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Analysis of Local Plate Buckling Experimental Data

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1. Introduction

In imperfect systems, because of initial geometric deviations, deflections begin to grow with the commencement of load. Local buckling by bifurcation of a real (and therefore imperfect) plate cannot occur in a strict theoretical sense. But a defined local buckling stress for a plate on the basis of experimental observations is meaningful to an engineering designer in a manner similar to a defined yield stress for materials which have no sharp yield stress. At stresses considerably larger than the ideal critical stress, the elastic load versus out-of-plane deflection behaviour of an imperfect plate or plate component belonging to the stable symmetric class of structures approaches the behaviour of the perfect plate. From this point of view, knowledge of the ideal critical stress is useful in that it facilitates the description of elastic post buckling behaviour.

In this paper, various methods in common use, including Southwell's method [10], for determining local buckling loads of stiffened plates or plate composites are briefly discussed. A useful and very simple alternative to the Southwell plot for systems which exhibit a symmetrical post-buckling behaviour is presented. The initial post-buckling curve in this case is of parabolic form when no imperfections are present. This parabola can be plotted as a straight line under suitable transformation of variables. The

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curve for the imperfect structure will be a close approximation to this straight line when subjected to the same transformation. This leads to a useful technique for analysing experimental results in an effort to obtain the critical load. Experimental work is presented to substantiate this technique for plate composites. Experimental evidence is also presented which shows the problems arising from the usual Southwell method. In addition, critical stresses resulting from the theory presented in Ref. [13] are compared with experimental values using the technique discussed above and also the "vertical tangent method".

2. Various Methods for Determining Critical Loads from Test Data

The experimental methods for determining critical loads, including the Southwell procedure, primarily use load-deflection and stress-strain plots corresponding to mid-width of the plate elements. Details of each method are available in the literature. They are briefly examined below as applied to plates subjected to edge compression.

i) Inflexion point method

The correspondence of the inflexion point of the load-deflection curve to the critical load seems plausible because the minimum slope (zero) of the load-deflection curve of a perfect system occurs at the instant of buckling. According to Coan [2], if the geometric imperfection is not excessive, this method appears to be very satisfactory.

ii) 'Top of the knee' method

The critical stress resulting from the so-called "top-of-the-knee" method is essentially the stress corresponding to the knee of a curve of stress against out-of-plane deflections or stress against the strain-difference, in the direction of loading, on the two faces of the buckle. Although such a critical stress is significant as an indication of the rapidly growing out-
of-plane deflections, it is based on an inadequately defined point whose location is left to the personal judgement of the observer. It appears that this method leads to values that are appreciably lower than ideal critical values.

iii) Strain reversal method

In the strain-reversal method, the local buckling stress corresponds to the stress at which the extreme fibre strain on the convex side of the buckle stops increasing and begins to decrease. Critical stresses obtained by this method may lie roughly on the knee of the stress-deflection curve. This method generally gives more conservative values than the 'top-of-the-knee' method [7]. It gives values well below theoretical ones and is probably the least reliable of the methods examined here [2].

iv) Vertical tangent method

The vertical tangent to the load-average axial strain curve has been adopted in [5] as the criterion to estimate the local buckling stress. On the basis of the large deflection theory of plates Coan [2] found that, although this method overestimates the critical stress by 2 to 3%, it gives the most reliable indication of the critical load. Figures 3 and 5 demonstrate that this technique is indeed a simple one.

v) Load vs. bending strain method

The out-of-plane deflection is related only to the difference between the extreme fibre strains or simply the "bending strain", and not to the average extreme fibre axial strain. By a reasoning parallel to that used in connection with the load-deflection plot, the critical load can be determined by the inflexion point of the "load-bending strain" curve. The accuracy of this method is of the same order as that of the inflexion point method.
vi) Modified strain-reversal method

A modified form of the strain-reversal method discussed above was employed by Divakaran [4]. In this procedure, the onset of nonlinearity in the load versus average axial strain plot is taken to correspond to the critical stress. However, from the preceding discussion it appears that this method is conservative and as unreliable as the strain-reversal method itself.

Of the methods discussed above, it can be concluded that the vertical tangent method gives a reliable estimate of the critical load and the method itself is a simple one.

It should be kept in mind that all criteria for experimental determination of the elastic critical stresses of plate elements and plate composites are somewhat arbitrary. The critical stress which is determined by using these criteria is in the neighbourhood of the true value, but may not be the true value itself.

3. The Southwell Method

Southwell [10] whose analysis is based on small deflection theory, demonstrated that the load-deflection curve of an initially imperfect pin-ended column is a rectangular hyperbola in the neighbourhood of the smallest critical load $P_{cr}$, the asymptotes of the hyperbola being the load axis and the horizontal line $P = P_{cr}$. By suitable change of coordinates (variables), this hyperbola may be transformed into a straight line whose slope is the reciprocal of the smallest critical load of the column. In practice, the lateral deflection $\delta$ and load $P$ are measured during a column test, and by plotting $\delta/P$ versus $\delta$, $P_{cr}$ can be calculated from the gradient of the best straight line fitted to the test points. In this way, it is possible to determine the ideal critical load from a test on an imperfect structure.
Timoshenko [11] has suggested the use of the Southwell method in plate buckling experiments. In a theoretical investigation undertaken by Hu, Lundquist, Batdorf [7] to evaluate the effect of small deviations from flatness on the behaviour of simply supported square plates under compression, it was shown that the stress-deflection curve usually does not approximate a rectangular hyperbola at stresses near the critical stress unless the initial deviation from flatness happens to be very small. Laboratory experience appeared to indicate that the initial deviation from flatness and the inevitable inaccuracy of loading were not sufficiently small to guarantee this close approximation. Even if these factors were sufficiently small, difficulty was encountered in measuring with sufficient accuracy the very small deflections used to make the Southwell plot. They conclude that the Southwell plot method may not yield, in general, satisfactory results when applied to flat plates.

In a study of the elastic stability of uniformly compressed thin-walled sections, Chilver [1] indicated that if low deflection values were used, then the straight lines in the plot similar to those predicted by Southwell for columns were, in fact, obtained experimentally. Walker [14] applied the Southwell technique to the results of tests on eccentrically loaded plates and thin-walled sections. The method has also been applied to a number of other elastic buckling problems, such as frames and shells. However, bearing in mind that the Southwell approach is based on small deflection theory, Roorda [9], argued that in most stability experiments, the initial geometric imperfections are of such magnitude that to obtain meaningful results, experimental observations must be taken at values of deflections well beyond the small deflection range. It would appear, therefore, that to be consistent, a large deflection theory which takes into account nonlinearities in the structure, has to be used as a basis of the
Southwell plot. In the nonlinear theory of elastic stability, the post-buckling behaviour is generally not of the neutral type, which was the basis of Southwell's plot, but takes one of the three forms described as "asymmetric", "stable symmetric", and "unstable symmetric". The load-deflection curves for the corresponding imperfect systems are not rectangular hyperbolas but have the perfect equilibrium curves as asymptotes. These load-deflection curves lead to curved Southwell lines rather than straight lines as in the case of neutral characteristic. A qualitative analysis [9] of the nature of these more general Southwell lines indicates that the best straight line fitted to the experimental points in a Southwell plot of elastic plate buckling data would over-estimate the critical stress for that plate.

Attempts [12] were made to use the Southwell method for determining local buckling loads of concentrically, and eccentrically loaded thin-walled sections, but without much success. Two examples are given below. Figure 1 shows the experimental load-deflection data related to the web plate components of a concentrically loaded thin-walled test specimen using the Southwell method. It is apparent that straight lines of different slopes could be constructed through the data corresponding to low deflection values. One such straight line shown in dotted form yields a value of 5.84 ksi for the critical stress. This value is far from the actual value of 9.72 ksi.

Figure 2 shows the interpretation of the Southwell plot for the web plate component of an eccentrically loaded thin-walled test section. It is seen from Figure 2 that the Southwell plot yields at least two values for critical stress, 2.11 ksi and 12.85 ksi, depending upon which subset of test data, related to small deflection range, is considered. Furthermore, neither of these two values are close to the actual value of 9.2 ksi.
Thus it appears that a "trial type" judgement has to be used in determining the critical loads and the whole process seems to inspire no confidence in the end results. It could be concluded, therefore, that the Southwell plot is not well suited to plate type structures possessing a nonlinear post-buckling behaviour. A useful alternative to the Southwell method which employs load and deflection data seems, therefore, very desirable.

4. The alternative \((P - w^2)\) method

For systems which exhibit a symmetric post-buckling behaviour, it has been shown in Appendix 1 that the initial post-buckling curve is of parabolic form when no imperfections are present (Fig. 7a). When \(P\) is plotted against \(w^2\), this parabola transforms into a straight line which intersects the \(P\)-axis at a value of \(P_{cr}\). The curve for the imperfect structure will be in close approximation to this straight line when plotted in a similar way (Figure 7b). If a tangent is drawn to this imperfect curve at an arbitrary value of deflection beyond the small deflection range, this tangent intersects \(P\)-axis at a value \(P_i\), which is a lower bound to \(P_{cr}\) (See Appendix 1). Experimental data plotted in this manner for web and flange components of concentrically and eccentrically loaded thin-walled channel sections are shown in Figure 4 and 6, respectively. These plots are typical of the many tests performed and indicate sharp changes in slopes of the \(P-w^2\) curves near the level of critical loads. It is apparent that the values of \(P_i\) resulting from these plots for the web plate components are close to the corresponding values determined by "the vertical tangent method" that uses the load-average strain plots (Figures 3 and 5). A comparison of Figure 4 with Figure 1 and a comparison of Figure 6 with Figure 2 substantiates the reliability, simplicity and suitability of the \((P-w^2)\)
method over that of the Southwell method.

5. **Comparison of Experimental and Theoretical Critical Stresses**

The theoretical critical loads used for comparison here are those derived on the basis of the theory presented in Ref. [13] for stiffened channel sections of the type shown in Fig. 8(a) and (b). The theory treats the web plate as being elastically restrained by the flanges as it buckles (Fig. 8(c)). The theory is capable of treating the stiffener-flange junction in two ways: a) as an elastically supported edge in which the stiffener acts as an elastic beam, or (b) as a simply supported edge. It is shown in [13] that the critical load is relatively insensitive to the stiffener size beyond a certain level.

For the case of simply supported flanges (case b), the variations in the local buckling constants for the web \( k_w \) and the flange \( k_f \) for a range of flange width to web width ratios \( b_f/b_w \) are given in Fig. 9. In this figure \( \gamma = 0 \) is a concentric loading and \( \gamma = 1.0 \) an eccentric loading with zero stress on the stiffener. These results show that the web buckling constant, \( k_w \), is relatively insensitive to both \( b_f/b_w \) and \( \gamma \) for low values of \( b_f/b_w \), and may, with a high degree of confidence, be set equal to the constant value \( k_w = 5.6 \) in this range.

Compression tests were done on 72 thin-walled, stiffened channels with \( b_f/b_w \) ranging from 0.375 to 0.500, both with eccentric (\( \gamma = 1 \)) and concentric (\( \gamma = 0 \)) loading patterns to determine elastic critical loads. Edge stiffeners were provided on each test specimen in conformity with, or in excess of, design code requirements. For the purpose of obtaining elastic critical loads the flange stiffener junction may therefore be assumed to be simply supported without great inaccuracy. On the basis of the above arguments, therefore, all test specimens can be compared to the single theoretical relationship
The test specimens were formed of galvanized sheet steel of a wide range of thicknesses. The overall nominal dimensions of the cross-section and details such as the ratios \( b_w/t \) and the number of specimens tested are given in Table 1. The behaviour of the web plate, rather than that of the flanges, was measured since the web initiates instability in the test specimen (see Fig. 9), and is therefore likely to exhibit more nonlinearity in the region of the critical load (see Figs. 3 and 5). Electrical strain gauges and/or direct current displacement transducers were mounted on each specimen at mid-width and mid-height of the web plate. The length of the test specimens was such that local instability of the cross-sections preceded their final collapse.

Experimental critical loads were determined on the basis of either or both of the following method:

i) the vertical tangent (to the load versus average axial strain) method, and

ii) the alternative \((P - w^2)\) method that uses the plots of load versus the square of the lateral deflection.

The theoretical critical stress and the experimentally determined values are tabulated in Table 1. The results of the vertical tangent method are plotted in Fig. 10 and those of the alternate method in Fig. 11. In these figures the experimental results are shown as vertical bars indicating the range of results for each \( b_w/t \) ratio, with the average value indicated by the cross line. The theoretical curve is given by eqn. (1). In both cases the experiments tend to give slightly higher results than the theory, more so in the case of the vertical tangent method than the \((P - w^2)\) method. A

\[
\sigma_{cr} = \frac{5.6 \pi^2 E}{12(1 - \nu^2)(b_w/t)^2} \tag{1}
\]
linear regression analysis performed on the 53 data points obtained by
the vertical tangent method shows the best fit curve to take the form

\[ \sigma_{cr(EXP)} = \frac{6.40 \pi^2 E}{12(1 - \nu^2)(b_w/t)^2} \]  
(2)

A similar analysis on the 59 data points from the \((P - w^2)\) method gives

\[ \sigma_{cr(EXP)} = \frac{5.58 \pi^2 E}{12(1 - \nu^2)(b_w/t)^2} \]  
(3)

Relative to each other, these two equations fall in line with theoretical
expectations. The vertical tangent method is known to over-estimate the
actual critical stress by a few percent [2] and the alternate \((P - w^2)\)
method is known to give a lower bound to the actual critical stress (Appendix
1). Comparison of eqs. (2) and (3) with eqn (1) leads one to the conclusion
that the use of a web buckling constant \(k_w = 5.6\) gives predicted critical
stresses on the conservative side. The overall correlation between theoret-
ical and experimental values appears to be very good in essence and form.
Thus the theory presented in Ref. [13] is substantially confirmed.

It should be pointed out, however, that the alternative method proposed
here still involves some personal judgement in selecting the location of the
tangent line on the \((P - w^2)\) curve, thus directly influencing the values of
critical stresses. The main advantage of the \((P - w^2)\) method is that the
deflections beyond the small deflection range are involved. From a practical
viewpoint, the \((P - w^2)\) method is simple to use. Unlike the Southwell method,
the effect of personal judgment is comparatively small. The present experi-
ence suggests that the deflections in the close neighbourhood and just
beyond those related to the critical load are the most appropriate for the
purpose of the suggested alternative method. The marked deviation of \((P - w^2)\)
curves, in the region of critical loads, is a definite advantage in favour of this method. On the other hand, the vertical tangent method requires very little subjective judgement in determining critical stresses. The results from this method are thought to be more reliable.

6. Conclusions

1. The vertical tangent to the load-average strain plot gives a reliable estimate of the critical load. This method is a very simple and direct one.

2. The use of the Southwell method for plate type structures involves a "trial type" judgment. It leads to unreliable and erroneous results. Its application to such systems is discouraged.

3. The load plotted against the square of the out-of-plane deflections of an imperfect plate system having a symmetric post-buckling behaviour leads to a straight line in the region just beyond the critical load. The intercept on the load axis resulting from a tangent drawn in this region yields a lower-bound value to the critical load. This method is simple to use, and gives consistent results.

4. The experimental results, analysed by the vertical tangent method and by the alternate \( (P - w^2) \) method, substantially confirm the theory presented in Ref. [13].

5. The experimental and theoretical comparisons indicate that for edge-stiffened thin-walled channels with small \( b_w/b_f \) ratios, a local buckling constant of \( k_w = 5.6 \) gives critical loads that are on the conservative side.
REFERENCES


13. Venkataramaiah, K.R. and Roorda, J., "Local buckling of thin walled channels", Recent Research and Developments in Cold-Formed Steel Structures, Proc. Fourth Int. Specialty Conference on Cold-Formed Structures, University of Missouri-Rolla, pp. 363-408, 1978.
APPENDIX 1

ANALYSIS OF EXPERIMENTAL OBSERVATIONS IN PROBLEMS OF ELASTIC LOCAL INSTABILITY

Koiter's [8] stability equation for load (P)-deflection (w) relationship applicable to symmetric systems could be written as

\[ w_0 \beta_1 + 2(P-P_{cr})A_2'w + n A_n w^{n-1} = 0. \]  \hspace{1cm} (A.1)

For symmetric systems the first non-trivial term of importance corresponds to \( n = 4 \). Therefore

\[ w_0 \beta_1 + 2(P-P_{cr})A_2'w + 4 A_4 w^3 = 0. \]  \hspace{1cm} (A.2)

In Equation (A.2), \( \beta_1`, A_2' \) and \( A_4 \) are functionals which are related to the system under consideration. For stable symmetric systems (e.g., plate type problems), \( A_2' < 0 \) and \( A_4 > 0 \). Solving for the quantity, \( P-P_{cr} \), from Equation (A.2) yields

\[ P_{cr} - P = \frac{4 A_4 w^2}{2 A_2'} + \frac{w_0 \beta_1}{2 A_2' w}, \]  \hspace{1cm} (A.3)

where \( P_{cr} \) is the critical load of the perfect system. Let

\[ \frac{4 A_4}{2 A_2'} = -C_1, \quad \text{and} \quad \frac{w_0 \beta_1}{2 A_2'} = C_2 \]

where \( C_1 \) is a positive quantity. Then

\[ P = P_{cr} + C_1 w^2 - \frac{C_2}{w}. \]  \hspace{1cm} (A.4)

For a perfect case \( w_0 = 0 \); hence \( C_2 = 0 \) and

\[ P = P_{cr} + C_1 w^2. \]  \hspace{1cm} (A.5)
Equation (A.5) suggests a parabolic relationship between $P$ and $w$. Setting $w^2 = Z$ changes (A.4) for an imperfect case into

$$P = P_{cr} + C_1 Z - C_2 Z^{-1/2} \quad (A.6)$$

The slope, $dP/dZ$, is

$$\frac{dP}{dZ} = C_1 + \frac{1}{2} C_2 Z^{-3/2} \quad (A.7)$$

For large values of $Z$ or for $|w_0| \ll 1$, Equation (A.7) implies a near straight line relationship between $P$ and $Z$. The slope given by (A.7) approaches the value of the perfect case, $C_1$, where the $P-Z$ relationship is of straight line form (see Figure 7b).

If a point $Z_1$ is chosen arbitrarily on the straight line portion given by (A.7) and a straight line tangent is drawn at this point, then this line may be described by

$$P = (C_1 + \frac{1}{2} \frac{C_2}{Z_1^{3/2}})Z + C_3 \quad (A.8)$$

where $C_3$ is a constant.

Simultaneous solution of the tangent (A.8) and the curve (A.6) at the point $Z = Z_1$ yields

$$C_3 = \frac{P_{cr} - \frac{3}{2} C_2 Z_1^{-1/2}}{Z_1}$$

and hence the tangent line is given by

$$P = (C_1 + \frac{1}{2} \frac{C_2}{Z_1^{3/2}})Z + P_{cr} - \frac{3}{2} C_2 Z_1^{-1/2} \quad (A.9)$$

For the perfect case

$$P = P_{cr} + C_1 Z \quad (A.10)$$
It is readily seen that as $Z_1$ becomes large or $C_2$ becomes small, the tangent line obtained from (A.9) approaches that of the perfect case. In practical situations, only the load versus deflection curve emanating from the zero load point has some significance, although other complementary curves exist in theory. It is therefore readily shown that the combination $C_2 Z_1^{-1/2}$ should always be taken as a positive quantity. The intercept of the tangent line with the vertical axis (Z = 0)

$$P_i = P_{cr} - \frac{3}{2} C_2 Z_1^{-1/2}, \quad (A.11)$$

therefore represents a lower bound on the critical load of the perfect case, and it approaches this critical load as $Z_1$ becomes large or $C_2$ becomes small.

If this procedure is followed graphically on the experimental load-deflection curves (with appropriate axes) it follows that the intercept with the vertical axis of an appropriate tangent line, drawn on a straight line portion, will yield a close estimate of the theoretical critical load.

It must be remembered, however, that the equation developed by Koiter is valid only in the region of the critical load, for small deflections and for small imperfections. For this reason, the point for drawing the tangent should be chosen with care. It is, however, appropriate to mention that the experimental $P$ versus $w^2$ plots for compressed thin-walled sections shown in Figures 4 and 6 indicate that Koiter's theory may be applicable in the range beyond small deflection. It is thus felt that the technique outlined will lead to a close (lower bound) approximation to the ideal critical load.
### TABLE 1. Local Buckling Stresses of Test Sections

<table>
<thead>
<tr>
<th>Section geometry $b_w \times b_f$</th>
<th>Load type</th>
<th>Web thickness ratio ($b_w/t$)</th>
<th>Theor. critical stress (ksi)</th>
<th>Experimental Critical stress (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of tests</td>
<td></td>
<td>Vertical tangent method</td>
<td>$(P - w^2)$ method</td>
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<tr>
<td>8 x 4 in.</td>
<td>C</td>
<td>11</td>
<td>298</td>
<td>1.71</td>
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<td>8 x 3 in.</td>
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### TABLE 1 (continued)

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<th>Section geometry $b_w \times b_f$</th>
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<th>No. of tests</th>
<th>Web thickness ratio $(b_w/t)$</th>
<th>Theor. critical stress (ksi)</th>
<th>Experimental Critical stress (ksi)</th>
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$E = \text{eccentric} \quad C = \text{concentric}$
FIG. 1 - SOUTHWELL PLOT ON WEB DEFLECTIONS FOR CONCENTRICALLY LOADED CHANNEL SPECIMEN - TOR 2
FIG. 2 - SOUTHWELL PLOT ON WEB DEFLECTIONS FOR ECCENTRICALLY LOADED CHANNEL SPECIMEN - G16LL1
FIG. 3 - AVERAGE STRESS VS. AXIAL STRAIN FOR DIFFERENT LOCATIONS IN CROSS SECTION AT MID-HEIGHT OF COLUMN-TOR2
FIG. 4 - LOAD VS. SQUARE OF DEFLECTIONS OF WEB AND FLANGE COMPONENTS AT MID-HEIGHT OF COLUMN-TOR2
FIG. 5 - UNIFORM STRESS ON WEB VS. AXIAL STRAIN FOR DIFFERENT LOCATIONS IN CROSS SECTION AT MID-HEIGHT OF COLUMN-G16L1
FIG. 6 - LOAD VS. SQUARE OF DEFORMATIONS OF WEB AND FLANGE COMPONENTS AT MID-HEIGHT OF COLUMN-G16LL1
FIG. 7 - POSTBUCKLING BEHAVIOUR
EXPERIMENTAL DATA ON LOCAL BUCKLING

(a) Loading Diagram

(b) Equivalent Loading Systems

(c) Free Body Diagrams of Web and Flange Plate Components

FIG. 8 - LOADING DIAGRAMS OF CHANNEL SECTION
FIG. 9 - VARIATION OF LOCAL BUCKLING CONSTANTS WITH GEOMETRY FOR CHANNEL SECTIONS
THE VERTICAL TANGENT METHOD
53 DATA POINTS

THEORY
Eq. (1)

HIGHEST VALUE
MEAN VALUE
LOWEST VALUE

EXPERIMENT

CRITICAL STRESS, $\sigma_{cr}$ (ksi)

WEB THINNESS RATIO ($b_w/t$)

FIG. 10
THE \((p-w^2)\) METHOD
59 DATA POINTS

THEORY
Eq. (1)

HIGHEST VALUE
MEAN VALUE
LOWEST VALUE

CRITICAL STRESS, \(\sigma_{cr}\) (ksi)

WEIGHT THINNESS RATIO \((b_w/t)\)

FIG. 11