \right)_1 = 16.2 \text{ ksi}\]
b. For Torsional-Flexural Buckling:

\[
(F_n)_2 = \left(\frac{1}{2\beta}\right) \left(\sigma_{ex} + \sigma_t - \sqrt{(\sigma_{ex} + \sigma_t)^2 - 4\beta\sigma_{ex}\sigma_t}\right) \tag{Eq. 3.4.3-1}
\]

where

\[
\sigma_{ex} = \left[\left(\pi^2E_o\right)/(K_xL_x/r_x)^2\right](E_t/E_o) \tag{Eq. 3.4.3-3}
\]

\[
\sigma_t = \left[\frac{1}{(Ar_t^2)}\right]G_o + \left(\pi^2G_oC_w\right)/(K_tL_t)^2](E_t/E_o) \tag{Eq. 3.4.2-1}
\]

\[
G_o = 10500 \text{ ksi (Table A4 of the Standard)}
\]

Similar to the determination of flexural buckling stress, the plasticity reduction factor of \(E_t/E_o\) used for determining the torsional-fleural buckling stress depends on the assumed stress value. For the first approximation, assume a buckling stress of \(f=20\) ksi. The value of \(E_t/E_o\) is found to be equal to 1.0, which is obtained from Table A10 or Figure A7 of the Standard. Thus,

\[
\sigma_{ex} = \left[\left(\pi^2E_o\right)/(128.17)^2\right]x(1.0)
\]

\[
= 16.2 \text{ ksi}
\]

\[
\sigma_t = \left[\frac{1}{(1.284x2.329)}\right][10500x0.0078 + 0] \times (1.0)
\]

\[
= 27.4 \text{ ksi}
\]

Therefore,

\[
(F_n)_2 = \left(\frac{1}{2\beta}\right) \left(\sigma_{ex} + \sigma_t - \sqrt{(\sigma_{ex} + \sigma_t)^2 - 4\beta\sigma_{ex}\sigma_t}\right)
\]

\[
= \left[\frac{1}{(2x0.648)}\right] \left[(16.2+27.4)
\right.
\]

\[
- \sqrt{(16.2+27.4)^2 - 4x0.648x16.2x27.4}
\]

\[
= 12.5 \text{ ksi < assumed stress = 20 ksi}
\]

Because the computed stress \((F_n)_2\) is less than the assumed value of \(f=20\) ksi, the second approximation will be assumed that a stress of \(f=12.5\) ksi and \(E_t/E_o = 1.0\). Thus,
\[(F_n)_2 = 12.5 \text{ ksi} \quad \text{OK}\]

\[F_n \text{ should be the smaller of } (F_n)_1 \text{ and } (F_n)_2. \quad \text{Thus,} \]
\[F_n = 12.5 \text{ ksi}\]

13. Determination of \(A_e\):  
Flanges: \((k = 0.5)\)
\[w = 1.678 \text{ in.}\]
\[w/t = 1.678/0.135 = 12.43\]
\[
\lambda = (1.052/\sqrt{k})(w/t)\sqrt{\frac{f}{E_o}}, \quad f = F_n
\]
\[= (1.052/\sqrt{0.5})(12.43)\sqrt{\frac{12.5}{27000}}
\[= 0.398 < 0.673\]
\[b = w\]

Stem: \((k = 0.5)\)
\[w = 2.678 \text{ in.}\]
\[w/t = 2.678/0.135 = 19.84\]
\[
\lambda = (1.052/\sqrt{k})(w/t)\sqrt{\frac{f}{E_o}}, \quad f = F_n
\]
\[= (1.052/\sqrt{0.5})(19.84)\sqrt{\frac{12.5}{27000}}
\[= 0.635 < 0.673\]
\[b = w\]
\[= 2.678 \text{ in.}\]

Since flanges and stem are fully effective, the effective area is the same as the full section area, i.e.,
\[A_e = A = 1.284 \text{ in.}^2\]

14. Determination of \(\Phi F_n\): (Section 3.4 of the Standard)
\[P_n = A F_n \quad (\text{Eq. 3.4-1})\]
\[
\phi_c = 0.85
\]

The design axial strength is

\[
\phi_c P_n = 0.85 \times 16.05
\]

\[
= 13.64 \text{ kips}
\]
EXAMPLE 19.2  T-SECTION (ASD)

Determine the allowable axial load for the T-section used in Example 19.1.

Solution:

1. Basic parameters used for calculating the sectional properties:

   See Example 19.1 for calculation of sectional properties of the T-section.

2. Determination of $F_n$

   The following results are obtained from Example 19.1.

   a) For Flexural Buckling:

   $$(F_n)_1 = \frac{(n^4 E_t)}{(K_x L_x/r_x)^2}$$
   $$(F_n)_1 = \frac{(n^4 \times 26000)}{(128.17)^2}$$
   = 16.2 ksi

   b) For Torsional-Flexural Buckling:

   $$(F_n)_2 = \frac{(1/2\beta)[(\sigma_{ex} + \sigma_t) - \sqrt{(\sigma_{ex} + \sigma_t)^2 - 4\beta \sigma_{ex} \sigma_t}]}{\sqrt{1 + (\sigma_{ex} + \sigma_t)^2}}$$

   where

   $$\sigma_{ex} = \left[\frac{(n^4 E_o)}{(K_x L_x/r_x)^2}\right] \left(\frac{E_t}{E_o}\right)$$

   $$\sigma_t = \left[\frac{1}{(A_r^2)}\right] \left[\frac{G_o J^2 + (n^4 E_o C_w)}{(K_t L_t)^2}\right] \left(\frac{E_t}{E_o}\right)$$

   $G_o = 10500$ ksi (Table A4 of the Standard)

   $$(F_n)_2 = \left[\frac{1}{(2 \times 0.648)}\right] \left[(16.2 + 27.4) - \sqrt{(16.2 + 27.4)^2 - 4 \times 0.648 \times 16.2 \times 27.4}\right] \times 1.0$$

   = 12.5 ksi (control)

   Then, $F_n$ should be the smaller of $(F_n)_1$ and $(F_n)_2$.

   $F_n = 12.5$ ksi

3. Determination of $A_e$: 246
The effective area is the same as the full section area, i.e.,

\[ A_e = A = 1.284 \text{ in.}^2 \text{ (see Example 19.1)} \]

4. Determination of \( P_a \):

\[
P_n = A_e F_n
\]

\[
= 1.284 \times 12.5
\]

\[
= 16.05 \text{ kips}
\]

\[
\Omega = 2.15
\]

The allowable axial load is

\[
P_a = P_n / \Omega = 16.05 / 2.15
\]

\[
= 7.47 \text{ kips}
\]
EXAMPLE 20.1 TUBULAR SECTION - SQUARE (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design axial strength for section shown in Figure 20.1. Use Type 301 stainless steel, 1/4-Hard.

Figure 20.1 Section for Example 20.1

Given:
1. Section: 4.0" x 4.0" x 0.065" Square Tube.
2. $K_{L_x} = K_{L_y} = 10$ ft.

Solution:
1. Properties of 90° Corners:
   \[ r = R + t/2 = 1/16 + 0.065/2 = 0.095 \text{ in.} \]
   Length of arc, \[ u = 1.57r = 1.57 \times 0.095 = 0.149 \text{ in.} \]
   Distance of c.g. from center of radius, \[ c = 0.637r = 0.637 \times 0.095 = 0.061 \text{ in.} \]
\[ I_x = I_y = I \] (doubly symmetric section)

<table>
<thead>
<tr>
<th>Element</th>
<th>( L ) (in.)</th>
<th>( \text{Distance to Center of Section} ) (in.)</th>
<th>( L_y^2 ) (in.(^3))</th>
<th>( I' ) About Own Axis (in.(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flanges</td>
<td>2 x 3.744 = 7.488</td>
<td>2 - 0.065/2 = 1.968</td>
<td>29.001</td>
<td>--</td>
</tr>
<tr>
<td>Corners</td>
<td>4 x 0.149 = 0.596</td>
<td>3.744/2 + 0.061 = 1.933</td>
<td>2.227</td>
<td>--</td>
</tr>
<tr>
<td>Web</td>
<td>2 x 3.744 = 7.488</td>
<td>--</td>
<td>--</td>
<td>8.747</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td><strong>15.572</strong></td>
<td></td>
<td><strong>31.228</strong></td>
<td><strong>8.747</strong></td>
</tr>
</tbody>
</table>

\[ \frac{w}{t} = \frac{3.744}{0.065} = 57.60 < 400 \text{ (Section 2.1.1-(1)-(ii))} \]

\[ A = Lt = 15.572 \times 0.065 = 1.012 \text{ in.}^2 \]

\[ I' = L_y^2 + I'_1 = 31.228 + 8.747 = 39.975 \text{ in.}^3 \]

\[ I = I't = 39.975 \times 0.065 = 2.598 \text{ in.}^4 \]

\[ r = \sqrt{I/A} = \sqrt{2.598/1.012} = 1.602 \text{ in.} \]

\[ KL/r = 10 \times 12/1.602 = 74.91 < 200 \text{ (Section 3.4-(5))} \]

2. Since the square tube is a doubly symmetric closed section, provisions of Section 3.4.1 of the Standard apply, i.e., section is not subjected to torsional-flexural buckling.

\[ F_n = \frac{nE_t}{(KL/r)^2} \] (Eq. 3.4.1-1)

In the determination of the flexural buckling stress, it is necessary to select a proper value of \( E_t \) from Table A13 or Figure A11 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of \( f = 24.0 \text{ ksi.} \)
From Table A13, the corresponding value of $E_t$ is found to be equal to 17000 ksi. Thus,
\[
F_n = \frac{n^2 x 17000}{(74.91)^2} \text{ ksi}
\]
\[
= 29.90 \text{ ksi} > \text{ assumed stress } f=24.0 \text{ ksi}
\]
Because the computed stress is larger than the assumed value, further successive approximations are needed. For the second approximation, assume $f=26.33$ ksi, and
\[
E_t = 14960 \text{ ksi}
\]
\[
F_n = \frac{n^2 x 14960}{(74.91)^2} \text{ ksi}
\]
\[
= 26.31 \text{ ksi} \approx \text{ assumed stress OK}
\]
3. Determination of the Effective Width:

\[
k = 4.0
\]
\[
\lambda = \frac{1.052/\sqrt{k}(w/t)\sqrt{f/E_o}}{f = F_n} \quad \text{(Eq. 2.2.1-4)}
\]
\[
= (1.052/\sqrt{4})(3.744/0.065)\sqrt{26.31/27000} = 0.946 > 0.673
\]
(Section not fully effective)
\[
\rho = \frac{1-0.22/\lambda}{\lambda} \quad \text{(Eq. 2.2.1-3)}
\]
\[
= (1-0.22/0.946)/0.946 = 0.811
\]
\[
b = \rho w \quad \text{(Eq. 2.2.1-2)}
\]
\[
= 0.811 x 3.744 = 3.036 \text{ in.}
\]
\[
A_e = A - 4(w-b)t
\]
\[
= 1.012 - 1.4(3.744-3.036) x 0.065
\]
\[
= 0.828 \text{ in.}^2
\]
4. Determination of the Design Axial Strength:
\[
P_n = A_e F_n \quad \text{(Eq. 3.4-1)}
\]
\[
= 0.828 x 26.31
\]
\[
= 21.80 \text{ kips}
\]
\[ \phi_c = 0.85 \]

\[ \phi_c P_n = 0.85 \times 21.80 = 18.53 \text{ kips} \]
EXAMPLE 20.2 TUBULAR SECTION - SQUARE (ASD)

Determine the allowable axial load for tubular section used in Example 20.1.

Solution:

1. Basic parameters used for calculating the section properties:
   
   See Example 20.1 for section properties of tubular section.

2. Determination of $F_n$

   The following results are obtained from Example 20.1.

   For Flexural Buckling Only:
   
   $F_n = \frac{E_t}{K} \left(\frac{L}{r_y}\right)^2$
   
   $F_n = \frac{(\pi^2) x 14960}{(74.91)^2}$
   
   $F_n = 26.31 \text{ ksi}$

3. Determination of $A_e$:

   The effective area is obtained from Example 20.1 as follows:

   $A_e = A = 0.828 \text{ in.}^2$

4. Determination of $P_a$:

   $P_n = A_e F_n$
   
   $P_n = 0.828 \times 26.31$
   
   $P_n = 21.80 \text{ kips}$

   $\Omega = 2.15$

   $P_a = P_n / \Omega = 21.80 / 2.15$
   
   $P_a = 10.14 \text{ kips}$
EXAMPLE 21.1 TUBULAR SECTION - ROUND (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design axial strength for the tubular section shown in Figure 21.1. Use Type 316 stainless steel, 1/4-Hard.

![Section for Example 21.1](image)

**Figure 21.1 Section for Example 21.1**

Given:
1. Section: Shown in sketch above.
2. Height: \( L = 10'-0'' \), simply supported at each end.

Solution:
1. Full section properties:
   \[
   I = \frac{1}{4} \pi \left[ (O.R.)^4 - (I.R.)^4 \right]
   \]
   \[
   = \frac{1}{4} \pi \left[ (8)^4 - (3.875)^4 \right]
   \]
   \[
   = 23.98 \text{ in.}^4
   \]
   \[
   A = \frac{1}{4} \pi \left[ (O.D.)^2 - (I.D.)^2 \right]
   \]
   \[
   = \frac{1}{4} \pi \left[ (8)^2 - (7.75)^2 \right]
   \]
   \[
   = 3.093 \text{ in.}^2
   \]
   \[
   r = \sqrt{I/A}
   \]
   \[
   = \sqrt{23.98/3.093}
   \]
2. Determination of Design Axial Strength:

Ratio of outside diameter to wall thickness,

\[ \frac{D}{t} = \frac{8.000}{0.125} = 64.00 \]

\[ \frac{D}{t} < 0.881 \frac{E_o}{F_y} = 0.881(27000/50) = 475.7 \text{ OK} \]

The design axial strength, \( \Phi_c F_n \), for cylindrical tubular member is determined in accordance with Section 3.6.2 of the Standard as follows:

\[ \Phi_c = 0.80 \]

\[ F_n = \frac{F_e}{A_n} \quad (\text{Eq. 3.6.2-1}) \]

\[ F_n = \frac{\pi E_c}{(KL/r)^2} \quad (\text{Eq. 3.4.1-1}) \]

where \( F_n \) is the flexural buckling stress determined according to Section 3.4.1 of the Standard.

\[ A_e = \frac{1-(1-(E_c/E_o)^2)(1-A_o/A)}{A} \quad (\text{Eq. 3.6.2-2}) \]

\[ A_o = K_c A \quad (\text{Eq. 3.6.2-3}) \]

\[ K_c = (1-C)(E_o/F_y)/\left(\left(8.93-A_c\right)(D/t)\right) + 5.882C/(8.93-A_c) \quad (\text{Eq. 3.6.1-3}) \]

\[ C = \frac{F_{pr}/F_y}{\lambda_c} \]

\[ \lambda_c = 3.048C \]

From Table A17 of the Standard, the ratio of \( F_{pr}/F_y \) is equal to 0.5 in longitudinal compression for Type 301, 1/4-Hard stainless steel.

Therefore,

\[ K_c = (1-0.5)(27000/50)/\left(\left(8.93-3.048x0.5\right)(64.0)\right) \]

\[ + (5.882x0.5)/\left(8.93-3.048x0.5\right) \]

\[ = 0.967 \]

\[ A_o = 0.967 A_e \]
In the determination of the flexural buckling stress, it is necessary to select a proper value of $E_t$ from Table A13 or Figure A11 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of $f=40.0$ ksi. From Table A13, the corresponding value of $E_t$ is found to be equal to 8370 ksi. Thus,

$$F_n = \frac{n^2 \times 8370}{(10 \times 12/2.784)^2}$$

$$= \frac{n^2 \times 8370}{(43.10)^2}$$

$$= 44.46 \text{ ksi} > \text{assumed stress } f=40.0 \text{ ksi}$$

Because the computed stress is larger than the assumed value, further successive approximations are needed.

Assume $f=41.83$ ksi, and

$$E_t = 7876 \text{ ksi}$$

$$F_n = \frac{n^2 \times 7876}{(43.10)^2}$$

$$= 41.84 \text{ ksi} = \text{assumed stress } f=41.83 \text{ ksi} \; \text{OK}$$

For the compressive stress of $F_n = 41.83$ ksi, the corresponding value of $E_t/E_o$ is equal to 0.292, which is obtained from Table A10 of Figure A7 of the Standard. Therefore,

$$A_e = [1-(1-(E_t/E_o)^2)(1-A_o/A)]A$$

$$= [1-(1-(0.292)^2)(1-0.967)]A$$

$$= 0.97xA = 3.00 \text{ in.}^2$$

$$P_n = F_n A_e$$

$$= (41.83)(3.00)$$

$$= 125.50 \text{ kips}$$

$$\phi_c = 0.80$$

$$\phi_c P_n = 0.80 \times 125.50$$

$$= 100.40 \text{ kips}$$

255
EXAMPLE 21.2 TUBULAR SECTION - ROUND (ASD)

Determine the allowable axial load for tubular section used in Example 21.1.

Solution:

1. Basic parameters used for calculating the section properties:

   See Example 21.1 for section properties of tubular section.

2. Determination of $F_n$

   The following results are obtained from Example 21.1.

   \[
   F_n = \frac{(n^2 E_t)}{(K_L \eta y/r_y)^2} \\
   F_n = \frac{(n^2 \times 7876)}{(43.10)^2} \\
   = 41.84 \text{ ksi}
   \]  

3. Determination of $A_e$:

   The effective area is obtained from Example 21.1 as follows:

   \[A_e = 3.00 \text{ in.}^2\]

4. Determination of $P_a$:

   \[
   P_n = A_e F_n \\
   = 3.00 \times 41.83 \\
   = 125.50 \text{ kips} \\
   \Omega = 2.15 \\
   P_a = \frac{P_n}{\Omega} = \frac{125.50}{2.15} \\
   = 58.37 \text{ kips}
   \]
EXAMPLE 22.1 C-SECTION (LRFD)

By using the Load and Resistance Factor Design (LRFD) criteria, check the adequacy of a channel section (Fig. 22.1) to be used as a compression member which is subjected to eccentrically axial loads of \( P_{DL} = 0.35 \) kips and \( P_{LL} = 1.75 \) kips. Consider the following two loading cases: (A) axial loads are applied 2 in. to the left of the c.g. of the full section at both ends, (B) axial loads are applied 2 in. to the left and 4 in. above the c.g. of the full section at both ends. Assume that the effective length factors \( K_x = K_y = K_t = 1.0 \), and that the unbraced lengths \( L_x = L_y = L_t = 16 \) ft. Use Type 304, 1/4-Hard, stainless steel. Assume dead to live load ratio \( D/L = 1/5 \) and \( 1.2D + 1.6L \) governs the design.

![Figure 22.1 Section for Example 22.1](image)

Solution: Part (A)
The following equations used for computing the sectional properties for channel with lips are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.

1. Full section properties:

\[ r = R + \frac{t}{2} = \frac{3}{16} + 0.105/2 = 0.240 \text{ in.} \]
\[ a = A' - (2r + t) = 8.000 - (2 \times 0.240 + 0.105) = 7.415 \text{ in.} \]
\[ \bar{a} = A' - t = 8.000 - 0.105 = 7.895 \text{ in.} \]
\[ b = B' - (2r + t) = 3.000 - (2 \times 0.240 + 0.105) = 2.415 \text{ in.} \]
\[ \bar{b} = B' - t = 3.000 - 0.105 = 2.895 \text{ in.} \]
\[ c = C' - (r + t/2) = 0.800 - (0.240 + 0.105/2) = 0.508 \text{ in.} \]
\[ \bar{c} = C' - t/2 = 0.800 - (0.105/2) = 0.748 \text{ in.} \]
\[ u = 1.57r = 1.57 \times 0.240 = 0.377 \text{ in.} \]

Distance of corner's c.g. from center of radius = 0.637r
\[ = 0.637(0.240) = 0.153 \text{ in.} \]
\[ A = t(a + 2b + 2c + 4u) = 0.105[7.415 + 2 \times 2.415 + 2 \times 0.508 + 4 \times 0.377] \]
\[ = 1.551 \text{ in.}^2 \]
\[ I_x = 2t[0.0417a^3 + b(a/2 + r)^2 + 2u(a/2 + 0.637r)^2 + 0.298r^3 \]
\[ + 0.0833c^3 + (c/4)(a-c)^2] \]
\[ = 2 \times 0.105[0.0417(7.415)^3 + 2.415(7.415/2 + 0.240)^2 \]
\[ + 2 \times 0.377(7.415/2 + 0.637 \times 0.240)^2 + 0.298(0.240)^3 \]
\[ + 0.0833(0.508)^3 + (0.508/4)(7.415 - 0.508)^2] \]
\[
\[ = 15.108 \text{ in.}^4 \]
\[ \bar{x} = \frac{(2t/A)[b(b/2 + r) + u(0.363r) + u(b + 1.637r) + c(b + 2r)]}{(2 \times 0.105/1.551) \times 2.415(2.415/2 + 0.240) + 0.377(0.363 \times 0.240) \times 0.377(2.415 + 1.637 \times 0.240) + 0.508(2.415 + 2 \times 0.240)} \]


\[ I_y = 2t(b(b/2+r)^2 + 0.0833b^3 + 0.505r^2 + c(b+2r)^2 + u(b+1.637r)^2 - A(\bar{x})^2 \]

\[ = 2x0.105[(2.415(2.415/2+0.240)^2 + 0.0833(2.415)^3 + 0.505(0.240)^3 + 0.377(2.415+1.637x0.240)^2] - 1.551(0.820)^2 \]

\[ = 1.786 \text{ in.}^4 \]

\[ m = (bt/12I_x)\{6\bar{c}(\bar{a})^2 + 3\bar{b}(\bar{a})^2 - 8(\bar{c})^3 \} \]

\[ = [((2.895x0.105)/(12x15.108x0.748)](6x0.748(7.895)^2 + 3x2.895(7.895)^2 - 8(0.748)^3) \]

\[ = 1.371 \text{ in.} \]

\[ x_0 = -(\bar{x}+m) = -(0.820+1.371) \]

\[ = -2.191 \text{ in.} \]

\[ J = (t^2/3) a+2b+2c+4u \]

\[ = [(0.105)^2/3](7.415+2x2.415+2x0.508+4x0.377) \]

\[ = 0.005699 \text{ in.}^4 \]

\[ C_w = \left( t^2/A \right) \left[ (\bar{x}(\bar{a})^2)/t \right] \left[ (\bar{b})^2/3+m^2-m\bar{b} \right] + (A/3t) \left[ (m)^2(\bar{a})^3 \right] + (b)(\bar{c}^2(2\bar{c}+3\bar{a}) - (I_x n^2/t)(2\bar{a}+4\bar{c}) + [m(\bar{c})^3/3] [8(\bar{b})^2(\bar{c}) \right.

\[ + 2m(2\bar{c}(\bar{c}-\bar{b}) + \bar{b}(2\bar{c}-3\bar{a})) + [(\bar{b})(\bar{a})^2/6] [3\bar{c}+\bar{b})(4\bar{c}+a)-6(\bar{c})^2] \]

\[ - m^2(\bar{a})^4/4 \right] \]

\[ = [(0.105)^2/1.551] \left[ [0.820x1.551x(7.895)^3/0.105]((2.895)^2/3 \right.

\[ + (1.371)^2-1.371x2.895) + 1.551/(3x0.105)\left[(1.371)^2(7.895)^3 \right.

\[ + (2.895)^2(0.748)^2(2x0.748+3x7.895)] \]

\[ - [15.108x(1.371)^2/0.105](2x7.895+4x0.748) \]

\[ + [1.371(0.748)^2/3][8(2.895)^2(0.748) \]

\[ + 2x1.371(2x0.748(0.748-7.895)+2.895(2x0.748-3x7.895))] \]

\[ + ((2.895)^2(7.895)^2/6)(3x0.748+2.895)(4x0.748+7.895) \]

259
\[ \beta_w = -\{0.0833 [t(x-a)^3] + t(x-a)^3 \} \]
\[ = -\{0.0833 [0.105x0.820(7.895)^3] + 0.105(0.820)^3 \times 7.895 \} \]
\[ = -3.987 \]

\[ \beta_f = (t/2) [(b-x)^4 - (6-x)^4] + [t(a/2)^3] \]
\[ = (0.105/2) [(2.895-0.820)^4 - (0.820)^4] \]
\[ + [0.105(7.895)^3] [(2.895-0.820)^2 - (0.820)^2] \]
\[ = 6.894 \]

\[ \beta_i = 2\pi \delta (b-x)^3 + (2/3)\pi \delta (b-x) \left[ (a/2) - (a/2-\delta)^3 \right] \]
\[ = 2\pi \delta x_0 0.748 \times 0.105(2.895-0.820)^3 + (2/3) \times 0.105(2.895-0.820) \left( \frac{0.895}{2} - \frac{2.191}{2} \right)^3 \]
\[ = 5.581 \]

\[ j = \frac{1}{2} \pi \rho \left( \beta_w + \beta_f + \beta_i \right) - x_0 \]
\[ = \left[ \frac{1}{2} (2x1.786) \right] (-3.987+6.894+5.581) - (-2.191) \]
\[ = 4.567 \]

\[ r_x = \sqrt{I_x / A} = \sqrt{15.108/1.551} = 3.121 \text{ in.} \]
\[ K_{x}L_{x} / r_x = 1(16x12) / 3.121 = 61.52 \]

\[ r_y = \sqrt{I_y / A} = \sqrt{1.786/1.551} = 1.073 \text{ in.} \]
\[ K_{y}L_{y} / r_y = 1(16x12) / 1.073 = 178.94 < 200 \text{ (Section 3.4-(5))} \]

\[ r_o = \sqrt{r_x^2 + r_y^2 + r_o^2} \]
\[ = \sqrt{(3.121)^2 + (1.073)^2 + (-2.191)^2} = 3.961 \text{ in.} \]

\[ \beta = 1 - (x_0 / r_o)^2 \]
\[ = 1 - (-2.191/3.961)^2 = 0.694 \]

2. Determination of $P_n$ (Section 3.4):

Since the channel is singly symmetric, $F_n$ shall be taken as
the smaller of $F_n$ calculated according to Section 3.4.1 or $F_n$ calculated according to Section 3.4.2.

a. For Flexural Buckling:

$$F_n = \frac{\pi^2 E_t}{(K_L/r)^2}$$  \hspace{1cm} (Eq. 3.4.1-1)

In the determination of the flexural buckling stress, it is necessary to select a proper value of $E_t$ from Table A13 or Figure A11 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of $f=20$ ksi. From Table A13, the corresponding value of $E_t$ is found to be equal to 27000 ksi. Thus,

$$F_n = \frac{\pi^2 \times 27000}{(178.94)^2}$$

= 8.322 ksi < assumed stress $f=20$ ksi

Because the computed stress is less than the assumed value, no further approximation is needed. The section is subject to the elastic flexural buckling.

Therefore, $(F_n)_1 = 8.322$ ksi

b. For Torsional-Flexural Buckling:

$$F_n = \frac{1}{2} \left[ \frac{\pi^2 E_t}{(K_L/r)^2} \right] \left( \frac{G_o}{(n^2 E_o C_w) / (K_L/r)^2} \right) \left( \frac{E_t}{E_o} \right)$$  \hspace{1cm} (Eq. 3.4.2-1)

where

$$\sigma_{ex} = \frac{\pi^2 E_t}{(K_L/r)^2} \left( \frac{E_t}{E_o} \right)$$  \hspace{1cm} (Eq. 3.4.3-3)

$$\sigma_t = \left[ \frac{1}{2} \right] \left[ \frac{G_o}{(n^2 E_o C_w) / (K_L/r)^2} \right] \left( \frac{E_t}{E_o} \right)$$  \hspace{1cm} (Eq. 3.4.2-1)

$G_o = 10500$ ksi (Table A4 of the Standard)

Similar to the determination of flexural buckling stress, the plasticity reduction factor of $E_t/E_o$ depends on the assumed stress value. For the first approximation, assume a buckling stress of $f=20$ ksi. The value of $E_t/E_o$ is found to be equal to 1.0, which is obtained from Table A10 or Figure A7.
of the Standard. Thus,

\[ \sigma_{ex} = \left( \frac{n^2 \times 27000}{(16 \times 12 / 3.121)^2} \right) \times (1.0) \]
\[ = 70.41 \text{ ksi} \]

\[ \sigma_t = \frac{1}{\left(1.551 \times 15.69\right)} \left[ \frac{10500 \times 0.005699 + n^2 \times 27000 \times 23.468}{(16 \times 12)^2} \right] \times (1.0) \]
\[ = 9.43 \text{ ksi} \]

Therefore,

\[ F_{n2} = \frac{1}{2} \left( (\sigma_{ex} + \sigma_t) - \sqrt{(\sigma_{ex} + \sigma_t)^2 - 4 \sigma_{ex} \sigma_t} \right) \]
\[ = \frac{1}{2 \times 0.694} \left[ \frac{70.41 + 9.43}{2} - \sqrt{(70.41 + 9.43)^2 - 4 \times 0.694 \times 70.41 \times 9.43} \right] \]
\[ = 9.024 \text{ ksi} < \text{ assumed value } f=20 \text{ ksi OK} \]

(Eq. 3.4.3-1)

This section is subject to elastic torsional-flexural buckling, and \( F_{n2} = 9.024 \text{ ksi} \)

Then, \( F_n \) should be the smaller of \( F_{n1} \) and \( F_{n2} \).

\[ F_n = 8.322 \text{ ksi} \]

For element 1:

\[ w = 7.415 \text{ in.} \]

\[ \frac{w}{t} = 7.415 / 0.105 = 70.62 < 400 \text{ OK (Section 21.1-(1)-(ii))} \]

\[ k = 4.0 \text{ (Since connected to two stiffened elements)} \]

\[ \lambda = \frac{1.052 / \sqrt{k} \times \sqrt{f/E_o}} {w/t} \]
\[ = \frac{1.052 / \sqrt{4.00} \times (70.62) \times \sqrt{8.322 / 27000}} {0.652 < 0.673} \]
\[ = 0.652 < 0.673 \]

\[ b = w \]
\[ = 7.415 \text{ in. (Element 1 fully effective)} \]

For element 2:
\[ w = 2.415 \text{ in.} \]
\[ w/t = 2.415/0.105 = 23.00 \]
\[ S = 1.28\sqrt{\frac{E}{f}}, \quad f = F_n \]  
\[ = 1.28\sqrt{27000/8.322} = 72.91 \]  
\[ S/3 = 24.30 \]
\[ w/t = 23.00 < S/3 = 24.30 \]
\[ b = w \]  
\[ = 2.415 \text{ in. (Element 2 fully effective)} \]

For element 3:
\[ d = 0.508 \text{ in.} \]
\[ d/t = 0.508/0.105 = 4.84 \]
\[ k = 0.50 \text{ (unstiffened compression element)} \]
\[ \lambda = (1.052/\sqrt{0.50})(4.84)/8.322/27000 \]
\[ = 0.126 < 0.673 \]
\[ d' = d = 0.508 \text{ in.} \]
\[ d_s = d' \]  
\[ = 0.508 \text{ in. (Element 3 fully effective)} \]

Thus the whole section is fully effective.
\[ A_e = A = 1.551 \text{ in.}^2 \]
\[ P_n = A \frac{F_n}{e} \]  
\[ = 1.551 \times 8.322 \]
\[ = 12.91 \text{ kips} \]
\[ \Phi_c = 0.85 \]
\[ \Phi_c P_n = 0.85 \times 12.91 \]
\[ = 10.97 \text{ kips} \]
3. \( P_u = 1.2 \times 0.35 + 1.6 \times 1.75 = 3.22 \text{ kips} \)

\[
P_u/\phi_c P_n = 3.22/10.97 = 0.294 > 0.15
\]

Must check both interaction equations (Eq. 3.5-1) and (Eq. 3.5-2).

4. Determination of \( \phi_c P_n \) (Section 3.4 for \( F_n = F_y \)):

For element 1:
\[
\lambda = (1.052/\sqrt{4.00})(70.62/\sqrt{50/27000} = 1.599 > 0.673
\]
\[
\rho = (1-0.22/\lambda)/\lambda \tag{Eq. 2.2.1-3}
\]
\[
= (1-0.22/1.599)/1.599 = 0.539 \tag{Eq. 2.2.1-2}
\]
\[
b = \rho w
\]
\[
= 0.539 \times 7.415 = 4.000 \text{ in.}
\]

For element 2:
\[
S = 1.28\sqrt{27000/50.0} = 29.74
\]
\[
S/3 = 9.91
\]
\[
S/3 = 9.91 < w/t = 23.00 < S = 29.74
\]
\[
I_a = 399t^4 \left[ ((w/t)/S)-0.33 \right]^3 \tag{Eq. 2.4.2-6}
\]
\[
= 399(0.105)^4 [(23/29.74)-0.33]^3
\]
\[
= 0.004227 \text{ in.}^4
\]
\[
I_s = d^3t/12 = (0.508)^3(0.105)/12
\]
\[
= 0.001147 \text{ in.}^4
\]
\[
I_s/I_a = 0.001147/0.004227 = 0.271
\]
\[
D/w = 0.8/2.415 = 0.331
\]
\[
n = 1/2
\]
\[
k = [4.82-5(D/w)](I_s/I_a)^n+0.43 < 5.25-5(D/w) \tag{Eq. 2.4.2-9}
\]
\[
(4.82-5(0.331))(0.271)^{1/2}+0.43 = 2.078
\]
\[
5.25-5(0.331) = 3.595 > 2.078
\]

264
\[ k = 2.078 \]
\[ \lambda = \frac{1.052}{\sqrt{2.078}}(23.00)\sqrt{50/27000} = 0.722 > 0.673 \]
\[ \rho = \frac{1-0.22/\lambda}{\lambda} \]
\[ = \frac{1-0.22/0.722}{0.722} = 0.963 \]
\[ b = \rho w \]
\[ = 0.963 \times 2.415 = 2.326 \text{ in.} \]

For element 3:
\[ \lambda = \frac{1.052}{\sqrt{0.50}}(4.84)\sqrt{50/27000} = 0.310 < 0.673 \]
\[ d'_s = d = 0.508 \text{ in.} \]
\[ d_s = d' (I_s/I_a) \leq d'_s \]
\[ \text{Since } I_s/I_a = 0.271 < 1.0 \]
\[ d_s = 0.508(0.271) = 0.138 \text{ in.} \]
\[ A_e = 1.551-0.105(7.415-4.000)-0.105(0.508-0.138)x2 \]
\[ -0.105(2.415-2.326)x2 \]
\[ = 1.096 \text{ in.}^2 \]
\[ P_{no} = 1.096 \times 50 = 54.80 \text{ kips} \]
\[ \phi_c = 0.85 \]
\[ \phi_c P_{no} = 0.85 \times 54.80 \]
\[ = 46.58 \text{ kips} \]

5. Determination of \( M_{uy} \) (required flexural strength about y-axis): (\( M_{ux} = 0 \) since \( e_y = 0 \))

\( M_{uy} \) will be with respect to the centroidal axes of the effective section determined for the required axial strength alone.

\[ A_e = 1.551 \text{ in.}^2 \] under required axial strength alone

Since \( A_e = A \), the centroidal axes for the effective section are
the same as those for the full section. Therefore, $e_x$ did not change.

$$M_{uy} = 3.22(2.00) = 6.44 \text{ kips-in. (Required Flexural Strength)}$$

The interaction equations (Eq. 3.5-1) and (Eq. 3.5-2) reduce to the following:

$$P_u/\phi_c P + C_{my} M_{uy} / \Phi_b M_{ny} \leq 1.0 \quad \text{(Eq. 3.5-1)}$$

$$P_u/\phi_c P + M_{uy} / \Phi_b M_{ny} \leq 1.0 \quad \text{(Eq. 3.5-2)}$$

6. Determination of $\Phi_b M_{ny}$ (Section 3.3.1):

$\Phi_b M_{ny}$ shall be taken as the smaller of the design flexural strengths calculated according to sections 3.3.1.1 and 3.3.1.2:

a. Section 3.3.1.1: $M_{ny}$ will be calculated on the basis of initiation of yielding.

Here it is evident that the initial yielding will not be in the compression flange, rather it will be in the tension flange.

The procedure is iterative: one assumes the actual compressive stress $f$ under $M_{ny}$. Knowing $f$ one proceeds as usual to obtain

266
\( x_{cg} \) (measured from top fiber) to neutral axis. Then one obtains
\[ f = F_y \frac{x_{cg}}{(3-x_{cg})} \]
and checks if it equals to the assumed value. If not, one reiterates by assuming another \( f \) until finally it checks. Then for this condition one obtains
\[ I_y \text{ and } M_{ny} = f(I_y/x_{cg}) \]
\[ = F_y \frac{I_y}{(3-x_{cg})}. \]
For the first iteration assume a compressive stress \( f = 20 \) ksi in the top compression fibers and that the webs are fully effective.

Compression flange:

\[ k = 4.00 \]
\[ \frac{w}{t} = 7.415/0.105 = 70.62 \]
\[ \lambda = (1.052/\sqrt{4.00})(70.62)/27000 = 1.011 > 0.673 \]
\[ \rho = [1-(0.22/1.011)]/1.011 = 0.774 \]
\[ b = 0.774 \times 7.415 = 5.739 \text{ in.} \]

To calculate effective section properties about y-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>( L )</th>
<th>Distance from Top Fiber (in.)</th>
<th>( L_x ) (in.²)</th>
<th>( L_x^2 ) (in.³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Webs</td>
<td>2x2.415 = 4.830</td>
<td>1.500</td>
<td>7.245</td>
<td>10.868</td>
</tr>
<tr>
<td>Upper Corners</td>
<td>2x0.377 = 0.754</td>
<td>0.140</td>
<td>0.106</td>
<td>0.015</td>
</tr>
<tr>
<td>Lower Corners</td>
<td>2x0.377 = 0.754</td>
<td>2.860</td>
<td>2.156</td>
<td>6.167</td>
</tr>
<tr>
<td>Compression Flange</td>
<td>5.739</td>
<td>0.053</td>
<td>0.304</td>
<td>0.016</td>
</tr>
<tr>
<td>Tension Flanges</td>
<td>2x0.508 = 1.016</td>
<td>2.948</td>
<td>2.995</td>
<td>8.830</td>
</tr>
<tr>
<td>Sum</td>
<td>13.093</td>
<td>12.806</td>
<td>25.896</td>
<td>2.347</td>
</tr>
</tbody>
</table>

267
Distance from top fiber to y-axis is

\[ x_{cg} = \frac{12.806}{13.093} = 0.978 \text{ in.} \]

\[ f = F_y \left( \frac{x_{cg}}{3-x_{cg}} \right) \]

\[ = 50 \left[ \frac{0.978}{(3.00-0.978)} \right] = 24.18 \text{ ksi} > 20 \text{ ksi} \]

need to do another iteration.

For the second iteration assume a compressive stress

\[ f = 25.50 \text{ ksi} \]

in the top compression fibers, and that the webs are fully effective.

Compression flange:

\[ \lambda = \frac{1.052/\sqrt{4.0} \times 70.62}{\sqrt{25.5/27000}} = 1.142 > 0.673 \]

\[ \rho = \frac{1-(0.22/1.142)}{1.142} = 0.707 \]

\[ b = 0.707 \times 7.415 = 5.242 \text{ in.} \]

Effective section properties about y-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>( x ) Distance from Top Fiber (in.)</th>
<th>( L ) Effective Length (in.)</th>
<th>( L_x ) (in.(^2))</th>
<th>( L'_x ) (in.(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Webs</td>
<td>2x2.415 = 4.830</td>
<td>1.500</td>
<td>7.245</td>
<td>10.868</td>
</tr>
<tr>
<td>Upper Corners</td>
<td>2x0.377 = 0.754</td>
<td>0.140</td>
<td>0.106</td>
<td>0.015</td>
</tr>
<tr>
<td>Lower Corners</td>
<td>2x0.377 = 0.754</td>
<td>2.860</td>
<td>2.156</td>
<td>6.167</td>
</tr>
<tr>
<td>Compression Flange</td>
<td>5.242</td>
<td>0.053</td>
<td>0.278</td>
<td>0.015</td>
</tr>
<tr>
<td>Tension Flanges</td>
<td>2x0.508 = 1.016</td>
<td>2.948</td>
<td>2.995</td>
<td>8.830</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td><strong>12.596</strong></td>
<td><strong>12.780</strong></td>
<td><strong>25.895</strong></td>
<td><strong>2.347</strong></td>
</tr>
</tbody>
</table>

Distance from top fiber to y-axis is

268
\[ x_{cg} = \frac{12.780}{12.596} = 1.015 \text{ in.} \]

\[ f = 50 \left( \frac{1.015}{(3.00 - 1.015)} \right) = 25.57 \text{ ksi (close enough)} \]

Thus actual compressive stress \( f = 25.50 \text{ ksi} \)

To check if the webs are fully effective (Section 2.2.2):

\[ f_1 = \left( \frac{(1.015 - 0.293)}{1.985} \right) (50) = 18.17 \text{ ksi (compression)} \]

\[ f_2 = -\left( \frac{(1.985 - 0.293)}{1.985} \right) (50) = -42.62 \text{ ksi (tension)} \]

\[ \Psi = \frac{f_2}{f_1} = \frac{-42.62}{18.19} = -2.343 \]

\[ k = 4 + 2(1 - \Psi)^2 + 2(1 - \Psi) \] (Eq. 2.2.2-4)

\[ = 4 + 2[1 - (-2.343)]^2 + 2[1 - (-2.343)] \]

\[ = 85.406 \]

\[ h = w = 2.415 \text{ in.} \]

\[ w/t = \frac{2.415}{0.105} = 23.00 < 200 \text{ OK (Section 2.1.2-(1))} \]

\[ \lambda = \frac{(1.052/\sqrt{85.406})(23.00)/\sqrt{18.19/27000} = 0.068 < 0.673} \]

\[ b_e = 2.415 \text{ in.} \]

\[ b_2 = \frac{b_e}{2} \] (Eq. 2.2.2-2)

\[ = \frac{2.415}{2} = 1.208 \text{ in.} \]

\[ b_1 = \frac{b_e}{(3 - \Psi)} \] (Eq. 2.2.2-1)

\[ = \frac{2.415}{3 - (-2.343)} = 0.452 \text{ in.} \]

Compression portion of each web calculated on the basis of the effective section = \( x_{cg} - 0.293 = 1.015 - 0.293 = 0.722 \text{ in.} \)

Since \( b_1 + b_2 = 1.660 \text{ in.} > 0.722 \text{ in.} \), \( b_1 + b_2 \) shall be taken as 0.722 in. This verifies the assumption that the web is fully effective.

\[ I'_y = I_x + I'_1 - Lx_{cg}^2 \]

\[ = 25.895 + 2.347 - 12.596(1.015)^2 \]
Actual $I_y = I'_y$

$= 15.265 \times 0.105 = 1.603 \text{ in.}^3$

$S_e = \frac{I_y}{(3.000-x_{cg})}$

$= 1.603/(3.000-1.015)$

$= 0.808 \text{ in.}^3$

$M_{ny} = \frac{S_F}{e_y}$

(Eq. 3.3.1.1-1)

$= 0.808(50)$

$= 40.40 \text{ kips-in.}$

$\Phi_b = 0.90$

$\Phi_b M_{ny} = 0.90 \times 40.40 = 36.36 \text{ kips-in.}$

b. Section 3.3.1.2: $M_{ny}$ will be calculated on the basis of the lateral buckling strength. (y-axis is the axis of bending).

$M_n = S_c (M_c/S_f)$

(Eq. 3.3.1.2-1)

$M_c = C_b A_s \sigma_{ex} \left[ j + C_s \sqrt{j^2 + r_o^2 (\sigma_t/\sigma_{ex})^2} \right]$

(Eq. 3.3.1.2-5)

where

$\sigma_{ex} = \frac{(m^2 E_o)/(K_x L_x/r_x)^2}{(E_t/E_o)}$

(Eq. 3.4.3-3)

$= 70.41 \times (E_t/E_o) \text{ ksi (from item 2.b of this example)}$

$\sigma_t = 1/(A r_o^4) \left[ GO_j (m^2 E_o C_w)/(K_t L_t)^2 \right](E_t/E_o)$

(Eq. 3.4.2-1)

$= 9.43 \times (E_t/E_o) \text{ ksi (from item 2.b of this example)}$

$C_b = 1.75+1.05(M_1/M_2)+0.3(M_1/M_2)^2$

$= 1.75+1.05(-1.0)+0.3(-1.0)^2 = 1.0$

$C_s = 1.0$

$r_o = 3.961 \text{ in.}$

$j = 4.567$

$M_c = 1.0 \times 1.0 \times (1.551)(70.41) \times 4.567$
\[
+1.00\sqrt{(4.567)^2+(3.961)^2(9.431/70.41)}
\]
\[
= 1022.0 \left(\frac{E_t}{E_o}\right) \text{ kips-in.}
\]

\[
M_n = S_c \left(\frac{M}{S_f}\right)
\]

\[
M_n = S_f
\]

\[
f = M_c/S_f = 1022.0 \left(\frac{E_t}{E_o}\right)/2.046 = 499.5 \left(\frac{E_t}{E_o}\right) \text{ ksi}
\]

In the determination of the lateral buckling stress, it is necessary to select a proper ratio of \(\frac{E_t}{E_o}\) from Table A10 or Figure A7 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of \(f = F_y = 50 \text{ ksi}\). From Table A10, the corresponding value of \(\frac{E_t}{E_o}\) is found to be equal to 0.19. Thus,

\[
f_1 = 499.5 \times 0.19
\]

\[
= 94.9 \text{ ksi} > \text{assumed stress } f = 50 \text{ ksi}
\]

Because the computed stress is larger than the maximum yield strength, the lateral buckling stress shall be limited to 50 ksi. Therefore,

\[
f = M_c/S_f = 50.0 \text{ ksi}
\]

To calculate effective section properties to obtain \(S_c\) at a stress of 50.0 ksi, we assume that the webs are fully effective.

**Compression flange:**

\[A = (1.052/\sqrt{4.00})(70.62)/\sqrt{50.0/27000} = 1.599 > 0.673\]

\[\rho = [1-(0.22/1.599)]/1.599 = 0.539\]

\[b = 0.539 \times 7.415 = 3.997 \text{ in.}\]
Effective section properties about y-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L Effective Length (in.)</th>
<th>x Distance from Top Fiber (in.)</th>
<th>Lx (in.²)</th>
<th>Lx² (in.⁴)</th>
<th>I'_{1} About Own Axis (in.⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Webs</td>
<td>2x2.415 = 4.830</td>
<td>1.500</td>
<td>7.245</td>
<td>10.868</td>
<td>2.347</td>
</tr>
<tr>
<td>Upper Corners</td>
<td>2x0.377 = 0.754</td>
<td>0.140</td>
<td>0.106</td>
<td>0.015</td>
<td>--</td>
</tr>
<tr>
<td>Lower Corners</td>
<td>2x0.377 = 0.754</td>
<td>2.860</td>
<td>2.156</td>
<td>6.167</td>
<td>--</td>
</tr>
<tr>
<td>Compression Flange</td>
<td>3.997</td>
<td>0.053</td>
<td>0.212</td>
<td>0.011</td>
<td>--</td>
</tr>
<tr>
<td>Tension Flanges</td>
<td>2x0.508 = 1.016</td>
<td>2.948</td>
<td>2.995</td>
<td>8.830</td>
<td>--</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td><strong>11.351</strong></td>
<td><strong>12.714</strong></td>
<td><strong>25.891</strong></td>
<td><strong>2.347</strong></td>
<td></td>
</tr>
</tbody>
</table>

Distance from top fiber to y-axis is

\[ x_{cg} = \frac{12.714}{11.351} = 1.120 \text{ in.} \]

To check if the webs are fully effective (Section 2.2.2):

\[ f_1 = \frac{[(1.120-0.293)/1.120](50.0)}{50.0} = 36.92 \text{ ksi (compression)} \]

\[ f_2 = -\left[\frac{(1.880-0.293)/1.120}{50.0}\right] = -70.85 \text{ ksi (tension)} \]

\[ \psi = -70.85/36.92 = -1.919 \]

\[ k = 4+2\left[1-(-1.919)\right]+2\left[1-(-1.919)\right] \]

\[ = 59.581 \]

\[ \lambda = (1.052/\sqrt{59.581})(23.00)\sqrt{36.92/27000} = 0.116 < 0.673 \]

\[ b_1 = 2.415 \text{ in.} \]

\[ b_2 = 2.415/2 = 1.208 \text{ in.} \]

\[ b_1 = 2.415/[3-(-1.919)] = 0.491 \text{ in.} \]

Compression portion of each web calculated on the basis of the effective section = 1.120-0.293 = 0.827 in.
Since $b_1 + b_2 = 1.699 \text{ in.} > 0.827 \text{ in.}, b_1 + b_2$ shall be taken as 0.827 in.. This verifies the assumption that the web is fully effective.

$$I_y' = 25.891 + 2.347 - 11.351(1.120)^2$$
$$= 13.999 \text{ in.}^3$$

Actual $I_y = 13.999(0.105) = 1.470 \text{ in.}^4$

$$S_c = \frac{I_y}{x_{cg}} = 1.470/1.120 = 1.313 \text{ in.}^3$$

$$M_{ny} = \frac{M_{sc}}{S_f}$$

(Eq. 3.3.1.2-1)

$$= 102.30(1.313)/2.046 = 65.65 \text{ kips-in.}$$

$$\Phi_b = 0.85$$

$$\Phi_b M_{ny} = 0.85 \times 65.65 = 55.80 \text{ kips-in.}$$

$\Phi_b M_{ny}$ shall be the smaller of 36.36 kips-in. and 55.80 kips-in.

Thus

$$\Phi_b M_{ny} = 36.36 \text{ kips-in.}$$

7. $C_{my} = 0.6 - 0.4 \left( \frac{M_1}{M_2} \right) \geq 0.4$

$M_1/M_2 = -1.00$ (single curvature)

$0.6 - 0.4(-1.00) = 1.00 > 0.4$

$C_{my} = 1.00$

8. Determination of $1/a_{ny}$:

$$\Phi_c = 0.85$$

$$P_E = n^2 E_o I_y/(K_{yy} L_y)^2$$

(Eq. 3.5-5)

$$I_y = 1.786 \text{ in.}^4$$

$$K_{yy} = 1.0(16\times12) = 192 \text{ in.}$$

$$P_E = \left[ n^2(27000)(1.786) \right]/(192)^2 = 12.91 \text{ kips}$$

273
1/\( \alpha_{ny} \) = \( 1/[1-P_u/\Phi_{c'n}P] \)  
\( \alpha_{ny} = 0.707 \)  

9. Check interaction equations:

\( P_u/\Phi_{c'n} + C_{my}M_{uy}/\Phi_{b'n} \alpha_{ny} \leq 1.0 \)  
\( 3.22/10.970 + 1.00x6.44/(36.36x0.707) = 0.294 + 0.251 \)  
\( = 0.545 < 1.0 \) OK  

\( P_u/\Phi_{c'n} + M_{ux}/\Phi_{b'n} \alpha_{ny} \leq 1.0 \)  
\( 3.22/46.58 + 6.44/36.36 = 0.069 + 0.177 = 0.246 \leq 1.0 \) OK

Therefore the section is adequate for the applied loads.

Solution: Part (B)

1. Full section properties are the same as previously calculated in part (A.1).

2. \( \Phi_{c'n}P = 10.970 \) kips (calculated in part (A)).

3. \( P_u/\Phi_{c'n} \) = 3.22/10.970 = 0.294 > 0.15

Therefore the following interaction equations must be satisfied.

\( P_u/\Phi_{c'n} + C_{mx}M_{ux}/\Phi_{b'n} \alpha_{nx} + C_{my}M_{uy}/\Phi_{b'n} \alpha_{ny} \leq 1.0 \)  
\( P_u/\Phi_{c'n} + M_{ux}/\Phi_{b'n} \alpha_{nx} + M_{uy}/\Phi_{b'n} \alpha_{ny} \leq 1.0 \)

4. \( \Phi_{c'n} = 46.58 \) kips (calculated in part (A.4)).

5. Determination of \( M_{ux} \) (Section 3.5):

The centroidal x-axis is the same for both the full and effective
6. Determination of $bM_{nx}$ (Section 3.3.1):

$\Phi bM_{nx}$ shall be taken as the smaller of the design flexural strengths calculated according to Sections 3.3.1.1 and 3.3.1.2.

a. Section 3.3.1.1: $M_{nx}$ will be calculated based on the initiation of yielding.

First approximation:

* Assume a compressive stress of $f = F_y = 50$ ksi in the top fiber of the section.

* Assume that the web is fully effective.

Compression flange:

\[
\begin{align*}
  w &= 2.415 \text{ in.} \\
  \frac{w}{t} &= \frac{2.415}{0.105} = 23.00 \\
  S &= 1.28 \sqrt{\frac{E_o}{f}} \\
  &= 1.28 \sqrt{27000/50.0} = 29.74 \\
  \text{For } S/3 &= 9.91 < \frac{w}{t} = 23.00 < S = 29.74 \\
  I_a &= t^3 399 \left[ \left( \frac{w}{t} \right) / S \right]^{-0.33} \\
  &= (0.105)^3(399) \left[ \left( \frac{23.00}{29.74} \right) - 0.33 \right]^3 \\
  &= 0.004227 \text{ in.}^4 \\
  I_s &= \frac{d't}{12} \\
  &= (0.508)^2(0.105)/12 = 0.001147 \text{ in.}^4 \\
  I_s / I_a &= 0.001147/0.004227 = 0.271
\end{align*}
\]
\[ D = 0.800 \text{ in.} \]

\[ \frac{D}{w} = \frac{0.800}{2.415} = 0.331 \]

\[ \frac{w}{t} = 23.00 < 50 \text{ OK (Section 2.1.1-(1)-(iii))} \]

For \( 0.25 < \frac{D}{w} = 0.331 < 0.8 \)

\[
k = \left( 4.82 - 5\left(\frac{D}{w}\right) \right)^{1/2} + 0.43 \leq 5.25 - 5\left(\frac{D}{w}\right) \quad \text{(Eq. 2.4.2-9)}
\]

\[
\left( 4.82 - 5(0.331) \right)^{1/2} + 0.43 = 2.078
\]

\[ 5.25 - 5(0.331) = 3.595 \]

\[ k = 2.078 \]

\[
\lambda = \left( \frac{1.052}{\sqrt{2.078}} \right) \left( \frac{23.00}{\sqrt{50.0/27000}} \right) = 0.722 > 0.673
\]

\[ \rho = \left[ 1 - \left( \frac{0.22}{0.722} \right) \right] / 0.722 = 0.963 \]

\[ b = 0.963 \times 2.415 = 2.326 \text{ in.} \]

Compression stiffener:

\[ d = 0.508 \text{ in.} \]

\[ \frac{d}{t} = \frac{0.508}{0.105} = 4.84 \]

\[ k = 0.50 \]

Assume the maximum stress in element, \( f = F_y = 50 \text{ ksi} \) although it will be actually less.

\[
\lambda = \left( \frac{1.052}{\sqrt{k}} \right) \left( \frac{w}{t} \right) \sqrt{\frac{f E_o}{E_o}} \quad \text{(Eq. 2.2.1-4)}
\]

\[
= \left( \frac{1.052}{\sqrt{0.50}} \right) (4.84) \sqrt{50.0/27000} = 0.310 < 0.673
\]

For \( \lambda < 0.673 \)

\[ b = w \quad \text{(Eq. 2.2.1-1)} \]

\[ d'_{s} = 0.508 \text{ in.} \]

\[ d_{s} = d'_{s} \left( \frac{I_s}{I_{a}} \right) \leq d'_{s} \quad \text{(Eq. 2.4.2-11)}
\]

\[ = d'_{s} (0.271) = 0.138 \text{ in.} \]

276
Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>L</th>
<th>Distance from Top Fiber (in.)</th>
<th>Ly (in.)</th>
<th>Ly^2 (in.^3)</th>
<th>I'_1 About Own Axis (in.^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression Flange</td>
<td>2.326</td>
<td>0.053</td>
<td>0.123</td>
<td>0.007</td>
<td>--</td>
</tr>
<tr>
<td>Compression Stiffener</td>
<td>0.138</td>
<td>0.362</td>
<td>0.050</td>
<td>0.018</td>
<td>--</td>
</tr>
<tr>
<td>Compression Corners Web</td>
<td>2x0.377 = 0.754</td>
<td>4.000</td>
<td>29.660</td>
<td>118.640</td>
<td>33.974</td>
</tr>
<tr>
<td>Tension Flange</td>
<td>2.415</td>
<td>7.948</td>
<td>19.194</td>
<td>152.557</td>
<td>--</td>
</tr>
<tr>
<td>Tension Stiffener</td>
<td>0.508</td>
<td>7.453</td>
<td>3.786</td>
<td>28.218</td>
<td>0.011</td>
</tr>
<tr>
<td>Tension Corners</td>
<td>2x0.377 = 0.754</td>
<td>7.860</td>
<td>5.926</td>
<td>46.582</td>
<td>--</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td>14.310</td>
<td>58.845</td>
<td>346.037</td>
<td>33.985</td>
<td>--</td>
</tr>
</tbody>
</table>

Distance from neutral axis to top fiber,

\[ y_{cg} = \frac{L_y}{L} = \frac{58.845}{14.310} = 4.112 \text{ in.} \]

Since the distance from the neutral axis to the top compression fiber is greater than half the depth of the section, a compressive stress of \( F_y = 50 \text{ ksi} \) governs as assumed.

\[
I'_x = L_y^2 + I'_1 - L_y^2 c_g
= 346.037 + 33.985 - 14.310(4.112)^2
= 138.06 \text{ in}^3
\]

Actual \( I'_x = t I'_x \)

\[ = (0.105)(138.06) = 14.50 \text{ in}^4 \]

Check Web

\[
w/t = \frac{7.415}{0.105} = 70.62 < 200 \text{ OK (Section } 2.1.2-(1))
\]

\[
f_1 = \left[ \frac{(4.112-0.293)}{4.112} \right](50) = 46.44 \text{ ksi(compression)}
\]
\[ f_2 = \frac{(3.888-0.293)}{4.112} (50) = -43.71 \text{ ksi (tension)} \]
\[ \psi = \frac{f_2}{f_1} = \frac{-43.71}{46.46} = -0.941 \]
\[ k = 4 + 2[1-(-0.941)]^2 + 2[1-(-0.941)] = 22.51 \]
\[ \lambda = \frac{1.052}{\sqrt{k}} \left(\frac{w}{t}\right) \frac{1}{\sqrt{E_o}} \] (Eq. 2.2.1-4)
\[ = \frac{1.052}{\sqrt{22.51}} \left(\frac{70.62}{46.44}\right) = 0.649 < 0.673 \]

For \( \lambda < 0.673 \)
\[ b = w \] (Eq. 2.2.1-1)
\[ b_e = 7.415 \text{ in.} \]
\[ b_2 = 7.415/2 = 3.708 \text{ in.} \]
\[ b_1 = 7.415/[3-(-0.941)] = 1.882 \text{ in.} \]
\[ b_1 + b_2 = 1.882 + 3.708 = 5.590 \text{ in.} > 3.785 \text{ in. (compression portion of web)} \]

Therefore web is fully effective as assumed.

Check Compression Stiffener

Actual maximum stress in stiffener = 46.44 ksi
\[ \lambda = \frac{1.052}{\sqrt{0.50}} \left(\frac{4.84}{46.44}\right) = 0.299 < 0.673 \]

For \( \lambda < 0.673 \)
\[ d'_s = 0.508 \text{ in.} \]

Since \( I_s/I_a \) is unchanged
\[ d_s = 0.138 \text{ in.} \]

Conservative assumption OK

\[ S_e = I_x/y_{cg} = 14.50/4.112 = 3.526 \text{ in.}^3 \]
\[ M_{nx} = S_e F_y \] (Eq. 3.3.1.1-1)
\[ = (3.526)(50) = 176.30 \text{ kips-in.} \]

\[ \phi_b = 0.90 \]
\[ \Phi_b M_{nx} = 0.90 \times 176.30 = 158.67 \text{ kips-in.} \]

b. Section 3.3.1.2: \( M_{nx} \) will be calculated based on the lateral buckling strength.

For the full section:
\( I_x = 15.108 \text{ in.}^4 \)
\( y_{cg} = 4.000 \text{ in.} \)
\( S_f = I_x / y_{cg} = 15.108 / 4.000 = 3.777 \text{ in.}^3 \)
\( M_y = S_f F_y \)
\[ = 3.777(50) = 188.85 \text{ kips-in.} \]
\( C_b = 1.00 \) (for members subject to combined axial load and bending moment)
\( r_o = 3.961 \text{ in.} \)
\( A = 1.551 \text{ in.}^2 \)
\( \sigma_{ey} = \left( \frac{n^2 E_o}{(K_y L_y / r_y)^2}(E_t / E_o) \right) \)
\[ = \left( \frac{n^2(27000)}{(178.94)^2}(E_t / E_o) \right) \]
\[ = 8.322 \left( \frac{E_t}{E_o} \right) \text{ ksi} \]
\( \sigma_t = 9.43 \left( \frac{E_t}{E_o} \right) \text{ ksi} \) (from part (A))
\( M_c = C_b r_o A \sigma_{ey} \sigma_t \) (Eq. 3.3.1.2-5)
\[ = (1.000)(3.961)(1.551)\sqrt{(8.322)(9.430)} \left( \frac{E_t}{E_o} \right) \]
\[ = 54.42 \left( \frac{E_t}{E_o} \right) \text{ kips-in.} \]

Let \( f = M_c / S_f \)
\[ = 54.42 \left( \frac{E_t}{E_o} \right) / 3.777 = 14.41 \left( \frac{E_t}{E_o} \right) \text{ ksi} \]

For the stress \( f \) less than 20 ksi, the plasticity reduction factor of \( E_t / E_o \) is equal to 1.0. The section is subject to elastic lateral buckling. Therefore,
\( M_c = 54.42 \text{ kips-in.} \)
\[ f = 14.41 \text{ ksi} \]

Determine \( S_e \), the elastic section modulus of the effective section calculated at a stress of \( \frac{M_c}{S_f} \) in the extreme compression fiber.

For compression flange:

\[ w = 2.415 \text{ in.} \]
\[ \frac{w}{t} = 2.415/0.105 = 23.00 \]
\[ s = 1.28\sqrt{\frac{E_y}{f}}, \quad f = F_n \quad \text{(Eq. 2.4-1)} \]
\[ s = 1.28\sqrt{\frac{27000}{14.41}} = 55.41 \]
\[ \frac{S}{3} = 18.47 < \frac{w}{t} = 23.00 < s = 55.41 \]
\[ I_a = 399(0.105)^4[(23.00/55.41)-0.33]^3 \]
\[ = 0.000030 \text{ in.}^4 \]
\[ I_s = 0.001147 \text{ in.}^4 \]
\[ \frac{I_s}{I_a} = 0.001147/0.000030 = 38.23 \]
\[ [4.82-5(0.331)](38.23)^{1/2}+0.43 = 20.00 > 3.595 \]
\[ k = 3.595 \]
\[ \Lambda = (1.052/\sqrt{3.595})(23.00)\sqrt{14.41/27000} = 0.295 < 0.673 \]
\[ b = w = 2.415 \text{ in.} \quad \text{(compression flange fully effective)} \]

For compression stiffener:

\( f \) is taken conservatively as 14.41 ksi as used in the top compression fiber.

\[ d/t = 4.84 \]
\[ \Lambda = (1.052/\sqrt{0.50})(4.84)\sqrt{14.41/27000} = 0.166 < 0.673 \]
\[ d_s' = d = 0.508 \text{ in.} \]
And since $I_s/I_a = 38.23 > 1.0$

\[ d_s = d' = 0.508 \text{ in. (compression stiffener fully effective)} \]

And since the web was fully effective at the stress $f = F_Y = 50 \text{ ksi}$, it will be fully effective for $f = 14.41 \text{ ksi}$.

Thus the whole section is fully effective at $M_c/S_f = 15.71 \text{ ksi}$

Therefore

\[ S_c = S_f = 3.777 \text{ in.}^3 \]

\[ M_{nx} = \frac{M_c S_c}{S_f} = \frac{54.42(3.777)}{3.777} = 54.42 \text{ kips-in.} \]

\[ \phi_b = 0.85 \]

\[ \phi_b M_{nx} = 0.85 \times 54.42 = 46.26 \text{ kips-in.} \]

\[ \phi_b M_{nx} \] shall be the smaller of 158.67 kips-in. and 46.26 kips-in.

Therefore

\[ \phi_b M_{nx} = 46.26 \text{ kips-in.} \]

7. Determination of $C_{mx}$ (Section 3.5):

\[ M_1/M_2 = -1.00 \text{ (single curvature)} \]

\[ C_{mx} = 0.6 - 0.4(-1.0) = 1.00 > 0.4 \text{ OK} \]

8. Determination of $\alpha_{nx}$ (Section 3.5):

\[ P_u = 3.22 \text{ kips} \]

\[ P_E = \frac{n^2E_0 I_x/(K_x L_x)^2}{[n^2(27000)(15.108)]/[1(16)x12]^2} = 109.21 \text{ kips} \]

\[ \phi_c = 0.85 \]
1/\alpha_{nx} = 1/(1-P_u/(\phi_{c}(P_E))) \quad (Eq. 3.5-4)
= 1/(1-3.22/(0.85x109.21)) = 1.036
\alpha_{nx} = 0.965

9. \text{M}_{uy} = 6.44 \text{kips-in. (calculated in part (A.5))}

10. \phi_{b,M_{ny}} = 36.36 \text{kips-in. (calculated in part (A.6))}

11. \text{C}_{my} = 1.0 \quad (calculated in part (A.7))

12. \delta_{ny} = 0.707 \quad (calculated in part (A.8))

13. Interaction equations (Section 3.5):

\begin{align*}
P_u/\phi_{c,M_{mx}} & + \text{C}_{M_{nx}} \alpha_{nx} + \text{C}_{M_{my}} \alpha_{ny} \phi_{b,M_{ny}} \alpha_{ny} \leq 1.0 \\
& (Eq. 3.5-1) \\
3.22/10.970 + 1.0 \times 12.88/(46.26 \times 0.965) + 1.0 \times 6.44/(36.36 \times 0.707) \\
& = 0.294 + 0.289 + 0.251 = 0.834 < 1.0 \text{ OK}
\end{align*}

\begin{align*}
P_u/\phi_{c,M_{no}} & + M_{nx} \phi_{b,M_{ny}} \phi_{b,M_{ny}} \leq 1.0 \\
& (Eq. 3.5-2) \\
3.22/46.58 + 12.88/46.26 + 6.44/36.36 \\
& = 0.069 + 0.278 + 0.177 = 0.524 < 1.0 \text{ OK}
\end{align*}

Therefore the section is adequate for the applied loads.
EXAMPLE 22.2 C-SECTION (ASD)

Rework Example 22.1 by using the Allowable Stress Design (ASD) method to check the adequacy of a channel section (Fig. 22.1) to be used as compression member.

Solution: Part (A)

1. Full section properties:

   The section properties (A, Ix, etc.) are the same as those calculated in Example 22.1.(1).

2. Determination of $P_a$:

   The following results are obtained from Example 22.1.(2).

   a) For Flexural Buckling:

      $$(F_n)_1 = \frac{(n^2 E_t)}{(K_{L_y} r_y)^2}$$  \hspace{1cm} (Eq. 3.4.1-1)
      $$(F_n)_1 = \frac{(n^2 \times 27000)}{(178.94)^2}$$  \hspace{1cm} = 8.322 ksi

      The section is subject to the elastic flexural buckling.

   b) For Torsional-Flexural Buckling:

      $$(F_n)_2 = \frac{1}{2\beta} \left[ (\sigma_{ex} \tau_0 - \sqrt{(\sigma_{ex} \tau_0)^2 - 4\beta \sigma_{ex} \tau_0}) \right]$$  \hspace{1cm} (Eq. 3.4.3-1)

      where

      $$\sigma_{ex} = \frac{(n^2 E_o)}{(K_{L_x} r_x)^2}(E_t/E_o)$$  \hspace{1cm} (Eq. 3.4.3-3)
      $$\tau_0 = 1/(A r_o^4) \left[ G_o J + (n^2 E_o C_w)/(K_{L_{t}})^2 \right](E_t/E_o)$$  \hspace{1cm} (Eq. 3.4.2-1)
      $$G_o = 10500 \text{ ksi (Table A4 of the Standard)}$$

      $$(F_{n2}) = \frac{1}{2\beta} \left[ (\sigma_{ex} \tau_0 - \sqrt{(\sigma_{ex} \tau_0)^2 - 4\beta \sigma_{ex} \tau_0}) \right]$$  \hspace{1cm} (Eq. 3.4.3-1)

      $$(F_{n2}) = \frac{1/(2 \times 0.694)}{(70.41+9.43)} - \sqrt{(70.41+9.43)^2 - 4 \times 0.694 \times 70.41 \times 9.43}$$  \hspace{1cm} = 9.024 ksi

      This section is subject to elastic torsional-flexural buckling.
Then, \( F_n \) should be the smaller of \((F_n)_1\) and \((F_n)_2\).

\[
F_n = 8.322 \, \text{ksi}
\]

Therefore,

\[
P_n = A F_n = 1.551 \times 8.322
\]

\[
= 12.91 \, \text{kips}
\]

\[
\Omega = 2.15
\]

\[
P_{a_o} = P_n / \Omega = 12.91 / 2.15 = 6.0 \, \text{kips}
\]

3. \( P = 0.35 + 1.75 = 2.10 \, \text{kips} \)

\[
P / P_{a_o} = 2.10 / 6.0 = 0.350 > 0.15
\]

Must check both interaction equations as follows:

\[
P / P_{a_o} + C_{mx} M_x / (M_{ax} \sigma_x) + C_{my} M_y / (M_{ay} \sigma_y) \leq 1.0
\]

\[
P / P_{a_o} + M_x / M_{ax} + M_y / M_{ay} \leq 1.0
\]

4. Determination of \( P_{a_o} \) (for \( F_n = F_y \)):

\[
A_e = 1.096 \, \text{in.}^2 \text{(from Example 19.1.4)}
\]

\[
P_{no} = 1.096 \times 50 = 54.80 \, \text{kips}
\]

\[
\Omega = 2.15
\]

\[
P_{a_o} = P_{no} / \Omega = 54.8 / 2.15
\]

\[
= 25.49 \, \text{kips}
\]

5. Determination of \( M_y \) (required flexural strength about y-axis):

\( (M_x = 0 \text{ since } e_y = 0) \)

\( M_y \) will be with respect to the centroidal axes of the effective section determined for the required axial strength alone.

\[
A_e = 1.551 \, \text{in.}^2 \text{ under required axial strength alone}
\]

Since \( A_e = A \), the centroidal axes for the effective section are
the same as those for the full section. Therefore, \( e_x \) did not change.

\[
M_y = 2.10(2.00) = 4.20 \text{ kips-in.} \quad \text{(Required Flexural Strength)}
\]

The interaction equations reduce to the following:

\[
P/P_a + C_{MY} \frac{M}{M_y} \leq 1.0
\]

\[
P/P_{ao} + M/M_y \leq 1.0
\]

6. Determination of \( M_{ay} \)

\( M_{ay} \) shall be taken as the smaller of the allowable flexural strengths calculated according to Sections 3.3.1.1 and 3.3.1.2:

a. Section 3.3.1.1: \( M_{ay} \) will be calculated on the basis of initiation of yielding.

\[
S_e = \frac{1.603}{(3.000-1.015)} = 0.808 \text{ in.}^3 \quad \text{(from Example 22.1)}
\]

\[
M_{ny} = S F \frac{e_y}{y} = 0.808(50) = 40.40 \text{ kips-in.}
\]

\[
\Omega = 1.85
\]

\[
M_{ay} = \frac{M_{ny}}{\Omega} = 40.40/1.85 = 21.84 \text{ kips-in.}
\]

b. Section 3.3.1.2: \( M_{ay} \) will be calculated on the basis of the lateral buckling strength. (y-axis is the axis of bending).

\[
M_n = S_c \left( \frac{M_c}{S_f} \right) \quad \text{(Eq. 3.3.1.2-1)}
\]

\[
M_n = S_c f
\]

\[
f = M_c/S_f = 50.0 \text{ ksi}
\]

\[
S_c = I_y / x_{cg} = 1.470/1.120 = 1.313 \text{ in.}^3
\]

\[
M_{ny} = M_c S_c / S_f \quad \text{(Eq. 3.3.1.2-1)}
\]
\[ M_{ay} = \frac{M_{ny}}{\Omega} = \frac{65.65}{1.85} = 35.49 \text{ kips-in.} \]

\( M_{ay} \) shall be the smaller of 21.84 kips-in. and 35.49 kips-in.

Thus

\[ M_{ay} = 21.84 \text{ kips-in.} \]

7. \( C_{my} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) \geq 0.4 \)

\( \frac{M_1}{M_2} = -1.00 \) (single curvature)

\[ 0.6 - 0.4(-1.00) = 1.00 > 0.4 \]

\( C_{my} = 1.00 \)

8. Determination of \( 1/\alpha_{ny} \):

\[ \Omega = 2.15 \]

\[ P_{cr} = \frac{n^2 \varepsilon_0 L_y}{(K L_y)^2} \]

\[ I_y = 1.786 \text{ in.}^4 \]

\[ K L_y = 1.0(16\times12) = 192 \text{ in.} \]

\[ P_{cr} = \frac{n^2(27000)(1.786)}{(192)^2} = 12.91 \text{ kips} \]

\[ 1/\alpha_{ny} = 1/[1-(\Omega P/P_{cr})] \]

\[ = 1/[1-(2.15\times2.1/12.91)] = 1/0.650 \]

\( \alpha_{ny} = 0.650 \)

9. Check interaction equations:

\[ \frac{P}{P_a} + C_{my} \frac{M_y}{M_{ay} \alpha_{ny}} \leq 1.0 \]

\[ 2.1/6.0 + 1.00 \times 4.2/(21.84 \times 0.650) = 0.350 + 0.296 \]

\[ = 0.646 < 1.0 \text{ OK} \]
\[
\frac{P}{P_a} + \frac{M_y}{M_{ay}} \leq 1.0 \\
2.1/25.49 + 4.2/21.84 = 0.082 + 0.192 = 0.274 < 1.0 \text{ OK}
\]

Therefore the section is adequate for the applied loads.

**Solution: Part (B)**

1. Full section properties are the same as previously calculated in Part (A.1).

2. \( P_a = 6.0 \text{ kips (calculated in Part (A))} \)

3. \( P = 0.35 + 1.75 = 2.10 \text{ kips} \)

\[
\frac{P}{P_a} = 2.10/6.0 = 0.350 > 0.15
\]

Must check both interaction equations as follows:

\[
\frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} \leq 1.0
\]

4. Determination of \( P_{ao} = 25.49 \text{ kips (calculated in Part (A))} \)

5. Determination of \( M_x \)

The centroidal x-axis is the same for both the full and effective sections.

\[ e_y = 4.00 \text{ in.} \]

\[ M_x = 2.10(4.00) = 8.40 \text{ kips-in. (Required Flexural Strength)} \]

6. Determination of \( M_{ax} \)

\( M_{ax} \) shall be taken as the smaller of the allowable flexural strengths.
calculated according to sections 3.3.1.1 and 3.3.1.2:

a. Section 3.3.1.1: $M_{ax}$ will be calculated on the basis of initiation of yielding.

$$S_e = \frac{14.50}{4.112} = 3.526 \text{ in.}^3 \text{ (from Example 22.1 Part (A))}$$

$$M_{nx} = S_e F_y$$

$$= 3.526(50) = 176.30 \text{ kips-in.}$$

$$\Omega = 1.85$$

$$M_{ax} = \frac{M_{nx}}{\Omega} = \frac{176.30}{1.85} = 95.30 \text{ kips-in.}$$

(Eq. 3.3.1.1-1)

b. Section 3.3.1.2: $M_{ax}$ will be calculated on the basis of the lateral buckling strength.

$$M_n = S_c \left( \frac{M_c}{S_f} \right)$$

(Eq. 3.3.1.2-1)

$$M_n = S_c f$$

$$f = M_c / S_f = \frac{54.42}{3.777} = 14.41 \text{ ksi}$$

$$S_c = S_e = 3.777 \text{ in.}^3$$

$$M_{nx} = \frac{M_c S_c}{S_f}$$

$$= 14.41 \times 3.777$$

$$= 54.42 \text{ kips-in.}$$

$$\Omega = 1.85$$

$$M_{ax} = \frac{M_{nx}}{\Omega} = \frac{54.42}{1.85} = 29.42 \text{ kips-in.}$$

Thus

$$M_{ax} = 29.42 \text{ kips-in.}$$

$M_{ax}$shall be the smaller of 95.30 kips-in. and 29.42 kips-in.

Thus

$$M_{ax} = 29.42 \text{ kips-in.}$$

7. $C_{mx} = 0.6 - 0.4(M_1/M_2) \geq 0.4$
8. Determination of $1/\alpha_{nx}$:

$$\Omega = 2.15$$

$$P_{cr} = \frac{\pi^2EI_x}{(KL_x)^2} = \frac{\pi^2(27000)(15.108)}{(192)^2} = 109.21 \text{ kips}$$

$$1/\alpha_{ny} = \frac{1}{[1-(\Omega_cP/P_{cr})]}$$

$$\alpha_{ny} = 0.650$$

9. $M_{uy} = 4.2 \text{ kips-in.}$

10. $M_{ay} = 21.84 \text{ kips-in.}$

11. $C_{my} = 1.0$

12. $\alpha_{ny} = 0.650$

13. Check interaction equations:

$$\frac{P/P_{a} + C_{mx}M_x/(M_{ax}\alpha_{nx}) + C_{my}M_y/(M_{ay}\alpha_{ny})}{\leq 1.0}$$

$$\frac{2.1/6.0 + 1.00 \times 8.4 / (29.42 \times 0.959) + 1.00 \times 4.2 / (21.84 \times 0.650)}{= 0.350 + 0.298 + 0.296 = 0.944 < 1.0 \text{ OK}}$$

$$\frac{P/P_{ao} + M_{ax}/M_{ax} + M_{ay}/M_{ay}}{\leq 1.0}$$

$$\frac{2.1/25.49 + 8.4/29.42 + 4.2/21.84}{= 0.082 + 0.286 + 0.192 = 0.560 < 1.0 \text{ OK}}$$

Therefore the section is adequate for the applied loads.
EXAMPLE 23.1 TUBULAR SECTION (LRFD)

By using the Load and Resistance Factor Design (LRFD) criteria, check the adequacy of a tubular section (Fig. 23.1) to be used as compression member which is subjected to an eccentrically axial load. The service axial load is $P = 15$ kips. Consider the following loading case: the eccentricity of axial load at each end of member, $e_y$, is 4 in. and member is bent in single curvature about x-axis, and $e_x = 0$. Assume that the effective length factors $K_x = K_y = 1.0$, and that the unbraced lengths $L_x = L_y = 10$ ft. Use Type 304, 1/4-Hard, stainless steel. Assume dead to live load ration $D/L=1/5$ and $1.2D+1.6L$ governs the design.

\[ r = R + \frac{t}{2} = \frac{3}{16} + 0.105/2 = 0.240 \text{ in.} \]
Length of arc, \( u = 1.57r = 1.57 \times 0.240 = 0.377 \text{ in.} \)

Distance of c.g. from center of radius, 
\[ c = 0.637r = 0.637 \times 0.240 = 0.153 \text{ in.} \]

\[ I_x = I_y \] (doubly symmetric section)

<table>
<thead>
<tr>
<th>Element</th>
<th>( L ) (in.)</th>
<th>( y ) Distance to Center of Section (in.)</th>
<th>( L y^2 ) (in.²)</th>
<th>( I'_1 ) About Own Axis (in.³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flanges</td>
<td>2 ( \times ) 7.414 = 14.828</td>
<td>3.948</td>
<td>231.120</td>
<td>--</td>
</tr>
<tr>
<td>Corners</td>
<td>4 ( \times ) 0.377 = 1.508</td>
<td>3.860</td>
<td>22.469</td>
<td>--</td>
</tr>
<tr>
<td>Webs</td>
<td>2 ( \times ) 7.414 = 14.828</td>
<td>--</td>
<td>--</td>
<td>67.921</td>
</tr>
<tr>
<td>Sum</td>
<td>31.164</td>
<td>253.589</td>
<td>67.921</td>
<td></td>
</tr>
</tbody>
</table>

\[ A = L\tau = 31.164 \times 0.105 = 3.272 \text{ in.}^2 \]

\[ I'_1 = L y^2 + I'_1 = 253.589 + 67.921 = 321.510 \text{ in.}^3 \]

\[ I_x = I_y = I'_1 = 321.510 \times 0.105 = 33.759 \text{ in.}^4 \]

\[ r_x = r_y = \frac{33.759}{3.272} = 3.212 \text{ in.} \]

\[ S_x = \frac{I_x}{4.000} = \frac{33.759}{4.000} = 8.440 \text{ in.}^3 \]

\[ K_x L_x / r_x = 1.0(10 \times 12)/3.212 = 37.36 < 200 \text{ OK (Section 3.4-(5))} \]

2. Determination of \( \phi P_c n \) (Section 3.4):

Since the square tube is a doubly symmetric closed section, provisions of Section 3.4.1 apply, i.e., section is not subjected to torsional flexural buckling.

For Flexural Buckling:
\[ F_n = \left( \frac{n^2 E_t}{(K L / r_y)^2} \right) \]  
(Eq. 3.4.1-1)

In the determination of the flexural buckling stress, it is necessary to select a proper value of \( E_t \) from Table A13 or Figure A11 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of \( f=44 \) ksi. From Table A13, the corresponding value of \( E_t \) is found to be equal to 7200 ksi. Thus,

\[ F_n = \left( \frac{n^2 \times 7200}{(37.36)^2} \right) \]

\[ = 50.91 \text{ ksi} > \text{assumed stress } f=44 \text{ ksi} \quad \text{NG} \]

Because the computed stress is larger than the assumed value, further successive approximations are needed.

Assume \( f=46.66 \) ksi, and

\[ E_t = 6600 \text{ ksi} \]

\[ F_n = \left( \frac{n^2 \times 6600}{(37.36)^2} \right) \]

\[ = 46.67 \text{ ksi} = \text{assumed stress } \text{ OK} \]

\( w = 7.414 \text{ in.} \)

\( w/t = 7.414/0.105 = 70.61 < 400 \text{ OK (Section 2.1.1-(1)-(ii))} \)

\( k = 4.00 \text{ (Section 2.2.1-(1))} \)

\( \lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_0}, f = F_n \)  
(Eq. 2.2.1-4)

\( = (1.052/\sqrt{4.00})(70.61)\sqrt{46.44/27000} = 1.544 > 0.673 \)

\( \rho = (1-0.22/\lambda)/\lambda \)  
(Eq. 2.2.1-3)

\( = (1-0.22/1.544)/1.544 = 0.555 \)

\( b = \rho w \)  
(Eq. 2.2.1-2)

\( = 0.555 \times 7.414 = 4.115 \text{ in.} \)

\( A_e = A - 4(w-b)t \)

\( = 3.272-4(7.414-4.115)(0.105) = 1.886 \text{ in.}^2 \)

\[ P_n = A F_n \]  
(Eq. 3.4-1)
1.886 \times 46.66 = 88.00 \text{ kips}

\Phi_c = 0.85

\Phi_c \cdot P_n = 0.85 \times 88.00 = 74.80 \text{ kips}

3. P_{DL+P_{LL}} = (P_{DL}/P_{LL} + 1)P_{LL}

= (1/5+1)P_{LL} = 1.2P_{LL} = P

P_{LL} = P/1.2 = 15/1.2 = 12.5 \text{ kips}

P_u = 1.2P_{DL} + 1.6P_{LL}

= (1.2P_{DL}/P_{LL} + 1.6)P_{LL}

= [1.2(1/5)+1.6](12.5) = 23 \text{ kips}

where

P_{DL} = Axial load determined on the basis of nominal dead load

P_{LL} = Axial load determined on the basis of nominal live load

P_u/\Phi_c \cdot P_n = 23/74.80 = 0.307 > 0.15

Must check both interaction equations (Eq. 3.5-1), (Eq. 3.5-2).

4. Determination of \( \Phi_c \cdot P_{no} \) (Section 3.4 for \( F_n = F_y \))

\begin{align*}
\lambda &= (1.052/\sqrt{4.00})(70.61)/\sqrt{50.0/27000} = 1.544 > 0.673 \\
\rho &= (1-0.22/1.598)/1.598 = 0.540 \\
b &= 0.540 \times 7.414 = 4.004 \text{ in.} \\
A_e &= 3.272 - 4(7.414 - 4.004)(0.105) = 1.840 \text{ in.}^2 \\
P_{no} &= 1.840 \times 50.00 = 92.00 \text{ kips} \\
\Phi_c \cdot P_{no} &= 0.85 \times 92.00 = 78.20 \text{ kips}
\end{align*}

5. Determination of \( M_{ux}, M_{uy} \) (Section 3.5):

Since the section is doubly symmetric, the centroidal axes of the
effective section at $\Phi_P$ are the same as those of the full section.

$M_{ux} = P_e y = 23 \times 4 = 92$ kips-in.

$M_{uy} = P_e x = 0$

Since $M_{uy} = 0$, the interaction equations (Eq. 3.5-1) and (Eq. 3.5-2) reduce to the following:

$$P_u/\Phi_P + C_m x_{ux}/\Phi_b M_{nx} \leq 1.0$$  (Eq. 3.5-1)

$$P_u/\Phi_P + C_m x_{ux}/\Phi_b M_{nx} \leq 1.0$$  (Eq. 3.5-2)

6. Determination of $\Phi_b M_{nx}$ (Section 3.3.1):

$\Phi_b M_{nx}$ shall be taken as the smaller of the design flexural strengths calculated according to Sections 3.3.1.1 and 3.3.1.2:

a. Section 3.3.1.1: $M_{nx}$ will be calculated on the basis of initiation of yielding.

Computation of $I_x$:

For the first approximation, assume a compression stress of $f = F_y = 50$ ksi in the compression flange, and that the web is fully effective.

Compression flange: $k = 4.00$ (stiffened compression element supported by a web on each longitudinal edge)

$w/t = 7.414/0.105 = 70.61 < 400$ OK (Section 2.1.1-(1)-(ii))

$\lambda = (1.052/\sqrt{4.00})(70.61)/\sqrt{50.0}/27000 = 1.598 > 0.673$

$\rho = (1-0.22/1.598)/1.598 = 0.540$

$b = 0.540 \times 7.414 = 4.004$ in.
Effective section properties about x-axis:

<table>
<thead>
<tr>
<th>Element</th>
<th>( L ) Effective Length (in.)</th>
<th>( y ) Distance from Top Fiber (in.)</th>
<th>( L_y ) Ly (in.(^2))</th>
<th>( L_y^2 ) Ly^2 (in.(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Webs</td>
<td>14.828</td>
<td>4.000</td>
<td>59.312</td>
<td>237.248</td>
</tr>
<tr>
<td>Upper Corners</td>
<td>0.754</td>
<td>0.140</td>
<td>0.106</td>
<td>0.015</td>
</tr>
<tr>
<td>Lower Corners</td>
<td>0.754</td>
<td>7.860</td>
<td>5.926</td>
<td>46.582</td>
</tr>
<tr>
<td>Compression Flange</td>
<td>4.004</td>
<td>0.053</td>
<td>0.212</td>
<td>0.011</td>
</tr>
<tr>
<td>Tension Flange</td>
<td>7.414</td>
<td>7.948</td>
<td>58.926</td>
<td>468.348</td>
</tr>
<tr>
<td>Sum</td>
<td>27.754</td>
<td>124.482</td>
<td>752.204</td>
<td>67.921</td>
</tr>
</tbody>
</table>

Distance from top fiber to x-axis is

\[ y_{cg} = \frac{L_y}{L} = \frac{124.482}{27.754} = 4.485 \text{ in.} \]

Since the distance of top compression fiber from neutral axis is greater than one half the section depth (i.e., 4.485 > 4.000), a compression stress of 50 ksi will govern as assumed (i.e., initial yielding is in compression).

To check if the web is fully effective (Section 2.2.2)

\[ f_1 = \left(\frac{(4.485 - 0.293)}{4.485}\right)(50) = 46.73 \text{ ksi (compression)} \]

\[ f_2 = -\left(\frac{(3.515 - 0.293)}{4.485}\right)(50) = -3.592 \text{ ksi (tension)} \]

\[ \Psi = \frac{f_2}{f_1} = \frac{-3.592}{46.73} = -0.0769 \]

\[ k = 4 + 2\left[1 - (-0.769)\right]^2 + 2\left[1 - (-0.769)\right] \]

\[ = 18.610 \]

\[ h = w = 7.414 \text{ in.}, \quad \frac{h}{t} = \frac{w}{t} = \frac{7.414}{0.105} = 70.61 \]

\[ \frac{h}{t} = 70.61 < 200 \text{ OK (Section 2.1.2-(1))} \]
\[ \lambda = \frac{(1.052/\sqrt{18.610})(70.61)\sqrt{46.73/27000}}{0.716} > 0.673 \]
\[ \rho = \frac{(1-0.22/0.716)}{0.716} = 0.968 \]
\[ b_e = 0.968 \times 7.414 = 7.177 \text{ in.} \]
\[ b_2 = \frac{b_e}{2} \quad \text{(Eq. 2.2.2-2)} \]
\[ = \frac{7.177}{2} = 3.589 \text{ in.} \]
\[ b_1 = \frac{b_e}{(3-\Psi)} \quad \text{(Eq. 2.2.2-1)} \]
\[ = \frac{7.177}{3-(-0.769)} = 1.904 \text{ in.} \]

Compression portion of the web calculated on the basis of the effective section:
\[ y_{cg} = -0.293 = 4.485 - 0.293 = 4.192 \text{ in.} \]

Since \( b_1 + b_2 = 5.493 \text{ in.} > 4.192 \text{ in.} \), \( b_1 + b_2 \) shall be taken as 4.192 in.

This verifies the assumption that the web is fully effective.

\[ I'_{x} = Ly^2 + I'_{1x} - Ly_{cg}^2 \]
\[ = 752.204 + 67.921 - 27.754(4.485)^2 \]
\[ = 261.847 \text{ in.}^3 \]

Actual \( I_x = tI'_{x} \)
\[ = (0.105)(261.847) = 27.494 \text{ in.}^4 \]
\[ S_e = \frac{I_x}{y_{cg}} = \frac{27.494}{4.485} = 6.130 \text{ in.}^3 \]
\[ M_{nx} = S_e F_y \quad \text{(Eq. 3.3.1.1-1)} \]
\[ = (6.130)(50) = 306.50 \text{ kips-in.} \]
\[ \Phi_b = 0.90 \]
\[ \Phi _b M_{nx} = 0.90 \times 306.50 = 275.85 \text{ kips-in.} \]

b. Section 3.3.1.2: \( M_{nx} \) will be calculated on the basis of lateral buckling strength. However for this square tube (closed box-type member) the provisions of Section 3.3.1.2 do not apply.
Therefore
\[ \phi_b M_{nx} = 275.85 \text{ kips-in.} \]

7. \[ C_{mx} = 0.6-0.4(M_1/M_2) \]
\[ M_1/M_2 = -(92/92) = -1.0 \text{ (single curvature)} \]
\[ 0.6-0.4(M_1/M_2) = 0.6-0.4(-1.0) = 1.0 \]

8. Determination of \(1/\alpha_{nx}\):
\[ \phi_c = 0.85 \]
\[ P_E = \frac{n^2E I_x}{(K L x)^2} \]  
(Eq. 3.5-5)
\[ I_x = 33.759 \text{ in.}^4 \]
\[ K L x = 1.0(10\times12) = 120 \text{ in.} \]
\[ P_E = \frac{n^2(27000)(33.759)}{(120)^2} = 624.73 \text{ kips} \]
\[ 1/\alpha_{nx} = 1/(1-P_u/\phi_p P_E) \]  
(Eq. 3.5-4)
\[ = 1/[1-23/(0.85\times624.73)] = 1.045 \]
\[ \alpha_{nx} = 0.957 \]

9. Check interaction equations:
\[ P_u/\phi_p P + C_{mx} M_{ux} / \phi_b M_{nx} \alpha_{nx} \leq 1.0 \]  
(Eq. 3.5-1)
\[ 23/74.80+1\times92/(275.85\times0.957) = 0.307+0.349 = 0.656 < 1.0 \text{ OK} \]
\[ P_u/\phi_p P + M_{ux} / \phi_b M_{nx} \leq 1.0 \]  
(Eq. 3.5-2)
\[ 23/78.20+92/275.85 = 0.294+0.334 = 0.628 < 1.0 \text{ OK} \]

Therefore the section is adequate for the applied loads.
EXAMPLE 23.2

Rework Example 23.1 by using the Allowable Stress Design (ASD) method to check the adequacy of a tubular section (Fig. 23.1) to be used as a compression member.

Solution

1. Full section properties are the same as those calculated in Example 23.1.

2. Determination of $P_a$

   The following results are obtained from Example 23.1.(2).
   
   $F_n = \frac{(n^2 \times 6600)}{(37.36)^2} = 46.67$ ksi
   
   $A_e = 1.886 \text{ in.}^2$
   
   $P_n = F_n A_e = 46.67 \times 1.886 = 88.0$ kips
   
   $\Omega = 2.15$
   
   $P_a = \frac{P_n}{\Omega} = \frac{88.0}{2.15} = 40.93$ kips

3. $P = 15$ kips

   $P/P_a = \frac{15.0}{40.93} = 0.366 > 0.15$

   Must check both interaction equations as follows:
   
   $\frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} \leq 1.0$
   
   $\frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} \leq 1.0$

4. Determination of $P_{ao}$

   $P_{no} = A F_n e y$

298
= 1.84x50 = 92.0 kips

Ω = 2.15

\[ P_{ao} = P_{no}/Ω = 92.0/2.15 = 42.79 \text{ kips} \]

5. Determination of \( M_x \) and \( M_y \)

\( \varepsilon_y = 4.00 \text{ in.}, \varepsilon_x = 0 \)

\[ M_x = 15.0(4.00) = 60.0 \text{ kips-in. (Required Flexural Strength)} \]

\[ M_y = 0 \]

6. Determination of \( M_{ax} \)

\( M_{ax} \) shall be taken as the smaller of the allowable flexural strengths calculated according to sections 3.3.1.1 and 3.3.1.2:

a. Section 3.3.1.1: \( M_{ax} \) will be calculated on the basis of initiation of yielding.

\[ S_e = 27.494/4.485 = 6.130 \text{ in.}^3 \text{ (from Example 23.1)} \]

\[ M_{nx} = S_e F_y = 6.130(50) = 306.50 \text{ kips-in.} \]

\[ Ω = 1.85 \]

\[ M_{ax} = M_{nx}/Ω = 306.50/1.85 = 165.68 \text{ kips-in.} \]

b. Section 3.3.1.2: \( M_{ax} \) will be calculated on the basis of the lateral buckling strength. However for this square tube (close box-type member) the provision of Section 3.3.1.2 do not apply.

Therefore,

\[ M_{ax} = 165.68 \text{ kips-in.} \]

299
7. \( C_{m} = 0.6 - 0.4 \left( \frac{M_{1}}{M_{2}} \right) \geq 0.4 \)

\[ \frac{M_{1}}{M_{2}} = -1.00 \text{ (single curvature)} \]

0.6 - 0.4(-1.00) = 1.00 > 0.4

\( C_{m} = 1.00 \)

8. Determination of 1/\( a_{nx} \):

\[ \Omega = 2.15 \]

\[ P_{cr} = \frac{n^{2}E_{I_{x}}}{(K_{x}L_{x})^{2}} \]

\[ = \left( \frac{n^{2}(27000)(33.759)}{(120)^{2}} \right) = 624.73 \text{ kips} \]

\[ 1/a_{nx} = 1/\left[1-(\Omega_{c}P/P_{cr})\right] \]

\[ = 1/\left[1-(2.15\times2.1/624.73)\right] = 1/0.948 \]

\[ a_{nx} = 0.948 \]

13. Check interaction equations:

\[ 15.0/40.93+1.00\times60.0/(165.68\times0.948) \]

\[ = 0.366+0.382 = 0.748 < 1.0 \text{ OK} \]

\[ P/P_{ao} + M_{x}/M_{ax} \leq 1.0 \]

\[ 15.0/42.79+60.9/165.68 = 0.351+0.362 = 0.713 < 1.0 \text{ OK} \]

Therefore the section is adequate for the applied loads.
EXAMPLE 24.1 FLAT SECTION w/BOLTED CONNECTION (LRFD)

Determine the maximum design strength, $\phi P_n$, for the bolted connection shown in Fig. 24.1. Use two 1/2 in. diameter hot-finished, Type 316 bolts with washers under both bolt head and nut. The plates are Type 304, 1/16-Hard, stainless steel.

Solution:

1. Design strength based on spacing and edge distance (Section 5.3.1)

$$P_n = \phi t e F_u$$

(Eq. 5.3.1-1)
e = 1.0 in.

\[ F_u = 80 \text{ ksi (from Table A16 of the Standard)} \]

\[ P_n = 0.105(1)(80) = 8.40 \text{ kips/bolt} \]

\[ \Phi P_n = 0.7(2 \text{ bolts})(8.40 \text{ kips/bolt}) = 11.76 \text{ kips} \]

Distance between bolt hole centers must be greater than 3d.

\[ 3d = 3(0.5) = 1.5 \text{ in.} < 2 \text{ in. OK} \]

Distance between bolt hole center and edge of connecting member must be greater than 1.5d.

\[ 1.5d = 1.5(0.5) = 0.75 \text{ in.} < 1 \text{ in. OK} \]

2. Design strength based on tension on net section.

Required tension strength on net section of bolted connection shall not exceed \( \phi_T T_n \) from Section 3.2:

\[ A_n - \text{ based on Table 5} \]

\[ A_n = 0.105 \cfrac{4-2(1/2+1/16)}{2(1/2+1/16)} = 0.302 \text{ in.}^2 \]

\[ F_y = 45 \text{ (from Table A1 of the Standard)} \]

\[ T_n = A_n F_y \]

\[ = (0.302)(45) = 13.59 \text{ kips} \]

\[ \phi_T = 0.85 \]

\[ \phi_T T_n = 0.85(13.59) = 11.55 \text{ kips} \]

or \( \Phi P_n \) from Section 5.3.2:

\[ P_n = (1.0-r+2.5rd/s)F_u A_n \leq F_u A_n \]

where in this case:

\[ r = 2(\phi P_n/2)/\phi P_n = 1 \]

\[ d = 0.5 \text{ in.} \]
\[ s = 2 \text{ in.} \]
\[ P_n = \left[1.0 - (1 + 2.5(1)(0.5)/2\right](80)(0.302) \]
\[ = 15.10 \text{ kips} < 80(0.302) = 24.16 \text{ kips} \text{ OK} \]
\[ \phi = 0.70 \text{ for single shear connection} \]
\[ \phi P_n = 0.70(15.10) = 10.57 \text{ kips} \]

Therefore, design strength based on tension on net section is 10.57 kips.

3. Design strength based on bearing (Section 5.3.3)

For single shear with washers under bolt head and nut, the design bearing strength \( \Phi P_n \) is:
\[ \phi = 0.65 \]
\[ P_n = 2.00F_u dt = 2.00(80)(0.5)(0.105) = 8.4 \text{ kips/bolt} \]
\[ \phi P_n = 0.65(2 \text{ bolts})(8.4 \text{ kips/bolt}) = 10.92 \text{ kips} \]

4. Design strength based on bolt shear (Section 5.3.4)

\[ P_n = A_b F_n \quad (\text{Eq. 5.3.4-1}) \]
\[ A_b = (\pi/4)(0.5)^2 = 0.196 \text{ in.}^2 \]
\[ F_n = F_{nv} = 45 \text{ ksi (Table 6, for no threads in shear plane)} \]
\[ P_n = (45)(0.196) = 8.82 \text{ kips/bolt} \]
\[ \phi = 0.65 \]
\[ \phi P_n = 0.65(2 \text{ bolts})(8.82 \text{ kips/bolt}) = 11.47 \text{ kips} \]

5. Comparing the values from 1, 2, 3, and 4 above, the design tensile strength on the net section of the connected part controls and thus,
\[ \phi P_n = 10.57 \text{ kips} \]
EXAMPLE 24.2 FLAT SECTION w/BOLTED CONNECTION (ASD)

Rework Example 24.1 to determine the maximum allowable load, \( P_a \)

Solution:

1. Allowable load based on spacing and edge distance

\[ P_n = 0.105(1)(80) = 8.40 \text{ kips/bolt (from Example 24.1.(1))} \]

\[ \Omega = 2.40 \text{ (Table E of the Standard)} \]

\[ P_a = \frac{(2 \text{ bolts})(8.40 \text{ kips/bolt})}{(2.40)} = 7.0 \text{ kips} \]

Distance between bolt hole centers must be greater than 3d.

\[ 3d = 3(0.5) = 1.5 \text{ in.} < 2 \text{ in. OK} \]

Distance between bolt hole center and edge of connecting member must be greater than 1.5d.

\[ 1.5d = 1.5(0.5) = 0.75 \text{ in.} < 1 \text{ in. OK} \]

2. Allowable load based on tension on net section.

Required tension strength on net section of bolted connection shall not exceed \( \phi_T T_n \) from Section 3.2:

\[ T_n = A_n F_y = (0.302)(45) = 13.59 \text{ kips (Example 24.1)} \]

\[ \Omega = 1.85 \]

\[ T_a = \frac{13.59}{1.85} = 7.35 \text{ kips} \]

or \( P_n \) from Section 5.3.2:

\[ P_n = (1.0-r+2.5rd/s)F_{u_n} \leq F_{A_n} \]

\[ = \frac{[1.0-(1)+2.5(1)(0.5)/2](80)(0.302)}{2.40} \]

\[ = 15.10 \text{ kips} < 80(0.302) = 24.16 \text{ kips (Example 24.1)} \]

\[ \Omega = 2.40 \]

\[ P_a = \frac{15.10}{2.40} = 6.29 \text{ kips} \]
Therefore, allowable load based on tension on net section is 6.29 kips.

3. Allowable load based on bearing

For single shear with washers under bolt head and nut, the design bearing strength $\Phi P_n$ is: (Example 24.1)

\[
P_n = 2.00F_u \frac{d}{t} = 2.00(80)(0.5)(0.105) = 8.4 \text{ kips/bolt}
\]

\[
\Omega = 2.40
\]

\[
P_a = \frac{(2 \text{ bolts})(8.4 \text{ kips/bolt})}{2.40} = 7.0 \text{ kips}
\]

4. Allowable load based on bolt shear

\[
P_n = A_b F_n
\]

\[
= (45)(0.196) = 8.82 \text{ kips/bolt (Example 24.1)}
\]

\[
\Omega = 3.0
\]

\[
P_a = \frac{(2 \text{ bolts})(8.82 \text{ kips/bolt})}{3.0} = 5.88 \text{ kips}
\]

5. Comparing the values from 1, 2, 3, and 4 above, the allowable load based on bolt shear strength controls and thus,

\[
P_a = 5.88 \text{ kips}
\]
EXAMPLE 25.1 FLAT SECTION w/LAP FILLET WELDED CONNECTION (LRFD)

Using the Load and Resistance Factor Design (LRFD) criteria, check to see if longitudinal fillet welded connection shown in Fig. 25.1 is adequate to transmit a factored load \( F = 4.5 \) kips. Assume that Type 301, 1/4-Hard, stainless steel sheet and E308 electrode are to be used.

Solution:

1. Design Strength for Weld Sheet.

\[
L/t = \frac{2}{0.06} = 33.33 > 30
\]

For \( L/t \geq 30 \),

\[
\phi = 0.55
\]
\[ P_n = 0.43tLF \]  
\[ \Phi P_n = 0.55(4.64) = 2.55 \text{ kips/weld} \]  
\[ (2.55 \text{ kips/weld})(2 \text{ welds}) = 5.1 \text{ kips} > 4.5 \text{ kips OK} \]

2. Design Strength for Weld Metal.

\[ \Phi = 0.55 \]

\[ P_n = 0.75t_wLF_{xx} \]  
\[ t_w = 0.707(0.0625) = 0.044 \text{ in.} \]

\[ F_{xx} = 80 \text{ ksi (from Table A15 of the Standard)} \]

\[ P_n = 0.75(0.044)(2)(80) = 5.28 \text{ kips} \]

\[ \Phi P_n = 0.55(5.28) = 2.90 \text{ kips/weld} \]

\[ (2.90 \text{ kips/weld})(2 \text{ welds}) = 5.80 \text{ kips} > 4.5 \text{ kips OK} \]
EXAMPLE 25.2 FLAT SECTION w/LAP FILLET WELDED CONNECTION (ASD)

Using the Allowable Stress Design (ASD) method, check to see if longitudinal fillet welded connection shown in Fig. 25.1 is adequate to transmit a total load $F = 3.5$ kips. Assume that Type 301, 1/4-Hard, stainless steel sheet and E308 electrode are to be used.

Solution:

1. Allowable load for Weld Sheet.

$$P_n = 0.43tLF_{ua} \quad \text{(Example 25.1)}$$

$$= 0.43(0.06)(2)(90) = 4.64 \text{ kips/weld}$$

$$\Omega = 2.50 \quad \text{(Table E of the Standard)}$$

$$P_a = 4.64 \times 2 / 2.50 = 3.71 \text{ kips} > 3.5 \text{ kips OK}$$

2. Allowable load for Weld Metal.

$$\Omega = 2.50$$

$$P_n = 0.75(0.044)(2)(80) = 5.28 \text{ kips/weld} \quad \text{(Example 25.1)}$$

$$P_a = 5.28 \times 2 / 2.50 = 4.22 \text{ kips} > 3.5 \text{ kips OK}$$
EXAMPLE 26.1 FLAT SECTION w/GROOVE WELDED CONNECTION IN BUTT JOINT (LRFD)

Determine the design tensile strength, $\phi P_n$, normal to the effective area of the groove welded connection as shown in Fig. 26.1. Use Type 304, annealed, stainless steel and E308 electrode.

![Diagram of welded connection](image)

$\text{Figure 26.1 Welded Connection for Example 26.1}$

Solution:

Determination of the design tensile strength, $\phi P_n$, normal to the effective area provided that the effective throat equal to the thickness of the welded sheet. (Section 5.2.1).

$$P_n = LtF_{ua}$$  (Eq. 5.2.1-1)

$F_{ua} = 75 \text{ ksi (Table A16 of the Standard)}$

$F_{xx} = 80 \text{ ksi (Table A15 of the Standard)}$

The minimum tensile strength for weld metal is larger than that the base metal. OK
\[ P_n = (8.000)(0.135)(75) \]
\[ = 81.00 \text{ kips} \]
\[ \phi = 0.60 \]
\[ \phi(P_n) = 0.60 \times 81.00 \]
\[ = 48.60 \text{ kips} \]
EXAMPLE 26.2 FLAT SECTION w/GROOVE WELDED CONNECTION IN BUTT JOINT (ASD)

Rework Example 26.1 to determine the allowable tensile load, $P_a$, normal to the effective area of the groove welded connection.

Solution:

Determination of the allowable tensile load, $P_a$, normal to the effective area.

$P_n = (8.000)(0.135)(75)$

$= 81.00$ kips (Example 26.1)

$F_{xx} = 80$ ksi $> F_{ua} = 75$ ksi OK

$\Omega = 2.50$ (Table E of the Standard)

$\left( P_a \right)_1 = 81.00/2.50 = 32.4$ kips
EXAMPLE 27.1 BUILT-UP SECTION - CONNECTING TWO CHANNELS (LRFD)

By using the LRFD criteria, determine the maximum permissible longitudinal spacing of connectors joining two channels to form an I-section (Fig. 27.1) to be used as a compression member with unbraced length of 12 ft. Also design resistance welds connecting the two channels to form an I-section used as a beam with the following load, span, and support conditions: (a) Span: 10'-0"; (b) Total uniformly distributed factored load including factored dead load: 0.520 kips per lin. ft.; and (c) Length of bearing at end support: 3 in. Use Type 304, 1/4-Hard, stainless steel.

Solution:

1. Maximum longitudinal spacing of connectors for compression member

   Section 4.1.1(1).

   For compression members, the maximum permissible longitudinal spacing of connectors is

   \[ s_{\text{max}} = \frac{Lr_{cy}}{(2r_f)} \]  

   (Eq. 4.1.1-1)
where
\[ r_{cy} = \text{radius of gyration of one channel about its centroidal axis parallel to web.} \]
\[ r_i = \text{radius of gyration of I-section about axis perpendicular to direction in which buckling would occur for given conditions of end support and intermediate bracing.} \]

The following equations used for computing the sectional properties for channel with lips are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.

Basic parameters used for calculating the section properties of a channel section with lips: (For parameter designations, see Fig. 22.1)

\[ r = R+t/2 = \frac{3}{32}+0.060/2 = 0.124 \text{ in.} \]

From the sketch \( a = 5.692 \text{ in.}, \ b = 1.317 \text{ in.}, \ c = 0.296 \text{ in.}, \)
\( A' = 6.0 \text{ in.}, \ B' = 1.625 \text{ in.}, \ C' = 0.45 \text{ in.}, \)
\( a = 1.00 \) (Since the section has lips)
\[ \bar{a} = A'-t = 6.0-0.060 = 5.94 \text{ in.} \]
\[ \bar{b} = B'-(t/2+a/2) = B'-t = 1.625-0.06 = 1.565 \text{ in.} \]
\[ \bar{c} = a(C'-t/2) = C'-t/2 = 0.45-0.06/2 = 0.42 \text{ in.} \]
\[ u = 1.57r = 1.57 \times 0.124 = 0.195 \text{ in.} \]

a. Area:
\[ A = t[a+2b+2u+a(2c+2u)] = t[a+2b+2c+4u] \]
\[ = 0.06[5.692+2\times1.317+2\times0.296+4\times0.195] \]
\[ = 0.582 \text{ in.}^2 \]
b. Moment of inertia about x-axis:
\[
I_x = 2t\left[0.0417a^3 + b(a/2+r)^2 + u(a/2+0.637r)^2 + 0.149r^3 \right.
+ \alpha \left[0.0833c^3 + (c/4)(a-c)^2 + u(a/2+0.637r)^2 + 0.149r^3 \right]
\]
\[
= 2t\left[0.0417a^3 + b(a/2+r)^2 + 2u(a/2+0.637r)^2 + 0.298r^3 
+ 0.0833c^3 + (c/4)(a-c)^2 \right]
\]
\[
= 2x0.06\left[0.0417(5.692)^3 + 1.317(5.692/2+0.124)^2
+ 2x0.195(5.692/2+0.637x0.124)^2 + 0.298(0.124)^3
+ 0.0833(0.296)^3 + (0.296/4)(5.692-0.296)^2 \right]
\]
\[
= 2.976 \text{ in.}^4
\]

c. Distance from centroid of section to centerline of web:
\[
\bar{x} = \frac{(2t/A)\left[b(b/2+r) + u(0.363r) + \alpha \left[u(b+1.637r) + c(b+2r) \right] \right]}{\left[ (2x0.06)/0.582 \right][1.317(1.317/2+0.124) + 0.195(0.363x0.124)
+ 0.195(1.317+1.637x0.124) + 0.296(1.317+2x0.124)]}
\]
\[
= 0.371 \text{ in.}
\]

d. Moment of inertia about y-axis:
\[
I_y = 2t\left[b(b/2+r)^2 + 0.0833b^2 + 0.356r^2
+ \alpha \left[c(b+2r)^2 \right. \right.
+ u(b+1.637r)^2 + 0.149r^3 \left. \right]
\]
\[
= 2x0.06\left[1.317(1.317/2+0.124)^2 + 0.0833(1.317)^3
+ 0.356(0.124)^2 + 0.296(1.317+2x0.124)^2
+ 0.195(1.317+1.637x0.124)^2 + 0.149(0.124)^3 \right]-0.582(0.371)^2
\]
\[
= 0.181 \text{ in.}^4
\]

e. Distance from shear center to centerline of web:
\[
\bar{m} = \left(\frac{bt}{12I_x} \right)6\bar{c}(\bar{a})^2 + 3\bar{b}(\bar{a})^2 - 8(\bar{c})^3
\]
\[
= \left[ (1.565x0.06)/(12x2.976) \right][6x0.42(5.94)^2
\]
\[
= 314
\]
Based on the above information, the section properties of I-section composed of two channels can be determined as follows:

\[ I = 2 \times 2.976 = 5.952 \text{ in}^4 \]
\[ A = 2 \times 0.582 = 1.164 \text{ in}^2 \]
\[ r_x = \sqrt{\frac{I}{A}} = \sqrt{\frac{5.952}{1.164}} = 2.26 \text{ in} \]
\[ I_y = 2 \times 0.81 + 0.582 \times (0.371 + 0.06/2)^2 = 0.549 \text{ in}^2 \]
\[ r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{0.549}{1.164}} = 0.687 \text{ in} < r_x \]

Therefore, \( r = r_y = 0.687 \text{ in} \)

\[ s_{max} = (12 \times 12) \times 0.558 / (2 \times 0.687) = 58.48 \text{ in.} \]

Therefore, the maximum spacing of connectors used for connecting these two channels as a compression member is 58 in.

2. Design resistance welds connecting the two channels to form an I-section used as a beam Section 4.1.1(2).

a. Spacing of welds between end supports:

The maximum permissible longitudinal spacing of welds for a flexural member is

\[ s_{max} = \frac{L}{6} \quad \text{(Eq. 4.1.1-2)} \]

\[ = 12 \times 10 / 6 = 20 \text{ in.} \]

Maximum spacing is also limited by
\[ s_{\text{max}} = \frac{2gT_s}{mq} \]  
\text{(Eq. 4.1.1-3)}

\begin{align*}
\text{in which} \\
g &= 5.0 \text{ in. (assumed for 6 in. deep section)} \\
T_s &= 0.60 \times 2.27 \times 0.25 = 0.341 \text{ kips (Section 5.2.3)} \\
m &= 0.668 \text{ in. (from above-calculated value)} \\
q &= 3 \times 0.520/12 = 0.130 \text{ kips per lin. in.}
\end{align*}

Therefore
\[ s_{\text{max}} = \frac{2 \times 5 \times 0.341}{(0.668 \times 0.130)} = 39.27 \text{ in.} \]

\[ s_{\text{max}} = L/6 \text{ controls. Use a spacing of 20 in. throughout the span.} \]

b. Strength of welds at end supports:

Since the weld spacing is larger than the bearing length of 3.0 in., the required design strength of the welds directly at the reaction is

\[ T_s = \frac{P_m}{2g} \]  
\text{(Eq. 4.1.1-5)}

\[ = 0.520 \times 5 \times 0.668/(2 \times 5) = 0.174 \text{ kips} \]

which is less than 0.341 kips as provided. OK
EXAMPLE 27.2 BUILT-UP SECTION - CONNECTING TWO CHANNELS (ASD)

Rework Example 27.1 for the same given data by using the ASD method. Assume that the applied uniform load is 0.4 kips/ft for the I-section used as a beam.

Solution:

1. Maximum longitudinal spacing of connectors for compression member

   Section 4.1.1(1).

   For compression members, the maximum permissible longitudinal spacing of connectors is

   \[ s_{\text{max}} = \frac{Lr_{\text{cy}}}{2r_I} \]

   \[ = \frac{(12 \times 12) \times 0.558}{2 \times 0.687} = 58.48 \text{ in.} \]

   Refer to Example 27.1 for the section properties used to calculate \( s_{\text{max}} \). The maximum spacing of connectors used for connecting these two channels as a compression member is 58 in.

2. Design resistance welds connecting the two channels to form an I-section used as a beam Section 4.1.1(2).

   a. Spacing of welds between end supports:

      The maximum permissible longitudinal spacing of welds for a flexural member is

      \[ s_{\text{max}} = \frac{L}{6} = \frac{12 \times 10}{6} = 20 \text{ in.} \]

      Maximum spacing is also limited by

      \[ s_{\text{max}} = \frac{2gT_s}{(mq)} \]

      in which

      \[ g = 5.0 \text{ in. (assumed for 6 in. deep section)} \]

      \[ T_s = \frac{(0.25 \times 2.27)}{2.50} = 0.227 \text{ kips} \]

      \[ m = 0.668 \text{ in. (from Example 24.1)} \]
\[ q = 3 \times 0.40 / 12 = 0.10 \text{ kips per lin. in.} \]

Therefore

\[ s_{\text{max}} = 2 \times 5.0 \times 0.227 / (0.668 \times 0.10) = 33.98 \text{ in.} \]

\[ s_{\text{max}} = L/6 \text{ controls. Use a spacing of 20 in. throughout the span.} \]

b. Strength of welds at end supports:

Since the weld spacing is larger than the bearing length of 3.0 in., the required design strength of the welds directly at the reaction is

\[ T_s = P_m / (2g) \]

\[ = 0.40 \times 5 \times 0.668 / (2 \times 5) = 0.134 \text{ kips} \]

which is less than 0.227 kips as provided. OK