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FLEXURAL BUCKLING IN COLD-FORMED STEEL STRUCTURAL MEMBERS

by

PETER LÖRINCZ*

1. INTRODUCTION

Thin-walled steel members can be manufactured by cold-rolling or cold-forming, using Abkant presses.

The present article analyzes standard cold-rolled steel members manufactured at Intreprinderea Metalurgica Iasi, Romania's largest manufacturer in this field. Although a number of studies by foreign authors, particularly from the USA, are considered, in the paper the author mainly deals with Romanian research work concerning the importance and distribution of the effect of strain-hardening as well as the influence of this effect on the critical flexural buckling strength.

Thorough theoretical and experimental studies by members of the Department of Steel Structures, The "Traian Vuia" Polytechnic Institute of Timisoara, Romania, have led to the revision and subsequently to the drafting of the first Romanian standard concerning the design of cold-formed steel members (1).

2. FLEXURAL BUCKLING

Analysis for stability generally starts from two distinct models:

(a) the ideal bar, which is perfectly elastic and straight and has no imperfections whatever, and
(b) the real bar, presenting both geometrical and structural imperfections.

For the ideal bar, Euler's formula is accepted with correction in the elastic-plastic range by the average tangent modulus of elasticity, in the form

\[ E_{ta} = \frac{\sum E_i I_i}{I} \]  

(1)

American specifications recommend a unique safety coefficient (2) of 23/12 for the whole range of slenderness, though geometrical imperfections are of greater importance in slender bars and structural flaws in studs.

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For hot-rolled bars there have been attempts to introduce the influence of imperfections into the analysis; thus, Dutheil (3) established the expression of an equivalent deviation in the form:

\[
\delta_1 = \frac{a(1 + b) \sigma_{cr}}{\sigma_E - (1 + b)\sigma_E} \frac{S_y}{N}
\]  

(2)

Admitting part of the cross-section to be plasticized, the theoretical elastic buckling stress under concentric loading, \(\sigma_{cr}\), represented by the coefficient \(\psi\), is obtained by an analysis for strength of the second order:

\[
\sigma_{cr} = \sigma_1 - \sqrt{\frac{2}{\sigma_1} - \frac{\sigma_c \sigma_E}{\psi} + (\psi-1)b}
\]  

(3)

where

\[
\sigma_1 = \frac{\psi \left[ \sigma_E + (1 + b) \sigma_c \right]}{2[\psi + (\psi-1)b]}
\]  

(4)

In the French standards CM 66 (4), the partial plasticizing of the section is not admitted, so that \(\psi = 1\); the value of the constant \(b\) is found experimentally and is \(b = 0.3\).

DIN 4114, the German standard, accepts a reduced initial eccentricity \(\overline{e_o}\):

\[
\bar{e}_o = e_o \frac{A}{S_y} = \frac{e_o}{r_s} \quad r_s = \frac{S_y}{A}
\]  

(5)

where

\(S_y\) is the modulus of resistance with respect to the most compressed fiber;

\(r_s\) is the radius of the central core on the side opposite to the centre of gravity \(G\) and the plasticized area.

The relation between the theoretical elastic buckling stress \(\sigma_{cr}\) and \(\lambda\) is:

\[
\lambda^2_{cr} = \left(\frac{KL}{r}\right)^2 \cdot \frac{\pi^2 \frac{E}{\sigma_{cr}}}{\left[1 - \frac{\bar{e}_o \cdot \sigma_{cr}}{F_y \sigma_{cr}} + a_1 \left(\frac{\bar{e}_o \cdot \sigma_{cr}}{F_y - \sigma_{cr}}\right)^2 + a_2 \left(\frac{\bar{e}_o \cdot \sigma_{cr}}{F_y - \sigma_{cr}}\right)^3\right]}
\]  

(6)

For the unfavorable \(I\)-shaped cross-section consisting of two angle irons, the values \(a_1 = 0.25, a_2 = -0.005\) are used.
Starting from experiments on hot-rolled structural members, ECCS (5) have set up the following analytical expressions for the three buckling curves a, b and c:

\[ \phi = \sigma_{cr}/F_y \]
\[ \phi = 1/[0.5 + \alpha \cdot \lambda^2 + \sqrt{(0.5 + \alpha \cdot \lambda^2)^2 - \beta \cdot \lambda^2}] \]  

(7)

where the coefficients \( \alpha \) and \( \beta \) are taken from Table 1. Romanian standards also admit these expressions for the computation of hot-rolled steel sections.

Table 1

<table>
<thead>
<tr>
<th>Buckling curve</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a ( \alpha )</td>
<td>0.514</td>
<td>0.554</td>
<td>0.532</td>
</tr>
<tr>
<td>b ( \beta )</td>
<td>0.795</td>
<td>0.738</td>
<td>0.377</td>
</tr>
</tbody>
</table>

\[ \bar{\lambda} = \frac{1}{\pi} \sqrt{\frac{F_y}{\pi} \cdot \lambda} \]  

(8)

The new European curves plotted by R. Maquoi and J. Rondal (5) recommended Eq. 9:

\[ \phi = \left(1 + \alpha_1 \sqrt{\lambda^2 - 0.04 + \lambda^2} / 2\lambda^2 - \sqrt{(1 + \alpha_1 \lambda^2 - 0.04 + \lambda^2) - 4\lambda^2 / 2\lambda^2}\right) \]  

(9)

The coefficient \( \alpha_1 \) is determined as a function of the shape of the buckling curve in Table 2.

Table 2

<table>
<thead>
<tr>
<th>Buckling curve</th>
<th>( \alpha_1 )</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>0.093</td>
<td>0.158</td>
<td>0.281</td>
<td>0.384</td>
<td>0.587</td>
</tr>
</tbody>
</table>
At present, in the computation of thin-walled cold-formed steel members no account is taken of the influence of strain-hardening on flexural buckling. Geometrical imperfections are considered to be more important than structural flaws; with bars that are not welded, structural imperfections are insignificant because of the very thin walls of the members. This paper proposes a simplified method of calculation for unwelded bars having \( Q = 1 \) (i.e., having an "active" cross-section).

According to H. Beer (6), an increase of the average yield point of the section \( F_y \), as a result of strain-hardening, leads to an increase in the critical buckling stress along both axes, particularly with members of small slenderness.

A non-uniform variation of the yield point over the cross-section of the bar has little influence on the critical buckling stress (7), as it is only the average yield point value that matters.

Since cold-rolled members have very supple walls, the cross-sections cannot plasticize before local buckling occurs. The loss in stability due to the divergence of equilibrium, which is characteristic of the buckling of real bars, can be approximated by a second-order analysis limiting the maximum edge stress \( \sigma_{\text{max}} \) to a value given as \( \sigma_{\text{lim}} \).

Since in a tensile test the fibers gradually yield Fig. 1, the value of \( \sigma_{\text{lim}} \) should be limited to the minimum \( F_y \) value over the cross-section.

![Characteristics curves for steel and entire member](image)

\( F_u = 53.65 \text{ Ksi}(3700 \text{ daN/cm}^2) \)
\( F_y = 34.80 \text{ Ksi}(2400 \text{ daN/cm}^2) \)
\( F_{y_a} = 46.95 \text{ Ksi} \)
\( 1 \text{ Ksi} = 68.96 \text{ daN/cm}^2 \)
Since the yield point is higher in the extreme fibers than it is in the initial material, which is due to both strain-hardening of the plane sections and to cutting with cutter, it would be more proper to take $F_p$ from the extreme fiber. However, as this value is difficult to assess, we suggest a limitation to the values of the elastic limit of the entire member, which is taken by considering the ratio $F_p/F_y = 0.8$ of the non-strainhardened steel.

$$ F_p = 0.8 \frac{F_y}{E} $$

For un-welded bars, structural imperfections are neglected, while the geometrical ones take the shape of a sinusoidal curve:

$$ \delta = \frac{\sigma_E}{\sigma - \sigma_E} \cdot \delta_o \cdot \sin \frac{\pi \cdot x}{L} $$

$$ \delta_{max} = \frac{\sigma_E}{\sigma - \sigma_E} \cdot \delta_o $$

$$ \sigma_{max} = \frac{N}{A} + \frac{N \cdot \delta_{max}}{S_y} $$

Denoting

$$ r_s = \frac{S_y}{A} \quad \text{and} \quad M = \frac{\delta_o}{r_s} = \delta_o \cdot \frac{A}{S_y} $$

we have:

$$ \sigma_{max} = \frac{N}{A} \cdot (1 + \frac{m \cdot \sigma_E}{\sigma - \sigma_E}) $$

As soon as buckling occurs, we have $\sigma = \sigma_{cr}$ and $\sigma_{max} = \sigma_{lim}$.

From Eq. 15 we obtain:

$$ \sigma_{cr}^2 \cdot [\sigma_{lim} + (1 + m)\sigma_E] \cdot \sigma_{cr} - \sigma_{lim} \cdot \sigma_E = 0 $$

Introducing the coefficient of buckling $\phi$ we have:

$$ \phi = \frac{\sigma_{cr}}{\sigma_{lim}} $$

and we obtain:

$$ \phi^2 \cdot [1 + (1 + m)\sigma_E/\sigma_{lim}] \cdot \phi + \sigma_E/\sigma_{lim} = 0 $$

The maximum stress that the bar can take is obtained from:

$$ \frac{N}{\phi \cdot A} \leq \sigma_{lim} $$

$$ N = \phi \cdot A \cdot \sigma_{lim} $$
3. EXPERIMENTAL DETERMINATION OF THE MECHANICAL CHARACTERISTICS

The Department of Steel Structures of the Timisoara Polytechnic have carried out a number of experiments (8) concerning the determination of the importance and distribution of strain-hardening and its influence on flexural buckling. The present paper is concerned with a number of studies on channel shaped members [4.72x2.36x0.157 in.] (120x60x4 mm) that were cold-rolled at I.M. Iasi.

The yield point at the corners of the members was determined experimentally on 200 samples artificially aged for 2.5 hours at 250°C. The result, obtained by statistics, was $F_{yc} = 62.57$ ksi (4,315 daN/cm$^2$).

An analysis using Eq. 3.1.1-2 from (2) for the Romanian steel grade OL 37, $F_u = 53.65$ ksi (3,700 daN/cm$^2$), $F_y = 34.80$ ksi (2,400 daN/cm$^2$) at a ratio $R/t = 1.5$ yields:

$$F_y = 1.779 \times F = 1.779 \times 2,400 = 4269.6 \text{ daN/cm}^2 = 61.914 \text{ ksi}$$

i.e., a difference of 1.05 per cent as compared to the experimental value.

An average value $F_{ua} = 59.71$ ksi (4118 daN/cm$^2$), $F_{ya} = 46.95$ ksi (3,238 daN/cm$^2$) resulted from the tensile test done on strips (Fig.2); the elastic limit, which was defined in a similar way as in the case of non-strainhardened Romanian steel OL 37, was seen to be $F_p = 0.8 \times 46.95 = 37.56$ ksi (2625 daN/cm$^2$) i.e. less than the smaller yield point in the strips (Fig. 2).

The analytical calculation of $F_y$ by means of Eq. 3.1.1-1 from (2) where $C=0.1127$, yields the value $F_y = 37.92$ ksi (2615 daN/cm$^2$) i.e., 80.76 per cent of the experimental value; by using Eq. 20 from (9), we get a value of $44.11$ ksi (3042 daN/cm$^2$) i.e., a difference of 6.05 per cent.
Yield point and failure variations over the cross-section and average values.

\[ F_{ya} = F_y + \Delta F_y \]  \hspace{1cm} (21)
\[ \Delta F_y = 180 \frac{n \cdot t^2}{A} \]  \hspace{1cm} (22)
4. BUCKLING TESTS

In order to obtain the compression stress, a testing machine (Fig. 3) was used amplifying a gravitational weight due to water in Tank 1 by means of Levers 3, 4, 5 so as to give a maximum force of 11,221 kips (25,000 daN).

Bar 6 rests on a supporting system (Fig. 4) patented as invention at OSIM (10). 12 identical, centrically loaded bars were tested. All these bars had hinged supports at both ends and on each bar tensometric marks (from 1 to 12) and flexometers were fixed (Fig. 5).

Since geometrical imperfections are of considerable importance, deformations were measured at points 1.968 in (50 mm) apart, with the horizontal reference plane passing through point 1, row IV (Table 3) for which it was assumed that \( \delta_0 = 0 \). The initial deformations were assumed to be positive when they were oriented towards the outside, and negative when oriented towards the inside of the member.

Measurements showed that the deformation was of an approximately sinusoidal shape, having a maximum amplitude \( \frac{\delta_{\text{max}}}{L} = 1/831 \); this value differed from the one recommended by various specification for hot-rolled members.

For the initial maximum deviation a value of \( \delta_0 = L/800 \) was assumed, which leads to fairly close values.

\[
P_{ya} = 24 + 180 \frac{2.4^2}{896} = 30.42 \text{ daN/mm}^2 = 44.11 \text{ ksi} \quad (23)
\]
FIG. 4.

Universal supporting system
Experiments (Fig. 6) showed that flexural buckling occurred without any local buckling of the walls. Fig. 7 shows the dependence experimentally established between the centric load $N$ and the maximum deflection $\delta_{\text{max}}$.

Table 3

<table>
<thead>
<tr>
<th>File</th>
<th>BARS</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
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<td>II</td>
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<td>VI</td>
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<tr>
<td>VII</td>
<td>VII</td>
</tr>
</tbody>
</table>

Initial deformations of the member walls

<table>
<thead>
<tr>
<th>File</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
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<tr>
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<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td></td>
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<td>3</td>
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<td>5</td>
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<td>7</td>
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<td>14</td>
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<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
</tr>
</tbody>
</table>

$\delta_{\text{max}} = \frac{0.83}{690} = \frac{1}{831}$
It is suggested that cold-rolled steel bars subject to centric compression should be checked for a loss in stability due to flexural buckling, if all walls are active \((Q=1)\) in accordance with Eq. 18, by applying the method presented in Eqs. 13–20. For \(\sigma_{\text{lim}}\), the proportional limit is admitted according to Eq. 10 in which \(F_{\text{pr}}\) is determined applying the method described in (2); for the yield stress \(F_{\text{y}}\) of the plane walls, strain hardening is taken into account, i.e.,

\[
F_{\text{yf}} \approx 0.15 xF_y
\]  

\((24)\)

**Fig 5**

Arrangement of tensometric marks and flexometers
Table 4

Maximum initial deviation, $\delta_0/0.039$ in.

<table>
<thead>
<tr>
<th>Bar</th>
<th>Dutheil France</th>
<th>Skaloud CSN 31402</th>
<th>NBN 152 Belgium</th>
<th>ECCS Europe</th>
<th>P. LORINCE I.P. Timisoara</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_0$</td>
<td>$0.425 \cdot 10^{-4} \lambda^2 r_s$</td>
<td>$0.3(\lambda/100)^2$</td>
<td>L/500</td>
<td>L/530-L/3360</td>
<td>L/800</td>
</tr>
<tr>
<td>$B_1$</td>
<td>0.469</td>
<td>0.399</td>
<td>1.38</td>
<td>1.30-0.205</td>
<td>0.81</td>
</tr>
<tr>
<td>$B_2$</td>
<td>0.986</td>
<td>0.839</td>
<td>2.00</td>
<td>1.88-0.297</td>
<td>1.25</td>
</tr>
<tr>
<td>$B_3$</td>
<td>2.369</td>
<td>2.017</td>
<td>3.10</td>
<td>2.92-4.610</td>
<td>1.93</td>
</tr>
</tbody>
</table>

Table 5 compares the principal international design specifications, the experimentally determined values and the calculation procedure proposed in the present paper. The calculations were carried out both for the yield stress of the virgin steel $F_v = 34.80$ ksi (2400 daN/cm²) and for the yield point increased due to strain-hardening, $F_{y_a} = 46.95$ ksi (3238 daN/cm²). The results obtained by the application of the proposed procedure are closest in value to the results that were obtained experimentally.

Fig. 6 Flexural buckling of the bar
FLEXURAL BUCKLING IN STRUCTURAL MEMBERS

![Graph showing flexural buckling in structural members with force (N) and load (kips) on the x-axis and deflection (cm) and deflection (in) on the y-axis. The graph includes load values such as 9,156 (20,400), 8,045 (17,925), and 6,378 (14,210) kips. The cross-section of the structural member is also shown with dimensions 4.72 x 2.36 x 0.157 in and 120 x 60 x 4 mm.](image)
Table 5:
Critical stresses as compared to experimental ones
Channel members £ 4.72×2.36×0.157 (120×60×4 mm)

<table>
<thead>
<tr>
<th>Bars</th>
<th>KL K=1</th>
<th>$\lambda_y = \frac{K \cdot L}{F_y}$</th>
<th>Units</th>
<th>AISI 1980 USA 1</th>
<th>DIN 4114 GERMANY 2</th>
<th>CM 66 FRANCE 3</th>
<th>ECCS 4</th>
<th>ECCS R. MAQUOI 5</th>
<th>P. Lörincz Experimental Kips (daN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>47.00</td>
<td>34.80 (2,400) Kips (daN)</td>
<td>%</td>
<td>98.67</td>
<td>77.71</td>
<td>95.84</td>
<td>88.54</td>
<td>86.12</td>
<td>90.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>46.97 (3239) Kips (daN)</td>
<td>%</td>
<td>129.95</td>
<td>105.24</td>
<td>123.88</td>
<td>117.06</td>
<td>110.23</td>
<td>96.80</td>
</tr>
<tr>
<td>B2</td>
<td>63.49</td>
<td>34.80 (2,400) Kips (daN)</td>
<td>%</td>
<td>105.96</td>
<td>78.57</td>
<td>97.82</td>
<td>103.53</td>
<td>87.28</td>
<td>93.51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>46.97 (3,238) Kips (daN)</td>
<td>%</td>
<td>136.37</td>
<td>102.03</td>
<td>119.62</td>
<td>112.65</td>
<td>136.32</td>
<td>99.94</td>
</tr>
<tr>
<td>B3</td>
<td>92.59</td>
<td>34.80 (2,400) Kips (daN)</td>
<td>%</td>
<td>113.78</td>
<td>75.47</td>
<td>88.51</td>
<td>81.71</td>
<td>83.56</td>
<td>91.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>46.97 (3,238) Kips (daN)</td>
<td>%</td>
<td>135.82</td>
<td>89.78</td>
<td>97.00</td>
<td>98.00</td>
<td>94.74</td>
<td>95.55</td>
</tr>
</tbody>
</table>

SIXTH SPECIALTY CONFERENCE
From the graph of Fig. 8 showing the variations in the deviation of calculated critical stresses determined in accordance with various specifications, it can be seen that there are considerable differences in all bars and particularly in Bar B. All calculation methods give better results than do the experimental ones.

5. CONCLUSIONS

The simplified method of computation proposed by the author takes account of the initial geometrical imperfections of the axis of the bar, which are assimilated with a sinusoidal variation, and of the increased yield point due to strain-hardening, by approximating the divergence of equilibrium by means of a second-order strength analysis. The maximum unit stress is limited to the elastic (proportional) limit of a material having the same behaviour as the non-strain-hardened steel from which it is derived, i.e., maintaining the ratio $F_p/F_y = 0.8$.

It should be considered that all walls of the member need to be kept active as a result of the increase of the average point over the entire member.

**Fig. 8.**

Calculated vs. experimental stresses
FLEXURAL BUCKLING IN COLD-FORMED STEEL STRUCTURAL MEMBERS

by

PETER LORINCI

- Summary -

The paper presents theoretical and experimental researches concerning the magnitude and the distribution of cold working as well as its influence over flexural buckling in cold-formed steel structural members in Romanian.

APPENDIX - REFERENCES


2. AISI Specification for the design of cold-formed steel structural members, 1980.

3. J. Dutheil Théorie de l'instabilité par divergence d'équilibre. 4 Congress de l'AIPG, London.


J. Rondal


G. Schultz

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>M. Prudhomme</td>
<td></td>
</tr>
<tr>
<td>I. Caraba</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX - NOTATION

A  Cross-sectional area, \( \text{in}^2 \) (\( \text{cm}^2 \))

a  Constant

\( a_1 \)  Constant

\( a_2 \)  Constant

b  Constant

E  Modulus of elasticity of steel, 29,500 ksi

\( E_{ta} \)  Average tangent modulus of elasticity, ksi

\( E_{ti} \)  Tangent modulus of the \( i \)-th sub-area, ksi

\( e_o \)  Eccentricity of load along the \( x \) axis, in.

\( e_{o_i} \)  Defined by Eq. 5

\( F_{u} \)  Ultimate tensile strength of virgin steel, ksi

\( F_{ua} \)  Average ultimate tensile strength of sections, ksi

\( F_{y} \)  Tensile yield point of virgin steel, ksi

\( F_{p} \)  Tensile proportional limit of virgin steel, ksi

\( F_{ya} \)  Average yield point of sections, ksi

\( F_{yc} \)  Tensile yield point of corner, ksi

\( I_{i} \)  Moment of inertia of the sub area about the neutral axis of the total cross section, \( \text{in}^4 \)

\( I_{y} \)  Moment of inertia of the section about \( y \) axis, \( \text{in}^4 \)

\( i \)  Number of sub-areas

K  Effective length factor

L  Length of member, in.

m  Defined by Eq. 14

N  Axial load constant along the length of the members, Kips

\( N_{cr} \)  Flexural buckling load, Kips

n  Number of corner
Appendix - Notation (cont.)

Q  Area factor to modify allowable axial stress
R  Inside bend radius, in.
rs  Defined by Eq. 5, in.
ry  Radius of gyration of cross section about centroidal principal axis, in.
Sy  Section modulus in³
r  Steel thickness of the member, in.
α  Empirical constant defined by Table 1
αl  Empirical constant defined by Table 2
β  Empirical constant defined by Table 1
δ  Deviation from flatness, in.
δo  Initial deviation from flatness, in.
δl  Equivalent deviation, in.
δmax  Maximum deviation, in.
λ  Slenderness ratio
λ  Slenderness ratio defined by Eq. 8
λcr  Slenderness ratio limit, defined by Eq. 6
λy  Slenderness ratio about y axis
σ  Axial stress, ksi
σl  Axial stress defined by Eq. 4
σcr  Flexural buckling stress, ksi
σe  Euler buckling stress, ksi
σlim  Limit tensile strength, ksi
σmax  Maximum edge stress, ksi
ϕ  Buckling coefficient defined by Eq. 7
ψ  Coefficient of the partial plasticizing