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Dynamic behavior of locally buckled frames

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INTRODUCTION

The dynamic analysis of framework structural systems has been a subject of investigation for the past decade due to the rapid development of high speed computers and matrix methods of structural analysis. However, the inclusion of local buckling effects on the response of structural frameworks is lacking. Furthermore, the American Iron and Steel Institute (Ref. 1) specification governing the design of cold-formed steel structural systems is based upon static loading conditions. Consequently, the influence of the reduced stiffness caused by local buckling on the dynamic response of framework structural systems requires investigation.

It is the purpose of this paper to present a finite element formulation to determine the dynamic response of frame structures composed of locally buckled members. Local buckling is incorporated into the dynamic analysis using the effective width concept. Previous research on beams and columns (Refs. 4, 5, 8 and 11) has shown that the effective width concept for determining the post-local-buckling strength in the locally buckled compression plate components is valid in dynamic analysis.

The post-local-buckling behavior introduces elastic (material) nonlinearity into the dynamic equilibrium equations. Consequently, a nonlinear solution strategy must be used. In this paper, an incremental implementation of the implicit Wilson-\( \theta \) step-by-step time integration scheme is used (Refs. 2, 3 and 6) to solve the nonlinear dynamic equilibrium equations. The finite element formulation of the dynamic equilibrium equations uses a consistent mass formulation and the effects of damping are neglected. Static and dynamic example problem results are presented.

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Postbuckling Strength of Locally Buckled Frames - The effective width equation developed by Wang, Errera and Winter (9) is used to account for the post-local-buckling strength of the buckled compression plate elements, i.e.,

\[
\frac{b}{t} = 0.95 \sqrt{\frac{KE}{\sigma_{\text{max}}}} \left(1 - 0.95 \frac{\xi}{w} \frac{t}{\sqrt{\sigma_{\text{max}}}}\right)
\]

for

\[
\frac{w}{t} \geq 0.64 \sqrt{\frac{KE}{\sigma_{\text{max}}}}
\]

in which \(b\) = effective width of compression plate element; \(t\) = plate thickness; \(E\) = modulus of elasticity; \(w\) = flat width of the compression plate element; \(\sigma_{\text{max}}\) = maximum edge stress; \(K\) = coefficient determined by boundary conditions and aspect ratio of the compression plate element; and \(\xi\) = modification factor based on experimental evidence and engineering judgement. For values of \(w/t\) smaller than 0.64 \(\sqrt{KE/\sigma_{\text{max}}\right), b = w. Eq. 1 has been shown, through experimental verification, to be applicable to both stiffened and unstiffened plate elements if \(K\) is appropriately adjusted. The value of \(K\) can be evaluated by considering the relative dimensions of the section. For sections under uniform compression, \(K\) varies from 4.00 to 6.97 for stiffened plate elements and from 0.425 to 1.28 for unstiffened plate elements. For design considerations, \(\xi\) may be taken as 0.22 and \(K\) may be taken as 0.50 and 4.0 for unstiffened and stiffened elements, respectively.

Consider the rigid plane frame shown in Fig. 1(a). The compression plate elements of the members in the frame will buckle locally and the neutral axis will shift away from the buckled compression plate element as shown in Fig. 1(c) if the compression
element stress is larger than the local buckling stress, \( \sigma_{\text{cr}} \). The local buckling stress is derived from Eq. 2 by replacing \( \sigma_{\text{max}} \) with \( \sigma_{\text{cr}} \) and solving for \( \sigma_{\text{cr}} \), i.e.,

\[
\sigma_{\text{cr}} = 0.41 \frac{KE}{(w/t)^2} \tag{3}
\]

For the regions along the member length with compression elements stressed at levels larger than \( \sigma_{\text{cr}} \), the reduced effective flexural rigidity \( (EI)_{\text{eff}} \) varies along the member length depending upon the stress magnitude. Consequently, in the post-local-buckling range the frame is composed of nonprismatic members as shown schematically in Fig. 1(d).

Finite Element Formulation - The finite element formulation of the dynamic equilibrium equations can be derived using Hamilton's principle to give

\[
[m][\ddot{\text{v}}^e] + [k(t)][\text{v}^e] = \{f\} \tag{4}
\]

where \([m]\) is the element mass matrix, \([k(t)]\) is the nonlinear element stiffness matrix, \([\text{v}^e]\) is the element acceleration vector, \([\ddot{\text{v}}^e]\) is the element displacement vector, \([f]\) is the element load vector, the overdot ' \(\cdot' \) signifies time differentiation and the effects of damping have been excluded from Eq. 4. The element mass and load vector are evaluated analytically whereas the element stiffness matrix is evaluated using a composite Simpson rule, i.e.,

\[
[k] = E \int_0^L [N^{\cdot\cdot}]^T \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix} [N^{\cdot\cdot}] \ dx \\
= \frac{EA\ell}{6} \sum_{j=1}^{n} w_j [N^{\cdot\cdot}(x_j)]^T \begin{bmatrix} A(x_j) & 0 \\ 0 & I(x_j) \end{bmatrix} [N^{\cdot\cdot}(x_j)] 
\tag{5}
\]
where \( \ell \) is the beam element length, \( \Delta \lambda \) is the segment length for Simpson rule evaluation, \( A \) is the cross sectional area, \( I \) is the moment of inertia, \( n \) is the number of integration points, \( w_j \) is the \( j^{th} \) weighting coefficient, \( x_j \) is the \( j^{th} \) integration point coordinate, \([N]\) is the matrix of shape functions, i.e.,

\[
[N] = \begin{bmatrix}
N_1 & 0 & 0 & N_4 & 0 & 0 \\
0 & N_2 & N_3 & 0 & N_5 & N_6
\end{bmatrix}
\]  

(6)

\( T \) signifies transpose and the prime denotes differentiation with respect to \( x \). The six shape functions of Eq. 7 are defined in Fig. 2. Due to local buckling, the cross sectional area and the moment of inertia vary along the length of the member. Consequently, they are calculated exactly at the integration points, \( x_j \).

Performing standard coordinate transformations on the elemental mass and stiffness matrices and using the direct stiffness assembly procedure, the dynamic equilibrium equations for the structure can be represented as

\[
[M]\{\ddot{v}\} + [K(t)]\{v\} = \{p\}
\]  

(7)

where \([M]\) is the global mass matrix, \([K(t)]\) is the global nonlinear stiffness matrix, \(\{\ddot{v}\}\) is the global acceleration vector, \(\{v\}\) is the global displacement vector and \(\{p\}\) is the global force vector.

**Incremental Wilson-\( \Theta \) Method** - The Wilson-\( \Theta \) method for integrating the dynamic equilibrium equations of Eq. 7 is essentially an extension of the linear acceleration method (linear variation of acceleration from time \( t \) to time \( t + \Delta t \) is assumed, \( \Delta t \) is the time increment). The Wilson-\( \Theta \) method assumes that the acceleration varies linearly from time \( t \) to time \( t + \Theta \Delta t \) where \( \Theta \geq 1 \) (Ref. 2). For \( \Theta = 1.0 \), the Wilson-\( \Theta \) method reduces to the linear acceleration scheme, but for unconditional stability, it is necessary to use
LOCALLY BUCKLED FRAMES

θ > 1.37 and usually a value of θ = 1.40 is chosen.

The solution of the nonlinear dynamic equilibrium equations given by Eq. 7 using the incremental Wilson-θ method can be summarized as (Refs. 3 and 6):

1. Initialization
   (a) Set initial values for displacements \( \{ v_0 \} \), velocities \( \{ \dot{v}_0 \} \) and forces \( \{ P_0 \} \).
   (b) Calculate the initial accelerations \( \{ \ddot{v}_0 \} \):

\[
[M] \{ \ddot{v}_0 \} = (\{ P_0 \} - [K] \{ v_0 \})
\]

(c) Select a time step \( Δt \), the factor θ (usually 1.4) and calculate the time integration constants:

\[
τ = θΔt; \quad a_1 = \frac{6}{τ^2}; \quad a_2 = \frac{6}{τ}
\]

2. For each time step
   (a) Calculate the effective stiffness matrix \( \bar{K} \):

\[
\bar{K} = \{ K_1 \} + a_1[M]
\]

(b) Calculate using linear interpolation the incremental load \( \{ \Delta P_1 \} \) for the time interval \( t_i \) to \( t_i + τ \):

\[
\{ \Delta P_1 \} = θ(\{ P_{i+1} \} - \{ P_i \})
\]

(c) Calculate the effective incremental load \( \{ \Delta \bar{P}_i \} \) for the time interval \( t_i \) to \( t_i + τ \):

\[
\{ \Delta \bar{P}_1 \} = \{ \Delta P_1 \} + a_2[M] \{ \dot{v}_1 \} + 3[M] \{ v_1 \}
\]
(d) Solve for the incremental displacement \( \{ \Delta v_1 \} \):

\[
[\bar{K}]{\Delta v_1} = \{ \Delta P_1 \}
\]

(e) Calculate the incremental acceleration for the extended time increment \( \tau \):

\[
\{ \Delta \dot{v}_1 \} = a_1 \{ \Delta v_1 \} - a_2 \{ \ddot{v}_1 \} - 3\{ \dddot{v}_1 \}
\]

(f) Calculate the incremental acceleration for the time interval \( \Delta t \):

\[
\{ \Delta \ddot{v}_1 \} = \frac{1}{6} \{ \Delta \dot{v}_1 \}
\]

(g) Calculate the incremental velocity \( \{ \Delta \dot{v}_1 \} \) and the incremental displacement \( \{ \Delta v_1 \} \) for the time increment \( \Delta t \):

\[
\{ \Delta \dot{v}_1 \} = \{ \dot{v}_1 \} \Delta t + \frac{1}{2} \{ \Delta \dot{v}_1 \} \Delta t
\]

\[
\{ \Delta v_1 \} = \{ v_1 \} \Delta t + \frac{1}{2} \{ \dot{v}_1 \} \Delta t^2 + \frac{1}{6} \{ \Delta \dot{v}_1 \} \Delta t^3
\]

(h) Accumulate the displacement and velocity vectors at time \( t_{i+1} = t_i + \Delta t \):

\[
\{ v_{i+1} \} = \{ v_i \} + \{ \Delta v_i \}
\]

\[
\{ \dot{v}_{i+1} \} = \{ \dot{v}_i \} + \{ \Delta \dot{v}_i \}
\]

(i) Calculate the static member end forces, \( \{ f^e_s \} \), at \( t_{i+1} \):

\[
\{ f^e_{s_{i+1}} \} = \{ f^e_{s_i} \} + [k_i] \{ \Delta v^e_i \}
\]
Calculate the incremental stiffness force vector, \( \Delta F_{s1} \), for the time increment \( \Delta t \):

\[
\{ \Delta F_{s1} \} = [K_i] \{ \Delta v_{i} \}
\]

Accumulate the stiffness force vector at time \( t_{i+1} \):

\[
\{ F_{s1} \} = \{ F_{s1} \} + \{ \Delta F_{s1} \}
\]

Calculate the acceleration at time \( t_{i+1} \) directly from the dynamic equations of equilibrium:

\[
[M] \{ \ddot{u}_{i+1} \} = \{ p_{i+1} \} - \{ F_{s1} \}
\]

The numerical analysis procedure given in the above algorithm possesses two significant assumptions: (1) the acceleration varies linearly between \( t \) and \( t + \tau \), and (2) the stiffness properties remain constant during the time interval. In general, neither of these assumptions is correct, even though the errors are small if the time interval is short. Therefore, errors will generally arise in the incremental-equilibrium relationships which might tend to accumulate from step to step. Consequently, steps \( j-1 \) were introduced into the algorithm to impose total dynamic equilibrium at the end of each time step.

The solution to the system of equations appearing the the Wilson-\( \Theta \) incremental algorithm were obtained using the Gauss-Crout profile solver of Taylor (7).

**NUMERICAL RESULTS**

The results for the static and dynamic analysis of the braced frame shown in Fig. 3 are discussed in this section. All analyses
were performed on the IBM 370/165 computer available at the University of Kentucky and 15 point composite Simpson rule quadrature was used to evaluate the element stiffness matrix.

**Static Analysis** - A static analysis was performed on the braced frame shown in Fig. 3(a) to verify the post-local-buckling analysis portion of the dynamic analysis program (a static analysis option was implemented into the dynamic program). For static analyses, the dynamic program uses an incremental tangent stiffness procedure to incorporate the nonlinear behavior. The braced frame of Fig. 3(a) was analyzed by Wang and Blandford (Ref. 10) using a step-iterative secant stiffness procedure. The Wang and Blandford analysis of the post-local-buckling behavior was based on a moment-curvature table which neglected the axial stress contributions to local buckling. The member end moment numbers for the braced frame are shown in Fig. 3(b) and the rectangular tube cross section dimensions used are shown in Fig. 3(c). For the given loading conditions shown, the bending moments at the ends of several members are shown in Table 1. The results attributed to Wang and Blandford (column 3 of Table 1) correspond to the converged post-local-buckling results. The incremental tangent stiffness approach of this study are shown in columns 4, 5 and 6 of Table 1 corresponding to 10, 20 and 40 load steps, respectively. The non-locally buckled frame results are also shown in Table 1 for comparison purposes. Table 1 shows that results obtained in this study exhibit less post-local-buckling behavior than the results reported in Ref. 10. Due to the insensitivity on the number of load steps, the differences in the computer member end moments is probably due to the fact that the implemented finite element procedure does not include fixed-end force redistribution caused by the nonprismatic beam geometry in the post-local-buckling range. Whereas, the formulation used by Wang and Blandford is based on an exact matrix
formulation of the nonprismatic beam geometry which neglects axial deformation. Furthermore, the post-local-buckling results of Ref. 10 were obtained using a moment-curvature table which neglected the axial stress as compared with exact cross sectional area and moment of inertia calculations at the integration points using the total stress (bending + axial) that was used in this investigation.

Dynamic Analysis - The braced frame of Fig. 3(a) was also dynamically analyzed subjected to the time dependent uniform load of Fig. 3(d). A time increment of $\Delta t = 0.001$ second and $\vartheta = 1.40$ were used in the dynamic analysis. The rotation and moment versus time results for member end moment number 20 of Fig. 3(b) are shown in Figs. 4 and 5, respectively. The results labelled "linear elastic" means that no local buckling was considered in the analysis whereas the results labelled "locally buckled" means that post-local-buckling behavior was included in the dynamic analysis. Figs. 4 and 5 show that the rotation and member end moments, corresponding to location 20, increased when post-local-buckling behavior was included in the analysis. Figs. 4 and 5 also reveal that the maximum post-locally-buckled results occur at slightly larger time levels than the corresponding linear elastic results and that the post-locally-buckled peaks are flatter. Furthermore, both the linear elastic and locally buckled results of Figs. 4 and 5 exhibit amplitude decay which is to be expected when using the Wilson-$\vartheta$ method.

CONCLUSIONS

An incremental Wilson-$\vartheta$ procedure for the dynamic analysis of plane frames in the post-local-buckling range based on the finite element method and the effective width concept has been presented. It has been found that the solution scheme converges rapidly and that the inclusion of axial stress in the computation of the exact
area and moment of inertia at the integration points does not add significantly to the analysis costs. Based on the results obtained, it appears that the method is well suited for the type of problem considered.

Due to the weakening effects of local buckling, it has been shown that some member end moments increase in magnitude as compared with the linear elastic results for both static and dynamic load cases. The increase/decrease in the member forces is dependent on the amount of force distribution in the post-local-buckling range. The amount of post-local-buckling strength of the frame depends on the dimensions, types of members and the frame considered.

Further studies are underway to develop the exact nonprismatic stiffness matrix including axial deformation, to include beam-column and P-delta geometric nonlinearity, damping effects and earthquake loading on the dynamic response of frames in the post-local-buckling range.

ACKNOWLEDGEMENTS

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Table 1. - Comparison of Braced Frame Moments for Static Case (Fig. 3).

<table>
<thead>
<tr>
<th>Moment Number</th>
<th>Moment of Elastic Prismatic Frame; in Pound-inches</th>
<th>Wang and Blandford Load Steps</th>
<th>Moment of Locally Buckled Frame in lbs-in (1-lb-in = 0.113 N-m)</th>
<th>This Study Load Steps</th>
</tr>
</thead>
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<td></td>
<td></td>
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<td>20</td>
<td>40</td>
</tr>
<tr>
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<td>-4743.5</td>
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</tr>
<tr>
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<td>3022.3</td>
<td>2974.0</td>
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<tr>
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</tr>
</tbody>
</table>
Fig. 1. - Rigid Frame in the Post-Local-Buckling Range
(a) Element Displacements

\[ N_1 = 1 - \xi \quad ; \quad N_4 = \xi \]
\[ N_2 = 1 - 3\xi^2 + 2\xi^3 \quad ; \quad N_5 = 3\xi^2 - 2\xi^3 \]
\[ N_3 = x(1 - 2\xi + \xi^2) \quad ; \quad N_6 = x(\xi - \xi^2) \]

(b) Element Shape Functions ($\xi = x/\ell$)

Fig. 2 - Beam Finite Element
Fig. 3 - Post-Local-Buckling Analysis of Braced Frame

(a) Braced Frame
\[ w = 4.8 \text{ lb/in} \]
\[ l = 100" \]

(b) Member End Moment Numbers

(c) Dimensions for Rectangular Tubular Section

(d) Dynamic Load

\[ (1 \text{ lb/in} = 175 \text{ N/m}, 1 \text{ lb} = 4.45 \text{ N}, 1 \text{ in} = 25.4 \text{ mm}) \]
Fig. 4 - Rotation-Time Variation for the Braced Frame
Fig. 5 - Moment-Time Variation for the Braced Frame
APPENDIX I - REFERENCES


APPENDIX II - NOTATION

\[ a_1, a_2 = \text{time integration constants} \]
\[ b = \text{effective width of compression plate element} \]
\[ i = \text{increment number, or time step number} \]
\[ \lambda = \text{element length} \]
\[ \bar{m} = \text{mass density per unit length} \]
\[ n = \text{number of Simpson integration points} \]
\[ t = \text{plate thickness, or time} \]
\[ \Delta t = \text{time increment} \]
\[ w = \text{flat width of the compression plate element exclusive of fillets, or magnitude of uniform load} \]
\[ w_j = j^{th} \text{ Simpson quadrature weighting coefficient} \]
\[ A = \text{cross sectional area} \]
\[ E = \text{modulus of elasticity} \]
\[ (EI)_{\text{eff}} = \text{effective flexural rigidity} \]
\[ I = \text{moment of inertia} \]
\[ K = \text{buckling coefficient} \]
\[ \xi = \text{modification factor based on experimental evidence and engineering judgement, or nondimensionalized interpolation coordinate} \]
\[ \xi_j = j^{th} \text{ Simpson quadrature coordinate} \]
\[ \sigma_{\text{cr}} = \text{critical local buckling stress} \]
\[ \sigma_{\text{max}} = \text{maximum edge stress} \]
\[ \theta = \text{time weighting coefficient used in Wilson-} \theta \text{ method} \]
\[ \tau = \theta \Delta t; \text{ extended time increment} \]
\[ \{f\} = \text{element force vector} \]
\[ \{f_s^e\} = \text{static member end forces for time step } i \]
\[ \{\Delta f_s^i\} = \text{incremental static force vector for time increment } i \]
\[ \{F_s^i\} = \text{static force vector for time step } i \]
\[ \{p\}^i = \text{force vector} \]
\[ \{\Delta p_i\} = \text{incremental load vector for extended time increment } i \]
effective incremental load for extended time increment $i$

$\{\Delta P_i\}$ = element displacement vector

$\{\Delta V_i\}$ = displacement vector for time step $i$

$\{\Delta V_i\}$ = velocity vector for time step $i$

$\{\Delta V_i\}$ = acceleration vector for time step $i$

$\{\Delta \bar{V}_i\}$ = incremental displacement vector for time increment $i$

$\{\Delta \bar{V}_i\}$ = incremental velocity vector for time increment $i$

$\{\Delta \bar{V}_i\}$ = incremental acceleration vector for time increment $i$

$\{\Delta \bar{V}_i\}$ = incremental displacement vector for extended time increment $i$

$\{\Delta \bar{V}_i\}$ = incremental acceleration vector for extended time increment $i$

$[k(t)]$ = element nonlinear stiffness matrix for time $t$

$[K(t)]$ = global nonlinear stiffness matrix for time $t$

$[\bar{K}(t)]$ = effective nonlinear stiffness matrix for time $t$

$[m]$ = element consistent mass matrix

$[M]$ = global mass matrix

$[N]$ = matrix of element shape functions