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Interactive Buckling in Thinwalled Columns

by

Srinivasan Sridharan
Rafael Benito

Introduction

The prediction of ultimate strength of thinwalled columns made of coldformed steel necessarily involves consideration of local buckling and its interaction with overall buckling. The considerable volume of literature on the subject of interactive buckling has established the fact of the severity of imperfection-sensitivity of "naive" optimum designs based on the equality of the local and overall critical stresses. The problem of stiffened panels and box columns has received particular attention. The analytical approaches developed have been geared to the study of particular cases and lack generality. The use of an effective width given by an appropriate formula has its attractions from a design standpoint, but the effective width formulae do not incorporate the actual elastic rotational restraints available at the plate junctions - a difficulty which can become serious for the more complex cross-sectional geometries. Thus it would appear that the effective width approach is in need of further improvement.

A comprehensive analytical approach which can deal with a variety of cross-sectional geometries and still be sufficiently simple appears to be the need of the hour.

Little information is available on the behavior of column susceptible to local buckling when subjected to suddenly applied end compression. The interaction of local buckling with overall buckling under this dynamic loading is of considerable practical interest. Predictably, under conditions of near coincident critical stresses, the suddenly applied load cannot but reduce the load carrying capacity of the structure.

This paper presents some of the results of the first phase of a research program on the static and dynamic interactive buckling problem, currently in progress in Washington University in St. Louis. Here local and Euler buckling interaction of columns with symmetric cross-sections loaded uniformly and columns with one axis of symmetry under uniform end compression is considered. The theory of mode interaction in conjunction with the finite-strip method is employed to investigate the problem. The theory of mode interaction makes it possible to condense the degrees

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of freedom into as many as there are participating modes of buckling. But
this requires solution of several component problems which are facilitated
by the adoption of the finite strip technique\textsuperscript{9,10}. For the solution of the
dynamic interaction problem, an extension of the theory of mode interac­
tion suggested by Budiansky and Hutchinson\textsuperscript{2,3} is employed.

The present studies indicate that structures with slender unstiffened
elements are particularly vulnerable to interactive buckling - a conclusion
already foreshadowed by the Cornell test results of 1 columns\textsuperscript{6} and the work
of Van der Neut\textsuperscript{16}. Thus there would appear to be two parameters that appear
to govern the interactive buckling process; the ratio of local and overall
critical stresses and b/t (slenderness) of the (unstiffened) stiffening
elements. The dynamic analysis predicts some significant reduction of the
carrying capacity of the imperfect column under near coincident critical
stresses. The imperfections in the overall mode are of particular impor­
tance here.

Theory

In essence the present approach can be simply described as solving
for the following displacement fields (each satisfying equilibrium condi­
tions), viz.

(i) Local buckling - Eigenvalue problem \((u_1,v_1,w_1)\)

(ii) Post-local-buckling - second order displacement field \((u_{11},v_{11},w_{11})\)

(iii) Overall buckling - Eigenvalue problem \((u_2,v_2,w_2)\)

(iv) Post-overall buckling - second order displacement field \((u_{22},v_{22},w_{22})\)

(v) Mixed second order field which arises as a result of mode
interaction \((u_{12},v_{12},w_{12})\)

and then combining these solutions to produce a single potential energy
functional in terms of \(\xi_1\) and \(\xi_2\) the degrees of freedom corresponding to
local and overall buckling, which are chosen to be the maximum amplitudes
of buckling at key locations in the cross-section. Thus the problem is
reduced to one with two degrees of freedom which can be studied with great
ease. Note that our prebuckling state is one of uniform uniaxial compression.

Mode of loading:

For the case of simply supported end conditions, the loading is
applied by prescribing an axial displacement at the centroid at one of
the end sections with respect to the other. The assumed symmetry of the
cross-section makes it possible to choose a simple description of the
local buckling effects, based on uniform end compression without violating
equilibrium conditions at the end. For the case of columns with clamped
ends, the loading is applied by uniformly compressing one end section
with respect to the other.
Description of displacement components in the x direction

Figure 1 shows a typical prismatic plate structure divided into an appropriate number of strips together with the coordinate axes for a typical strip. The displacement components are taken in the form:

\[
\begin{pmatrix}
    u \\
    v \\
    w
\end{pmatrix} = \begin{pmatrix}
    u_0 \\
    v_0 \\
    w_0
\end{pmatrix} + \xi_1 \begin{pmatrix}
    v_1 \\
    w_1
\end{pmatrix} + \xi_2 \begin{pmatrix}
    v_2 \\
    w_2
\end{pmatrix} + \xi_1^2 \begin{pmatrix}
    v_{11} \\
    w_{11}
\end{pmatrix} + \xi_2^2 \begin{pmatrix}
    v_{22} \\
    w_{22}
\end{pmatrix} + \xi_1 \xi_2 \begin{pmatrix}
    v_{12} \\
    w_{12}
\end{pmatrix}
\]

(1)

where \( u_0, v_0 \) and \( w_0 \) are the prebuckling displacements given by \( \varepsilon_x = -\lambda, \varepsilon_y = \nu \lambda, \) and \( w_0 = 0 \). \( \xi_1 \) and \( \xi_2 \) are the nondimensional amplitudes of buckling and play the role of the degrees of freedom of the structure, once \( u_1, u_{ij} \) etc. are evaluated.

The description of the first order terms is taken in the standard form of the finite strip analysis:

\[
\begin{align*}
    u_1 &= \bar{u}_1(y) \cos(m_1 \pi x/\xi) \\
    v_1 &= \bar{v}_1(y) \sin(m_1 \pi x/\xi) \\
    w_1 &= \bar{w}_1(y) \sin(m_1 \pi x/\xi)
\end{align*}
\]

(2a, 2b, 2c)

where 'i' takes on values 1 or 2 and thus identifies a buckling mode (the subscript '1' referring to local and '2' referring to overall mode), \( m_1 \) gives the number of halfwaves of the buckling mode and \( \bar{u}_1(y) \ldots \) etc are appropriate functions of 'y', the transverse coordinate of the plate. The overall buckling of a clamped column is modelled by taking \( m_2 = 2 \) and by a suitable shift of the origin (Fig. 2) - a valid procedure when the cross-section does not suffer significant distortions.

The second order displacement field for each mode can be extracted from the governing differential equations with the aid of the perturbation technique\(^9,10\) in the form:
Similarly, the mixed second order displacement field be obtained in the form:

\[
\begin{align*}
\bar{u}_{i2} &= \bar{u}_{i2}(y) \sin \left( \frac{2m_1 \pi x}{\ell} \right) + \bar{u}_{i2}(y) \sin \left( \frac{2m_2 \pi x}{\ell} \right) \\
\bar{v}_{i2} &= \bar{v}_{i2}(y) \cos \left( \frac{2m_1 \pi x}{\ell} \right) + \bar{v}_{i2}(y) \cos \left( \frac{2m_2 \pi x}{\ell} \right) \\
\bar{w}_{i2} &= \bar{w}_{i2}(y) \cos \left( \frac{2m_1 \pi x}{\ell} \right) + \bar{w}_{i2}(y) \cos \left( \frac{2m_2 \pi x}{\ell} \right)
\end{align*}
\]

(3a) (3b) (3c)

In the foregoing the starred and barred quantities are again appropriate functions of 'y' to be determined using a finite strip analysis.

The displacement functions in eqn. 3(b-c) and 4(b-c) do violate the kinematic end conditions, but for the case of local-Euler buckling interaction, are justifiable approximations. In most practical cases, the number of half waves of local buckling (m1) are so great that the influence of the displacement boundary conditions is localized near the ends. Similar arguments apply for eqn. 4(b-c) when m2 takes relatively small values say 1 or 2. (Some minor errors that are introduced due to this violation are discussed in one of the later sections of the paper).

Again for the case of Euler buckling, the cross-sections remain sufficiently undistorted as to make the contributions of v22 and w22 in eqn. 3(b-c) insignificant. Thus it is seen that the equations 2(a-c), 3(a-c) and 4(b-c) are appropriate choices for the representation of the displacement fields.

Displacement functions in the transverse direction

The functions describing the variation of the inplane displacement contributions, in the transverse direction, are taken to be linear while cubic polynomials are employed to describe the out of plane displacement contributions. Thus there are 8 degrees of freedom associated with any harmonic m, (m\#0). (For the case of m=0, there are only 6 degrees of freedom).
Solution Procedure

Making use of the appropriate strain displacement and stress-strain relations, the total potential energy for the perfect structure can be expressed in the form:

$$\Pi = \xi_1^2 \left[ (a_{ij}^{(1)} - \lambda b_{ij}^{(1)}) q_{i,1} q_{j,1} \right] + \xi_2^2 \left[ (a_{ij}^{(2)} - \lambda b_{ij}^{(2)}) q_{i,2} q_{j,2} \right] + \xi_1^4 \left[ c_{ij}^{(11)} - \lambda d_{ij}^{(11)} \right] q_{i,11} q_{j,11} + a_{ijk}^{(11)} q_{i,11} q_{j,1} q_{k,1} + a_{ijkl}^{(11)} q_{i,1} q_{j,1} q_{k,1} q_{l,1} + \xi_2^4 \left[ c_{ij}^{(22)} - \lambda d_{ij}^{(22)} \right] q_{i,22} q_{j,22} + a_{ijk}^{(22)} q_{i,22} q_{j,2} q_{k,2} + a_{ijkl}^{(22)} q_{i,2} q_{j,2} q_{k,2} q_{l,2}$$

where $q_{i,1} \ldots q_{i,12}$ are the global degrees of freedom defining the appropriate displacement fields. Note that the cubic terms in the energy function have vanished because of the symmetry of the chosen system. The nature of interactions thus, assumes the character of being "doubly symmetric" whose features have been studied by Supple in detail. "$\lambda$" is the end compression parameter and is the average strain of the entire structure; $a_{ij}^{(1)} \ldots a_{ijkl}^{(12)}$ are the coefficients which are obtained by integration over the area of the strips and by transformation to the global coordinate system.

By differentiating this function w.r.t. to $q_{i,1}$ and $q_{i,2}$ and considering the lowest order terms, the corresponding eigenvalue problems are set up. The corresponding buckling modes are given by $\xi_1 q_{i,1}$ and $\xi_2 q_{i,2}$, where $\xi_1$ and $\xi_2$ can be viewed as scaling factors as yet unknown. Substitution of $q_{i,1}$ and $q_{i,2}$ and differentiation with respect to each of $q_{i,11}, q_{i,22}$ and $q_{i,12}$ produce sets of nonhomogeneous equations which give the respective unknowns. For the case of distinct (and well separated) critical stresses, the second order fields are evaluated with $\lambda$ set to $\min \{ \lambda c_1, \lambda c_2 \}$ where $\lambda c_1$ and $\lambda c_2$ are the respective critical stresses. Setting up and solving these equations is greatly simplified because of uncoupling of each of the two harmonics $\{ [0,2m_1], [0,2m_2], and \{ (m_1+m_2),(m_1-m_2) \} \}$ in each of the second order problems.
The expression for $\Pi$ (eqn. 5) can now be looked upon as the potential energy function in terms of $\xi_1$ and $\xi_2$. Differentiation of this with respect to each of $\xi_1$ and $\xi_2$, produces two simultaneous cubic equations which may be solved simultaneously. Lowest order effect of initial imperfections in the mode of buckling $\xi_1(0)$ and $\xi_2(0)$ can be easily taken into account by appending a linear term in each of the equations as shown by Koiter. The final equations after some rearrangement take the form:

\begin{align*}
(1 - \frac{\lambda}{\lambda c_1}) \xi_1 + b_{11} \xi_1^3 + b_{12} \xi_1 \xi_2^2 &= \frac{\lambda}{\lambda c_1} \xi_1(0) \\
(1 - \frac{\lambda}{\lambda c_2}) \xi_2 + b_{21} \xi_2^3 + b_{12} \xi_2 \xi_1^2 &= \frac{\lambda}{\lambda c_2} \xi_2(0)
\end{align*}

(6a) 

(6b)

where $\lambda c_1$ and $\lambda c_2$ are the critical values of $\lambda$.

The higher order effects of initial imperfections can be taken into account by writing a typical strain component in the form:

\begin{align*}
\varepsilon &= \varepsilon_0 + \varepsilon_{11} \xi_1 + \varepsilon_{22} \xi_2 + \varepsilon_{11} \{\xi_1^2 + 2\xi_1 \xi_1(0)\} \\
&+ \varepsilon_{22} \{\xi_2^2 + 2\xi_2 \xi_2(0)\} + \varepsilon_{12} \{\xi_1 \xi_2 + \xi_1(0) \xi_2 + \xi_2(0) \xi_1\} \\
&+ \varepsilon_{112} \{\xi_1^2 \xi_2 + 2\xi_1 \xi_1(0) \xi_2 + \xi_1(0) \xi_2(0)\} \\
&+ \varepsilon_{221} \{\xi_2^2 \xi_1 + 2\xi_2 \xi_2(0) \xi_1 + \xi_2(0) \xi_1(0)\} \\
&+ \varepsilon_{2212} \{\xi_2^2 \xi_1 + 2\xi_2 \xi_2(0) \xi_1 + \xi_2(0) \xi_1(0)\}
\end{align*}

(7)

and incorporating the additional terms in the expression $\Pi$. These terms are of importance when there occurs interaction of two local modes, but not, in general, in the local-Euler buckling interaction. However, these terms have been retained in the static part of the present analysis.

**Interactive buckling under suddenly applied load**

The effect of suddenly applied load is tackled by setting up the Lagrange's equations of motion written in terms of $\xi_1$ which are now functions of time. An expression for the kinetic energy is set up assuming that the excitation of the structure occurs solely in the modes of buckling. (This implies neglecting the effects of excitation of the higher order fields which are triggered by the excitation of the buckling modes). Thus the kinetic energy expression takes the form:
\( T = \{m_{ij}^{(1)} q_{i,1} q_{j,1}\} \xi_1^2 + \{m_{ij}^{(2)} q_{i,2} q_{j,2}\} \xi_2^2 \) \tag{8}

In the foregoing a dot denotes differentiation with respect to time, \( t \). The axial compression parameter \( \lambda \) takes the form of the Heaviside step function, which vanishes for \( t \leq 0 \) and equals unity for \( t > 0 \). Lagrange's dynamic equations of equilibrium take the form:

\[
\frac{1}{\omega_1^2} \ddot{\xi}_1 + (1 - \frac{\lambda}{\lambda c_1}) \xi_1 + b_{11} \dot{\xi}_1^3 + b_{12} \dot{\xi}_1 \dot{\xi}_2^2 = \frac{\lambda}{\lambda c_1} \xi_1^{(0)} \tag{9a}
\]

\[
\frac{1}{\omega_2^2} \ddot{\xi}_2 + (1 - \frac{\lambda}{\lambda c_2}) \xi_2 + b_{22} \dot{\xi}_2^3 + b_{21} \dot{\xi}_2 \dot{\xi}_1^2 = \frac{\lambda}{\lambda c_2} \xi_2^{(0)} \tag{9b}
\]

where

\[
\frac{1}{\omega_n^2} \ddot{w}_n + \sum_{m=1}^{(n)} a_{ij}^{(n)} q_{i,n} q_{j,n} \quad (n=1,2)
\]

and \( \omega_n \) is the "natural" frequency of vibration of the buckling mode. For a chosen value of \( \lambda \), these equations are solved by the Newmark's \( \beta \) method to obtain \( \xi_1 \)'s for any given levels of initial imperfections.

**Examples**

Examples are presented in this section to illustrate

(i) the accuracy and convergence of the numerical solutions,

(ii) the imperfection-sensitivity of 'clamped' stiffened panels under static prescribed end compression,

and (iii) the response of a simply supported I-section under static and dynamic axial compression.

**Convergence of the solution**

In order to illustrate the accuracy and convergence of the solution, the problem of interaction of two local modes in a rectangular plate is studied. This problem has been studied in considerable detail by Supple and his results provide a valuable basis for comparison. The plate considered is simply supported with the longitudinal edges allowed to move but held straight. Interaction between two modes, one having two halfwaves and the other with three halfwaves in the longitudinal direction (i.e., \( m_1=2, m_2=3 \)) is studied (Fig. 3). Supple has shown that in this case apart from the two uncoupled equilibrium paths emanating from the respective critical stresses, there exists a coupled equilibrium path branching...
from the secondary buckling path i.e. the equilibrium path corresponding to the higher critical stress (Fig. 4a). When the initial imperfections in the form of either mode are present, the buckling mode is a coupled one from the start of the loading, but the influence of one of the modes tends to dominate as loading progresses.

These conclusions are confirmed by the present study which apparently has taken a different route. Fig. 5(a-c) shows a comparison of the results of the present formulation obtained with 8 elements and 24 elements in half the plate for four sets of initial imperfections in the two modes. Attention has been restricted to natural loading paths and the complementary paths are not indicated. The convergence of the solution is seen to be remarkably good with the coarser discretization yielding results of sufficient accuracy.

Interaction of local and overall buckling in wide stiffened panels with clamped end conditions

Fig. 6 shows a wide integrally stiffened plate. Because of the symmetry with respect to the longitudinal centre lines of each panel, only the action of a typical panel included between two successive centre lines is considered.

Three types of panels are considered in the present study: Table 1 summarizes the details of the panels studied. Of these Panel A has a considerably slender web (d_w/t=25) which is vulnerable to torsional buckling; Panel B has a stiffer web (d_w/t=15) typically used in offshore construction. These panels exemplify the cases of near coincident buckling (\(\alpha c_1/\alpha c_2 \sim 1.0\)). The panel C is identical to Panel A but with the span reduced such that \(\lambda c_2/\lambda c_1 = 1.52\). In all these cases the overall mode is assumed to consist of two halfwaves to correspond with the boundary conditions indicated in Fig. 2. The length of the structure is so chosen that the local mode consists of even number of halfwaves to ensure that no cubic terms appear in the potential energy function. 24 strips were employed in the finite strip analysis to represent the panel.

Fig. 7(a-c) show the imperfection-sensitivity surfaces for the panels. These surfaces give the maximum load carried by the structure as a fraction of the lower of the critical loads of the participating modes. In contrast to the earlier investigations the maximum load is attained under prescribed end shortening. A common feature of the behavior of the panels A, B and C is the fact that there exists a limit on the end-shortening on the natural loading path in presence of imperfections. For the structure with imperfection in one mode only, a bifurcational type of instability is observed. The coupled equilibrium path takes the form of a descending hyperbola branching from the primary buckling path rendering the latter unstable (Fig. 4b). Thus a catastrophic failure occurs by a snap through to a remote equilibrium path.
The more acute imperfection sensitivity of panel A in comparison to B is in conformity with the general view that the greater the slenderness of the web, the greater would be the imperfection-sensitivity. The extent of participation of the web in the local buckling process and consequently its effective width are both controlled by its slenderness. The other factor which controls the imperfection sensitivity is, of course the ratio $\lambda c_2/\lambda c_1$ (the ratio of the overall to local critical stresses.) A comparison of the imperfection-sensitivity of panel A and C illustrates the effect of an increased $\lambda c_2/\lambda c_1 (= 1.52)$ for a panel with a slender stiffener. The imperfection-sensitivity is still pronounced though it has diminished and the maximum loads are higher. For a perfect panel the increase in the maximum load is about 15%.

**Interaction in a simply-supported I-section column**

Fig. 8a indicates the geometric proportions of an I-section column with near-coincident critical stresses; Fig. 8b shows the local buckling mode across the section. The Euler buckling occurs by bending about the web. The example chosen typifies the I-sections tested at Cornell University, discussed in the next section.6 24 strips were employed over half of the cross-section in the finite strip analysis. The corresponding imperfection sensitivity surface is shown in Fig. 9.

The explanation for the severe imperfection sensitivity of the column lies in the significant modification in the initial buckling mode over the flanges signified by the displacement $w_{12}$ which aggravates the displacements on the compression side of the Euler buckling (Fig. 8c). The severity of the imperfection sensitivity appears to be governed by a key term

$$\frac{\partial^2 u_2}{\partial x^2} \cdot \frac{\partial w_1}{\partial x} \cdot \frac{\partial w_{12}}{\partial x}$$

term in the energy function - a term ignored in the previous investigations.

**Interactive buckling under dynamic loading**

The column response was studied under suddenly applied end compression with different sets of assumed imperfection magnitudes. Some of the results are displayed in Fig. 10(a-b) which compares the static and the (maximum) dynamic amplitudes as $\lambda$, the prescribed compression is increased.

In general, the amplitude of vibration increases with $\lambda$, at first gradually but the rate of increase picks up sharply at a certain value of end compression. At a certain value of the end compression (viz. $\lambda_D$) the rate of increase of $\lambda$ with $\xi_2^{\text{max}}$ reaches a practical zero (Fig. 10a) and at this point dynamic buckling is deemed to have occurred.

Table 2 gives the values of static and dynamic buckling values of $\lambda$ as a fraction of $\lambda c_1$. It is evident from the table, that the additional loss of capacity to carry step loading results essentially from overall imperfections. The small differences in $\lambda_D/\lambda_S$ for equal $\xi_2^{(0)}$ are probably due to numerical difficulties in the exact definition of $\lambda_D$. For an overall imperfection of magnitude of 1/1000 of the length of the column the drop
In the limit end compression is about 8% from the static case. In many practical cases, plastic yielding would be triggered due to huge oscillations that build up for end compressions which may be significantly smaller than $\lambda_D$.

In Fig. 10b the variation of the local buckling amplitude ($\xi_{1,\text{max}}$) with $\lambda$ is illustrated for various levels of $\xi_1(0)$ and $\xi_2(0)$. The continuous curves apply for a short column, for which no interaction of overall buckling need be considered. The discontinuous curves apply for the column studied. In the absence of overall imperfections, the two curves for a given $\xi_1(0)$ coincide until bifurcation occurs for the longer column signalled by the sudden appearance of overall deflections (see Fig. 10a). These are accompanied by a sharp increase in the local buckling deflections as seen in Fig. 10b. On the other hand, in the presence of overall imperfections, the local buckling amplitudes are considerably larger for a given $\xi_1(0)$ and these build up at an increasingly rapid rate as $\lambda$ approaches $\lambda_D$.

**Comparison with Experimental Results**

The tests on I-section columns performed by Kalyanaraman et al. at Cornell University provide a valuable basis for the evaluation of the theoretical model. I-section column specimens were fabricated by glueing two channels back to back by an epoxy resin. The channels were formed out of high strength cold formed sheet steel. The failure occurred at average stresses well below the yield point thus justifying the use of an elastic theory. The full details of the tests are documented in ref. 6. Imperfection measurements are not, however, available. It is therefore decided to use standard values of imperfections which are taken to be

$$\xi_1(0) = 0.2$$

and

$$\xi_2(0) = \frac{1}{3000} \cdot \frac{k}{t}$$

for all the specimens: Note that the maximum amplitude of local imperfection is taken to be 0.2 of the thickness of the sheet and this occurs at the tips of the flanges of the I-section (Fig. 9).

Table 3 gives the experimentally observed ultimate loads and the theoretical predictions; it also gives the slenderness of flange outstands ($b/t$), the ratio $\sigma c_2/\sigma c_1$ and $m_1$, the number of halfwaves of local buckling. It is of interest to note that for the I-sections with high slendernesses of flange outstands (e.g. LCI, Table 3) the increase in strength beyond local buckling for the perfect case is only about 50% even for a case with $\sigma c_2/\sigma c_1 \approx 13$. This indicates the seriousness of the interaction problem.

In the majority of cases the theoretical predictions based on assumed imperfections tend to underestimate the collapse load. This is particularly true for the columns LCI, LCII and LCIII. In cases where the theoretical predictions were lower by more than 15% of the experimental results, the collapse loads were computed once again now assuming no imperfections.
In the cases of relatively short columns (viz. LCI-1, LCII-1, LCIII-1) where the local mode consisted only of 12 or 13 halfwaves it was suspected that the lack of satisfaction of the displacement boundary conditions at the ends in the mixed second order field tended to underestimate the stiffness of the structure. This effect was corrected by neglecting the displacements over a quarter of the wavelength of the mixed second order fields from either end in the calculation of the potential energy. This made a difference of about 4% in the three cases mentioned. The final results for the perfect structure are shown in the brackets in the last column of Table 3. With the exception of the columns LCI-1, LCI-2 and LCII-1, the collapse stresses of the perfect column provide an upper bound for the experimental results. One possible explanation for the generally high values of the experimental stresses lies in the mode of fabrication of the I column by glueing the channels by an epoxy resin. If the thickness of the epoxy resin material is of the order of thickness of the sheets glued (i.e. 0.05 in.), the flexural stiffness of the web can be considerably enhanced and a 10% increase in the local critical stresses and a similar increase in the ultimate loads could easily occur. Another source of discrepancy stems from the fact that the theoretical predictions are based on the first loss of stability on the natural loading path. In actuality, the structure may jump to a remote equilibrium path and continue to resist some extra compression. Keeping in view the above sources of error the agreement between the theory and experiment must be considered satisfactory.

Conclusions

A relatively simple semi-analytical technique is employed to study the interaction of local and Euler type buckling under static and dynamic loading conditions. The approach is shown to be capable of dealing with symmetric cross-sections simply supported at its ends and loaded concentrically as well as cross-sections with one axis of symmetry with clamped end conditions subjected to uniform end compression.

From a study of the sample problems of stiffened panels and I-section column, the imperfection-sensitivity and the collapse strength are governed by two parameters viz. slenderness (b/t or d/t) of the stiffening elements and the ratio of Euler to local critical stresses. Under conditions of the sudden application of load, the load carrying capacity of the structure is diminished by about 8% and possibly more.

Good agreement is found to exist between the theoretical predictions and the results of tests on I-section columns made of cold formed steel.

Acknowledgments

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Appendix - Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>Kinetic energy of the structure</td>
</tr>
<tr>
<td>$a_{ij}, a_{ijk}, \ldots$</td>
<td>The coefficients of the potential energy function of the component field problems</td>
</tr>
<tr>
<td>$d_s$</td>
<td>Depth of stiffener outstand</td>
</tr>
<tr>
<td>$\ell$</td>
<td>Length of the structure</td>
</tr>
<tr>
<td>$m_{ij}$</td>
<td>The coefficients of the mass matrix for vibration in local or overall mode</td>
</tr>
<tr>
<td>$m_1, m_2$</td>
<td>The number of halfwaves constituting the local and Euler mode respectively</td>
</tr>
<tr>
<td>$q_i$</td>
<td>A typical global degree of freedom of the structure</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness of plate</td>
</tr>
<tr>
<td>$t_f$</td>
<td>Flange thickness</td>
</tr>
<tr>
<td>$t_w$</td>
<td>Web thickness</td>
</tr>
<tr>
<td>$u, v, w$</td>
<td>The displacement components at the middle surface</td>
</tr>
<tr>
<td>$u_o, v_o, w_o$</td>
<td>Prebuckling displacements</td>
</tr>
<tr>
<td>$u_i, v_i, w_i$</td>
<td>Displacement components corresponding to the Eigenvalue problem</td>
</tr>
<tr>
<td>$u_{ij}, v_{ij}, w_{ij}$</td>
<td>Displacement components of the second order fields</td>
</tr>
<tr>
<td>$x, y, z$</td>
<td>The coordinate directions (longitudinal, transverse and normal)</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>The generic strain</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Prescribed end compression</td>
</tr>
<tr>
<td>$\lambda_c_1, \lambda_c_2$</td>
<td>The critical values corresponding to the local and overall modes respectively</td>
</tr>
<tr>
<td>$\lambda_D$</td>
<td>Dynamic buckling end compression</td>
</tr>
<tr>
<td>$\lambda_S$</td>
<td>Static buckling end compression</td>
</tr>
<tr>
<td>$\xi_1, \xi_2$</td>
<td>Scaling factors of the buckling amplitudes (Amplitude of buckling at a chosen location in the cross section divided by the plate thickness)</td>
</tr>
<tr>
<td>$\xi_{1,\text{max}}, \xi_{2,\text{max}}$</td>
<td>Maximum amplitudes of oscillation in the modes of buckling</td>
</tr>
</tbody>
</table>
In the modes 1 and 2, the maximum value of the average stress carried by the plate structure.

Appendix — References


Table 1

Geometry and Initial Buckling Data of Stiffened Panels

<table>
<thead>
<tr>
<th>Identification of Panel</th>
<th>Geometry</th>
<th>Buckling Modes and Critical Stresses*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>b</td>
<td>t (=ts)</td>
</tr>
<tr>
<td>A</td>
<td>50</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>50</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>50</td>
<td>1</td>
</tr>
</tbody>
</table>

*The subscripts 1 and 2 refer to the local and Euler modes.
Table 2
Dynamic Buckling Compression Values of the I-section Column

<table>
<thead>
<tr>
<th>Initial Imperfections</th>
<th>$\xi_2^{(0)}$</th>
<th>$\xi_1^{(0)}$</th>
<th>$\lambda_S/\lambda_{c_1}$</th>
<th>$\lambda_D/\lambda_S$</th>
<th>$\lambda_D/\lambda_{c_1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.1</td>
<td></td>
<td>0.92</td>
<td>1.0</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td></td>
<td>0.85</td>
<td>1.0</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td></td>
<td>0.75</td>
<td>1.0</td>
<td>0.75</td>
</tr>
<tr>
<td>1.0</td>
<td>0.1</td>
<td></td>
<td>0.80</td>
<td>0.96</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td></td>
<td>0.75</td>
<td>0.97</td>
<td>0.73</td>
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<tr>
<td>2.0</td>
<td>0.1</td>
<td></td>
<td>0.74</td>
<td>0.92</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td></td>
<td>0.70</td>
<td>0.93</td>
<td>0.65</td>
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## Table 3

Comparison Between Theory and Experiment

<table>
<thead>
<tr>
<th>Column Designation</th>
<th>Geometry (in)</th>
<th>Parameters of Interaction</th>
<th>Ultimate Load (P&lt;sub&gt;ult&lt;/sub&gt;)</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Experimental (kip)</td>
</tr>
<tr>
<td></td>
<td>l</td>
<td>b</td>
<td>d&lt;sub&gt;w&lt;/sub&gt;</td>
</tr>
<tr>
<td>LCI-1</td>
<td>63.25</td>
<td>2.866</td>
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<td>92.25</td>
<td>2.869</td>
<td>4.031</td>
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<td>LCI-3</td>
<td>123.25</td>
<td>2.876</td>
<td>4.006</td>
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<tr>
<td>LCII-1</td>
<td>57.19</td>
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<td>4.007</td>
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<tr>
<td>LCII-2</td>
<td>90.20</td>
<td>2.501</td>
<td>4.018</td>
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<tr>
<td>LCII-3</td>
<td>123.25</td>
<td>2.509</td>
<td>4.008</td>
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<tr>
<td>LCIII-1</td>
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<td>2.113</td>
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<td>3.990</td>
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<td>LCIV-2</td>
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<td>LCIV-3</td>
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<td>LCV-2</td>
<td>53.88</td>
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<tr>
<td>LCV-2</td>
<td>75.28</td>
<td>1.490</td>
<td>3.027</td>
</tr>
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</table>
Fig. 1 Finite strip configuration and the local coordinate system
Initial buckling mode of a column clamped at its ends

Inplane buckling of a stiffener and a possible loading arrangement

Fig. 2 The idealization of end boundary conditions for a uniformly compressed plate structure
Plate edges simply supported but held straight

Plate under study

Participating modes

Fig. 3 Details of the plate investigated
Fig. 4 Two possible types of behavior of a doubly symmetric system with 2 degrees of freedom

(a) STABLE
(b) UNSTABLE

PROJECTION ON $\xi_1$-$\xi_2$ PLANE
Fig. 5(a). Relationship between the deflection amplitudes (divided by $t$) and the average stress carried by the plate for imperfections in the secondary buckling mode

(i) $\xi_2^{(0)} = 0.02$, $\xi_1^{(0)} = 0.0$; (ii) $\xi_2^{(0)} = 1.0$, $\xi_1^{(0)} = 0.0$
Fig. 5(b-c). Relationship between the deflection amplitudes (divided by $t$) and the average stress carried by the plate for case (i) $\xi_1^{(0)} = 0.25$, $\xi_2^{(0)} = 0.20$ and case (ii) $\xi_1^{(0)} = 0.25$, $\xi_2^{(0)} = 0.125$
Fig. 6  Wide Integrally Stiffened Plate  (a) Local buckling mode
(b) Dimension of a typical panel
Fig. 7(a). Imperfection-Sensitivity Surface of Panel A
(vide Table 1)
(Imperfection-Amplitudes are rendered dimensionless by division by plate thickness)
Fig. 7(b). Imperfection-Sensitivity Surface of Panel B

(vide Table 1)
Fig. 7(c). Imperfection-sensitivity surface of Panel C'

(vide Table 1)
Fig. 8(a) Details of the I-section column investigated (l = 2000, \( \sigma_{c1} = 0.6211 \times 10^{-3} \), \( \sigma_{c2} = 0.6565 \times 10^{-3} \))

(b) The local buckling mode and initial local imperfection across the section

(c) Modification of the local mode
Fig. 9 Imperfection-sensitivity of the I-section column
Fig. 10(a) The variation of dynamic (the maximum) and static amplitude of overall buckling with $\lambda$. 
Fig. 10(b) The variation of dynamic (the maximum) local buckling amplitude with $\lambda$. 

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LOCAL BUCKLING ALONE

OVERALL & LOCAL BUCKLING