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Probability Analysis of Cold-formed Steel Structures

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PROBABILITY ANALYSIS

OF

COLD-FORMED STEEL STRUCTURES

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INTRODUCTION

Ultimate and serviceability limit states are examined for cold-formed steel floor joists. The analysis is based on an assumed set of tolerances, probabilistic models for loads and strength as well as allowable levels for deflections. The design criteria used in this paper were kept simple in order to demonstrate the procedure but the principles illustrated can be extended to more complex design situations involving the consideration of multiple failure modes. The information on the relative importance of strength and stiffness as well as the effect of tolerances on various parameters is expected to be useful in the design of cold-formed steel structures.

Design conditions usually require to satisfy inequalities of the type $D \leq C$ where D denotes the demand such as load effects, deflections or levels of vibration and C is the capacity such as strength or compliance threshold for deflection or vibration. Since D and C are generally uncertain, the design condition cannot be satisfied with certainty. Thus other criteria are needed for design. Probabilistic studies require that the inequality $D \leq C$ be validated with a specified probability. Various approximations have been developed to measure the probability, $P(D \leq C)$,

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of $D \leq C$. Reliability and serviceability indices are the most frequently applied measures. These indices can be obtained from Ref. 2:

$$\beta = \frac{m_C - m_D}{\sqrt{\sigma_C^2 + \sigma_D^2}} \tag{1}$$

depending only on the means, m, and the standard deviations, σ, of D and C, or from Ref. 8:

$$\beta = \min_x \sqrt{[\Phi^{-1}(F_D(x))]^2 + [\Phi^{-1}(F_C(x))]^2} \tag{2}$$

in which F denotes distributions of D and C and Φ is the distribution of the standard Gauss variable. Other formulations are also available for finding reliability. The index in Eq. 1 can be in great error when the capacity or the demand has skewed distributions since $\Phi(\beta)$ may differ significantly from $P(D \leq C)$ but is exact for Gaussian capacities and demands. On the other hand, the index of Eq. 2 is superior to that of Eq. 1 but is less simple. Typical values of reliability indices are in the range 3 to 4 and correspond to probability of failures of the order 10^{-3} to 10^{-4}.

In this study, the reliability and serviceability indices are determined for simply supported joists from Eq. 2. Not all failure modes and serviceability requirements were accounted for. The analysis of the ultimate limit states considers only flexural failure and is based on the condition

$$\frac{Q\ell^2}{8} \leq SF_y P^* \tag{3}$$

in which ℓ is the joist span, Q is the uniformly distributed load, S is the section modulus, F_y is the yield stress, and P* is the professional factor that corrects the flexure formula in Eq. 3 to fit test results. Other failure modes were not considered. The effects of torsion, continuity over supports, web crippling and local buckling were ignored. For example the consideration of torsion effects as is done for purlins in Ref. 7 would have been too complicated for the purposes of this study. Because the behavior of floor joists is similar to that of purlins, similar studies for floor joists are needed. Little is known about the behavior of continuous joists near the supports where the compression flange is laterally unbraced. The inclusion of such considerations in a probabilistic approach is planned for the future.

Serviceability limit states are assumed to be controlled by the deflection at midspan, that is:

$$\frac{5Q\ell^4}{384EI} \leq \delta \qquad (4)$$

in which I is the moment of inertia, δ is the allowable deflection and E is the modulus of elasticity. It is assumed that Q, S, I, F_y and P* are random variables. In the following sections, the statistics for these variables are determined and then used to find reliability and serviceability indices.

STATISTICS FOR MOMENT OF INERTIA AND FLEXURAL STRENGTH

Statistics will be determined for the moment of inertia, I, and the flexural strength, $R = S\,F_y\,P^*$, for the lipped channel joist shown in Fig. 1 using the findings of Ref. 4 and the tolerances specified in the Swedish Standards (9).

There are no studies available in literature on the dimensional accuracy of cold-formed steel members. This is perhaps due to the fact that such members are not standardized and that the industry practices vary. The dimensional accuracy depends on the condition of the rolls used to manufacture the section and the care used in fabrication.

The only requirement on this subject in the AISI Specification (Ref. 1) pertains to the thickness. It is stated that "the uncoated minimum steel thickness of the cold-formed product as delivered to the job site shall not at any location be less than 95 percent of the thickness used in its design." There are no other requirements in the AISI Specification on tolerances. The following values (all given in inches) were quoted by a North American manufacturer as their dimensional tolerances. The thickness of hot rolled sheets of 0.060" to 0.177" thickness are held within ± 0.007". The thickness of cold rolled sheets of 0.060" to 0.142" thickness is held within ± 0.005" to 0.006". The total section depth and the flange width joists are required to be within ± 0.003" to 0.004". The inner corner radius is required to be within ± 0.001" to 0.002" of the specified values. The depth of lips are to be accurate within ± 0.120" of the specified values. The corners are required to be within ± 2 degrees of the specified values.

In their studies on cold-formed steel members such as those reported in Ref. 7, dimensional inaccuracies an order of magnitude higher than those listed here were observed. It was therefore decided not to use these values. It was decided to use the tolerances specified in the Swedish Standard on Thin-Walled Construction (Ref. 9). In this Standard, it is required to have the following maximum deviations in order to use section properties based on nominal dimensions:

Sheet thickness .... -5%

Profile depth .... -1 mm for dimensions $\leq$ 50 mm

Profile depth .... -2% for dimensions > 50 mm

Width of single lip edge stiffener .... -5%

Depth of an intermediate stiffener .... -5%

Corner radius .... +1 mm

Angle .... ± 3 deg.

Table 1 shows the sensitivity of the moment of inertia to variations in the dimensions. In this table the perimeter is kept constant and each dimension is varied ± 10 percent. From this table it is seen that the variation in various dimensions influence the moment of inertia to different degrees.

The values of the geometric parameters specified in an American steel manufacturer's products catalog are b_s = 1.8125, d_s = 7.25, r_s = 0.094, p_s = 11.71 and t_s = 0.076, all in inches. According to Ref. 4, the mean of the actual, random thickness, T, exceeds t_s by 6 percent and the ratio T/t_s has a coefficient of variation of 0.053. Unfortunately, as stated above, such information is not available for other geometric parameters.

It has been assumed that the mean of the actual to specified value ratios is unity for B/b_s, D/d_s, R/r_s and P/p_s. The coefficient of variation for these variables were computed on the basis of the tolerances specified in the Swedish Standards (Ref. 9) and the assumption that the values of these geometric parameters have the same likelihood within the range of tolerances as follows. If the e1 and e2 are the absolute values of the tolerances about the specified value, xs, of a random geometric parameter, X, then the mean and the coefficient of variation of X are, respectively, xs + (e2 - e1)/2 and (e1 + e2)/($2\sqrt{3}$ (xs + (e2 - e1)/2)). According to the Swedish Standard (Ref. 9), the values of (e1, e2) are (0.02, 0.02), (0.02, 0.02), (0.02, 0.04) and (0.01, 0.01) for B/b_s, R/r_s and P/p_s. The means and coefficients of variations determined for the geometric parameters have been used to calibrate normal and lognormal distributions assumed for these parameters in the analysis of the moment of inertia and of the flexural strength.

The moment of inertia of the lipped channel section (about the x-x axis) shown in Fig. 1 can be written as

$$I = 2T\{0.0417\,A^3 + B^1(A/2 + R')^2 + U(A/2 + 0.637R')^2 + \\ + 0.149R'^3\,\alpha[0.0833\,C^3 + C(A - C)^2/4 + \\ + U(A/2 + 0.637R')^2 + 0.149R'^3]\} \quad (5)$$

where $R' = R + T/2$, $U = \pi R'/2$, $B' = B - 2R' - T$, $A = D - 2R' - T$, $C = ((P - (A + 2B' + 2U))/\alpha - 2U)/2$ and $\alpha = 1$.

It is possible to find simple approximations for the mean and variance of I from similar statistics of the geometric parameters if I is approximated by a linear equation in these parameters that can be obtained by first order Taylor expansion of Eq. 5 about the mean of the geometric parameters. However, this approach provides no information on the distribution of I and can be in great error because of the complex dependence of I on the geometric parameters. To overcome these difficulties, the statistics of I have been found by simulation. Nine hundred samples have been generated from the geometric parameters and used in Eq. 5 to obtain samples of I. Table 2 gives the means, coefficients of variation, coefficients of skewness, γ_3, and the coefficients of kurtosis, γ_4, of I. Fig. 2 provides histograms of I for normally and lognormally distributed geometric parameters.

A similar approach was used to develop statistics for the flexural strength, $R = SF_y P^*$. Samples of S have been obtained directly from samples of I since $S = 2I/D$. These samples were then combined with random values of F_y and P^* to determine the flexural strength. From Ref. 4, F_y is lognormally distributed with mean $1.17f_y$ and coefficient of variation of 0.10. The professional factor is assumed to be a normal variable with means and coefficients of variation of 1.02 and 0.06, respectively, or 0.98 and 0.10, respectively, as estimated based on our experience on cold-formed steel research. These statistics were used to examine the sensitivity of the strength to the professional factors in various studies. Table 3 gives statistics found for $SF_y P^*/f_y$ and Fig. 3 shows histograms of the flexural strength.

Findings in Tables 1 and 2, and Figs. 2 and 3 and results in Table 4 obtained from Ref. 5 show that the moment of inertia and the flexural strength can be modelled by normal or lognormal variables since they have positive but negligible skewness and kurtosis coefficients nearly equal to 3. It is also seen that the statistics postulated for the professional factor modify the flexural strength appreciably (Table 3).

PROBABILISTIC MODELS FOR LOADS

The floor joists examined in this study support dead and live loads. The dead load is assumed to be perfectly known and equal to 10 psf since the uncertainty in this load is not generally significant but probabilistic models are used for the live load. The live load involves two components, the sustained live load that is practically constant over the duration of any occupancy and the extraordinary live load that occurs infrequently and is active over short periods (Fig. 4).

It has been found (Refs. 3, 6) that the maximum live load in 64 years, L, can be represented by an extreme Type I random variable with mean $18.7 + 520/\sqrt{A}$ (psf) and variance $14.2 + 18900/A$ (psf)^2 in which the influence area, A, is twice the tributary area for beams (Refs. 3, 5). The mean and the variance of L must be reduced depending on the area only for areas larger than $A = 200$ ft^2 (Ref. 3). The distribution of L is

$$\text{Prob}(L \leq x) = \exp\{- \exp[-\alpha\ (x - u)]\} \qquad (6)$$

in which $\alpha = \pi/\sqrt{6(14.2 + 18{,}900/A)}$ and $\mu = 18.7 + 520/\sqrt{A} - 0.577216/\alpha$. The maximum live load, L, is used to check ultimate limit states.

Analysis of serviceability limit states is usually based on a different loading condition, the largest load in an occupancy (Ref. 10), because any serviceability failure during an occupancy is usually repaired before the beginning of another occupancy. This loading condition can be obtained from the sum of the instantaneous value of the sustained load, L_{apt}, and the maximum value of the extraordinary live load during an occupancy, L_E. The load L_{apt} follows a Gamma distribution with mean 11.6 psf and variance $26.2 + 14300/A$ psf^2 (Ref. 3) while L_E can be approximated by $\ell_E = F_E^{-1}(0.9253)$ because this load has a small variance, in which F_E = the distribution of the extraordinary load (Ref. 6).

INDICES OF RELIABILITY AND SERVICEABILITY

Reliability and serviceability indices have been determined for the joist section shown in Fig. 1, e.g., for spans of 230, 209 and 189 inches with spacing between the joists equal to 12, 16, and 24 inches, respectively, for type B sections.(Fig. 1).

The midspan bending moment can be expressed as $M = a(10 + L)$ kip-in where L is the maximum live load in psf and a has the values 0.55104, 0.60668 and 0.69768 for spans of 230, 209 and 183 inches, respectively. The design condition is then

$$R = SF_y \; P^* \geq M = a(10 + L) \tag{7}$$

where R is assumed to be a lognormal variable and L is extreme Type I distributed load. Table 5 gives reliability indices obtained from Eq. 2. These indices differ significantly from those obtained from Eq. 1 based on means and variables. For example, β from Eq. 1 is 5.05 for a span of 230 inches and a professional factor of 1.02. The table shows that there is a significant variation in safety of different designs. Some of the designs recommended in the manufacturer's literature appear to be somewhat unconservative.

The analysis of serviceability limit states is outlined in Eq. 4 and can be rewritten in the form

$$M_s \leq \frac{48E}{5\ell} \frac{\delta}{\ell} I \tag{8}$$

where $M_s = a(10 + L_{apt} + \ell_E)$, I is the moment of inertia with the statistics given in Table 1 and δ/ℓ is the allowable deflection assumed to be 1/500, 1/200, or 1/125. From Table 1 and the statistics of L_{apt} in Ref. 3, it can be assumed that M_s and L are lognormal random variables. The serviceability indices obtained from Eq. 2 are approximately 9 for δ/ℓ = 1/500 for all spacing between the joists. The large values obtained for the serviceability limit states indicate that serviceability limit states involving deflections are likely to be satisfied for the joists considered. It should be noted that the present serviceability analysis accounts for static deflections only. Effects of vibration of floor joists have not been investigated here.

CONCLUSIONS

Reliability and serviceability indices have been determined for a certain cold formed steel floor joist. It was found that:

(i). Flexural strength and stiffness depend significantly on the variation of the geometric parameters of the joists. There is then a need for a study of tolerances in the fabrication of cold-formed steel joists,

(ii). Reliability indices for flexure varies significantly from design to design and some designs appear to be unconservative. Since these indices account for only a mode of failure, they overestimate the actual level of reliability of these structures. Further studies are in order to find indices of reliability that account for all failure modes, and

(iii). Deflections appear to be well-controlled in most designs. Yet, other serviceability limit states, such as vibrations need to be investigated.

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TABLE 1

SENSITIVITY OF MOMENT OF INERTIA

TO DIMENSIONAL VARIATIONS

PARAMETER VARIED	RATIO* (for +10% variation)	RATIO* (for -10% variation)
B	1.017	.980
A	1.158	.826
R	1.003	.997
P	1.090	.865
T	1.100	.900

* I_{xv}/I_x

I_x for the section with B = 1.549, D = 7.25, R = 0.94, P = 11.44, T = 0.76 (all in inches) A was calculated from the given dimensions.

I_{xv} for the section with changed dimensions.

TABLE 2

STATISTICS FOR MOMENT OF INERTIA (TYPE B SECTION)

Moments	Normal Geometric Parameters	Lognormal Geometric Parameters
Mean	6.9355	6.9595
C.O.V.	0.0522	0.0500
γ_3	0.0290	0.0208
γ_4	2.7819	3.0737

TABLE 3

STATISTICS FOR NORMALIZED FLEXURAL STRENGTH, SF_yP^*/f_y (Type B Section)

Moments	Normal Geometric Parameters		Lognormal Geometric Parameters	
	$(m_p^*;v_p^*)=$ (1.02;.06)	(.98;.10)	(1.02;.06)	(.98;.10)
mean	1.9517	1.8746	1.9602	1.8750
c.o.v.	0.0767	0.1167	0.0765	0.1158
γ_3	0.0323	0.1444	0.1494	0.0929
γ_4	2.7967	3.1421	3.0500	3.1235

Note: m_p^*; v_p^* = mean; coefficient of variation of the professional factor P*.

TABLE 4

COEFFICIENTS OF SKEWNESS AND KURTOSIS FOR VARIOUS DISTRIBUTIONS

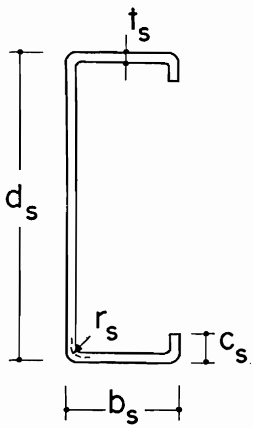
Moments	Gauss	Lognormal		Gamma		Exponential	Extreme Type I
		c.o.v.=0.1	=0.2	c.o.v.=0.1	=0.2		
γ_3	0	0.3010	0.6080	0.2000	0.4000	2	1.1395
γ_4	3	3.1615	3.6644	3.0600	3.2400	9	5.4000

TABLE 5

RELIABILITY INDICES

Spans (ft-in)	Spacing (in)	Normal Geometric Tolerances		Lognormal Geometric Tolerances	
		$(m_P*;v_P*)=$ (1.02;.06)	(.98;.10)	(1.02;.06)	.98;.10)
Type A Section					
13-6	12	3.30	2.98	3.28	2.94
12-3	16	2.92	2.61	2.90	2.57
10-8	24	2.38	2.08	2.36	2.04
Type B Section					
19-2	12	3.70	3.32	3.72	3.33
17-5	16	3.31	2.95	3.33	2.96
15-3	24	2.74	2.39	2.76	2.40
Type C Section					
23-5	12	3.50	3.12	3.52	3.15
21-3	16	3.12	2.75	3.14	2.78
18-7	24	2.55	2.20	2.57	2.23

$m_P*;v_P*$ = mean; coefficient of variation of the professional factor P*.



Section Type	b_s	d_s	p_s	r_s	t_s
A	1.8125	5.50	9.96	.094	.0495
B	1.8125	7.25	11.71	.094	.076
C	1.8125	9.25	13.71	.094	.076

Note: p_s = perimeter; f_y = 40 ksi; and c_s is derived from p_s and other parameters.

Fig. 1. Cold-Formed Steel Joists.

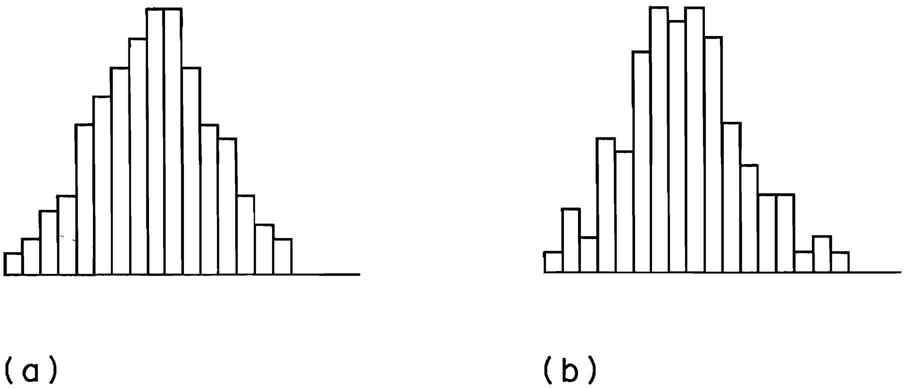


Fig. 2. Histograms of Moment of Inertia for (a) Normal and (b) Lognormal Geometric Parameters (Type B Sections).

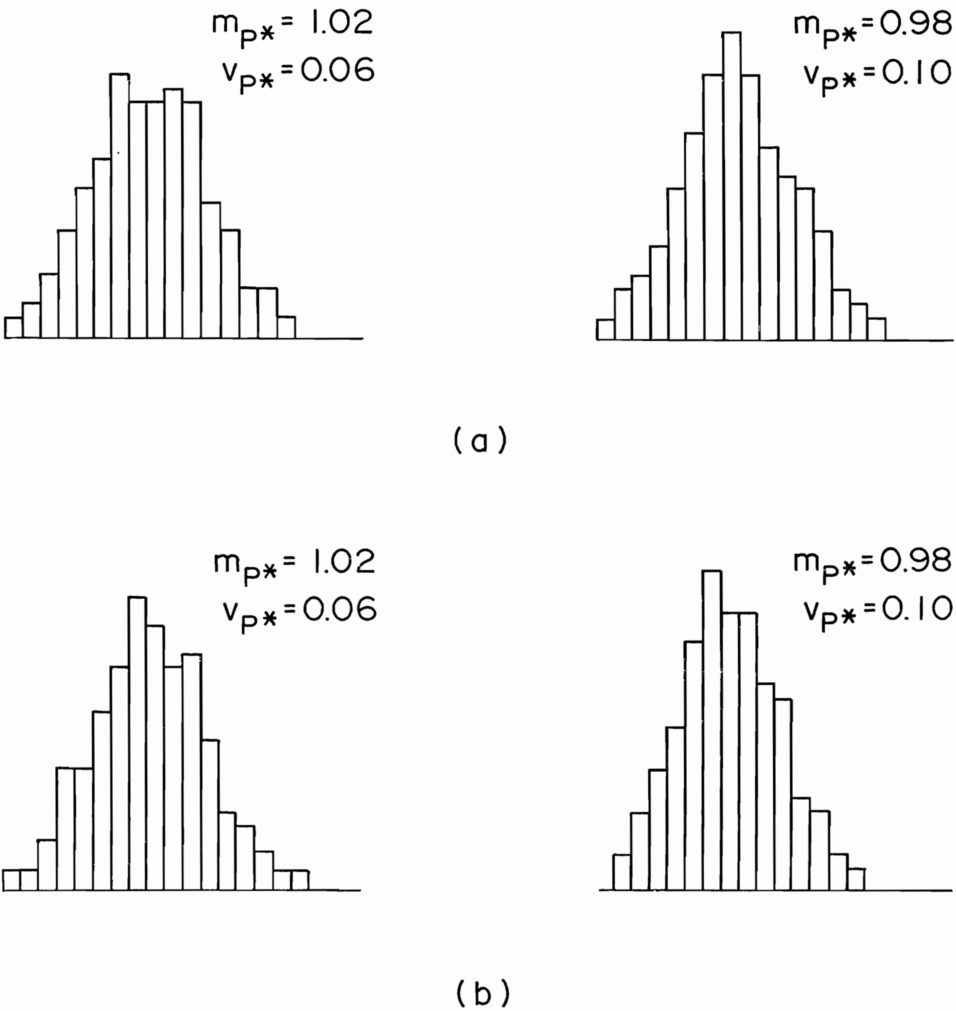


Fig. 3. Histograms of Normalized Flexural Strength, $SF_y P^*/f_y$, for (a) Normal and (b) Lognormal Geometric Parameters (Type B Sections).

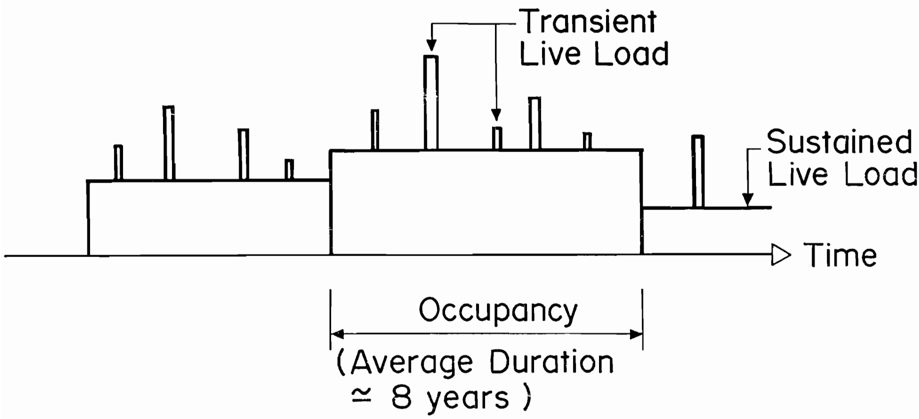


Fig. 4. Probabilistic Model for Live Loads.