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ANALYSIS OF LOCALLY BUCKLED
THIN-WALLED COLUMNS

By Gale P. Mul I Igan and Teoman Pekoz

ABSTRACT

An effective section method is presented for analyzing the effects of local buckling on the overall modes of behavior of singly symmetric, thin-walled columns and beam-columns. The post-local buckling strength of the component plate elements is recognized through effective width concepts. A single effective width equation is developed to represent the sub-ultimate behavior under service loads and the ultimate strength of uniformly compressed stiffened elements. Also, an effective width approach is proposed for predicting the response of eccentrically compressed stiffened elements. An experimental investigation is reported on cold-formed, lipped channel, steel columns with concentric or eccentric loading. Predictions obtained with the presented analysis method are in good correspondence with experimental sub-ultimate deflections and ultimate strength. In contrast to this, current design methods are shown to be inadequate for predicting the experimental results. In certain cases, laterally unbraced (edge) stiffened elements are prone to an elastic local-torsional failure mode. A limiting stress approach is proposed for this situation which provides fair agreement with limited experimental data.

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INTRODUCTION

Local buckling of component plate elements of thin-walled columns is a primary consideration for analysis. While this has an adverse effect on the overall structural behavior, it does not generally result in ultimate conditions, and the plate elements can sustain additional load into the post-buckling range before failure. This interaction between local buckling of plate elements and overall modes of behavior for columns is addressed in the present paper. The specific application considered is the analysis of cold-formed, singly symmetric, slender, steel columns under concentric or eccentric compressive axial load.

Previous research (5,12) has been conducted in this area, but the developed analytical methods proved impractical for routine application. Hence, researchers (3,8,15) proposed analysis methods wherein local buckling effects, including post-buckling strength, were accounted for through an effective width concept (21). Generally, these methods provided good correlation with experimental strengths but proved inadequate for predicting sub-ultimate conditions, e.g., service load deflections (2,9,10). Also, the methods had limited applicability because only one plate element, under uniform compression, was subject to local buckling.

A natural extension of the effective width concept to an effective (cross) section concept was proposed in Refs. (18,10) where all elements were subject to local buckling. This concept was employed with a rather complex analysis method and showed good correlation with experimental results for concentrically loaded columns.

The objective of the present paper is to propose a design-oriented effective section method for analyzing the effects of local buckling on the overall modes of behavior of thin-walled columns. While the method is general, it is
specialized here to lipped channel shapes. Effective width approaches are then developed for predicting the sub-ultimate behavior of uniformly compressed stiffened elements (plates supported along edges parallel to the direction of loading) and for predicting the response of eccentrically compressed stiffened elements. Next, an experimental investigation of concentrically and eccentrically loaded, cold-formed steel lipped channels is reported. Then, to justify the analysis method, a comparison is made between predicted and experimental results, for both sub-ultimate and ultimate conditions. Current design methods are also evaluated.

**ANALYSIS METHOD**

It is well known that local buckling reduces the compressive stiffness and strength of a column. However, local buckling can also influence the failure mode of a column; for example, it can cause a concentrically loaded, singly symmetric column to fail by flexural yielding, instead of by flexural buckling.

In the present study, these effects are accounted for in a beam-column analysis method and an effective section concept. The objective is to predict the behavior and strength of a locally buckled column under axial loading, or bending in the plane of symmetry.

**Problem Statement.** - Fig. 1 shows a cross section of a slender lipped channel column, of length L, and identifies the component plate elements. The plate flat widths are denoted as $w_n$, where $n$ equal to 1, 2, and 3 refers to the web, flange, and lip elements, respectively. For generality, the axial load $P$ is defined as eccentric with respect to the gross centroid (c.g.), at an eccentricity $e$. Only bending in the plane of symmetry ($x$ axis) is considered.

After local buckling, the section becomes partially effective, which is indicated in Fig. 1 by the solid portions of the section. Correspondingly the centroid or neutral axis shifts to an effective location (eff. c.g.) defined by $e'$. These changes occur gradually and are dependent on the applied load level.
Basic Assumptions and Equations. - To simplify the analysis, it is assumed that the effective section, see Fig. 1, is of prismatic form. Thus, from elastic beam theory and for pinned-end column conditions, the stresses at the midheight of the deformed beam-column, and along reference axes \( k \) (\( k = 1 \) and \( k = 2 \) correspond to an axis through the web and lip, respectively), are determined from the derived equilibrium equations (19)

\[
f_k = \frac{P}{A_e} \pm \left( \frac{P e c_k / l_e}{I_e} \right) \sec(pL/2) \quad \ldots k = 1,2
\]

where \( p = \sqrt{P/(E_l e)} \), \( E \) = Young's modulus, and the plus sign for \( k = 1 \). A compressive stress is positive, as is the eccentricity \( e_e \) when the load is positioned to the left of the effective center of gravity. The associated lateral deflection, \( \Delta \) (positive in the \( +x \) direction), at the midheight of the column is given by

\[
\Delta = e_e \left[ 1 - \cos(pL/2) \right] / \cos(pL/2)
\]

It is noted that effective section properties are used in Eqs. 1 and 2 to account for the effects of local buckling, namely \( A_e \) and \( I_e \) are the effective area and moment of inertia, respectively, and \( c_k \) defines the appropriate distance from the effective centroid to the reference axis \( k \).

The effective section is assumed to be uniquely dependent on the stresses \( f_k \), i.e., constant along the length. Nevertheless, because of this dependence and the general form of Eq. 1, the two equations generated by Eq. 1 are inherently nonlinear and require an iterative solution strategy, which is discussed subsequently.

Furthermore, the maximum load resistance is assumed to be attained when the maximum stress level \( f_k \) reaches the yield strength (5), \( f_y \), or when unstable equilibrium is experienced, i.e., when the load \( P \) decreases with increased elastic axial straining. Also, the flanges (elements 2 of Fig. 1) are assumed to be supported against local-torsional buckling when the flange-lip juncture deflects in the lateral (\( y \)) direction. Relaxation of this last assumption,
which eliminates dependence of the analysis method on lip or edge stiffener requirements (16,2), is investigated later in this paper.

Solution Strategy. - In order to obtain a unique combination of axial load and stress that satisfies Eq. 1, it is necessary to make a behavioral assumption with respect to one of the variables: $P$, $f_1$, or $f_2$. This is accomplished with the following approach.

In the present application, where the web element (along reference axis 1) dominates the local buckling response, the effective centroid, $e_e$, is presumed to be located at a position in the positive $x$ direction, see Fig. 1. (If this is not the case, a modification of the approach is necessary.) Accordingly, the load-strain, $P-\varepsilon$, response is assumed to follow the general behavior indicated in Fig. 2, depending on the position of applied load. Note that because the material is linearly elastic and all nonlinearities are included in the effective section properties, stress and strain are used interchangeably, i.e., the relation $f = E\varepsilon$ is assumed valid.

The characteristic parameter $x_y$, which has a positive value, delineates the response and is defined as the distance between the gross and effective centroids for a section under a uniform compressive yield stress, e.g., $f_1 = f_2 = f_y$.

For the cases of Figs. 2a and c, the strain on the concave side of the column increases continually with load until failure, which is indicated at the yield strain, $\varepsilon_y$. These cases admit the possibility of a reversal for the strain on the convex side of the column.

The remaining case of Fig. 2b covers negative eccentricities with an absolute value less than $x_y$. For this case, the strain along reference axis 2, $\varepsilon_2$, may reverse before yielding. Also, failure may occur in the elastic unloading mode discussed previously, which is not shown in the figure. The parameter $\varepsilon$ refers to the uniform strain state.
Given the behavioral assumptions of Fig. 2, it is possible to construct an incremental-iterative algorithm to solve Eq. 1 which allows the complete load-strain (stress) history to be traced. In general, the strain on the concave side of the column is chosen as the incremental strain, or independent variable, which is indicated by arrows in the figure. A slightly different approach is followed for load eccentricities in the range indicated in Fig. 2b. For this case, the incremental strain is switched from \( \varepsilon_2 \) to \( \varepsilon_1 \), at \( \varepsilon_2 = \varepsilon \), because of a possible strain reversal. The uniform compressive strain, \( \varepsilon \), is solved for using Eq. 1 and a bisection searching algorithm (6). In either case, the incremental strain is continually increased in increments corresponding to a stress of 1 ksi (6.89 N/mm²). Then, for a fixed total level for the incremental strain, the remaining unknown strain (stress) and load, \( P \), are calculated from Eq. 1 using direct iteration and the secant method (6), respectively. Once convergence is obtained in the iteration loop, for a fixed level of the incremental strain, the associated lateral deflection is obtained from Eq. 2.

**Effective Section.** - The effective section is defined from largely empirical effective width equations which are applied to each of the component plate elements. These equations are developed and discussed in the following sections according to the plate’s compressive (displacement) loading condition.

**Effective Width of Uniformly Compressed Stiffened Element.** - (Web element 1 of Fig. 1) The following equation forms the basis used in the American Iron and Steel Institute (AISI) Specification (16) for predicting the effective width of stiffened elements, \( w_{1e} \) (21).

\[
\frac{w_{1e}}{w_1} = \sqrt{\frac{f_{cr1}}{f_1}} (1 - 0.218 \sqrt{\frac{f_{cr1}}{f_1}})
\]

(3)

where \( f_1 \) = the edge stress. Eq. 3 is valid for

\[
f_1 > 0.461 f_{cr1}
\]

(4)

where the critical buckling stress, \( f_{cr1} \), is defined from

\[
f_{crn} = \frac{K_n t^2 E}{[12(1 - \nu^2)(w_n/t)^2]}
\]

(5)
with \( n = 1 \), the buckling coefficient \( K_1 = 4 \), and \( v \) = Poisson's ratio. Eq. 3 has proved very successful for predicting ultimate conditions, when \( f_1 = f_y \); however it has been shown to be inadequate in the sub-ultimate range, when \( f_1 < f_y \) (2,9,10). This problem has been studied (1,18), but the developed methods required two separate effective width equations to represent sub-ultimate and ultimate conditions.

In contrast, a single effective width equation is proposed below for predicting the sub-ultimate and ultimate response of uniformly compressed stiffened elements. The effective width \( w_{1e} \) is given by

\[
\frac{w_{1e}}{w_1} = C_1 \lambda^2 + C_2 / \lambda + C_3 + C_4 \lambda
\]

(6)

Eq. 6 is derived in Appendix 1, where the constants \( C_1 - C_4 \) are defined, following the approach of Ref. (18) to simulate theoretical and empirical behavior for imperfect plates.

Fig. 3 compares the response predicted with Eqs. 6 and 3 for a specific value of \( \lambda_L \) equal to ten, where

\[
\lambda_L = \sqrt{f_y / f_{cr1}}
\]

(8)

The predicted compressive stress-strain response of Fig. 3a is based on the following definition of effective width.

\[
\frac{w_{1e}}{w_1} = \frac{f_{av}}{f_1}
\]

(9)

where \( f_{av} \) = the average stress across the element. Fig. 3a shows that, in the sub-ultimate range, better agreement with theoretical elastic response for an imperfect plate is obtained from Eq. 6 than from Eq. 3. Also, this figure illustrates that Eq. 6 predicts ultimate conditions at the point, \( y \), of yielding, i.e., the maximum value of \( f_{av} \) occurs at \( \varepsilon_1 = \varepsilon_y \). Furthermore, Eq. 6 is demonstrated in Fig. 3 to be consistent with Eq. 3 at the transition point, \( f \), from fully to partially effective, see Eq. 4, and at the ultimate point, \( y \).
**Effective Width of Eccentrically Compressed Stiffened Elements.** - (Flange elements 2 of Fig. 1) A modification of empirical Swedish research (18) is proposed for predicting the effective width of stiffened elements under a stress gradient. Fig. 4 depicts the situation. The generalized stresses $f_1$ and $f_j$ refer to the maximum and minimum edge stresses, respectively, and the corresponding effective widths are defined as $w_{2el}$ and $w_{2ej}$. Two cases are considered depending on whether the stress $f_j$ is compressive (positive) or tensile.

The effective widths are determined from:

$$w_{2el} = \frac{w_{2e}}{2}$$  \hspace{1cm} (10)

where $w_{2e}/w_2 = \sqrt{f_{cr2}/f_1 (1 - 0.218 \sqrt{f_{cr2}/f_1})}$  \hspace{1cm} (11)

for $f_j \geq 0$; $w_{2ej} = w_{2el} (1.5 - 0.5 f_j/f_1)$  \hspace{1cm} (12)

for $f_j < 0$; $w_{2ej} = 1.5 w_{2el} + w_0$  \hspace{1cm} (13)

where $w_0$ is defined in Fig. 4, and $i = 1$ and $j = 2$ for $f_1 > f_2$, and vice versa for $f_2 > f_1$. It is noted that the effects of the stress gradient are included in the approach, Eqs. 12, 13, but are ignored for the calculation of the critical buckling stress $f_{cr2}$, e.g., $K_2 = 4.0$ in Eq. 5. Finally, the total effective width is, of course, constrained to satisfy

$$w_{2el} + w_{2ej} \leq w_2$$  \hspace{1cm} (14)

The above approach is consistent with current design approaches (16), Eq. 11, for the limiting state of uniform compression, i.e., for $f_j/f_1 = 1.0$. It is also very similar to an approach proposed independently in Ref. (20), which has a theoretical basis.

**Effective Width of Uniformly Compressed Unstiffened Elements.** - (Lip elements 3 of Fig. 1) A unified approach is adopted to calculate the effective width $w_{3e}$, e.g.,

$$w_{3e}/w_3 = \sqrt{f_{cr3}/f_2 (1 - 0.218 \sqrt{f_{cr3}/f_2})}$$  \hspace{1cm} (15)

where $f_{cr3}$ is obtained from Eq. 5 with $K_3 = 0.425$. Eq. 15 was originally developed for stiffened elements, see Eq. 3, but has been recently shown to be
LOCALLY BUCKLED COLUMNS

applicable for unstiffened elements (plates with one unloaded edge free) as well
(8,9).

Limits on Ultimate Load Capacity. - In addition to failure by flexural
yielding, as predicted by Eq. 1, the ultimate load capacity of the column may be
limited by flexural or torsional-flexural buckling. In the present paper, the
prediction of the flexural buckling strength is based on an effective section
method and employs the modified Structural Stability Research Council (SSRC)
approach (3,8) to define the effective buckling stress, \( f_e' \), where

\[
\begin{align*}
\text{for } f_e & \geq f_y / 2; \quad f_e' = f_y - \left( \frac{f_y}{4f_e'} \right)^2 \\
\text{for } f_e & < f_y / 2; \quad f_e' = \frac{f_y^2}{(L/r_e)^2}
\end{align*}
\]

where \( r_e \) = the effective radius of gyration, about the y axis (Fig. 1), and \( f_e' \)
= the elastic flexural buckling stress. Because the effective section pro-
properties are dependent on the unknown stress \( f_e' \), an iterative solution method is
required. Once the stress \( f_e \) is determined, the flexural buckling load, \( P_e \), is
obtained from

\[ P_e = A_e f_e \]  (17)

A conservative approach is adopted to calculate the effective section
where Eq. 3 is applied to the web element. The effective widths for the flange
and lip elements are calculated as discussed previously. In these calculations,
It is noted that \( f_e = f_1 = f_2' \).

The prediction of the torsional-flexural buckling strength, \( P_{tf} \), is more
complicated; therefore the details of the calculations are not presented here,
and reference is made to Ref. (13). In brief, the torsional-flexural buckling
strength is based on the \( Q \)-factor method (16,14,21).

Typical Results. - Next, typical results predicted with the analysis
method are discussed through application to the two lipped channel columns of
Table 1.
For column C 120x60, the axial load-lateral deflection, \( P-\Delta \), response is shown in Fig. 5 for several load eccentricities, \( e \). The results for the initially concentric loading, \( e = 0 \), show that local buckling transforms purely axial to beam-column behavior. Moreover, this figure illustrates that it is possible to increase the ultimate load capacity by applying the load eccentrically. This aspect is considered further in Fig. 6 where the influence of eccentricity on ultimate strength, \( P_u \), is studied. The vertical line at \( e = -0.3 \) inches (7.6 mm) delineates the response for yielding about reference axis 1 or 2. Also, the load capacity is shown to be limited by torsional-flexural buckling at \( P_{tf} \).

Slightly different behavior is predicted for the relatively slender column C 180x60. In this case, the column is prone to an elastic failure for certain eccentricities in the range indicated in Fig. 2b. This behavior is illustrated in Fig. 7 where the load decreases, after reaching its ultimate value at \( P_u \), to the load associated with yielding, \( P_y \). The elastic failure mode is caused by an interaction of post-local buckling effects and beam-column action. Actually, the load capacity is limited by inelastic flexural buckling, at \( P_e \), but this is irrelevant to the present discussion.

**CURRENT DESIGN METHOD**

A current design method for predicting flexural buckling of concentrically loaded compression members will now be reviewed. The AISI Specification (16) employs the Q-factor method (21) to determine the flexural buckling stress \( f_f \). Thus

For \( f_f > Qf_y/2 \);

\[
f_f = \frac{Qf_y - (Qf_y)^2}{(4f_f')}
\]

(18a)

For \( f_f < Qf_y/2 \);

\[
f_f' = \frac{\pi^2 E}{(L/r)^2}
\]

(18b)

where \( f_f' \) is the elastic buckling stress. Eq. 18 assumes simply supported end conditions, and \( r \) is the radius of gyration of the gross cross section. The
effects of local buckling are accounted for through the strength reduction factor $Q$,

$$Q = \frac{A_e}{A}$$  \hspace{1cm} (19)

where $A_e$ and $A$ refer to the effective and gross areas, respectively. The associated ultimate load capacity is given by

$$P_f = Af_f$$  \hspace{1cm} (20)

The above approach is directly applicable to singly symmetric columns, e.g., lipped channels (16). Also, $A_e$ in Eq. 19 is determined conservatively for a uniform compressive stress at yield.

**EXPERIMENTAL INVESTIGATION**

An experimental investigation was conducted to study the influence of local buckling on column behavior and to provide quantitative data to support the analysis method described previously. In total, twenty-two lipped channel steel columns were tested to failure under concentric and eccentric loading conditions. Several different cross sections and column lengths were considered. The response for corresponding stub columns is studied elsewhere (13).

**Test Specimens and Properties.** - All specimens were fabricated by press-braking out of thin sheets of 18 gage (1.2 mm) steel. The average dimensions of the specimens are reported in Table 2, along with calculated values for the slenderness ratio, $L/r$, and the gross area, $A$. Note that the numbers in the specimen designation, e.g., 120x60, refer to the approximate width-to-thickness ratios $w_1/t$ and $w_2/t$. Also the lip dimensions, $w_3$, were proportioned to be adequate as defined in Ref. (2). The yield strengths, $f_y$, were obtained from standard tensile tests of virgin material. All material was sharp yielding.

As indicated in Table 2, several specimens employed braces or ties, L 1/2 x 1/2 x 0.05 inches (12.7 x 12.7 x 1.2 mm), which were welded from lip-to-lip and spaced at $w_1$, along the length. This extra flange support was provided to study its influence on the behavior and strength of the section.
Experimental Procedures. - Technical Memorandum No. 4 of the SSRC, "Procedure for Testing Centrally Loaded Columns," (7) was followed in part for the tests. Also, special end fixtures (14) were employed which provided a pinned-end condition about the weak axis (y axis of Fig. 1) and a fixed-end condition about the strong axis. The tests were conducted in a hydraulic testing machine using the static method where the load was stabilized at each load increment.

All column specimens were instrumented with longitudinal strain and transverse dial gages placed at the midheight of the column. The instrumentation was continuously monitored with a computer data acquisition system.

Special procedures (13) were used for column alignment because of the sensitivity of the response to load eccentricity. The alignment was checked by monitoring the "corner" membrane strains for a uniform or eccentric strain condition. If the relative strain error was less than 15%, then the alignment was considered satisfactory. Only the two specimens indicated in Table 2 failed to satisfy this criterion.

Failure Modes. - Generally, the failure mode for the specimens was by a gradual lateral deflection, without twisting of the cross section, and the subsequent formation of a localized kink; usually near the midheight. There were, however, exceptions to this.

Several specimens; C2.1 180x60 and C2, C3, and C2.1 180x90; exhibited significant twisting of the cross section which indicated torsional-flexural buckling. In one case, C2.1 180x90, the failure was sudden. Another specimen, C2.2 180x60, failed suddenly due to improper alignment, which affected the response adversely. A sudden failure mode was also experienced for test C1 90x90 where the unbraced flanges failed by an elastic local-torsional instability.
COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

Ultimate Strength of Concentrically Loaded Columns. - The concentrically loaded columns are evaluated first in Table 3 where the parameter $x$ defines the location of the gross centroid (Fig. 1). From a comparison of the experimental ultimate loads, $P_{u_{exp}}$, for duplicate specimens tested with and without braces, it can be seen that, generally, the use of braces had a favorable effect on strength. Note that evaluation of specimen C1 90x90 is temporarily omitted.

The flexural buckling strengths $P_f$ and $P_e$, predicted with Eqs. 20 and 17, are tabulated in Table 3 to determine the accuracy of the respective $Q$-factor method of the AISI Specification (16) and the modified SSRC method (3), which was recently proposed for specification adoption. The statistical evaluation presented at the bottom of the table indicates that, for the twelve (N) columns, both methods give nearly identical and unconservative (15%) predictions of ultimate strength. The reason for this unconservatism is that these methods do not recognize the actual beam-column behavior exhibited by the locally buckled columns.

On the other hand, the analysis method based on Eq. 1 recognizes the actual behavior of the columns. In this case, the associated ultimate loads are slightly conservative; a mean strength ratio $P_{u_{exp}}/P_u$ of 1.136 is obtained which deviates on the average by about 4%. It is noted that an elastic failure is predicted for the two slender columns C3 and C4 120x60.

Ultimate Strength of Eccentrically Loaded Columns. - A corresponding evaluation of the eccentrically loaded columns is more complicated. Several of the columns were loaded such that the load carrying capacity was amplified, as seen in Fig. 6; consequently the possibility of flexural or torsional-flexural buckling was increased.

Moreover, eccentric loading toward the lip causes a larger proportion of the load to be carried by the flange and edge stiffener and exaggerates the
possibility of local-torsional failure for unbraced flanges. The basic problem
with unbraced flanges is that, when the aspect ratio \((L/w_2)\) is large, the ever
present local initial imperfections interact with overall modes of action to
lower their structural strength. Related problems have been experienced for
webs of beams \((4,11,17)\).

A simple limiting stress approach is proposed below for this situation.
In order to prevent local-torsional buckling of the flanges, it is necessary to
limit the edge stress \(f_2\) to values less than the critical buckling stress \(f_{cr2}\),
\[
f_2 < f_{cr2} \tag{21}
\]
where \(f_{cr2}\) is defined by Eq. 5 with \(n=2\). The associated buckling coefficient \(K_2\)
is taken conservatively as 4.0. This approach assumes that the edge stiffener
is adequate as defined in Ref. (2) and that the effects of local buckling inter­
action with the web are negligible. When the equality of Eq. 21 is imposed on
the analysis method, the limiting stress ultimate load \(P_{ul}\) is obtained.

The eccentrically loaded columns are evaluated in Table 4 where the ex­
perimental eccentricity \(e\) and location of the neutral axis \(\bar{x}\) are also defined.
Again, the beneficial effect of the braces is shown from a comparison of the
experimental ultimate loads, \(P_{uexp}\), for "duplicate" specimens. Also, test C2.2
180x60 is eliminated from consideration because of improper alignment.

Torsional-flexural buckling is adequately predicted by the Q-factor method
for test C2.1 180x90. However, for test C2.1 180x60, failure is
unconservatively predicted by either the Q-factor method for torsional-flexural
buckling or by Eq. 17 for flexural buckling, both are nearly identical. Because
the column failed in a twisting mode, this suggests that research is needed on
the interaction of local and torsional-flexural buckling. Work in this area is
currently under way at Cornell University.
As Table 4 shows, excellent correlation is obtained between the experimental results and the analysis method based on Eq. 1, for the specimens with a predicted failure mode of flexural yielding. The mean strength ratio $P_{uexp}/P_u$ is 1.046 for the four tests.

Local-torsional flange failure is predicted at the ultimate load $P_{ul}$ for the remaining three columns, including specimen C1 90x90 of Table 3. The average predicted load is slightly conservative (13%), and the relatively large standard deviation (18%) is due to the effect of local initial imperfections which are quite random in nature.

**Sub-Ultimate Response.** - The sub-ultimate response predicted with the analysis method, Eqs. 1 and 2, is evaluated next. The discussion is necessarily brief due to space limitations; however the presented results are representative of the more thorough evaluation of Ref. (13).

For the concentrically loaded column C2 120x60, the load-lateral deflection response of Fig. 8 indicates that good agreement between predicted and experimental results is obtained when the sub-ultimate effective width of Eq. 6 is applied to the web. In contrast to this, excessively conservative predictions are obtained with the effective width of Eq. 3.

Fig. 9 presents the results for the eccentrically loaded column C2.4 120x60. In this case, unconservative deflections are predicted using Eq. 3 for the web, and good correspondence with experimental results is again obtained with Eq. 6.

**SUMMARY AND CONCLUSIONS**

An effective section method is presented for analyzing the effects of local buckling on the behavior and strength of singly symmetric thin-walled columns and beam-columns. The method recognizes the post-local buckling strength of the component plate elements and the associated shift of the centroid which causes additional bending stresses. It employs effective width
approaches for stiffened elements that are developed for predicting the sub­{}
ultimate behavior under uniform compression and the response to eccentric compression. The analysis method requires an iterative solution strategy but is otherwise straightforward. Overall, it represents a step toward the development of more unified and rational design methods.

The results predicted with the analysis method are in good agreement with experimental strengths and deflections obtained from tests of thin-walled lipped channel steel columns and beam-columns. In contrast to this, existing design methods are shown to be inadequate for representing the actual strength and stiffness.

When loading conditions and/or cross-sectional geometry are such that relatively high stress levels are carried by laterally unbraced compression flanges, they are prone to an elastic local-torsional failure mode. A limiting stress approach is proposed for this situation which provides fair correlation with limited experimental data.

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APPENDIX I. DERIVATION OF EFFECTIVE WIDTH EQ. 6

The following relationship is assumed between the average stress, $f_{av}$, and the edge stress $f_1$.

$$S = C_1 + C_2 \frac{C_3}{\lambda} + C_4 \lambda^3$$

(22)

where $S = f_{av}/f_{cr1}$ and $\lambda$ is given by Eq. 7. The constants $C_1$-$C_4$ are determined from the boundary conditions at the transition point $f$ and the ultimate point $y$, (Fig. 10).

Eq. 3 is assumed valid for establishing two boundary conditions (21). This equation is written in terms of $S$ and $\lambda$ as
\[ S_w = \lambda - 0.218 \]  
(23)

The intercepts of Eqs. 22 and 23 follow immediately as

\[ S(\lambda_f) = S_w(\lambda_f) \]  
(24)

\[ S(\lambda_y) = S_w(\lambda_y) \]  
(25)

where \( \lambda_y \) is defined by Eq. 8 and \( \lambda_f \) is determined from the equality of Eq. 4, and Eq. 7.

The slope at the transition point \( f \) is assumed consistent with the fully effective curve \( (S = \lambda^2) \), or

\[ \left( \frac{dS}{d\lambda} \right)_{\lambda_f} = 2\lambda_f \]  
(26)

Lastly, the slope at the ultimate point \( y \) is assumed as

\[ \left( \frac{dS}{d\lambda} \right)_{\lambda_y} = 0 \]  
(27)

which simulates the theoretical behavior (5) of a vanishing slope at the point of yielding (13).

Eqs. 24–27 permit explicit determination of the constants of Eq. 22 as

\[ C_4 = 2[(1 - \lambda_f)\lambda_y - 0.218]/(\lambda_f - \lambda_y)^3 \]  
(28)

\[ C_3 = \lambda_f/(\lambda_f - \lambda_y) - 1.5C_4(\lambda_f + \lambda_y) \]  
(29)

\[ C_2 = -\lambda_y(2C_3 + 3C_4\lambda_y) \]  
(30)

\[ C_1 = \lambda_f^2(1 - C_2/\lambda_f - C_3 - C_4\lambda_f) \]  
(31)

The effective width Eq. 6 is obtained from Eqs. 9 and 22.

APPENDIX II. - REFERENCES


15. Rhodes, J., and Loughlan, J., "Simple Design Analysis of Lipped Channel Columns," presented at the 1980, Fifth International Specialty Conference of Cold-Formed Steel Structures held at St. Louis, Mo.


APPENDIX III. - NOTATION

The following symbols are used in this paper:

$A$ = area;

$C$ = constant;

$c$ = distance from centroid to reference axis;

$E$ = Young's modulus = 29,500 ksi (203 kN/mm$^2$);

$e$ = eccentricity;

$f$ = stress;

$I$ = moment of inertia;

$K$ = buckling coefficient of Eq. 5;

$L$ = length;
OR = outside radius of corners;

P = axial load;

p = parameter of Eq. 1;

Q = strength reduction factor of Eq. 19;

r = radius of gyration;

S = stress ratio of Eq. 22;

\( t \) = thickness;

w = flat width;

\( \bar{x} \) = location of centroid, see Fig. 1;

\( \Delta \) = lateral deflection;

\( \varepsilon \) = strain;

\( \bar{\varepsilon} \) = uniform strain;

\( \lambda \) = stress ratio of Eq. 7; and

\( \nu \) = Poisson's ratio = 0.3.

**Subscripts**

av = average;

cr = critical;

e = effective or flexural;

exp = experimental;

f = transition from fully to partially effective, or flexural;

l = limiting;

tf = torsional-flexural;

u = ultimate; and

y = yielding.
<table>
<thead>
<tr>
<th>Column</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
<th>( w_3 )</th>
<th>( t_1 )</th>
<th>( t_2 )</th>
<th>( t_3 )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( f_y' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 120x60</td>
<td>5.827</td>
<td>2.855</td>
<td>0.546</td>
<td>0.047</td>
<td>0.163</td>
<td>75.23</td>
<td>63.75</td>
<td>0.639</td>
<td>32.06</td>
<td></td>
</tr>
<tr>
<td>C 180x60</td>
<td>8.803</td>
<td>2.873</td>
<td>0.558</td>
<td>0.047</td>
<td>0.163</td>
<td>95.07</td>
<td>83.20</td>
<td>0.781</td>
<td>33.70</td>
<td></td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm; 1 ksi = 6.89 N/mm²; Refer to Fig. 1; OR = outside radius of corners.
<table>
<thead>
<tr>
<th>Specimen</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>t</th>
<th>OR</th>
<th>L</th>
<th>A</th>
<th>$f_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>in</td>
<td>in</td>
<td>in</td>
<td>in</td>
<td>in</td>
<td>in</td>
<td>in</td>
<td>square inch</td>
</tr>
<tr>
<td>C1.1</td>
<td>120x30</td>
<td>5.832</td>
<td>1.378</td>
<td>0.231</td>
<td>0.048</td>
<td>0.131</td>
<td>19.96</td>
<td>36.35</td>
</tr>
<tr>
<td>C1</td>
<td>120x60</td>
<td>5.856</td>
<td>2.885</td>
<td>0.559</td>
<td>0.045</td>
<td>0.154</td>
<td>63.00</td>
<td>53.08</td>
</tr>
<tr>
<td>C2</td>
<td>120x60</td>
<td>5.837</td>
<td>2.888</td>
<td>0.519</td>
<td>0.045</td>
<td>0.152</td>
<td>75.02</td>
<td>63.66</td>
</tr>
<tr>
<td>C2.1</td>
<td>120x60</td>
<td>5.836</td>
<td>2.835</td>
<td>0.542</td>
<td>0.047</td>
<td>0.168</td>
<td>75.00</td>
<td>63.78</td>
</tr>
<tr>
<td>C2.2</td>
<td>120x60</td>
<td>5.842</td>
<td>2.850</td>
<td>0.562</td>
<td>0.048</td>
<td>0.166</td>
<td>75.72</td>
<td>63.99</td>
</tr>
<tr>
<td>C2.3</td>
<td>120x60</td>
<td>5.816</td>
<td>2.851</td>
<td>0.546</td>
<td>0.045</td>
<td>0.152</td>
<td>75.00</td>
<td>63.32</td>
</tr>
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<td>C2.4</td>
<td>120x60</td>
<td>5.812</td>
<td>2.863</td>
<td>0.550</td>
<td>0.048</td>
<td>0.164</td>
<td>75.00</td>
<td>63.70</td>
</tr>
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<td>C3</td>
<td>120x60</td>
<td>5.872</td>
<td>2.884</td>
<td>0.517</td>
<td>0.046</td>
<td>0.154</td>
<td>121.1</td>
<td>102.8</td>
</tr>
<tr>
<td>C4</td>
<td>120x60</td>
<td>5.805</td>
<td>2.885</td>
<td>0.538</td>
<td>0.045</td>
<td>0.156</td>
<td>121.0</td>
<td>102.1</td>
</tr>
<tr>
<td>C5</td>
<td>120x60</td>
<td>5.836</td>
<td>2.841</td>
<td>0.556</td>
<td>0.048</td>
<td>0.164</td>
<td>75.00</td>
<td>63.70</td>
</tr>
<tr>
<td>C1</td>
<td>180x60</td>
<td>8.813</td>
<td>2.895</td>
<td>0.528</td>
<td>0.045</td>
<td>0.152</td>
<td>72.01</td>
<td>63.16</td>
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<td>C2</td>
<td>180x60</td>
<td>8.821</td>
<td>2.882</td>
<td>0.532</td>
<td>0.045</td>
<td>0.156</td>
<td>95.12</td>
<td>83.48</td>
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<td>180x60</td>
<td>8.852</td>
<td>2.865</td>
<td>0.551</td>
<td>0.048</td>
<td>0.156</td>
<td>95.05</td>
<td>83.70</td>
</tr>
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<td>C2.2</td>
<td>180x60</td>
<td>8.790</td>
<td>2.875</td>
<td>0.585</td>
<td>0.048</td>
<td>0.156</td>
<td>95.09</td>
<td>82.88</td>
</tr>
<tr>
<td>C3</td>
<td>180x60</td>
<td>8.841</td>
<td>2.894</td>
<td>0.534</td>
<td>0.044</td>
<td>0.150</td>
<td>118.0</td>
<td>103.6</td>
</tr>
<tr>
<td>C4</td>
<td>180x60</td>
<td>8.750</td>
<td>2.871</td>
<td>0.562</td>
<td>0.048</td>
<td>0.164</td>
<td>95.03</td>
<td>82.77</td>
</tr>
<tr>
<td>C1</td>
<td>90x90</td>
<td>4.184</td>
<td>4.148</td>
<td>0.612</td>
<td>0.048</td>
<td>0.166</td>
<td>99.16</td>
<td>60.14</td>
</tr>
<tr>
<td>C1</td>
<td>180x90</td>
<td>8.418</td>
<td>4.172</td>
<td>0.623</td>
<td>0.048</td>
<td>0.162</td>
<td>75.07</td>
<td>45.88</td>
</tr>
<tr>
<td>C2</td>
<td>180x90</td>
<td>8.441</td>
<td>4.144</td>
<td>0.600</td>
<td>0.048</td>
<td>0.166</td>
<td>99.07</td>
<td>61.00</td>
</tr>
<tr>
<td>C2.1</td>
<td>180x90</td>
<td>8.415</td>
<td>4.180</td>
<td>0.614</td>
<td>0.048</td>
<td>0.152</td>
<td>99.03</td>
<td>60.79</td>
</tr>
<tr>
<td>C2.2</td>
<td>180x90</td>
<td>8.422</td>
<td>4.164</td>
<td>0.609</td>
<td>0.048</td>
<td>0.164</td>
<td>99.19</td>
<td>60.79</td>
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<tr>
<td>C3</td>
<td>180x90</td>
<td>8.434</td>
<td>4.156</td>
<td>0.587</td>
<td>0.048</td>
<td>0.166</td>
<td>99.16</td>
<td>61.00</td>
</tr>
</tbody>
</table>

a Braces used.
b Poor alignment.

Note: 1 in. = 25.4 mm; 1 ksi = 6.89 N/mm²; Refer to Fig. 1; OR = outside radius of corners.
<table>
<thead>
<tr>
<th>Specimen</th>
<th>( \bar{x} ), in inches</th>
<th>( P_{u_{\text{exp}}} ), in kips</th>
<th>( P_{f} )</th>
<th>( P_{e} )</th>
<th>( P_{u} )</th>
<th>( P_{u_{\text{ul}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 120x60</td>
<td>1.047</td>
<td>9.80</td>
<td>0.846</td>
<td>0.845</td>
<td>1.057</td>
<td></td>
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<tr>
<td>C2 120x60</td>
<td>1.035</td>
<td>10.40</td>
<td>0.928</td>
<td>0.925</td>
<td>1.162</td>
<td></td>
</tr>
<tr>
<td>C3 120x60</td>
<td>1.032</td>
<td>8.20</td>
<td>0.816</td>
<td>0.822</td>
<td>1.108</td>
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<tr>
<td>C4 120x60</td>
<td>1.046</td>
<td>8.40</td>
<td>0.832</td>
<td>0.837</td>
<td>1.124</td>
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<tr>
<td>C5 120x60</td>
<td>1.039</td>
<td>11.80</td>
<td>0.929</td>
<td>0.926</td>
<td>1.157</td>
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<tr>
<td>C1 180x60</td>
<td>0.857</td>
<td>9.60</td>
<td>0.823</td>
<td>0.827</td>
<td>1.162</td>
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</tr>
<tr>
<td>C2 180x60</td>
<td>0.856</td>
<td>8.75</td>
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<td>C3 180x60</td>
<td>0.855</td>
<td>7.60</td>
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<td>0.768</td>
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</tr>
<tr>
<td>C4 180x60</td>
<td>0.871</td>
<td>10.80</td>
<td>0.847</td>
<td>0.855</td>
<td>1.223</td>
<td></td>
</tr>
<tr>
<td>C1 90x90</td>
<td>1.791</td>
<td>11.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.274</td>
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<tr>
<td>C1 180x90</td>
<td>1.403</td>
<td>12.30</td>
<td>0.886</td>
<td>0.884</td>
<td>1.135</td>
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<tr>
<td>C2 180x90</td>
<td>1.386</td>
<td>12.10</td>
<td>0.838</td>
<td>0.840</td>
<td>1.108</td>
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<td>0.830</td>
<td>0.833</td>
<td>1.088</td>
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</tr>
</tbody>
</table>

Mean (\( N = 12 \)) 0.843 0.847 1.136
Standard deviation 0.052 0.046 0.043

a Braces used.
b Not included in statistical evaluation.
c Predicted load does not control
d Duplicate tests, with and without braces, for similar cross sections.

Note: 1 in. = 25.4 mm; 1 k = 4.45 kN.
### TABLE 4 - Evaluation of Eccentrically Loaded Columns

<table>
<thead>
<tr>
<th>Specimen</th>
<th>e, inches</th>
<th>x, inches</th>
<th>Puexp, kips</th>
<th>Ptf, Pu, Pe</th>
<th>Puexp</th>
<th>Puexp</th>
<th>Puexp</th>
<th>Puexp</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td></td>
</tr>
<tr>
<td>C1.1 120x30</td>
<td>-0.203</td>
<td>0.380</td>
<td>8.00</td>
<td>-</td>
<td>-</td>
<td>0.911</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>C2.1 120x60a,d</td>
<td>-0.536</td>
<td>1.036</td>
<td>10.30</td>
<td>-</td>
<td>-</td>
<td>1.109</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>C2.2 120x60d</td>
<td>-0.534</td>
<td>1.047</td>
<td>8.75</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.929</td>
<td></td>
</tr>
<tr>
<td>C2.3 120x60a</td>
<td>0.982</td>
<td>1.043</td>
<td>6.75</td>
<td>-</td>
<td>-</td>
<td>1.142</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>C2.4 120x60a</td>
<td>-0.212</td>
<td>1.048</td>
<td>12.40</td>
<td>-</td>
<td>-</td>
<td>1.024</td>
<td>-</td>
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<tr>
<td>C2.1 180x60a,d</td>
<td>-0.424</td>
<td>0.854</td>
<td>10.40</td>
<td>0.820</td>
<td>0.821</td>
<td>-</td>
<td>-</td>
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<tr>
<td>C2.2 180x60d</td>
<td>-0.397</td>
<td>0.870</td>
<td>10.00</td>
<td>-</td>
<td>-</td>
<td>0.809b</td>
<td>-</td>
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</tr>
<tr>
<td>C2.1 180x90a,d</td>
<td>-0.521</td>
<td>1.394</td>
<td>12.50</td>
<td>0.924</td>
<td>-</td>
<td>-</td>
<td>-</td>
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</tr>
<tr>
<td>C2.2 180x90d</td>
<td>-0.515</td>
<td>1.396</td>
<td>8.75</td>
<td>-</td>
<td>-</td>
<td>1.192</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>1.046</th>
<th>1.132</th>
<th>0.103</th>
<th>0.180</th>
<th>3e</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Standard deviation</td>
<td>0.103</td>
<td>0.180</td>
<td>3e</td>
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<tr>
<td></td>
<td>N</td>
<td>4</td>
<td>4</td>
<td>3e</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a Braces used.
b Poor alignment, not included in statistical evaluation.
c Predicted load does not control.
d Duplicate tests, with and without braces, for similar cross sections and loading conditions.
e Includes C1 90x90 of Table 3.

Note: 1 in. = 25.4 mm; 1 k = 4.45 kN.
FIG. 1 - Effective Section
FIG. 2 - Behavioral Assumptions
LOCALLY BUCKLED COLUMNS

The theoretical Ref. (22)

(a) Compressive Stress-Strain Response

(b) Effective Width

FIG. 3 - Sub-Ultimate Approach
(a) Case I: $f_i > f_j > 0$

(b) Case II: $f_i > 0 > f_j$

**FIG. 4 - Effective Width of Eccentrically Compressed Flange**
FIG. 5 - Load Deflection Response of Lipped Channel Beam-Columns
(1 in. = 25.4 mm, 1 k = 4.45 kN)
FIG. 6 - Strength of Lipped Channel Beam-Columns
(1 in. = 25.4 mm, 1 k = 4.45 kN)
FIG. 7 - Load Deflection Response and Elastic Failure
(1 in. = 25.4 mm, 1 k = 4.45 kN)
FIG. 8 - Load-Deflection Response for Specimen C2 120x60
(1 in. = 25.4 mm, 1 k = 4.45 kN)
FIG. 9 - Load-Deflection Response for Specimen C2.4 120x60
(1 in. = 25.4 mm, 1 k = 4.45 kN)
\[ S = \frac{f_{av}}{f_{cr1}} \]

\[ \lambda = \left( \frac{f_1}{f_{cr1}} \right)^{1/2} \]

**FIG. 10 - Derivation of Sub-Ultimate Approach**