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Mechanical connections in cold-formed steel: comprehensive test procedures and evaluation methods

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MECHANICAL CONNECTIONS IN COLD-FORMED STEEL:
COMPREHENSIVE TEST PROCEDURES AND EVALUATION METHODS

by

John Fraczek

G. Winter and T. Pekoz
Project Directors

A Research Project Sponsored by
The American Iron and Steel Institute

Ithaca, New York May, 1976
PREFACE

The advice of Professors George Winter and Teoman Pekoz, Project Directors, and their contributions to this work are gratefully acknowledged. The author also wishes to express his deep appreciation to Professor Robert G. Sexsmith for his assistance and support in the probabilistic aspects of this work.

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A slightly modified version of this report was originally presented as a thesis to the Faculty of the Graduate School of Cornell University in partial fulfillment of the requirements for the Degree of Doctor of Philosophy.
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ABSTRACT

Design equations do not exist for connections in cold-formed steel formed with most types of mechanical fasteners, and such designs are presently based on test results. A lack of standard test procedures and corresponding evaluation methods impedes the acceptance of connection designs and the exchange of information vital to understanding the connection performance of a large number of mechanical fasteners with diverse properties.

Initial steps toward standardization are taken with the development of fixtures and procedures for single shear, pull-out and pull-over tests. The proposed single shear tests employ both a simple specimen formed with two overlapping straps and a more sophisticated fixture designed to simulate the behavior of a connection in a cold-formed steel shear diaphragm. A single test fixture applicable to any specimen shape and stiffness is developed for pull-out and pull-over tests. Test results, including results of large scale pull-over tests, obtained in the development and verification process are compared and discussed.

Twenty-five identical tests on typical specimens are performed in accordance with each of the proposed test procedures to provide data for the determination of probability distributions on ultimate strength. A beta probability law is assumed as the underlying model for the probability density function on connection strength, and Bayes' theorem is used with a diffuse prior to evaluate the joint posterior likelihood function.
on the beta parameters in digital form. Marginal density functions on the parameters are determined and the Bayesian distribution on connection strength is evaluated. Samples of various size are drawn from the distribution by simulation, and probability density functions on the sample mean are determined by a method of moments fit to a beta model.

The Bayesian distributions are compared with distributions obtained with maximum likelihood estimates of the beta parameters to ascertain the suitability of the computationally efficient maximum likelihood approach for model determination with limited sample sizes.

The test evaluation method is based on a load and resistance factor design criterion with a first order probabilistic code format. An expression is developed for the coefficient of variation of the resistance which includes the effects of strength variation, the size of the test sample and the degree of simulation of actual field conditions. Resistance factors are determined for connections in temporary, standard and vital structures made with different levels of workmanship and inspection. It is found that three tests are sufficient to estimate the mean resistance and that a single set of resistance factors is appropriate for use with the proposed test procedures.
CHAPTER 1

INTRODUCTION

Connections in cold-formed steel can be made by various means, including the use of mechanical fasteners, welds and adhesives. This investigation deals solely with connections formed with standard type mechanical devices employed by the cold-formed steel industry.

The large number of possibilities that exist in the design and performance of such connections has hindered the establishment of standard design formulae, and most connection designs in cold-formed steel are currently based on test results. Standard test procedures to determine fastener performance in connections, however, either do not exist or are not suitable for cold-formed steel. Many users have been compelled to perform private tests on proposed products and assemblies, and then demonstrate the adequacy of their connection designs to various authorities to gain approval for their use. Although the need for some prototype testing will always exist, much of the test duplication involved in gaining connection design acceptance could be eliminated with the establishment of recognized and accepted standard test procedures.

A number of benefits may be realized from the establishment of unified standard tests for fasteners and connections. Some of these advantages are:

1. Testing techniques and criteria required for the design and development of superior fasteners and connections will be available, eliminating the need for developing new test
procedures.

2. Fastener specifications may be easily produced and checked with a series of standard tests.

3. Duplication of tests by producers, users and authorities will be eliminated.

4. New fasteners and connections may be readily evaluated.

5. The degree of interchangeability, substitutability and replaceability of fasteners and connections may be determined by comparing standard test results.

Some of the benefits arising from the adoption of a standard test procedure are not realized without a rational method for evaluating the test results. A test evaluation method provides a consistent interpretation of test results, and thus complements the uniformity sought through the establishment of standard test procedures. There are several desirable features to a test evaluation method. The method should:

1. Dictate the number of tests to be performed or provide guidance to the user in determining the number.

2. Indicate when a test result is probably out of range, signifying faulty specimen preparation or testing.

3. Provide for the variability that is inherent in the test procedure and the physical phenomenon itself.

4. Account for the error in estimating statistical parameters such as the mean from a limited number of tests.

5. Indicate the manner in which the test results are to be interpreted and used, together with any limitations on their use. The interpretation should also have the goal of consistent reliability.
1.1 **Purpose of Investigation**

The purpose of this investigation is to:

(a) review and evaluate existing specifications and test procedures on mechanical fasteners and connections to determine their acceptability as standards for the cold-formed steel industry,

(b) adopt or modify existing test procedures selected as potential standards to meet the requirements of the industry or, if necessary, develop new test procedures and

(c) develop appropriate methods to evaluate the results obtained from the test procedures chosen as standards.

1.2 **Scope of Investigation**

This investigation is concerned with fasteners and test procedures that are important to the cold-formed steel industry. Tests of properties of slight importance to the industry, such as fatigue and high temperature characteristics, are excluded.

The investigation is conducted in three phases: the first consists of a literature review of mechanical fastener and connection specifications and test procedures; the second comprises experimental work designed to determine the suitability of some established test procedures for the cold-formed steel industry and, if necessary, to modify them or develop entirely new procedures; and the third deals with the establishment of test evaluation methods for the selected procedures.

1.2.1 **Literature Review**

The large variety of available mechanical fasteners prohibits a thorough discussion of each type. Consequently
they are classified in the broad categories of bolts and screws, tapping screws and cold rivets, and are limited to standard type devices employed in the cold-formed steel industry. The review includes both tests of the fastener itself and tests of the connection, and is based on information obtained from the open literature, design specifications and fastener manufacturers and users. It is presented in Chapter 2 of Reference 2.

1.2.2 Experimental Work

The experimental work consisted of steel-to-steel connection tests in shear, pull-over and pull-out. The tests were typically small scale, employing one or two test fasteners, except for a number of large scale tests performed with a vacuum type loading system designed to verify the uplift results. A series of twenty-five identical tests was performed in single shear, pull-over and pull-out to obtain data for the ultimate load frequency distributions used in the determination of the evaluation method.

1.2.3 Evaluation Method

The test evaluation method applies to the ultimate strength of connections as determined by the proposed test procedures. It assumes that each mode of failure, as defined by the appropriate test procedure, has its own characteristic coefficient of variation and is governed by a characteristic probability law.

The method is based on the principles of "Load and Resistance Factor Design," and employs the "safety index" concept.
Suggestions are provided for the treatment of designs based on deflection constraints and situations not covered by the proposed test procedures.

1.2.4 Proposed Test Procedures and Evaluation Methods

As a result of this investigation a set of proposed test procedures has been developed and is presented as Appendix A under the title "Recommended Procedures for Conducting Pull-over Pull-out and Single Shear Tests of Mechanical Connections." The corresponding evaluation method appears as Appendix B.
CHAPTER 2

PRELIMINARY INVESTIGATION OF FIXTURES AND PROCEDURES FOR SHEAR AND UPLIFT TESTS

The literature review, described in detail in Chapter 2 of Reference 2, reveals that standardized tests of most important fastener properties have either been established and are available, or can be instituted without much difficulty. Standardized tests of connections, on the other hand, are quite limited and appear to be rarely used; connection tests are typically devised to fulfill a particular requirement without regard to standardization. Although bolted connections in cold-formed steel are adequately defined, the same cannot be said of connections formed with other mechanical fasteners. The increasing interest in using screw fasteners in cold-formed steel heightens the need for standardized test fixtures and procedures for such connections. This need is further augmented by the fact that most failures in properly designed connections formed with mechanical fasteners are caused by a failure of the connected material rather than a failure of the fastener.

A preliminary investigation was conducted to determine the suitability of some test fixtures and procedures described in Reference 2 to cold-formed steel connections employing small diameter fasteners. The initial tests were a very cursory examination of single shear configurations and two uplift test fixtures. These tests were followed by a more extensive series of single and double shear tests together with uplift tests employing a different test fixture.
All of the tests in this investigation were conducted with #14 x 3/4" hex head Type A thread forming fasteners, assembled with 5/8" OD 20 gage washers of galvanized steel bonded to neoprene. This fastener was selected because of its availability and the fact that it is fairly representative of mechanical fasteners employed by the cold-formed steel industry. Despite the fastener industry's recommendation\(^3\) that Type A fasteners be supplanted by Type AB because of the latter's wider versatility of application, the use of Type A fasteners is still prevalent. The recommended practice that the washer thickness be at least that of the thinnest connected sheet was not followed so as to enable direct comparison of test results without accounting for fastener variability.

2.1 Initial Tests

The initial tests were all conducted with 16 gage galvanized mild steel sheet, with all specimens cut from a single sheet. The material properties are presented in Table 2.1. Following recommended procedure\(^4\), the hole in the sheet immediately under the fastener head assembly was drilled slightly larger than the basic screw diameter with a 1/4 inch high speed bit.

2.1.1 Single Shear Tests

The literature review\(^2\) indicates that the only procedure designated as a standard test to determine the single shear strength of joints is provided by the Aerospace Industries Association in ARTC Report No. 33\(^5\) under the title "Fastener Lap Joint Test Procedure." A slightly modified version of this
test procedure has been adopted as "Test 4, Joint Shear Strength" of MIL-STD-1312. These procedures specify a lap joint connected with two fasteners in line parallel to the direction of force as shown in Figure 2.1. The variable strap width specification is intended to prevent a transverse tension failure across the net section of the sheet.

A more sophisticated testing device had been designed at Cornell University to determine the single shear capacity of a connection in a cold-formed steel diaphragm. The device is depicted in Figure A15 and essentially consists of two sets of flat, heavy plates constrained by guide tracks to movement only in their own plane and in the direction of force. The two parts of the specimen whose connection is to be tested are each clamped between the plates with high strength, high torque bolts as shown in Figures 2.2 and A20. These bolts provide a friction type connection, thus eliminating stress concentrations which might be produced by the bolts bearing against the specimen. The load is applied by means of a hydraulic jack through bars pin-connected to the two sets of flat plates. The geometry of the device eliminates undesirable eccentricities and keeps the loads acting co-linearly. Eccentricity in the joint is removed through the use of spacer plates, or shims, of the same thickness as the materials being tested. Teflon pads are used on the guide tracks to reduce friction to a minimum.

A transverse tension failure was to be avoided in the lap joint tests to permit comparison with similar simulated diaphragm action tests. Hence the strap width was arbitrarily
selected as 2 1/2 inches for convenience. The majority of single shear lap joints tested previously by others had employed a single fastener; thus it was decided to conduct single fastener tests as well as the recommended two fastener tests. In order to observe the possible effects of edge distance greater than the specified minimum, edge distances of 1/2 inch (approximately twice the nominal fastener diameter, 2d, as specified), 1 1/4 inches (5d) and 2 1/2 inches (10d) were used in the single fastener lap joints. These same edge distances were used in the two fastener lap joints, with fastener separation taken as twice the edge distance. Figure 2.3 shows the single shear lap joint specimen configurations. To compare these tests directly to simulated diaphragm action tests, configurations with identical fastener separation were tested with that device.

The test specimens had the fastener hole in the connected sheet drilled with a #8 (0.199 inch) high speed bit, and the fasteners were hand torqued to approximately 35 in.-lbs. The lap joint specimens were tested in a Baldwin Southwark hydraulic testing machine, with the head travelling at a constant rate of 0.05 inch per minute. Load-deformation curves were obtained autographically. Specimens tested in the simulated diaphragm action fixture were loaded at displacement increments of 0.010 - 0.015 inch.

Table 2.2 summarizes the ultimate load per fastener realized by the specimens tested. The ultimate load of single fastener lap joints (designated as SS1A, -2A, -3A) increased
with edge distance, while that for two fastener lap joints (SS1B, -2B, -3B) was relatively constant regardless of edge distance or fastener spacing. The latter results were also in good agreement with the simulated diaphragm action results, which were consistent for both one and two fastener connections. The two fastener lap joint and simulated diaphragm action specimens all reached an ultimate load within 5 percent of the 1567 lb. average save for one specimen whose ultimate was approximately 7 percent above the average.

All of the specimens failed in bearing, with considerable elongation of the hole and "piling-up" of material in front of the fastener. Figure 2.4 shows a typical load-deformation curve of the two fastener lap joint compared to that of a similar simulated diaphragm action specimen. The similarities of the two curves are obvious, and the shape of these curves is representative of all curves obtained from these initial single shear tests. The fastener(s) remained relatively normal to the plane of the specimen up to the "yield plateau" caused by fastener tilting or rotation at essentially constant load. This rotation produced some interlocking of the holes and deformed the washer assembly against the sheet. The load then increased to its ultimate value. The lap joint specimens exhibited some out of plane distortion on the side of the fastener head assembly upon attainment of the ultimate load or immediately thereafter.

2.1.2 Uplift Tests

The Aerospace Industries Association also provides a
standard tension test in ARTC Report No. 33(5) under the title "Fastener Tension Test Procedure." This same procedure is "Test 8, Tensile Strength" of MIL-STD-1312. The procedure specifies a test fixture composed of two hardened steel plates, shown in Figure 2.5, which are placed together and positioned with one plate rotated 45 degrees from the other. The 0.375 inch holes of one plate are then concentric with the 0.750 inch holes of the other. The test fastener is installed and shouldered studs are placed through the concentric holes to produce the configuration shown in Figure 2.6. This assembly is tested between the compression heads of a testing machine, using a spherical seat compression plate on the top head of the machine. The test procedure states that the strength of the joint may be determined by using tension plates that are of identical material and thickness as employed in the actual joint, with the countersink shown in Figure 2.5 optional.

The nature of the tension plate fixture restricts its use primarily to sheet-to-sheet applications. A simple frame device that had been successfully employed in sheet-to-structural uplift tests by one steel producer is also described in Reference 2. This device is depicted in Figure 2.7.

Tests were conducted with both of these devices to determine their suitability as standard fixtures for the cold-formed steel industry. Three tests were performed with the tension plate fixture. To observe the possible effects of plate dimension and stud hole location, one specimen was tested with the specified dimension of 4.25 inches square, while the two remaining specimens had plate dimensions of 6.25 and 8.25 inches.
The specimen configuration is shown in Figure 2.8. The two tests with the frame device consisted of a 12 x 3.75 inch rectangular sheet connected to a 1 3/4 inch 14 gage wall stud. One test had the major dimension of the sheet parallel to the axis of the stud and the other had it normal to the stud axis.

The hole in the connected sheet of the tension plate fixture was drilled with a #12 (0.189 inch) high speed bit, while that in the wall stud was drilled with a 5/32 inch bit. The fastener was hand torqued to 60 in.-lbs., approximately half the strip-out torque in the 16 gage sheet. The tests were performed with a Baldwin Southwark hydraulic testing machine with the head travelling at a constant rate of 0.05 inch per minute. Deformations were measured with a dial gage.

The results of these uplift tests are presented in Table 2.2. The tension plate specimens all failed with the fastener pulling out of the connected plate. The ultimate load sustained by the connection together with the amount of plate distortion prior to failure increased with plate size, although the increase in load was less between the 6.25 and 8.25 inch specimens (7 percent) than between the 4.25 and 6.25 inch specimens (22 percent). There may have been some interference between the plates and the studs in the larger size plates. In the sheet-to-structural uplift tests, failure resulted from bending of the sheet when the longer axis of the sheet was perpendicular to the axis of the wall stud. Failure occurred with the fastener pulling out of the structural member when the longer axis of the sheet was parallel to that of the stud.
2.1.3 Conclusions from Initial Tests

Research at Cornell University has shown that even low ductility steels possess sufficient ductility to overcome the adverse effects of stress concentrations.\(^7\) Stress concentrations in cold-formed steel connections formed with small diameter fasteners, however, are probably very large because the ratio of fastener diameter to fastener spacing is typically quite small and the load is transferred fundamentally by bearing of the material against the fastener rather than by friction. It is well known that two or more holes in line with the direction of load provide a lower stress concentration factor than does a single hole.\(^8,9\) This fact, coupled with the observation in these initial tests that two fastener single shear connections had an ultimate capacity relatively independent of the edge distance and fastener spacing, provided they exceeded some minimum, while those formed with one fastener did not, suggested that single shear connection tests be performed with two fasteners in line parallel to the direction of load. This proposition was further supported by the good agreement between the lap joint test results and those obtained with the simulated diaphragm action fixture. In fact, the agreement between these two procedures suggested that the simpler lap joint tests could be used with sufficient accuracy provided that two fastener connections were employed.

The tension plate test fixture and procedure performed adequately, but appeared to have a number of potential drawbacks. One of these was the apparent inconsistency in results,
which may have been due to one or more of the following factors: an insufficient number of tests was performed to obtain an idea of the ultimate load distribution, and the single exceedingly low value may have had a small probability of occurrence; improper alignment of the studs may have produced varying amounts of interference between the studs and their respective holes; and the greater deformations undergone by the larger size specimens may have altered the fastener hole geometry and produced some wedging against the fastener. These problems might have been overcome with additional tests and greater care in specimen preparation, but the necessity for accuracy in specimen preparation, particularly in stud hole location, was a disadvantage. Other drawbacks were the need to employ two plates for each test and the inability to test structural members. Problems could also be anticipated in attempting to test thin or corrugated specimens in pull-over, where high membrane stresses would probably cause binding of the material against the studs. For these and probably other reasons, the tension plate fixture was not deemed suitable for connections in cold-formed steel.

The frame device used in the sheet-to-structural uplift tests had a number of favorable qualities. Chief among these was its simplicity both in construction and in application. Another was its use of all components of the connection to be tested. Its primary deficiency was its possible lack of universality, notably that a test sheet or section might possess insufficient stiffness to prevent a bending or buckling
failure. Such might well be the case for a thin section subjected to a loading that ought to produce a pull-over failure. Consequently a more suitable test fixture was desired.

2.2 Additional Tests

An additional series of tests based on the initial conclusions was undertaken to further examine two fastener single and double shear lap joint configurations and evaluate a test fixture considered suitable for uplift tests. The tests were performed with a range of material believed representative of that used by the cold-formed steel industry. The material ranged in thickness from 0.021 to 0.123 inch and consisted of 10, 16, 22 and 26 gage mild steel sheet, with all save the 10 gage material galvanized. Table 2.1 presents the average material properties and thickness of each gage employed.

The test equipment and procedures for the shear tests were identical to those used previously. All oversize holes were drilled with a 1/4 inch high speed bit, and the mating holes were drilled as follows: a #1 drill bit (0.228 in.) for 10 gage, #6 bit (0.204 in.) for 16 gage, #12 bit (0.189 in.) for 22 gage and a #13 bit (0.185 in.) for the 26 gage material.

2.2.1 Shear Tests

The single shear tests included forty lap joints tested with two fasteners in line parallel to the direction of force and the fastener heads on opposite sides of the joint. To determine the effects of oversize holes, half of these
connections had the hole immediately under the fastener head assembly drilled oversize and half did not. Twenty single shear lap joints were also tested with the fastener heads on the same side of the joint, together with an equal number of simulated diaphragm action tests of similarly configured specimens. These two test series were conducted to compare lap joint and simulated diaphragm action test results and to determine the effect of having the fastener heads on the same side or opposite sides of the joint. Finally, twenty-four double shear lap joints, half with the fastener heads on the same side and half with them on opposite sides of the joint, were tested to provide a comparison with single shear results.

2.2.1.1 Test Procedures

All single shear lap joints consisted of two straps fifteen inches long and two inches wide, connected with two fasteners at an edge distance of one inch (approximately four fastener diameters) and a fastener separation of two inches. These dimensions were selected rather arbitrarily and intended to prevent edge failures and transverse tension failures across the net section. A fastener separation of twice the edge distance appeared to be a logical choice. The two single shear configurations tested, designated as Groups I and II, are shown in Figure 2.9. The single shear tests conducted with both fastener heads on the same side of the joint always had the thinner sheet positioned under the fastener head assemblies. This procedure was observed for both the lap joint and the simulated diaphragm action tests.
The double shear lap joints consisted of two 15 by 2 inch 10 gage straps connected through two cover plates as shown in Figure 2.10. The edge distance was again maintained at one inch, and both cover plates were always of the same thickness. The cover plates were of 16, 22 and 26 gage material, with use of 10 gage material precluded by insufficient fastener length.

2.2.1.2 Shear Test Results

Table 2.3 presents the results of single shear tests performed on joints formed with two sheets of equal thickness. All of the specimens failed through yield in bearing except the joints formed with 10 gage material, which failed by shearing of the fasteners. Typical load-deformation curves were characteristic of this fastener type, i.e. an initial elastic portion followed by a "plateau" produced by tilting of the fasteners and a final increase to the ultimate. The lap joints tested exhibited very slight or negligible out of plane distortion. It should be noted that despite the loss of symmetry there was apparently little, if any, difference in Group I and Group II results. There was some indication that not drilling the top hole oversize might slightly increase the strength of the connection, although this procedure necessitates the stripping of the top hole threads to achieve a tight joint. In the stripping process the hole is deformed outward in the direction normal to the plane of the sheet. This concavity may produce some interlocking with the hole in the bottom sheet when that sheet is drawn tight through application of torque to the
fastener. Fair agreement existed between the simulated diaphragm action and lap joint test results, although there was considerable discrepancy in the 10 gage results. This inconsistency was probably produced by joint eccentricity in the lap joint tests, for the lap joints were tested without the use of shims while the simulated diaphragm action test required their use.

Table 2.4 presents the ultimate single shear strengths of joints formed with sheets of unequal thickness. All of the specimens failed through yield in bearing, but many of the Group II lap joints also displayed signs of transverse tension failure (16-10 gage and 26-16 gage combinations) and plate tearing combined with out-of-plane distortions (26-10 gage and 26-16 gage combinations). The most prominent feature of this table is the substantial difference in ultimate load between Group I and Group II lap joint configurations. Group II connections were provided with additional fastener rigidity and clamping force because both fasteners were driven into the thicker sheet. The effects of joint eccentricity were again visible, perhaps most dramatically in a comparison of the 22-10 gage and 22-16 gage combinations of the Group II lap joints, where connection to a thinner sheet produced an increase in ultimate load. If the effects of joint eccentricity were either eliminated or accounted for, the agreement between the lap joint and simulated diaphragm action tests might have been quite good.

The double shear test results are presented in Table 2.5,
with all specimens again failing through yield in bearing. The results with the fastener heads on the same side of the joint were consistently higher than the single shear results, primarily because the fastener was constrained to remain normal to the shear plane. These results also revealed the importance of the clamping force provided by the fastener head assembly, especially for the thinner material. With the fastener heads on opposite sides of the joint, movement of the connected plates could occur through yielding only at the point side of each fastener. This was precisely the failure mode for the 22 and 26 gage material. With the fastener heads on the same side, however, movement could only occur with yielding at both the point side and head side of the fastener. Thus the constraint afforded by the fastener head assembly resulted in the greater ultimate load.

2.2.2 Uplift Tests

The inadequacies of the test fixtures used in the initial tests required the evaluation of another fixture. Uplift tests basically consist of pulling a fastener either out of ("pull-out") or through ("pull-over") the material to be tested. Variations on uplift test procedures include the size of the specimen, the manner of support and the method of load application. A test fixture described in Reference 2 that was fairly representative of fixtures employed for uplift tests is depicted in Figures 2.11 and 2.12. This fixture also possessed several advantages over other fixtures, including simplicity of use and specimen preparation, and the incorporation of both
joint components (sheet material and structural member) in the test.

A number of uplift tests was conducted with this fixture. A series of tests was also performed with a smaller version of the same fixture. The smaller fixture had the 1/2 inch bolts used to restrain the test sheet located on a 3 inch square, as opposed to the 6 inch square used previously. In all of these tests the hole immediately under the fastener head assembly was drilled oversize.

2.2.2.1 Test Procedures

The test specimen configurations are shown in Figure 2.13. The channels were formed into the given shape from flat sheet with a brake press. The test channel was connected to the test sheet in two ways: either driving the fastener into the sheet as depicted in Figure 2.14a, or driving it into the channel as shown in Figure 2.14b. Failure would typically occur with the fastener being pulled out of the material into which it had been driven. With the channel attached, the sheet was positioned on the base of the test fixture and the four 1/2 inch bolts were hand tightened with washers under the heads. This portion of the fixture was placed in one grip of a testing machine, one end of the loading arm was attached to the test channel with a 1/2 inch pin and the other was placed in the other grip of the machine, producing the configuration shown in Figure 2.11.

These specimens were tested in a Tinius Olsen electric
testing machine, with the loading head traveling at rates of 0.02 to 0.10 inch per minute. The higher rates were used to continue deforming the material when the load remained constant, while the lowest rate was used in the vicinity of the ultimate load. Deformations were measured with a dial gage positioned between the loading arm and base plate of the test fixture.

2.2.2.2 Uplift Test Results

The results of the pull-out tests conducted with the sheet clamped on a 6 inch square are presented in Table 2.6. The unusually low values denoted by asterisks were probably due to partial or complete stripping of the formed threads while driving the fastener, and these values were excluded in computing the mean value. The test channels formed with lighter gage material underwent considerable distortion during the tests, with the 22 and 26 gage channels essentially straightening into a V-shape and only the 10 gage channels relatively free from distortion. The test sheets, however, suffered negligible permanent distortion except in the immediate vicinity of the hole. The pull-out values of the channels were higher than those of the sheets because the channels distorted and altered the fastener hole geometry, resulting in clamping and wedging action around the fastener threads. Another observation was the increase in the coefficient of variation with thinner sheets. The coefficient of variation varied from about 5 percent for the 10 and 16 gage material
to about 15 percent for the 22 and 26 gage. This increase was due to the higher sensitivity of the thinner material to tightening torque, with a greater risk of partial strip-out.

Table 2.7 contains the results of pull-out tests conducted on test sheets clamped on 3 inch and 6 inch squares. With premature failures again excluded, it is seen that the results were in reasonable agreement. Figure 2.15 shows the load-deformation curves for a pull-out failure from a 22 gage sheet, with the sheet clamped on a 6 inch and 3 inch square. Both curves are similar, with the curve for the 3 inch square much stiffer, as expected.

The pull-over results are presented in Table 2.8. Again the channel experienced higher ultimate loads because of distortion, but the agreement between the sheets clamped on a 3 inch and 6 inch square was good. This agreement is also evident in Figure 2.16, depicting load-deformation curves for a pull-over failure from a 22 gage sheet clamped on both size fixtures.

2.3 Conclusions

A number of conclusions bearing on the establishment of standardized test procedures and fixtures were drawn from this preliminary investigation, despite the relatively small number of specimens tested. The most important requirement, applicable to all types of tests, was that the materials, fastener assemblies and hole diameters employed in the test be identical to those used in the actual application, and that the assembly techniques, tools and torques be as closely
matched as practicable. The remaining conclusions applied to specific types of tests.

2.3.1 Single Shear Tests

With equal thickness sheets, there was apparently little, if any, difference in having the fastener heads on the same or on opposite sides of the joint. Connections in materials of different thickness, however, are the more prevalent and driving both fasteners into the thicker material is more representative. The increased anchorage provided by this action, resulting in greater fastener rigidity and clamping force, has a substantial effect on the ultimate load.

Oversize holes in all but the bottom sheet, as recommended by the industry, appeared to have slight effect on the ultimate strength of the joint in shear. Because it is often convenient not to use oversize holes in the actual application, the best approach would be to use the hole sizes actually employed.

Good agreement could probably have been obtained with the lap joint and simulated diaphragm action tests by using shims and/or longer strap lengths for lap joints formed with thicker material to reduce joint eccentricity and increasing the amount of overlap in the simulated diaphragm action specimens to reduce boundary effects. The simulated diaphragm action fixture, with its independence of edge distance and prevention of out of plane distortion, was believed to provide a good representation of the actual situation and was recommended for retention as a standard test fixture. It was realized
that this fixture is expensive to produce and difficult to use, and that the lap joint test would be adequate in most instances.

Washer assemblies had an important effect in shear as well as pull-over applications and should be selected with care.

2.3.2 Double Shear Tests

Double shear test results did not correlate well with single shear results because the fastener was constrained to remain normal to the shear plane. Use of the center sheet as the test sheet would not be appropriate because the clamping force could be varied by the selection of cover plates and the test sheet material would be constrained against any deformation. Since double shear connections are relatively uncommon in thin sheet construction, double shear tests should be performed only when a double shear connection actually exists and then essentially as a prototype test.

2.3.3 Uplift Tests

From the uplift tests conducted it appeared reasonable to use a substantially thick channel, with the fastener driven into the sheet for pull-out tests and into the channel for pull-over tests. For tests of actual channel and sheet combinations the length of the test channel could be increased from two inches to a length sufficient to provide the stiffness required to eliminate channel distortion, and thus better simulate actual conditions.
On the basis of the minor permanent deformation of the test sheet except in the immediate vicinity of the fastener and the reasonable agreement in results between sheets clamped on 3 inch and 6 inch squares, it appeared that both pull-out and pull-over were very localized in their effect and behavior and depended on the material properties in the immediate vicinity of the fastener. Support conditions consequently appeared to be of secondary importance when they provided for the existence of membrane stresses. In the interest of uniformity, however, the fixture with the text sheet clamped on a 6 inch square was recommended for future tests.
CHAPTER 3
COMMENTARY ON THE RECOMMENDED TEST FIXTURES AND PROCEDURES

The preliminary tests described in the previous chapter formed the basis for proposed test procedures by suggesting fixtures deemed suitable for single shear and pull-out/pull-over tests. These fixtures, together with others, were employed in a number of additional tests to verify their suitability, with modifications made where warranted. Large scale uplift tests designed to fail in pull-over were also conducted to establish the reliability of the pull-over test procedure. The proposed test procedures for single shear, pull-out and pull-over are presented as Appendix A.

The primary purpose of this commentary is to provide the potential user of the proposed test procedures access to the data and reasoning that formed the bases of the provisions. Knowledge of the rationale for the provisions will enable the user to make intelligent decisions in seeking solutions to problems that may arise in the application of those provisions. Additional test results, including tests with high strength, low ductility steels and pull-out/pull-over tests with an alternate test fixture, are described in detail in Chapter 6 of Reference 2.

3.1 Single Shear Tests

Probably the most frequent and important loading encountered in connections in cold-formed steel construction is that which produces shear stresses in the fastener. The importance of this type of connection has resulted in the execution of a
large number of shear tests. Bolted connections in cold-formed steel, in fact, have been sufficiently tested \(7,10,11\) to result in design specifications.\(^{(12)}\) The shear tests performed have largely been either prototype tests or tests designed to study the influence of various connection parameters, and as such have not employed a "standard" test specimen.

The proposed test procedures contain only single shear tests for two interdependent reasons. The large majority of shear connections in cold-formed steel involves single shear, and double shear tests are generally not representative of single shear connections, notably in those instances where fasteners which tend to rotate under single shear loads are employed. Double shear tests typically restrain such fasteners from rotation and consequently result in a higher load capacity. It is understood that where the need exists for conducting double shear tests such tests shall essentially be prototype tests of the proposed connection.

Two separate single shear tests are provided because the traditional lap joint strap test may lack the degree of accuracy desired or may not be readily amenable to the shape of the section to be tested. Possible eccentricity in the joint combined with some out of plane distortions may produce a test that does not well simulate the behavior of a connection in a cold-formed steel diaphragm, and for this reason the simulated diaphragm action test is included.

3.1.1 Single Shear (Lap Joint) Test

The single shear test of two overlapping straps joined
together with one or more fasteners is the most common test of shear connections. Its simplicity and almost universal acceptance make it a natural choice for selection as a standard.

The decision on fastener arrangement was based on the following factors: The use of multiple fasteners arranged in a line perpendicular to the direction of force serves principally to increase the average tensile stress across the net section, and the same effect can be obtained by reducing the width of the specimen. Connections formed with multiple fasteners in line parallel to the direction of force, however, offer a number of advantages over those formed with single fasteners. Multiple fasteners produce lower stress concentrations than does a single fastener. They also tend to smooth out, to some extent, test variations or scatter in results by providing an average load per fastener. Finally, multiple in line fastener connections are more representative of actual connections. Tests conducted in Sweden on single and multiple fastener screwed and riveted connections typically revealed a difference in ultimate load per fastener between connections formed with a single fastener and those formed with two or more in line fasteners.

The principal objection to connections formed with multiple in line fasteners is the complex load distribution across the joint while it is in the elastic range. This objection can be substantially diminished by limiting the number of in line connectors in the test specimen to two, and noting that the elastic load distribution effects will be eliminated with
yielding. All of the advantages of multiple in line fasteners mentioned above are valid for two in line fasteners.

The Department of Defense and the Aerospace Industries Association prescribe the two fastener single shear test specimen depicted in Figure 3.1a.\(^{(1,5)}\) The preliminary tests described in Chapter 2 revealed that the equal strap thickness requirement and the specification that the fastener heads be on opposite sides of the joint are inappropriate for the cold-formed steel industry. The resulting test specimen thus consists of two overlapping straps of the same thicknesses as employed in the actual application, joined together with two fasteners in line parallel to the direction of force, with both fastener heads on the same side of the joint as in actual practice. This configuration is shown in Figure 3.1b.

3.1.1.1 Specimen Parameters

The proposed test procedure is for mechanical fasteners in general, without regard to type or size, and so suggests the generally accepted practice of specifying the dimensions in terms of the nominal fastener diameter \(d\). This practice is followed herein. Minimum dimensions are established to provide a common reference for very small diameter fasteners and for convenience in specimen preparation and testing. The specimen dimensions determined are the edge distance \(e\), fastener spacing \(s\), specimen width \(w\) and component strap length \(L\).

3.1.1.1 Edge Distance \(e\)

The configuration of the test specimen (see Figure 3.1b)
is such that an edge failure can occur by longitudinal shearing of one or both component straps. Failure involving one strap, shown in Figure 3.2a, is not a true edge failure because the material around the fastener most distant from the edge fails by yield in bearing. The ultimate load per fastener is then the average of two individual fastener failure loads. A true edge failure must involve both straps and can only occur with longitudinal shearing at the head side of one fastener and the point side of another, as shown in Figure 3.2b. This constraint produces an edge failure load that is the average of a fastener head side failure and point side failure, which can be significantly different for fasteners displaying non-symmetric behavior.

These constraints result in an ambiguous edge failure load, and consequently the test specimen should be designed to avoid edge failures. Tests conducted on bolted connections employing high ductility steel\(^{(10)}\) concluded that the limiting edge distance for edge failure is 3.5 d. Similar tests with low ductility material\(^{(7)}\) revealed a limiting edge distance of 3.33 d. Design recommendations published by the National Swedish Institute for Building Research\(^{(14)}\) state that edge failures need not be considered in riveted and screwed connections with edge distances of 3 d or more. An upper limit on edge distance is suggested by MIL-HDBK-5A,\(^{(15)}\) which states in Section 1.4.7, "Bearing Properties," that bearing values for edge distances greater than 5.5 d must be substantiated by test. Consequently, an edge distance of 4 d was here selected for the test specimen.
This distance should conservatively prevent the occurrence of edge failures regardless of the material ductility or mechanical fastener employed.

Should the possibility of an edge failure exist in the actual application, the single shear test is to be performed employing a single test fastener and the edge distance to be used in the actual application.

3.1.1.1.2 Fastener Spacing $s$

To permit the advantage of obtaining an average load per fastener, the fastener spacing ought to be large enough to allow the independent behavior of each fastener. Both of the single shear joint test specifications that were consulted\textsuperscript{(1,5)} provide for a fastener spacing of twice the edge distance $e$. That provision is maintained here, and results in a fastener spacing of 8 $d$. This spacing has proven to be representative of the larger spacings often used by the cold-formed steel industry.

3.1.1.1.3 Strap Width $w$

If edge failures are prevented, the two major material failure modes that can occur are bearing failures and transverse tension failures. The parameter governing these failure modes is the strap width $w$ or, more properly, the dimensionless $d/w$ ratio.

Tests performed on bolted connections with two and three bolts in line parallel to the direction of force\textsuperscript{(16)} suggested the following formula for determining the ultimate tension load
\[ p_t: \]

\[ p_t = (1.0 - 0.9 r + 3 r \frac{d}{w}) A_{\text{net}} \sigma_t \leq A_{\text{net}} \sigma_t \]  

(3.1)

where \( r \) is the force transmitted by the bolt or bolts at the section considered divided by the tension force in the member at that section and \( \sigma_t \) is the ultimate tensile strength of the material. This formula, developed from tests with high ductility material, was found to be sufficiently valid for low ductility material. \(^{(7)}\)

Using Eq. 3.1 to compare the ultimate load for a single bolt connection to that for two in line bolts reveals that over the entire range of \( d/w \) which results in a transverse tension failure,

\[ p_1 \text{ Bolt} \leq p_2 \text{ Bolts} < 2 p_1 \text{ Bolt} \]  

(3.2)

This equation states that the ultimate load of a two bolt connection can never be twice that of a one bolt connection when transverse tearing is the mode of failure. For this reason the test specimen should be designed to prevent the occurrence of this type of failure. This restriction results in a specimen that should fail predominantly by yield in bearing.

Tests conducted on bolted connections in high ductility material \(^{(10)}\) suggested that the ultimate load in bearing could be adequately predicted by the expression

\[ p_b = C \sigma_y d t \]  

(3.3)
where \( \sigma_y \) is the yield strength and \( t \) the thickness of the material, and \( C \) is a constant equal to 4.9 for material with an ultimate to yield strength ratio \( \sigma_u/\sigma_y \) equal to or greater than 1.35. Similar tests with low ductility material\(^{(7)}\) revealed that \( C \) equals 3.0 for material with \( \sigma_u/\sigma_y \) less than or equal to 1.10, and suggested linear interpolation for intermediate \( \sigma_u/\sigma_y \) ratios. A more comprehensive formula developed in Reference 7 to predict bearing failures or the combination of bearing, shear and tension failures with greater accuracy is apparently valid only for single bolt connections. Application to two bolt in line connections resulted in negative and imaginary roots except for extremely high \( d/t \) ratios.

Equating twice the bearing load predicted by Eq. 3.3 with the transverse tension load for a two bolt connection given by Eq. 3.1 results in a limiting value of \( d/w \) above which failure occurs by transverse tension and below which by yield in bearing. A plot of \( (d/w)_{\text{lim}} \) versus \( \sigma_u/\sigma_y \) is presented as Figure 3.3. The minimum value of \( (d/w)_{\text{lim}} \) occurs at a \( \sigma_u/\sigma_y \) ratio of 1.35 and is

\[
(d/w)_{\text{lim}} = 0.085 = 1/11.8 \quad (3.4)
\]

This equation implies that assurance of a bearing failure in a two bolt connection requires a strap width \( w \) of at least 12 \( d \). Examination of the data on which Eq. 3.1 is based, however, reveals that the equation provides a conservative estimate of the transverse tension failure load. The data further
implies that fastener rotation during load transfer is conducive to bearing failures. A series of two bolt in line tests with the bolts finger tight and no washer under the bolt heads resulted in all specimens failing in bearing, despite a d/w ratio of 0.1875 in over half of the specimens. Bolted connections are more resistant to rotation than other mechanical fasteners, especially in torqued connections with high strength bolts and washers under both the head and nut, and they require a hole larger than their nominal diameter and hence provide a lower net area than their d/w ratio would indicate. They thus probably represent an extreme case in favoring a transverse tension failure.

On the bases of the conservative estimates of Eq. 3.1 and the tendency toward tension failures shown by bolted connections, the values predicted by Eq. 3.1 were increased by 20 percent and again equated to twice the bearing load. The minimum value of $(d/w)_{lim}$ is now

$$(d/w)_{lim} = 0.105 = 1/9.5 \quad (3.5)$$

A strap width $w$ equal to 10 $d$ was therefore selected for the test specimen, and has since proved adequate.

### 3.1.1.1.4 Component Strap Length $L$

The length of the component strap greatly influences the joint eccentricity, and this is the primary reason for the establishment of a minimum length. Another possible consequence of strap length is the effect of the grips or holding devices
of the testing machine on the stress distribution. This effect is immaterial in comparison with the joint eccentricity, for by St. Venant's principle this effect fades out in a distance approximately equal to one specimen width. An argument can be made for no minimum length requirement, as lengths that are too short will automatically draw penalties in the form of lower ultimate loads. Lack of a common base, however, makes the comparison of data obtained by different researchers difficult and hence a minimum length is specified.

The edge distance and fastener spacing requirements produce a minimum overlap length equal to the greater of 16 d or 4 inches. Allowing a free length from the grips to the overlap of twice the overlap length and a grip length that is the greater of 12 d or 3 inches results in the minimum strap length requirement of 60 d or 15 inches, whichever is greater. This requirement is so much more stringent than the specification of 28 d exclusive of grip length in MIL-STD-1312(11) that a grip length of 4 inches is suggested, with a longer length permitted if necessary.

3.1.1.2 Test Procedure

Fasteners and fastener accessories, driving and/or tightening torques and techniques, and hole diameters and tolerances have all proven to affect the strength and performance of connections formed with mechanical fasteners, and for this reason they should be the same as in the actual application.

The preliminary investigation described in Chapter 2 revealed that the effects of joint eccentricity can be very
substantial when relatively thick (approximately 0.10 inch) component straps are used. These effects can be diminished by the use of shims and/or longer strap lengths. Although somewhat awkward to use, shims are more efficient than longer straps and their use is confined to situations with a component strap thickness equal to or greater than 0.075 inch. It should be stressed that in such cases the use of shims is important and results in a considerably greater and more realistic ultimate load.

The speed of testing is specified both in terms of a load rate and a displacement rate because joints formed with different fastener types and different material thicknesses and ductilities can have markedly dissimilar stiffnesses and ultimate loads. Specification of a single load rate would thus result in almost impact loading for some connections and be far too slow for others. On the other hand, the rotation at nearly constant load that is characteristic of some fasteners produces large displacements which would take an inordinately long time to traverse at a constant or small displacement rate. Consequently a displacement rate is to be used for the initial "elastic" and subsequent "strain hardening" behavior range of the connection, and a load rate is to be employed for the "inelastic" range.

Many shear connections in cold-formed steel are characterized by excessive displacements at ultimate load, resulting in joints whose effective load-carrying capacity is defined by displacement rather than ultimate load. Hence measurement of
of the relative displacement of the two component straps is important. Two methods are suggested for obtaining a load-displacement curve to make acquisition of the curve readily attainable, but such acquisition is not a stipulated requirement. The measurement of the displacement at ultimate load is required, however, because it can be readily accomplished, offers a good indication of the joint stiffness and may prove useful in the design of the connection.

3.1.1.3 Summary

A two in line fastener test specimen configuration is more representative of in line connections than a single fastener configuration, and also tends to reduce the scatter in results by load averaging. The frequency of shear connections formed with unequal thickness components and the use of fasteners exhibiting distinct head side and point side behavior necessitates employing a test specimen with the thickness combination used in the actual application and both fastener heads on the same side of the joint as in actual practice. Edge failures occurring with this configuration will not be representative of actual edge failures, and thus are avoided. The prohibition of transverse tension failures because the failure load is dependent on the number of fasteners parallel to the direction of force results in a specimen designed to fail by yield in bearing.

Bolted connections have been examined rather extensively (7, 10, 11, 16) and formulas exist for predicting edge failures, transverse tension and bearing failures in both high and low
ductility material. They also represent the most severe case for edge and tension failures. Application of the established bolted connection formulas resulted in the specified edge distance and strap width of the proposed specimen.

The fastener spacing requirement was selected to assure the independent behavior of the two fasteners, and the strap length was determined to reduce and establish a common base for the joint eccentricity.

Preliminary tests indicated the need to use shims to reduce the joint eccentricity of specimens with a strap thickness equal to or greater than 0.075 inch. A speed of testing requirement expressed both in terms of a displacement rate and a load rate was dictated by the variable stiffness of connections formed with mechanical fasteners and the inelastic behavior produced by some of them. Finally, the possibility of designs based on displacement limits suggested the desirability of load-displacement measurements and a requirement to measure the displacement at ultimate load.

3.1.2 Single Shear (Simulated Diaphragm Action) Test

The apparatus and procedure for this test were developed at Cornell University to experimentally study the behavior of connections in cold-formed steel shear diaphragms. Development was necessitated because standard laboratory equipment and techniques did not lend themselves to the specimen shapes and loadings typical of diaphragms. The apparatus was specifically designed to determine the load-displacement characteristics and strength of connections under shear load.
The geometry of the test fixture was devised to eliminate undesirable eccentricities and to restrain out of plane deformations, restricting the movement to that which is likely to occur at a connection in an actual shear diaphragm. Tests revealed that the design produced connection failures that were completely similar to those occurring in actual shear diaphragms. The good simulation of behavior suggested that this test be adopted as a standard test procedure to be used in those instances where the specimen shape or degree of accuracy desired precludes the use of the lap joint test.

3.1.2.1 Test Fixture and Principles of Operation

The test fixture, shown in Figure A15, essentially consists of a base plate with guide tracks and a set of loading arms, each composed of two heavy steel plates. The form of the arms is such as to produce a system of co-linear self-equilibrated forces. The test specimen, depicted in Figure A16, is clamped between the shear plates with 1/2 inch high strength bolts. This technique transfers the load to the specimen by friction alone, avoiding stress concentrations or local distortion that could result from direct bearing of the bolts on relatively thin specimen components.

With one of the arms connected to a fixed point on a rigid support with a restraining rod, as indicated in Figure A21, load is applied to the other arm by pulling on a loading rod passing through a hollow hydraulic ram. The load is measured either with a load cell or by calibrating the restraining rod through
attachment of strain gages. The relative displacement of the two parts of the specimen is measured with two dial gages having an accuracy of 0.001 inch.

The use of guide tracks constrains the loading arms to move only in their own plane, in the direction of the load. Teflon pads used along the guide tracks reduce friction to a negligible minimum. Spacer plates of the same thicknesses as the specimen components aid in eliminating undesirable eccentricities. A vertical section view of the test fixture depicting these features is presented as Figure A20.

3.1.2.2 Specimen Configuration and Parameters

The general specimen configuration, shown in Figure A16, is largely dictated by the geometry and principles of operation of the test fixture. These factors determine the overall shape of the specimen and eliminate edge failures and transverse tension failures as possible failure modes. Initial tests indicated good agreement in ultimate load per fastener between connections formed with one and two fasteners, and a two fastener configuration was adopted for the benefit of load averaging.

The fastener spacing requirement of 8d that was selected for the lap joint test was also adopted here. This requirement is intended to assure the independent behavior of the fasteners, and has proven to be adequate.

The overlap length of 3/4 inch is to prevent interference from the loading arms during the relative displacement of the two specimen components. The loading arms have a minimum separation of one inch, and the proposed overlap allows for a 1/8
inch margin on each side. The fixture was originally designed to accommodate specimens with a 1 1/2 inch overlap, but it was found that one component of the specimen would often bind against the shear plates of the opposite arm during displacement. Should the fastener dictate a larger overlap than 3/4 inch, however, the overlap can be increased to a maximum of 1 1/2 inches provided that care is exercised to assure against such interference during the test.

3.1.2.3 Test Procedure

The test procedure is also largely determined by the geometry and principles of operation of the test fixture. After appropriate centering of the test specimen between the plates comprising the loading arms, the high strength bolts are installed and torqued to 80 ft.-lbs, thus securely clamping the specimen. The assemblage of arms and specimen is then placed on the guide tracks, the loading and restraining rods attached, and the dial gages positioned.

Use of a hydraulic ram as the loading device necessitates incremental loading, for it is almost impossible to apply and measure a smooth, continuously increasing load. Furthermore, loads applied with a hydraulic ram tend to be applied at a rather high load rate. The specified minimum of ten load increments produces an effective displacement rate that is sufficiently low to allow the initiation and propagation of yielding to occur gradually, and also provides a sufficient number of points for construction of a load-deformation curve.
3.1.3 Correlation of Lap Joint and Simulated Diaphragm Action Tests

A number of similarly configured lap joint and simulated diaphragm action tests were performed as part of the preliminary investigation described in Chapter 2. The specimens consisted of combinations of 10, 16, 22 and 26 gage mild steel sheet and conformed to the proposed procedures, except the lap joints employed a strap width of two inches and shims were not used. The lap joint configuration and dimensions are shown in Figure 2.9 as the Group II specimen. Details of these tests may be found in Section 2.2.1.

The test results are presented in Table 3.1. A comparison of the ultimate loads of the specimens, defined here as the maximum load attained during the test, shows the correlation between the lap joint and simulated diaphragm action tests to be reasonable. A better correlation, however, can be obtained by comparing "yield" load results. The "yield" load was defined as the load at which the slope of the load-deformation curve first becomes zero and may be a better indicator of connection strength than the ultimate load. The yield load is associated with yielding of the material due to bearing stresses, which in thinner materials and certain fastener types is accompanied by fastener rotation, and is independent of the subsequent complicated behavior such as wedging and "piling-up" of material which may produce an increase in load. It also occurs at displacement levels which are often considerably smaller, and hence more representative and reasonable, than those associated
with the ultimate load. In examining the yield values presented in Table 3.1, and excluding the 10 - 16 and 10 - 22 gage combinations because the thickness of the 10 gage material produced large joint eccentricities in the lap joint tests, the mean values of the lap joint results were all within ± 15 percent of the simulated diaphragm action results and all but the 16 - 22 and 26 - 26 gage combinations were within ± 10 percent. Neither the ultimate loads nor the yield loads were uniformly higher for one specimen type over the other.

A series of twenty-five identical tests was also conducted on both lap joint and simulated diaphragm action specimens to obtain estimates of their ultimate load distributions. The specimens were fabricated from 18 gage galvanized steel sheet, whose average material properties are given in Table 3.2, and had the same configuration and dimensions as the specimens described above. The holes were drilled in both components of the specimen simultaneously with a #9 (0.196 in.) high speed bit, and a tightening torque of 35 in.-lbs. was used. The proposed test procedures were followed.

The results of the two test series are presented in Table 3.3, which gives both the "yield" and the ultimate values. The "yield" load was defined in this case as the average load during fastener rotation. Table 3.3 indicates that the lap joint specimens had a mean "yield" load approximately 7 percent greater, and a mean ultimate load approximately 11 percent greater, than the simulated diaphragm action specimens. The lap joint specimens also displayed less scatter in results that the simulated diaphragm action specimens, as shown by their respective
coefficients of variation.

These two test series do not support the proposition that the variation between the lap joint and simulated diaphragm action tests was due to experimental scatter. This variation is believed to have been caused by two different factors and the use of fasteners subject to rotation under single shear loadings. Fastener rotation is substantially restrained when the connection involves a sheet thickness greater than approximately 0.060 inch (16 gage). Lap joint tests with sheets of such thickness involve a considerable amount of joint eccentricity when shims are not used. This eccentricity is absent in the simulated diaphragm action test and produces lower lap joint results. In shear connections where fastener rotation is prominent, the rotation deforms the material in the immediate vicinity of the fastener in the manner depicted in Figure 3.4. In these instances the two inch overlap width of the lap joint specimen provides greater restraint to this type of deformation than the 3/4 inch overlap of the simulated diaphragm action specimen. This greater restraint, coupled with the smaller joint eccentricity of the thin sheets, results in higher loads for the lap joint specimens. This conclusion is supported by the smaller variation in the two test fixtures in the determination of the fastener rotation load than the ultimate load.

In summary, the correlation between the lap joint and simulated diaphragm action tests is reasonable, and is better in the prediction of "yield" strength than ultimate strength. The use of shims in lap joint tests involving thicker material,
as specified in the proposed procedures, or an increase in the overlap width of simulated diaphragm action specimens to 1 1/2 inches for specimens exhibiting prominent fastener rotation will improve the agreement between the two tests. If shims are employed, the simpler lap joint test will well represent the behavior of shear connections, including those in shear diaphragms.

3.1.4 Verification Tests

Several series of five tests each were performed to verify that the proposed test procedures are acceptable for larger, more realistic, fastener spacings and to observe the effects of specimen configuration in lap joint tests.

The tests to check fastener spacing employed the same materials and procedures used for the ultimate load distribution tests described in the previous section. The only variation from the previous tests was a 10 inch, rather than a 2 inch, fastener spacing. A 10 inch spacing is the largest that can be employed in the simulated diaphragm action test and still maintain a one inch edge distance. The test results are given in Table 3.4, together with the mean values of the ultimate load distribution tests. A comparison reveals that there was virtually no difference in results between a 2 inch and 10 inch fastener spacing. This conclusion is supported by the initial tests with a variable spacing described in Chapter 2. Thus results with the two inch spacing specified in the proposed test procedures are valid for the larger fastener spacings that might actually be employed.
Similar materials and procedures were used for the tests of specimen configuration, although a slightly larger hole size was employed. The configurations depicted in Figure 3.5 were tested together with the standard lap joint and simulated diaphragm action configurations. The test results are presented in Table 3.5. All of the specimens failed through yield in bearing and, with the exception of Configuration C, had mean ultimate loads within 6 percent of each other. Configuration A proved to be the most susceptible to fastener rotation, with rotation occurring at a load of approximately 800 lbs. per fastener. This value compares with 900 lbs. for Configuration B and 1000 lbs. for the remaining specimens. Configurations A and B and the standard lap joint configuration all displayed out of plane distortions in the vicinity of the ultimate load. Despite fastener rotation occurring at 1000 lbs., Configuration C showed no out of plane distortion and failed at a mean load 21 percent greater than the other specimens. This increase in load is believed to have been caused by the restraint afforded by the stiffeners against deformation of the material immediate under the washer assembly in the direction normal to the plane of the material. This situation occurs during rotation of the fastener and is depicted in Figure 3.4. None of the other configurations tested, including the simulated diaphragm action configuration, offered this restraint.

In summary, the proposed lap joint configuration is slightly less flexible than configurations which transmit the load through a single fastener. The increased stiffness results
a higher fastener rotation load and less out of plane distor-
tion. Although configurations which transmit the load through a single fastener apparently give good results, the proposed configuration provides a better indication of the fastener rotation load and is to be preferred. Configurations which completely restrict out of plane deformations tend to produce artificially higher ultimate loads.

3.2 Pull-out/Pull-over Tests

A number of test fixtures have been developed to study the behavior of connections subject to pull-out and/or pull-over failures, with almost every investigator having employed a different fixture. Despite the variations in the design of such fixtures, they essentially consisted of a device or technique to hold the test specimen and an adaptor to pull the test fastener either out of or through the specimen. In the selection process for a proposed standard fixture, attention was focused on the following criteria: adaptability, i.e. the fixture should accommodate all cold-formed steel specimen shapes and as many other material shapes as possible without major modifications; simplicity of operation; ease of fabrication, both of the fixture and the test specimen; and, primarily, accuracy of results. It is felt that the fixture selected reasonably meets these criteria.

3.2.1 Test Fixture and Principles of Operation

The test fixture selected basically consists of a base plate assembly, shown in Figure A3, which serves as the holding
device for the specimen, and a loading assembly composed of a loading channel pin connected to a loading arm. These items are depicted in Figures A4 and A5. The test specimen, shown in Figure A2, is attached to the base plate with four 1/2 inch bolts. The entire assemblage, with a specimen positioned for a pull-over test, is shown in Figure A1.

Preliminary tests indicated that the test fixture must provide for the existence of membrane stresses in the specimen. The use of support systems which do not meet this requirement, e.g. systems in which the specimen is simply supported on two or four sides, results in flexure failures rather than pull-over failures whenever the specimen lacks sufficient stiffness to support the pull-over load. The accompanying deformations are also so large as to alter the fastener hole geometry. Details of tests with one such support system are provided in Chapter 6 of Reference 2. The provision for membrane stresses essentially dictates that the specimen be clamped either at several points or continuously. Bolting at the four corner points was chosen because it is both sufficient and convenient.

The size of the base plate was determined after initial tests indicated that pull-out and pull-over are apparently very localized in their effect and behavior. An 8 inch square base plate, allowing for a 6 inch square support system, provides for a convenient specimen size and relatively representative specimen deformations, and is still small enough and light enough to be readily manageable.

The dimensions of the base plate do not limit its use to
flat specimens or regular corrugated specimens with a pitch multiple of 6 inches. Use of spacer sleeves, i.e. tubular sections with an inside diameter slightly larger than 1/2 inch to accommodate the bolts, of the proper length should enable the use of this assembly with any regular corrugated specimen, although the specimen may have to be deformed in the vicinity of the 9/16 inch holes to provide a surface suitable for bolting. Figure A8 depicts a test specimen with a pitch multiple greater than 6 inches that is fastened eccentrically with respect to the corrugations. Spacer sleeves of a length equal to the depth of the corrugations are required when the 9/16 inch holes in the test specimen do not lie in the same plane, and may be required when the angle sections are employed.

The use of angle sections was adopted when corrugated specimens with a pitch multiple of less than 6 inches were tested in pull-over. Such sections have insufficient stiffness in the direction perpendicular to the corrugations, even when clamped, and distort to essentially a V-shape before pulling over the fastener head assembly. This distortion, which alters the fastener hole and material geometry and consequently affects the ultimate load, is adequately suppressed by the angle sections. The angle sections also permit the testing of structural shapes in pull-out, as depicted in Figure A12.

The location of the test fastener is important in both the pull-out and pull-over test. It is obvious that the fastener must be centered in the test specimen to avoid eccentricity during the test. The fastener should also have the
same location with respect to any corrugations that it has in the actual application. Cold working alters the material properties, and it is important that the material properties in the vicinity of the fastener be close to those in the actual application. Furthermore, fasteners positioned eccentrically with respect to the corrugations are subject to eccentric loading, both in the actual application and in the test specimen, because of the different stiffnesses on either side of them. In these situations, an example of which is shown in Figure A8, the specimen stiffness may be such that use of the angle sections is necessary to prevent the specimen from deforming into an eccentric V-shape prior to failure.

The channel shaped loading device can be readily connected to the test specimen and loading arm with the fastener in either the pull-out or pull-over configuration. A reasonably thick channel was selected to resist deformations and hence permit its reuse. If a portion of an actual channel is used in the test to employ all parts of the actual connection, it should be of a sufficient length to resist deformation into a V-shape prior to failure. With proper modifications, other materials and section shapes could be adapted for use in lieu of the test channel when it is desired to test all components of a connection simultaneously.

3.2.2 Pull-over Test

This test was designed to determine the strength of relatively thin panel sections connected to heavier members and subjected to loadings which produce tension in the connecting
fastener. Failure occurs with the panel material pulling over
the head assembly of the fastener. Although this test is be-
lieved to offer a reasonable simulation of this type of failure,
it ought to be used with caution when the actual member to which
the panel is connected is susceptible to rotation under load.
Such rotation places a highly eccentric load on the fastener
and tends to pry the fastener head assembly out of the panel
material. This action is difficult to simulate in the test,
for the actual rotation depends on the load, member geometry,
and the support and restraint conditions. Some idea of the de-
crease in pull-over load that results from support member rota-
tion can be obtained by positioning the fastener eccentrically
in both the test specimen and loading channel, but the accuracy
of this technique is in doubt. The proper evaluation of connec-
tions in members which tend to rotate under load is the major
limitation of this test.

3.2.2.1 Test Procedure

The dimensions and nature of the fastener head and washer
assembly have a significant effect on pull-over strength. The
tightening torque and presence of filler material such as insu-
lation determine the clamping force of the fastener and hence
are also important, as are the fastener hole tolerance and hole
location. These factors therefore should be identical in the
test specimen and the actual application.

The requirement for membrane stresses is met by bolting
the specimen to the base plate assembly. The use of washers
under the bolt heads and compliance with at least the specified
tightening torque are important in achieving proper clamping of the specimen without excessive bearing stresses or deformations.

The speed of testing is expressed in terms of both a load rate and a displacement rate because of the load-deformation characteristics of the test specimen. Initially the specimen can be relatively flexible in the direction normal to its plane, and a specified load rate is the logical choice. The specimen's stiffness increases with additional membrane stresses, and a specified displacement rate becomes the proper option. The displacement rate was selected to allow the gradual initiation and propagation of yielding, and it is crucial that the rate be low. At or near the ultimate load, high ductility materials experience plastic flow in the vicinity of the fastener. This process is not instantaneous, and it is essential that the load be maintained for at least the specified minimum time increment. The one minute increment selected is believed to be an absolute minimum, and some researchers recommend five or even ten minute increments.

The determination of a load-deformation curve is optional and such a curve should be used with caution. It will provide the general load-deformation shape, but the scale of the deformation axis is a variable determined by the support and boundary conditions of the actual application.

3.2.2.2 Comparison with Large Scale Uplift Tests

To check the accuracy of the proposed test fixture in its prediction of the failure load, a number of uplift tests were conducted on 8' x 6' and 8' x 5' roof panel sections and the
results compared to tests with the proposed test fixture.

The large scale loading system adopted consisted of a vacuum chamber formed with a wooden frame approximately one foot deep, with inside dimensions slightly larger than those of the specimen to be tested. The frame was braced at a sufficient number of points to prevent large lateral deflections under the application of a vacuum load. The test specimen, composed of roof panel sections attached to main members, was inverted and placed on top of the frame. The entire assembly was then covered with a polyethylene sheet which was taped to the floor in an airtight fashion around the perimeter of the frame. The loading system was completed with the attachment to the frame of a manometer line, air control valve and one or more air removal tubes. A schematic drawing of the entire system is presented as Figure 3.6.

A modification that improved this system was the placement of the polyethylene sheet over the test panel in such a manner that it conformed exactly to the panel surface prior to attaching the panel to the main members. This technique placed the polyethylene between the panel and main members and resulted in the direct loading of only the test panel, assuring a uniform load distribution.

The tests described herein were conducted primarily with 6 mil polyethylene sheet, although 4 mil sheet proved adequate. Polyethylene thicknesses in excess of 6 mil are neither necessary nor recommended as they are cumbersome to use. Commercial grade polyethylene pressure tape was suitable for taping the
film to the concrete floor, and industrial-type vacuum cleaners were used to evacuate the air from the chamber. Other useful procedures were the covering of all sharp edges with cloth base tape and the reinforcement of the film in the vicinity of the fasteners with polyethylene pressure tape.

The test specimen was loaded with a minimum of ten load increments. The vacuum cleaner was started and the air control valve gradually closed until the desired load, as indicated by the manometer, was achieved. After attainment of equilibrium, the load was maintained for a minimum period of one minute before proceeding to the next increment. The vacuum cleaner continued to run for the duration of the test, with load levels determined by manipulating the control value.

This loading system was inexpensive to construct and easy to employ. It provided reasonably accurate results for specimens subjected to uniform loadings, and was not limited by the size or shape of the test specimen. Although these tests were conducted on relatively small test sections, this loading system has been successfully used by others at Cornell University on 10' x 75' roof deck specimens employing channel and zee sections as the main members.

3.2.2.2.1 Large and Small Scale Uplift Test Procedures

The vacuum chamber was constructed by building an 8'6" x 6'6" frame with 2 x 12 inch lumber. The supporting members were three 12 gage lipped channels approximately 7 feet long. They were placed on 4 foot centers and braced against rotation by
welding similar members to them at the ends and midspan, as shown in Figure 3.7. The fasteners were positioned on 12 inch centers, with the fastener holes drilled simultaneously in the test panel and supporting members with a #6 bit. The supporting members and fastener spacing were chosen to result in a pull-over failure. Tests were performed with the polyethylene both draped over the entire assembly and placed between the panel and supporting members.

A mercury manometer, read with the aid of a magnifying glass, was used for load measurement. Manometers employing water or even lower density fluids are recommended for future tests because of their increased accuracy. The loading was conducted in approximate 10 psf increments, with the load level controlled by a 2 inch gate valve.

The corresponding small scale tests were performed in accordance with the proposed test procedures. The test fastener and washer assembly, fastener location relative to the ribs, hole diameter and tightening torque were identical to those used in the large scale tests.

### 3.2.2.2 Large and Small Scale Uplift Tests

Failure in the large scale tests always occurred with the panel material pulling over the head assembly of one or more fasteners located in the center support. In the discussion that follows, the load per fastener at failure was always determined by treating a one foot wide section of the panel as a two span, uniformly loaded, simply supported beam. Although
this procedure assumed rigid supports and did not account for
the vertical deflection of the center support in excess of that
of the outside supports, several factors tend to mitigate this
effect. First, the bracing welded to the supports at midspan
served to reduce the center support deflection by transferring
some of the load to the outside supports. Second, the major
deflection differential occurs at midspan and the panel sec­
tions that provided 30 and 36 inch coverage were joined to­
gether at that point, resulting in a double thickness of ma­
terial which always prevented failure at that location. Fin­
ally, the load on the panel remains uniform regardless of the
support deflections. Failure occurs at the fastener support­
ing the greatest load, and that fastener will not be located
near the midspan of the center support if a substantial part
of the load there has been transferred to the outside supports
because of deflection.

The first series of large scale uplift tests was performed
on three different roof panel sections. The panel configura­
tions and fastener locations are shown in Figure 3.8 and their
material properties are presented in Table 3.6.

The pilot test was performed on an 8' x 6' specimen of
22 gage galvanized low ductility steel, designated as Panel I,
with the polyethylene draped over the entire assembly. Failure
occurred with the panel pulling over the head and washer as­
sembly of three consecutuve fasteners at a load of 195 psf,
or 975 lbs. per fastener. Four corresponding specimens tested
in the pull-over test fixture failed at an average load of 910
lbs., as indicated in Table 3.7, but with considerable scatter and a failure mode that did not resemble that of the large scale test. Failure of the uplift specimen produced large, rough, jagged holes, while that in the pull-over specimens occurred with the material splitting in the direction parallel to the corrugations. The angle sections were not used with these specimens, and the splitting probably resulted from the greater membrane stresses in the direction perpendicular to the corrugations caused by the greater panel flexibility in that direction. The uplift test was also a pilot test, and hence not as well controlled as an established test.

The next three tests were performed on 8' x 5' specimens of 22 gage painted high ductility material, designated as Panel II in Figure 3.8. The first of these tests failed prematurely due to an excessive gap between the edge of the panel and the wooden frame, allowing the polyethylene to deform into the gap and place a large load on the edge of the panel. This situation was corrected with the placement of two 2 x 12's along the top of the frame to reduce the size of the gap. The second test had the polyethylene draped over the entire assembly and performed satisfactorily, failing at a load of 340 psf, or 1700 lbs. per fastener. During the course of this test it was noticed that some contact between the polyethylene and panel surface was lost as the panel deformed. This loss of contact occurred in the vicinities of the supporting members and the "valleys" of the panel surface. To observe these contact effects, the third specimen had the polyethylene positioned
between the panel and supporting members in such a manner that it conformed exactly to the panel surface and hence provided a very uniform load. This assembly failed at a load of 330 psf, or 1650 lbs. per fastener, only slightly below the failure load of the other specimen. The corresponding pull-over specimens failed at an average load of 1480 lbs. without much scatter in the results, as shown in Table 3.7. The specimens that failed at the three highest loads failed in a manner similar to the uplift test failures; the remainder failed predominantly by splitting in the direction parallel to the corrugations. The splitting failures are thought to be caused by the same mechanism as in the Panel I tests. Although this was nominally high ductility material, the fastener was placed through the relatively narrow rib, and both the strength and ductility in that region may have been affected by cold working.

The final test in this series was on an 8' x 6' specimen of 24 gage painted low ductility steel, designated as Panel III in Figure 3.8, with the polyethylene between the panel and the supporting members. This assembly failed at a load of 180 psf, or 900 lbs. per fastener, but the results are in some doubt as the unstiffened edge failed first, was braced and the specimen retested. The pull-over test specimens failed at an average load of 1060 pounds, as indicated in Table 3.7, with all failures occurring by splitting in the direction parallel to the corrugations.

The second series of tests was performed on 8' x 6' specimens of 24 gage painted high ductility material. The purpose
of these tests was to ascertain that polyethylene contact with the panel has only a slight influence on the ultimate load, to observe the effects of fastener location and eccentric loading, and to check the suitability of the pull-over test fixture for fasteners eccentrically loaded by the test specimen. The panel configuration and fastener locations used are shown in Figure 3.9 and the material properties are given in Table 3.6 under the designation of Panel IV.

Test A had the fastener located at the center of the rib and the polyethylene placed between the panel and the supporting members. This assembly failed at a load of 210 psf, or 1060 lbs. per fastener. Test B, identical to Test A save that the polyethylene was draped over the entire assembly, failed at a load of 230 psf, or 1140 lbs. per fastener, for an 8 percent increase over Test A. Tests with the pull-over fixture yielded a mean load of 1065 lbs. per fastener, as indicated in Table 3.8. These specimens did not exhibit much scatter and had the same failure mode as the large scale specimens.

Test C, with the fastener at the center of the flat and the polyethylene conforming to the contours of the panel, failed at a load of 160 psf, or 780 lbs. per fastener. Test D was identical to Test C save that the fastener position was eccentric by approximately one inch, placing it as close to the rib as the washer would allow. This section failed at a load of 190 psf, or 930 lbs. per fastener. The deflections in Test C were completely symmetric with respect to the centered
fasteners, while Test D showed a definitely eccentric deflection pattern. Test D also revealed local buckling at the mid-span of every rib adjacent to a fastener, with these buckles first appearing at a load approximately 70 percent of the ultimate. The higher load under eccentric loading produced some skepticism about the accuracy of the loading system, so a specimen with half of the fasteners in the Test C position and half in the Test D position was prepared and loaded to failure. A pull-over failure occurred at a fastener that was centered, not eccentric, confirming the higher strength of the eccentric fasteners.

Only two tests were initially conducted with the proposed pull-over fixture on the Test C and D configurations. The panel shape was such that one or two ribs would always lie between the support points of the fixture, greatly reducing the stiffness in the direction normal to the ribs. Thus with the fastener in the center of the flat (two ribs between supports), the specimen first straightened into a V-shape before carrying any substantial load, and finally failed at a load of 915 lbs. This load was 17 percent greater than that of the uplift specimen. With the fastener eccentric by one inch (one rib between supports), the portion of the specimen without the rib was immediately in tension while that with the rib had very little membrane stress. The specimen finally deformed into an eccentric V-shape and then reached its ultimate load of 820 lbs., tearing out of the tensile side of the specimen. This load was 13 percent below that of the uplift specimen.
Another fixture was designed for compatibility with the panel configuration. This fixture was a smaller version of the proposed fixture, with the support points positioned to form a 4 inch square. The problem again proved to be insufficient stiffness in the test specimen, with the Test C and D configurations failing at 1020 and 675 lbs., respectively. The smaller fixture, in fact, appeared to accentuate the shortcomings of the larger one.

Finally, the proposed fixture was modified with the addition of two angle sections as shown in Figure A9. Using this modified fixture and varying the distance between the angle sections from approximately 2 to 4 1/2 inches, it was noted that yield lines always formed from the fastener location to the points defined by the intersection of the angle sections and the ribs. From these observations and application of yield line theory it was concluded that the angle sections should be positioned such that 45 degree lines can be drawn from the fastener location to the points defined by the angle-rib intersection most distant from the fastener, as shown in Figure A10. Use of this technique resulted in average ultimate loads of 785 and 930 lbs. for the Test C and D configurations, respectively. The results of the second series of tests are tabulated in Table 3.8.

**3.2.2.2.3 Conclusions Drawn from Test Results**

A comparison of the large and small scale test results, presented in Table 3.7 and 3.8, reveals that the proposed test
fixture can adequately predict the pull-over strength of connections. In the first series of tests, the means of the predicted values were within 10 percent of the large scale results, with the exception of the doubtful results of Panel III. Use of the angle sections would probably have improved this agreement. The second series showed very good agreement with the uplift tests and little scatter in the results when the angle sections were properly employed. The success of this series, which included eccentric loading of the fastener by the specimen, also indicates the versatility in application of the proposed fixture.

The failure mode of the first test series, splitting in the direction parallel to the corrugations, was probably due to greater tensile stresses in one direction than the other. This situation did not exist in the large scale test, and could have been avoided in the small scale tests through the use of angle sections. These sections increase the stiffness in the weak direction and thus provide for comparable membrane stresses in both directions.

A comparison of Tests C and D in the second series shows a 19 percent increase in ultimate load when the fastener location was changed from the center of the flat to a position adjacent to the rib, despite the fact that in the latter location the fastener was loaded eccentrically. This difference was probably due to the increased material strength in the vicinity of the rib, where the material had been strain hardened. The effect of material strength is better illustrated
by comparing Tests A and C, which both involved concentric loading. Test A, with the fastener located in the center of a relatively narrow rib subjected to considerable cold working, failed at a load 36 percent greater than Test C.

3.2.3 Pull-out Test

This test was designed to determine the strength of connections subjected to loadings which produce tension in the connecting fastener and the mode of failure is extraction of the fastener from the material into which it was driven. This test procedure developed as a natural corollary to the pull-over test, and as a result the discussion on the development of that procedure is equally applicable. The fact that no problems were encountered in the performance of pull-out tests eliminated the need for significant development work on this test procedure.

The tightening torque and predrilled hole size and tolerance are critical in pull-out testing, especially when relatively thin materials are employed. For this reason additional care should be used to insure that these factors are identical to those in the actual application.

Provisions for membrane stresses are less important in pull-out than pull-over testing because failure usually occurs before significant membrane stresses are developed. Such provisions should still be made, however, especially in cases where the fastener is located eccentrically with respect to the corrugations.

The major limitation of the pull-over test, namely the
simulation of support members which exhibit torsional behavior under load, does not appear to be applicable to the pull-out test. Eccentric and concentric pull-out tests conducted on 12, 14 and 18 gage material yielded virtually identical results for the two types of loading. Thus, although the procedure prevents rotation of the specimen, the test results may be considered valid for members susceptible to rotation under load.

3.2.4 Summary

Pull-out/pull-over failures are normally associated with connections in relatively thin panel sections connected to more substantial supporting members. The primary reasons for the selection of the proposed pull-out/pull-over test fixture are its simplicity, adaptability to various panel configurations, and ability to provide for the presence of membrane stresses. A comparison with large scale uplift tests shows this fixture to be reasonably accurate in its prediction of the failure load when the prescribed procedures are followed, even when the fastener is eccentrically loaded by the panel. The primary limitation of this test is the accurate prediction of the pull-over load in those instances where the actual application involves supporting members subject to torsional behavior under load. The estimate of the pull-out strength from such members, however, will be reasonable.

One method of accurately treating supporting members subject to torsional behavior is the performance of large scale tests with a simple and inexpensive loading system. This system consists of a wooden frame of the desired size on which
the inverted specimen is placed, and a polyethylene covering capable of sealing the entire assembly. The air is evacuated with a vacuum cleaner or other convenient device, and the load determined by measurement of the air pressure differential.

The strength of connections subject to pull-over/pull-out failure is affected by the following features, which should be the same in the specimen as in the actual application: the fastener and fastener head assembly; fastener hole diameter, tolerance and location with respect to any corrugations; tightening torque; and the presence of filler material, such as insulation.

The proper management of membrane stresses is very important, especially in the pull-over test. Test specimens containing one or more corrugations are more flexible in the direction normal to the corrugations than in the direction parallel to them. This orthotropy can be reduced by the proper placement of angle sections. These sections should also be used when the fastener is positioned eccentrically with respect to the corrugations.

The speed of testing also has a significant effect on indicated strength, primarily for the pull-over test. The test rate was selected to allow the gradual initiation and propagation of yielding. High ductility materials tested in pull-over experience plastic flow in the vicinity of the fastener at or near the ultimate load. This process is not instantaneous, and it is essential that the load be maintained for at least the specified minimum one minute increment.
3.2.5 Tests to Estimate Ultimate Load Distributions

The proposed test fixture and procedures were used for a series of twenty-five identical pull-out and pull-over tests with materials of typical thickness to obtain estimates of the ultimate load distributions for these types of failures. The pull-out tests were conducted with 16 gage material and the pull-over tests with 26 gage material. These were galvanized sheets of average strength steels with material properties as given in Table 3.2. The fastener hole in the 16 gage material was drilled with a #8 (0.199 in.) high speed bit and the fastener was hand torqued to 35 in.-lbs. A #13 (0.185 in.) bit and a torque of 40 in.-lbs. were used for the 26 gage material.

The results of the two test series are presented in Table 3.9. The scatter in the pull-out tests was small, as is generally the case with reasonably thick materials, and the coefficient of variation is quite representative of other pull-out tests conducted. The coefficient of variation for the pull-over tests is also representative of this type of failure in thin materials.
CHAPTER 4
ULTIMATE STRENGTH PROBABILITY DISTRIBUTIONS

The test procedures were developed with the goal of standardization of test criteria and test interpretation. An important element of test interpretation involves the development of appropriate probabilistic models of the underlying processes responsible for the data, and the subsequent statistical procedures to properly characterize the models.\(^1\) In this chapter the construction of the models and interpretation procedures is developed.

The initial step in the construction of a probabilistic model is the selection of the type of model (e.g. normal, log-normal) to be used. Such selection may be based on assumptions about the basic nature of the underlying process, but is typically based on mathematical convenience. Once the model has been selected, several methods can be used to estimate the model parameters and hence fit the model to the data.

This investigation treats the model parameters as random variables whose values cannot be established with certainty. Different values are assumed for the model parameters, with each set of parameters characterizing a different probabilistic model. The probability that each model describes the process responsible for the data is calculated by the method of inverse inference. These probabilities serve as "weighting factors" in combining the different models to form a compound model of the underlying process. The resultant model incorporates both the uncertainty inherent in the process and the

\(^1\) A brief summary of the fundamentals of probability theory is presented as Appendix D.
uncertainty associated with the determination of the model parameters. A probabilistic model is also determined by using the method of maximum likelihood to calculate those values of the parameters which characterize the "most likely" single model of the process.

4.1 The Normal, Lognormal and Beta Distributions

Because the conclusions derived from the use of a particular probabilistic model are often dependent on the properties of the model, model selection is very important. For this reason the properties of two of the most common probabilistic models for continuous random variables, the normal and lognormal, are described below. The beta model, found to be very useful in this investigation, is also discussed.

4.1.1 The Normal Distribution

The normal is the most widely used model in probability theory. Its prominence is primarily due to the large body of statistical methods and tabulated results derived for the normal and often applicable in an approximate manner to other distributions. (18)

This distribution's great importance in statistics stems largely from the central limit theorem. The theorem essentially states that the distribution of the sum of random variables will approach the normal distribution as the number of variables in the sum becomes large. The variables may belong to any population, but are generally required to be statistically independent observations with finite variance from
the same population. They may belong to different populations provided that no one variable dominates, and under certain conditions the independence criterion may also be relaxed.\(^{(19)}\)

The normal distribution can thus be expected to represent those variables which are due to the sum of a number of random effects, no one of which dominates the total. As a result, the normal has been used to describe the experimental error in many types of measurements and to model systems whose failure depends on a number of parallel, random strength components. For example, it has been used by the American Concrete Institute to model concrete compressive strength.

The density function of a normal distribution with mean \(m_x\) and variance \(\sigma_x^2\) is given by the expression\(^{(20)}\)

\[
f(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x - m_x}{\sigma_x} \right)^2 \right] \tag{1.1}
\]

This is a two parameter distribution and is often abbreviated as \(N(m, \sigma^2)\). The normal curve with mean zero and standard deviation unity, or \(N(0,1)\), is referred to as the standard normal curve and is shown in Figure 4.1. All normal distributions can be transformed to this distribution through the substitution

\[
y = (x - m_x)/\sigma_x \tag{4.2}
\]

The transformed variable \(y\) is referred to as the standard normal deviate. The resulting density function is given by
\[ f(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \]  

and its properties are widely tabulated.

Most of the area under the normal curve is concentrated about the mean, with 99.9936 percent of the area lying within a range of 4σ from the mean.\(^{(21)}\) The implication is that the tails of the distribution can be ignored in a majority of cases. Changes in the value of the mean translate the midpoint but do not alter the shape of the curve. Changes in the standard deviation, on the other hand, greatly affect its shape. These characteristics are illustrated in Figure 4.2.

Various techniques are available for fitting data to a normal model, the most common being a fit by areas, a cumulative frequency fit and the method of moments. Tests for normality include a comparison of the asymmetry and kurtosis to the normal values of zero and three, respectively, the use of normal probability paper and the chi-square test.\(^{(22)}\)

Important limitations on the use of the normal to model physical phenomena are its symmetry and a non-zero probability for negative values. If necessary, however, the latter liability can be treated by truncation.

4.1.2 The Lognormal Distribution

The limitations of the normal distribution noted above can be removed by a simple transformation. The transformed distribution obtained by assuming the natural logarithms of a random variable to be normally distributed is called the
logarithmic normal or lognormal distribution.

It is always theoretically possible to determine a function which will transform a skew distribution into a normal one. In using a transformation function that does not contain parameters which have to be estimated from the sample, however, one incurs the advantage that the resulting distribution is fully described by two parameters, the mean and variance of the transformed variable. Thus one can use standard theory in determining estimates of these parameters to be used in further analysis of the distribution.\(^{(23)}\)

The lognormal probability law has been used in statistical studies of fatigue failures in metal members. It has also been employed to describe the distribution of earthquake magnitudes and interarrival times between earthquakes, as well as the distribution of the yield stress of steel reinforcing bars.\(^{(22)}\) Because it describes phenomena arising from a multiplicative mechanism acting on a number of factors, the lognormal distribution may be expected to find application in the area of structural engineering.

A random variable \(y\) whose logarithms, denoted by \(x\), are normally distributed has the following density function\(^{(22)}\)

\[
f(y) = \frac{1}{y \sigma_x \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \frac{(\ln(y) - m_x)}{\sigma_x}^2 \right] \quad y \geq 0
\]

\[(4.4)\]

where \(m_x\) and \(\sigma_x\) are the mean and standard deviation of \(x\) or \(\ln(y)\), not of \(y\) itself.
Figure 4.3 shows plots of lognormal distributions with equal means but different standard deviations. It can be seen that larger standard deviations result in more pronounced skewness. A small standard deviation produces a practically normal distribution. The mode is always less than the median, which in turn is less than the mean, with the disparity between these three characteristics increasing with the standard deviation. The lognormal distribution can be shifted and the skew reversed from right to left by simple linear transformations.

4.1.3 The Beta Distribution

Although it can be shown to arise from the consideration of various underlying mechanisms, the beta distribution acquires its importance primarily through its ability to describe many types of empirical data. It has finite upper and lower limits, and can assume a large variety of shapes between those limits by adjustment of its parameters.

In its basic form the beta is a two parameter distribution, abbreviated $BT(r,t)$, and is limited to the range zero to unity. For positive $r$ and $t-r$, the density function is defined by the expression

\[ f(x) = \frac{1}{B} x^{r-1} (1 - x)^{t-r-1} \quad 0 \leq x \leq 1 \]  

(4.5)

The constant $B$ normalizes the area under the curve to unity and is
\[ B = \frac{\Gamma(r) \Gamma(t-r)}{\Gamma(t)} \]  

where the symbol \( \Gamma(k) \) represents the gamma function, equal to \((k-1)!\) for integer values of \( k \), but more generally defined by the definite integral

\[ \Gamma(k) = \int_0^\infty e^{-u} u^{k-1} \, du \]  

The mean of the beta distribution is

\[ m_x = \frac{r}{t} \]  

while the variance is

\[ \sigma_x^2 = \frac{r(t-r)}{t^2(t+1)} \]

Examples of some beta distribution shapes are shown in Figure 4.4. It can be seen that the beta contains as special cases the uniform distribution \((r=1, t=2)\) and both right and left skew triangular densities \((r=1 \text{ or } 2, t=3)\). It is symmetrical if \( r = \frac{1}{2}t \), and skewed right or left for \( r \) less than or greater than \( \frac{1}{2}t \), respectively. It may be U-shaped, J-shaped or bell-shaped. The beta is unimodal and bell-shaped if \( r>1 \) and \( t>r+1 \), with the mode at \( x = (r-1)/(t-2) \) and with concentration about the mode increasing with increasing parameter values. Interchanging the parameters to \( r' = t-r \) and \( t' = t \) yields mirror images.
In the general case the beta may be defined over any interval with end points \( a \) and \( b \). It is then abbreviated \( BT(a,b,r,t) \) and its density function is provided by the expression (22)

\[
f(y) = \frac{1}{B(b-a)^{t-1}} (y-a)^{r-1} (b-y)^{t-r-1} \quad a \leq y \leq b
\]

(4.10)

with \( B \) as previously defined. The mean is now defined by

\[
m_y = a + \frac{r}{t} (b-a)
\]

(4.11)

and the variance by

\[
\sigma_y^2 = (b-a)^2 \frac{r(t-r)}{t^2(t+1)}
\]

(4.12)

4.2 Posterior Distributions with a Beta Model

At least three types of uncertainty arise in attempting to define a probabilistic model to describe a random variable. The first is simply the basic uncertainty that is characteristic of the phenomenon itself. The second type lies in the inability to precisely determine the model parameters. Finally there is uncertainty associated with the form of the model itself.

The generalized beta distribution is selected as the probabilistic model in this investigation in an effort to minimize the model uncertainty. Initial examinations of the data, in the form of histograms and cumulative frequency polygons, reveal that it is typically non-symmetric, with varying degrees and direction of skewness. The beta appears to be a natural
choice because it provides a reasonable fit to almost any desired shape and its upper and lower limits permit the exclusion of non-zero probabilities for negative or extremely large strength values.

The uncertainty associated with the determination of model parameters is handled by treating the unknown parameters as random variables. A probability distribution on the model parameters is assumed prior to conducting any tests. The method of inverse inference\(^{(24)}\) is used to obtain a posterior distribution on the parameters which incorporates both the subjective prior distribution and the test results. The posterior parameter distribution is then combined with the model distribution to define a compound distribution on connection strength. The basic advantage of this resultant distribution is that it simultaneously incorporates both the uncertainty of parameter estimation and the inherent uncertainty of the process itself. This procedure has been successfully employed by Sexsmith\(^{(25)}\) in a reliability analysis of concrete beams and columns.

4.2.1 The Method of Inverse Inference

The method of inverse inference uses Bayes' theorem to determine probabilities on the "causes" or relative truth of the "hypotheses" that underlie a probability model. The theorem and its components such as the sample likelihood function are of fundamental importance to this work and hence are described below.
4.2.1.1 Bayes' Theorem

The derivation of this theorem follows directly from the axioms of probability theory, and is essentially based on two concepts: conditional probability and the total probability theorem.

The conditional probability of event $A$ given that event $B$ has occurred, denoted $P[A|B]$, is defined as the probability of the intersection of $A$ and $B$ divided by the probability of $B$, or

$$P[A|B] = \frac{P[A \cap B]}{P[B]} \quad (4.13)$$

Consider now a set of mutually exclusive, collectively exhaustive events, $A_1, A_2, \ldots, A_n$, and another event $B$ in the same sample space. The probability of event $B$ can be expressed as

$$P[B] = P[A_1 \cap B] + P[A_2 \cap B] + \ldots + P[A_n \cap B]$$

$$= \sum_{i=1}^{n} P[A_i \cap B] \quad (4.14)$$

From Eq. 4.13


where the last term follows from the symmetry of $A$ and $B$. Using this relationship to expand each term in the sum of
Eq. 4.14,

$$P[B] = \sum_{i=1}^{n} P[B|A_i] P[A_i]$$  \hspace{1cm} (4.16)

This result is referred to as "the theorem of total probabilities."

Examine now the probability of some event $A_j$ given the event $B$. From Eq. 4.13

$$P[A_j|B] = \frac{P[A_j \cap B]}{P[B]} = \frac{P[B \cap A_j]}{P[B]}$$

Replacing the numerator with the product term given by Eq. 4.15 and expanding the denominator through the total probability theorem,

$$P[A_j|B] = \frac{\sum_{i=1}^{n} P[B|A_i] P[A_i]}{\sum_{i=1}^{n} P[B|A_i] P[A_i]}$$  \hspace{1cm} (4.17)

This result is the standard form of Bayes' theorem. It can be generalized by referring to the unknown classification $A_j$ as the "state", "cause" or "hypothesis" and the observed event $B$ as the "sample outcome." Eq. 4.17 is then

$$P[\text{state}|\text{sample outcome}] = \frac{\sum_{i=1}^{n} P[\text{sample outcome}|\text{state}] P[\text{state}]}{P[\text{sample outcome}|\text{state}] P[\text{state}]}$$

all states
The importance of Bayes' theorem is that it allows one to gather prior or unconditional probabilities on the state, and arrive at posterior probabilities on the state conditioned on the observation of a particular sample outcome. It thus provides for updating the probabilities of state as new information becomes available.

The sample likelihood function provides the relative likelihood of observing a particular sample outcome as a function of the state. Assume the parameters of a probability density function on a random variable X are \( \theta \). The relative likelihood of obtaining a particular sample D consisting of independent sample points \( x_1, x_2, \ldots, x_n \) is then

\[
L(D|\theta) = f(x_1|\theta) \cdot f(x_2|\theta) \cdots f(x_n|\theta) = \prod_{i=1}^{n} f(x_i|\theta) \quad (4.18)
\]

This expression is termed the sample likelihood function.

4.2.1.2 Posterior Likelihoods

The method of inverse inference determines the posterior probability density function on the parameters of the generalized beta probability law. The method involves postulating possible beta laws for the major random variable and employing Bayes' theorem to determine the probability that a particular law was responsible for the data.

Possible parameter sets for the generalized beta distribution are denoted by \( \theta_i \). Thus a particular \( \theta_i \) represents a specific beta law, \( BT(a_i,b_i,r_i,t_i) \). Although \( a_i, b_i, r_i \) and \( t_i \) assume discrete values for computational purposes, the
continuous case can be constructed from the discrete one in a straightforward manner. If the data, consisting of \( n \) observations \( x_1, x_2, \ldots, x_n \) of the process, is denoted by \( D \), Bayes' theorem is written as

\[
p[D|\theta_i] \frac{p[\theta_i]}{\sum_j p[D|\theta_j] p[\theta_j]} \tag{4.19}
\]

where \( j \) is the number of postulated beta laws.

The denominator in the above expression is simply a normalizing constant, and is denoted by \( C_N \). The probability of obtaining the sample given the parameters, \( p[D|\theta_i] \), is provided by the sample likelihood function, Eq. 4.18. Thus Bayes' theorem becomes

\[
p[\theta_i|D] = C_N L(D|\theta_i) p[\theta_i] \tag{4.20}
\]

There is very little prior information in this investigation with which to assign probabilities to the parameter sets. The most logical decision is therefore to assume the prior probabilities \( p[\theta_i] \) equal for all \( i \). Such a prior is termed "diffuse" or "flat" or "vague", and the resulting posterior distribution depends solely on the information contained in the sample. It is evident that use of a diffuse prior effectively reduces the resulting posterior distribution to a normalized sample likelihood function.

The use of relative likelihoods permits the dropping of the constant terms in Eq. 4.20, and Bayes' theorem is finally
When a beta law is used to describe a random variable $X$, the expression becomes

$$L(\theta_i | D) = L(D | \theta_i) \quad (4.21)$$

$$L(\theta_i | D) = \left[ \frac{\Gamma(t_i)}{\Gamma(r_i) \Gamma(t_i - r_i) (b_i - a_i)^{t_i - 1}} \right]^{n} \prod_{j=1}^{n} \left( \frac{1}{x_j - a_i} \right)^{r_i - 1} \cdot (b_i - x_j)^{t_i - r_i - 1} \quad (4.22)$$

where the parameters $a_i$, $b_i$, $r_i$ and $t_i$ define a particular beta law and the $x_j$'s are the observed sample points. This quantity is the joint posterior likelihood of $\theta_i$. Selection of appropriate ranges and increments on each of the parameters and the $n$ observations of the process are sufficient to evaluate this function.

Once the joint posterior likelihood is evaluated at all points $i$, a number of other functionals may be readily determined. Among the most important of these are the marginal density functions on each of the four parameters. The marginal density function on a given parameter, e.g. $a$, is by definition

$$f(a) = \int \int \int f(a, b, r, t) \, dt \, dr \, db \quad (4.23)$$

If the continuous joint density function is replaced by a joint likelihood function consisting of discrete points, this
equation becomes

\[ L(a_i) = \sum_{j \in a_i} L(\theta_j | D) \quad (4.24) \]

where the summation is over all points of the joint posterior likelihood of \( \theta \) for which \( a \) equals \( a_i \).

These marginal likelihoods may be plotted and a smooth curve passed through them. Adjustment of this curve to result in an area under it of unity, e.g. by direct scaling employing a form of Simpson's rule\(^{(26)}\), yields a continuous marginal density function on the parameter. The mean, mode, variance and other characteristics can then be determined, and a point estimate for the parameter selected if desired. Examination of this function will also reveal whether or not the chosen parameter range is appropriate, as well as the sensitivity of the posterior likelihood function to the parameter.

4.2.2 The Bayesian Distribution

The most important use of the joint posterior likelihood function is in the determination of a distribution on the major random variables \( X \). Combination of the model distribution on \( X \) with the posterior distribution on the model's parameters results in a compound distribution defined as

\[ f(x) = \int f(x | \theta) f(\theta) \, d\theta \quad (4.25) \]

It is called the Bayesian distribution on \( X \) as distinct from
the model distribution on $X$, $f(x|\theta)$.

The Bayesian distribution can be considered as a weighted average of all model distributions which are associated with different values of $\theta$. The unknown parameters do not appear in the resulting distribution, as they are "integrated out" of the expression. With increasing data the Bayesian distribution will approach the true distribution, for the distribution of the parameters will become increasingly concentrated about their true values. In general the Bayesian distribution will have a larger variance than the true distribution because it reflects the uncertainty in the model parameters as well as the uncertainty inherent in the process.

Expressed in the form of a discrete likelihood function, Eq. 4.25 becomes

$$L(x_i) = \sum_j P[x_i|\theta_j] L(\theta_j | D)$$

$$= \sum_j \frac{\Gamma(t_j)}{\Gamma(r_j) \Gamma(t_j - r_j)} \frac{(x_i - a_j)^{r_j-1} (b_j - x_i)^{t_j-r_j-1}}{(b_j - a_j)^{t_j-1}} L(\theta_j | D)$$

(4.26)

where the summation is over all points of the joint posterior likelihood function of $\theta$. The resulting likelihoods can be easily transformed to a continuous probability density function, which can be used to make probability statements.

4.2.3 Updating the Distribution

A diffuse prior is used in the determination of the
distributions on the parameters because without prior information the most logical decision is to consider every parameter set to have an equal "chance" of being the true set. As future information becomes available, however, there is no restriction on using the information produced by this investigation as the prior on a new posterior distribution. Although the distributions to be presented are not in close-form, possible closed-form approximations, however crude, may be used to facilitate computation.

In obtaining a new, posterior Bayesian distribution on the major random variable \( X \), Bayes' theorem must be applied to the prior distribution on the parameters. Thus given new data \( D' \), the posterior Bayesian distribution is given by

\[
f'(x) = \int f(x|\theta) f'(\theta) \, d\theta
= \int f(x|\theta) \, C_N \, L(D'|\theta) \, f(\theta) \, d\theta \tag{4.27}
\]

where \( f(\theta) \) is the new prior distribution on the parameters and \( C_N \) is a normalizing constant.

4.3 Bayesian Distributions on Connection Strength

The data to be used in the determination of the distributions on ultimate strength consists of the results of the four series of twenty-five identical tests on typical specimens described in Chapter 3. All of the data sets are treated in an identical manner. Under the assumptions that each failure mode defined by the proposed procedures is governed by a single
underlying probability law and has a characteristic value for its coefficient of variation, the distribution determined herein is valid for that failure mode in general.

Each data set is first normalized with respect to its mean value. This procedure is generally necessary when using the beta distribution, as large values of the argument lead to computational difficulties due to the exponential terms. Normalization results in a non-dimensional form of the data, with the standard deviation identical to the coefficient of variation.

The computations are performed with an IBM 370/168 computer. Conditioning problems in the form of overflow and underflow are resolved through the use of logarithms or judicious sequencing of the operations. The gamma function with arguments greater than 57.5, the approximate limit of the computer generated function, is computed by using Stirling's formula\(^{(27)}\) with a correction factor.

Ranges and increments on the parameters are essentially determined subjectively. Ten values are initially used for each of the four parameters, resulting in the determination of ten thousand points on the joint posterior likelihood function. In an attempt to reduce the amount of computation and facilitate the determination of parameter ranges, the value of the parameter \(t\) is computed directly from the requirement that the mean of the distribution equal unity. Such a constraint is reasonable in view of the data normalization, and the resulting Bayesian distribution differs negligibly from one computed
with a range on the parameter. The value of $t$ with this constraint is easily determined from Eq. 4.11 as

$$t_i = \frac{r_i (b_i - a_i)}{(1 - a_i)} \quad (4.28)$$

The marginal likelihoods of each of the remaining parameters are obtained through the use of Eq. 4.24, with transformation to a marginal density function achieved by numerical integration. Initially, the sole criterion for suitable parameter ranges is approximately bell-shaped marginal density functions on all three of the parameters. Failure to satisfy this criterion results in the adjustment of one or more of the ranges and a repetition of the entire process. Criterion satisfaction by this trial procedure soon proves to be both extremely difficult and tedious because of the interdependence of the parameters. Each of the parameters has at least one natural limit, namely

$$a \leq (x_i)_{min}$$
$$b \geq (x_i)_{max}$$
$$r \geq 1$$

The first two limits are self-evident, and the last stems from the requirement that the resulting distribution not be U-shaped. Each parameter is thus basically constrained to movement in the direction away from its limit. Such a move on the part of one parameter, however, favors similar moves by the others. For
example, an increase in the value of $r$, and hence a "sharpening" of the distribution, increases the likelihoods of those values of $a$ and $b$ that are more removed from the mean. Consequently an adjustment of one parameter range to improve its marginal density function typically impairs that of one or more of the other parameters, and simultaneous adjustment is more difficult.

The difficulty, and at times the apparent impossibility, of meeting the first criterion leads to the adoption of a second, to be used when the first cannot be satisfied. This criterion is based on the maximization of the sum of the joint posterior likelihoods of $\Theta$. The method, in effect, is to adjust the ranges in a manner which increases the value of the sum of the joint posterior likelihoods, given that ten equally spaced points comprise the range of each parameter. The rationale for this criterion is the selection of those one thousand points with the greatest relative likelihoods.

Upon attainment of suitable parameter ranges, the likelihood function on the major random variable is computed using Eq. 4.26. The range is the minimum value of $a$ to the maximum value of $b$, with increments of 0.01. This computation is quite lengthy, as all the points of the joint posterior likelihood function must be used for each value of the major random variable. Transformation of the likelihood function to the Bayesian distribution is again accomplished with numerical integration, which is also used for the determination of the cumulative distribution function.
Numerical integration is likewise employed in the determination of some of the characteristics of the Bayesian distribution. The two most important characteristics are the mean and variance. The mean is defined as

\[ m_x = \int x f(x) \, dx \]  \hspace{1cm} (4.29)

and the variance as

\[ \sigma_x^2 = \int (x - m_x)^2 f(x) \, dx \]  \hspace{1cm} (4.30)

4.3.1 Single Shear (Lap Joint) Results

The determination of the Bayesian distribution on single shear lap joint strength is presented in considerable detail to illustrate the procedure described above. The twenty-five test results which comprise this data set are presented in Table 4.1 in both basic and normalized form. A histogram of these results appears as Figure 4.5.

Following the procedure outlined in the previous section, with the values of \( t \) calculated from the constraint that the mean equal unity and ten values selected for each of the remaining parameters, one thousand points on the joint posterior likelihood of \( \theta \) are generated. A trial procedure leads to the ranges and increments on the parameters presented in Table 4.2 under the heading "Parameter Set 1." These values result in a marginal density function on \( a \) that is approximately bell-shaped, and one on \( b \) that is acceptable. The function on \( r \), however,
increases with the parameter and has its maximum value at the end of the parameter range, indicating that the range was unsatisfactory. The range of \( r \) is therefore extended, resulting in "Parameter Set 2" of Table 4.2. The effects of this adjustment are shown in Figures 4.6 - 4.8, which depict the marginal density functions of \( a \), \( b \), and \( r \), respectively, for the two parameter sets. Table 4.3 provides the characteristics of these distributions. It can be seen that although the marginal density function on \( r \) is improved by extending its range, those on \( a \) and \( b \) are impaired by an increase in the likelihoods of those points more removed from unity. In addition, the total sum of the joint posterior likelihoods decreases slightly.

The adjustment process is continued until the ranges and increments denoted as "Parameter Set 3" in Table 4.2 are attained. Although they produce marginal density functions that are less sharply defined than desired, these ranges appear to maximize the sum of the joint posterior likelihoods. They are also considered sufficiently broad to produce a Bayesian distribution which is probably conservative. Table 4.3 presents the characteristics of the marginal density functions.

The concern over the selection of parameter ranges is motivated by the desire to obtain a suitable distribution on the major random variable. Figure 4.9 shows this distribution for parameter sets 1 and 2. As expected, the unilateral increase in the values of \( r \) slightly "sharpened" the resulting distribution. The distribution obtained with parameter set 3 is depicted in Figure 4.10, which shows that this distribution
is not markedly different from the other two, despite the total dissimilarity in parameter ranges. Table 4.4 provides a summary of the characteristics of the three distributions. It is not surprising that the standard deviation of the distribution obtained with parameter set 3 lies between those from parameter sets 1 and 2; the values of $r$ in parameter set 1 are insufficiently large, producing a more dispersed distribution, while those in parameter set 2 may be considered to be slightly over-extended. Predictably, the standard deviations of all three distributions are larger than that of the data because the uncertainty in parameter estimation is also incorporated in the distribution. The cumulative distribution function from parameter set 3 is shown in Figure 4.11.

4.3.2 **Single Shear (Simulated Diaphragm Action) Results**

The test results to be used in the determination of the distribution on ultimate strength are presented in Table 4.1, and a histogram of these results appears as Figure 4.12.

Use of the trial procedure for the establishment of parameter ranges results in the ranges and increments denoted as parameter set 1 in Table 4.5, which also provides the characteristics of the marginal posterior density functions on these parameters. Although these ranges are considered to be adequate, the less restrictive ranges denoted as parameter set 2 are also employed. Despite the large differences in the expected values of the parameters, the variation in the resulting density functions on strength is insignificant.
This fact is evident in Table 4.6, which gives the characteristics of these functions from the two parameter sets. The Bayesian distribution from parameter set 2 is plotted in Figure 4.13 and the cumulative distribution function is shown in Figure 4.14.

4.3.3 Pull-over Results

The test results to be used in the establishment of the distribution on pull-over strength are presented in Table 4.7, and Figure 4.15 depicts a histogram of these results.

Some difficulty is encountered in the determination of parameter ranges. The problem arises in the inability to suitably bracket the value of the upper limit of the distribution, parameter b. Regardless of the range chosen for this parameter, the extreme value of the range always has the greatest posterior likelihood. In addition, the total sum of the joint posterior likelihood function increases as the range is extended, although at a decreasing rate. The selection process is terminated on reaching the ranges given in Table 4.8, where it can be seen that the mode of the marginal density function on b still coincides with its end point. These ranges, however, are considered sufficiently broad to result in an acceptable Bayesian distribution.

The posterior probability density function on pull-over strength is shown in Figure 4.16, and its characteristics are presented in Table 4.8. Figure 4.17 is a plot of the cumulative distribution function.
4.3.4 Pull-out Results

Table 4.7 also provides the results of the pull-out test series. A histogram of these results appears as Figure 4.18.

Two parameter sets again evolve from the selection process. The ranges and increments of these sets, together with the characteristics of the marginal posterior density functions on the parameters, are given in Table 4.9. It may be noted that the range of $b$ is identical in both sets. The likelihood of the minimum value of $b$ is an order of magnitude greater than the likelihoods of the remainder of the range, irrespective of the other parameters ranges, and thus the selected range of $b$ always proves suitable. In this case, a point estimate for the parameter would be sufficient.

The differences in the parameter ranges again has slight effect on the resulting Bayesian distribution, as is evident from Table 4.10. A plot of the probability density function on pull-out strength obtained from the second parameter set is presented as Figure 4.19, and the corresponding cumulative distribution function is shown in Figure 4.20.

4.4 Maximum Likelihood Estimates of the Beta Parameters

An alternative to the weighted average approach taken in the determination of the Bayesian distribution is the selection of point estimates for the beta parameters, resulting in the description of the major random variable by a single beta distribution. Several methods may be employed to obtain point estimates. Two of the most recognized and most widely used techniques are the method of moments and the method of
maximum likelihood. The method of maximum likelihood is better suited to the present work.

Maximum likelihood estimators possess several desirable properties. Although often biased for small sample sizes, they are asymptotically unbiased, i.e. their means approach the true parameter values as the sample size approaches infinity. They have, at least asymptotically, the minimum expected squared error of all unbiased estimators. They are consistent in that the error in estimation can be made arbitrarily small as the number of samples approaches infinity. And they provide maximum utilization of the information contained in the data. (22)

4.4.1 The Method of Maximum Likelihood

The method assumes that all possible parameter sets have an equal probability of being the true set. The maximum likelihood estimate of the parameters is that parameter set which maximizes the likelihood function. Any technique for determining the maximum of a function may be used in this process.

The likelihood function for a random variable distributed in accordance with a beta probability law is given by Eq. 4.22. The mathematical form of this function makes the standard maximization technique of calculus, i.e. equating the partial derivatives of the function with respect to each of the parameters to zero and solving for the parameters, intractable. The iterative technique employed in Reference 25 is thus used in combination with this method to seek the maximum.

It is often useful in seeking maximum likelihood estimators to deal with the logarithm of a function rather than the
function itself. Maximization of the logarithm also maximizes the function. The logarithm of Eq. 4.22 is

\[ \ln(L) = \ln[L(\theta|D)] = n \ln \rho - n(t-1) \ln(b-a) + \sum_{j=1}^{n} [(r-1) \ln(x_j-a) + (t-r-1) \ln(b-x_j)] \quad (4.31) \]

where \( \rho \) is a function of \( r \) and \( t \) only. Taking the partial derivatives of this equation with respect to \( a \) and \( b \) and equating them to zero,

\[ \frac{\partial \ln(L)}{\partial a} = n(t-1) \frac{1}{(b-a)} - \sum_{j=1}^{n} \frac{(r-1)}{(x_j-a)} = 0 \quad (4.32) \]

\[ \frac{\partial \ln(L)}{\partial b} = -n(t-1) \frac{1}{(b-a)} + \sum_{j=1}^{n} \frac{(t-r-1)}{(b-x_j)} = 0 \quad (4.33) \]

Letting

\[ v = (b-a) \sum_{j=1}^{n} \frac{1}{x_j-a} \]

\[ \mu = (b-a) \sum_{j=1}^{n} \frac{1}{b-x_j} \]

and denoting the ratio \( v/\mu \) by \( \delta \), Eqs. 4.32 and 4.33 may be solved for \( r \) and \( t \) to yield

\[ r = \frac{n\delta - v}{n(\delta + 1) - v} \quad (4.34) \]
and

\[ t = (r + 1) + \delta (r-1) \]

\[ = \frac{n(\delta + 1) - 2\nu}{n(\delta + 1) - \nu} \tag{4.35} \]

Hence given the values of \( a \) and \( b \) and the sample points, the maximum likelihood estimates of \( r \) and \( t \) can be readily calculated.

The procedure consists of selecting a sequence of values for \( a \) and \( b \) and, for each combination, calculating the values of \( r \), \( t \) and the likelihood function. The process is continued until the value of the likelihood function is a maximum, and the four parameter values producing the maximum are the maximum likelihood estimators for the beta distribution.

4.4.2 Computation of the Estimators

A computer program with an automated search technique is written to determine the maximum likelihood estimators. The search technique employs a variable, decreasing step size and hence permits any desired level of accuracy. Execution is typically terminated when \( a \) and \( b \) are determined to 0.001 or the value of the likelihood function is considered stationary, i.e. the normalized difference between iterations is less than \( 10^{-6} \). The data is that used for the Bayesian distribution, with both the data and the results normalized with respect to the mean.

Once the estimators are determined, the distribution on the major random variable is completely defined by Eq. 4.10.
Expressions for the mean and variance are given in closed form by Eqs. 4.11 and 4.12, respectively. Although the cumulative distribution function cannot be evaluated in closed form, it can be determined by the numerical integration of Eq. 4.10.

A more accurate method for the calculation of the cumulative distribution function is to express the density function as a power series and employ term by term integration. The basic form of the beta probability law is given by Eq. 4.5. Letting \( q = r-1 \) and \( p = t-r-1 \), the cumulative distribution function is defined as

\[
F(x) = \int_0^x f(x) \, dx = C_N \int_0^x x^q (1-x)^p \, dx \quad 0 \leq x \leq 1 \quad (4.36)
\]

where \( C_N \) is the normalizing constant for the probability density function. Expanding \( (1-x)^p \) by the binomial theorem, valid for \( |x| < 1 \) and all real values of \( p \), and applying term by term integration leads to the following expression for the indefinite integral:

\[
\int_0^x x^q (1-x)^p \, dx = \frac{x^{q+1}}{q+1} + \sum_{n=1}^{\infty} \frac{(-1)^n p(p-1)(p-2)\cdots(p-n+1)}{n! (q+n+1)} x^{q+n+1} \\
\quad 0 \leq x \leq 1 \quad (4.37)
\]

A general \( \text{BT}(a,b,r,t) \) variable \( y \) is related to a basic \( \text{BT}(0,1,r,t) \) variable \( x \) by the simple linear relationship

\[
y = a + (b-a) x \quad (4.38)
\]
Hence the cumulative distribution function for a BT\((a,b,r,t)\) variable can be evaluated to any desired accuracy by using Eqs. 4.37 and 4.36 with

\[
x = \frac{(y - a)}{(b - a)}
\]  

(4.39)

and

\[
C_N = \frac{1}{B}
\]  

(4.40)

where \(B\) is defined by Eq. 4.6. A computer program is written for this purpose. To hasten the convergence, the value of \(x\) is restricted to values less than or equal to the mean, using the mirror image of the density function for \(x\) values greater than the mean.

4.4.2.1 Single Shear (Lap Joint) Results

The lap joint test results produce rapid convergence to the maximum likelihood estimates of the beta parameters given in Table 4.11. The resulting probability density function is shown in Figure 4.21, and its characteristics are presented in Table 4.11. The cumulative distribution function for this distribution is plotted in Figure 4.22.

4.4.2.2 Single Shear (Simulated Diaphragm Action) Results

The results of the simulated diaphragm action tests yield the maximum likelihood estimators given in Table 4.12. In this instance, execution is terminated after very few cycles because the value of the likelihood function appears to be nearly
stationary. The resulting distribution is plotted in Figure 4.23, and its characteristics are given in Table 4.12. The cumulative distribution function is shown in Figure 4.24.

4.4.2.3 Pull-over Results

The difficulty experienced in determining the Bayesian distribution on pull-over strength is also encountered in the maximum likelihood approach. The problem, it will be recalled, arises in the inability to properly define the value of the upper limit of the distribution. Employment of the search technique continues to extend the value of parameter \( b \) at the maximum step size, with a corresponding gradual reduction in the value of parameter \( a \). Although convergence to maximum likelihood estimates is never attained, even after a very large number of cycles, it is found that the variations in the resulting beta distributions are negligible for values of \( b \) greater than about 2.5.

The maximum likelihood estimators finally used for this distribution are given in Table 4.13. These values are used largely to avoid the computational conditioning problems that arise when the range of the distribution is large with corresponding large values of \( t, r \) or their difference. The resulting beta distribution is plotted in Figure 4.25, and the cumulative distribution is plotted in Figure 4.26. The characteristics of the distribution are given in Table 4.13.

4.4.2.4 Pull-out Results

The test results produce convergence to the maximum
likelihood estimators presented in Table 4.14 after very few cycles. It may be noted that the maximum likelihood estimates of \( r \) and \( t \) are very close to the parameter values of a triangular distribution, \( r = 2 \) and \( t = 3 \). This fact is further evidenced in the plot of the probability density function, shown in Figure 4.27. The corresponding cumulative distribution function is presented as Figure 4.28, and Table 4.14 provides the characteristics.

4.4.3 **Comparison with Bayesian Distributions**

It is useful to compare the distributions on ultimate strength resulting from the posterior likelihood function with those obtained by maximum likelihood estimators. The method of maximum likelihood is much more direct, and requires a minimal computational effort compared to the posterior likelihood approach. Thus the two methods should be examined to ascertain whether the maximum likelihood technique can provide acceptable approximations to the method of posterior likelihoods.

It would make little sense to compare the expected values of the parameters obtained from the joint posterior likelihood function with their maximum likelihood estimates, for it was seen that widely different parameter ranges can produce nearly identical Bayesian distributions. Hence a comparison is made of the resulting distributions themselves.

Table 4.15 presents the characteristics of the distributions on single shear strengths, and Table 4.16 provides the same information on pull-over and pull-out strengths. An examination of these characteristics reveals reasonably good
agreement between the distributions resulting from the two methods. The dispersion, as measured by the standard deviation, is typically smaller for the distributions obtained from point estimates than for the Bayesian distributions, as expected. One important result is the relatively good agreement in the modes of the distributions, for reasonable estimates of the other characteristics can be obtained directly from the data, whereas an estimate of the mode is very difficult to make from the data.

Figures 4.29 - 4.32 show plots of the Bayesian and maximum likelihood distributions on single shear, pull-over and pull-out strengths, respectively. It can be seen that although the distribution shapes are generally similar, the distributions resulting from maximum likelihood estimators typically ignore the regions of the tails and concentrate more of the area about the mean.

4.5 Distributions of the Sample Mean

The determination of the probability distribution on strength is important because it permits the determination of the probability of failure. The ability to use the distribution, however, depends on the accurate determination of the mean. Economic or time constraints may preclude the testing of a sufficient number of specimens to accurately define the true mean. Thus a need exists for the ability to determine the uncertainty associated with a mean value obtained from a limited number of tests. This requirement is fulfilled by the distribution of the sample mean, given a particular sample
size. A separate distribution is required for each size sample of each test procedure.

4.5.1 Calculation of the Distributions

The central limit theorem states that for reasonably large sample sizes the distribution of the sample mean will be approximately normal, regardless of the form of the underlying distribution. Its mean will be the mean of the underlying distribution, and its variance will be the variance of the underlying distribution divided by the sample size. The sample sizes of concern, however, are those not large enough to invoke the central limit theorem. The distributions of the sample mean are therefore determined by simulation.

One thousand sample points are randomly selected from each of the four Bayesian density functions on connection strength. This process is performed with a computer, using an internal random number generator. The sample points are then grouped in lots of size two through ten, and the mean value of each lot is calculated. These mean values serve as the sample points in the calculation of the distributions of the sample mean, which are based on a beta probability law.

Given the values of the lower and upper limits, $a$ and $b$, of a general beta distribution, the values of the shape parameters $r$ and $t$ can be determined by either the method of maximum likelihood or the method of moments. The latter method is used in this instance. Eqs. 4.11 and 4.12, defining the mean and variance of a general beta distribution, may be solved for $r$ and $t$ to yield
where \( m \) and \( \sigma^2 \) are the mean and variance of the beta distribution. By the method of moments, these characteristics are approximated by the mean and variance of the sample, that is

\[
m = \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i
\]  
(4.43)

and

\[
\sigma^2 = s^2 = \frac{1}{n-1} \left( \sum_{i=1}^{n} x_i^2 - n \bar{x}^2 \right)
\]  
(4.44)

It may be noted that the method of moments estimates of \( r \) and \( t \) differ from the maximum likelihood estimates.

The procedure thus consists of using Eqs. 4.43 and 4.44 with the sample points taken as the mean values of each lot, and then calculating the shape parameters with Eqs. 4.41 and 4.42. Use of the limiting values of the parameters \( a \) and \( b \) of the Bayesian distributions as the lower and upper limits of the beta distributions on the sample means produce extremely large values of \( r \) and \( t \) (on the order of \( 10^3 \)), even for small sample sizes. These large parameter values produce conditioning problems, and consequently the values of \( a \) and \( b \) for the distributions on the sample means are taken as the \( 10^{-6} \) points.
of the Bayesian density functions.

4.5.2 Presentation of Results

Table 4.17 presents the values of the beta parameters of the distributions on the sample mean for samples of size two through ten for single shear lap joint strength. It also provides the standard deviation of the distributions. Tables 4.18 - 4.20 give similar information for single shear simulated diaphragm action, pull-over and pull-out strength, respectively. As expected, the distribution of the sample mean is "sharpened" about the mean of the underlying distribution as the sample size increases, and this fact is reflected in the increasing values of r and t and the decreasing values of the standard deviation. The mean is identical for each set of distributions because the same one thousand sample points are used to form the samples of varying size.

The maximum likelihood estimates of r and t are also determined in the performance of the calculations. These estimates are always very close to, although typically somewhat larger than, the estimates obtained by the method of moments. The implication is that the method of maximum likelihood usually produces a somewhat "sharper" probability density function than the method of moments when the end points of the distribution are specified. The variation, however, is typically negligible.

The "sharpening" of the distribution of the sample mean with increasing sample size is illustrated in Figure 4.33, which shows the distribution of pull-over strength for samples
of size one, two and five. The approximation to a normal distribution is evident. A more interesting example because of a nearly triangular underlying distribution is provided by Figure 4.34, which shows similar distributions for pull-out strength. The validity of the central limit theorem is apparent even for samples of size two.

4.6 Summary and Conclusions

Knowledge of the basic form of the underlying probability law governing a random process is extremely useful in establishing a rational method for the evaluation of observations of the process because it permits the extrapolation of information contained in a relatively small sample. Many mathematical models have been developed for probability laws. Among the most common are the normal, the distribution of processes whose components are additive, and the lognormal, a model of events caused by the product of random variables.

At least three types of uncertainty are encountered in the determination of a probability model. These are the inherent uncertainty of the process itself, the uncertainty associated with model selection, and the uncertainty related to the accurate determination of the model's parameters. In the establishment of the probability distributions on connection strength, the model uncertainty is minimized with the use of a generalized beta distribution. Its great flexibility in shape combined with finite upper and lower limits enables it to describe most empirical data without the constraints, e.g. symmetry, imposed by some models. The uncertainty connected
with parameter determination is handled by treating the parameters as random variables and using the method of inverse inference to obtain their joint density function. This distribution is combined with the model distribution to obtain a compound distribution on connection strength which combines both process and parameter uncertainty simultaneously.

The joint posterior density function on the parameters of the beta probability law is obtained by postulating possible beta laws for connection strength and using Bayes' theorem to ascertain the probability that a particular law is responsible for the data. Lack of sufficient information to assign probabilities to particular parameter combinations suggests the use of a diffuse prior on the parameters. Use of a diffuse prior effectively reduces the posterior distribution to a normalized sample likelihood function. With the advent of additional data, however, the posterior distributions on the parameters determined herein can be used as prior distributions to update the distributions on strength.

The resulting Bayesian distribution is apparently relatively insensitive to the parameter ranges used in determining the joint posterior density function on the parameters. Use of widely dissimilar parameter ranges produces slight variations in the ensuing compound distribution. Thus the only requirement for the selection of parameter ranges is that they be reasonably broad. Normalization of the data with respect to its mean value is almost a necessity to avoid computational difficulties with use of the beta law. Such normalization
permits the determination of the parameter $t$ from the constraint that the mean equal unity, with little loss of generality and considerable savings in computational effort.

Despite histograms that imply skewed distributions, the resulting probability density functions on single shear and pull-over strength are very nearly symmetrical. This result is not surprising due to the somewhat arbitrary nature of histograms. The single shear distributions can probably be approximated with normal or lognormal probability laws. The distribution on pull-over strength, with its mildly positive skew, might be approximated with a lognormal or shifted lognormal law. Such an approximation is supported by the inability to define the upper limit of the distribution with either the joint posterior likelihood or maximum likelihood approach. Both methods imply that the upper limit is infinite.

The existence of entirely different failure mechanisms in pull-over and pull-out is evident in a comparison of their probability density functions. The nearly triangular pull-out distribution indicates a definite upper bound on strength and its sensitivity to tightening torque. Torques other than the optimum result in a reduction in strength.

Maximum likelihood estimates of the beta parameters provide good estimates of the characteristics of the underlying distribution and give a general indication of its shape. A computer program for the automated determination of these parameters might thus be employed in a preliminary investigation to select the form of the model to be used. Such an
investigation, for example, would indicate that a lognormal distribution might be used to describe pull-over strength, while a triangular would be more appropriate for pull-out strength.

Maximum likelihood estimators with limited sample sizes generally ignore the regions of the tails and consequently should not be used for decisions sensitive to those regions. Use of the mode or mean of the posterior density functions on the parameters may provide better point estimates for the underlying distribution than maximum likelihood estimators. Use of reasonably broad ranges on the parameters would result in point estimates of $a$ and $b$ that produce a relatively wide range for the distribution, and hence better represent the region of the tails. This fact is illustrated by the maximum likelihood estimators for the distribution on pull-over strength, where a large value of the upper limit coupled with a small value of the lower limit produce a distribution that is similar in all respects to the one obtained by the method of inverse inference. Similarly, a point estimate of the upper limit could be used in the determination of the Bayesian distribution on pull-out strength.

Knowledge of the distribution of the sample mean is helpful in the determination of a rational sampling method because it permits the determination of the error incurred in estimating the mean strength from a limited number of samples. Although the distributions are obtained by simulation, it appears that application of the central limit theorem would be appropriate,
even for very small sample sizes. The validity of the theorem is evident even when the underlying distribution is triangular and samples of size two are drawn.
CHAPTER 5
FIRST ORDER PROBABILISTIC DESIGN CRITERIA

Structural design practice has traditionally treated the uncertainties associated with design through "safety factors" and more recently through "load factors" and "strength reduction factors." These factors, which evolved primarily from professional judgement and past performance, generally provided adequate margins of safety against failure. The neglect of the variation of uncertainty with different situations, however, produced designs with inconsistent safety levels.

A test evaluation method should provide an interpretation of test results that has the goal of consistent reliability. The inclusion of this feature was the motivation for the determination of probability distributions on ultimate strength and the sample mean in the previous chapter. Recent developments have resulted in first order probabilistic design criteria that offer a convenient framework for such an evaluation method.

The first steps toward a design procedure which explicitly reflects the uncertainties associated with the design variables were taken in this country with the 1963 ACI Code and subsequently more rigorously in 1967 with C. A. Cornell's proposal for a reliability-based code.\(^{(28)}\) His proposal was based on the concepts of load and resistance, and used only the mean, variance and related approximations to account for uncertainty. The method was independent of any assumed distributions and resulted in a code format that was very similar to the then existing ACI Code. The measure of safety could be based on
either the difference of resistance and loads or the natural logarithm of their ratio. Use of the logarithmic basis was subsequently promoted by Rosenblueth and Esteva.

The question of invariant safety measures was raised by Ditlevsen. The problem arises in the definition of load and resistance, where different definitions of resistance, all consistent with the laws of mechanics, can lead to different safety measures. Ditlevsen suggested a partial coefficient code, in which the resistance and load variables are reduced to their basic parameters and a separate coefficient is applied to each parameter.

Another method for calculating an invariant safety measure, which is independent of the shape of the "safe region" in the design space and hence does not require the identification of those variables which increase safety and those which decrease it, has been proposed by Hasofer and Lind.

5.1 Central Safety Factor Design Criteria

The central safety factor design criteria evolves directly from the code proposal by Cornell. The resistance is related to the load effects through a "safety index" $\beta$, using only the mean values and coefficients of variation of the relevant variables.

5.1.1 Basis for the Criteria

First order probabilistic design criteria employ first order approximations of the mean and variance. These are

$$m_f(x) \approx f(m_x)$$ (5.1)
and result from the first two terms of a Taylor series expansion of $f(x)$ about $m_x$. Such approximations are justified if the non-linearity of the function $f(x)$ in the region of the mean $m_x$ is not great and the coefficient of variation is not large. The advantage is that the moments of the dependent variables are always given in terms of functions of the moments of the independent variables.

Uncertainty is expressed solely through coefficients of variation. The coefficient of variation is believed to be less dependent on the mean than is the variance, and hence can be assumed constant for many factors. Furthermore, an approximation for the coefficient of variation of the product of random variables exists in simple form, and many engineering relationships involve product forms.

The resistance $R$ is assumed to be generally expressed as

$$ R = c M F E $$

(5.3)

where $c$ is a constant, $M$ is a material strength random variable, $F$ is a fabrication or workmanship factor and $E$ is a factor accounting for error in the prediction equation or mathematical model. Assuming $M$, $F$ and $E$ to be uncorrelated, the variance of $R$ is given by

$$ \sigma_f^2 = \left[ \left. \frac{d f(x)}{dx} \right|_{m_x} \right]^2 \sigma_x^2 $$

(5.2)
\[
\sigma_R^2 = c^2 \left( \frac{m_F^2}{m_M^2} \sigma_M^2 + \frac{m_E^2}{m_M^2} \sigma_M^2 \right) + m_F^2 \sigma_F^2 + m_E^2 \sigma_E^2 + m_M^2 \sigma_M^2 \sigma_F^2 + m_M^2 \sigma_M^2 \sigma_E^2 + m_E^2 \sigma_M^2 \sigma_E^2
\]

and the mean by
\[
m_R = c \cdot m_M \cdot m_F \cdot m_E
\]
The squared coefficient of variation is thus
\[
V_R^2 = V_M^2 + V_F^2 + V_E^2 + V_M^2 \cdot V_F^2 + V_M^2 \cdot V_E^2 + V_F^2 \cdot V_E^2 + V_M \cdot V_F \cdot V_E
\]
Since \(V_M, V_F\) and \(V_E\) are generally less than 0.2, the product terms can be neglected and
\[
V_R \approx \sqrt{V_M^2 + V_F^2 + V_E^2}
\]
The load effect \(Q\) has mean \(m_Q\) and coefficient of variation \(V_Q\), which reflects uncertainties produced by load idealizations, lack of sufficient load histories, etc., as well as measured statistical variation. The probability of failure for any member is the probability that the load will exceed the resistance, or
\[
P_F = P[R<Q] = P[R/Q<1] = P[\ln(R/Q)<0]
\]
Introducing the standardized variate \(U\),
\[ U = \frac{\ln \left( \frac{R}{Q} \right) - m \ln \left( \frac{R}{Q} \right)}{\sigma \ln \left( \frac{R}{Q} \right)} \]  

(5.8)

yields

\[ P_F = P \left[ U < \frac{-m \ln \left( \frac{R}{Q} \right)}{\sigma \ln \left( \frac{R}{Q} \right)} \right] = F_U \left[ \frac{-m \ln \left( \frac{R}{Q} \right)}{\sigma \ln \left( \frac{R}{Q} \right)} \right] \]  

(5.9)

in which \( F_U \) is the cumulative distribution function of the standardized variate.

The argument of this function defines the safety of the element and is referred to as the "safety index," denoted by \( \beta \). It explicitly defines the probability of failure if distribution assumptions are made, and is a relative measure of safety in the absence of such assumptions. The expression for \( \beta \) can be approximated as follows:

\[ m \ln \left( \frac{R}{Q} \right) \approx \ln \left( \frac{m_R}{m_Q} \right) = \ln \left( \frac{m_R}{m_Q} \right) \]  

(5.10)

and

\[ \sigma^2 \approx \left[ \frac{\partial \ln \left( \frac{R}{Q} \right)}{\partial R} \right]^2 \frac{\sigma^2}{m_R} + \left[ \frac{\partial \ln \left( \frac{R}{Q} \right)}{\partial Q} \right]^2 \frac{\sigma^2}{m_Q} = \frac{\sigma^2}{m_R} + \frac{\sigma^2}{m_Q} = V_R^2 + V_Q^2 \]  

(5.11)

Thus

\[ \beta = \frac{m \ln \left( \frac{R}{Q} \right)}{\sigma \ln \left( \frac{R}{Q} \right)} \approx \frac{\ln \left( \frac{m_R}{m_Q} \right)}{\sqrt{V_R^2 + V_Q^2}} \]  

(5.12)

Defining the central safety factor \( \theta \) as

\[ \theta = \exp \left( \beta \sqrt{V_R^2 + V_Q^2} \right) \]  

(5.13)
leads to the first order probabilistic design criterion

\[ m_R \geq \theta m_Q \] (5.14)

5.1.2 Advantages and Disadvantages of First Order Criteria

The first order probabilistic code format offers an improvement over deterministic codes while maintaining simplicity in form. It can be calibrated to present codes, and thus is very useful in the transition from deterministic to fully probabilistic design. It reflects the variability of factors affecting safety and permits the separation of factors so that individual factors can be chosen for the particular conditions of each design situation. Hence it permits different levels of safety for different types and locations of structures or assemblies. Conversely, it permits consistent reliabilities for different designs. It allows not only for statistical variation, but also for such factors as ignorance, model inaccuracies and neglected influences. Finally, it provides the possibility of rational updating with the advent of new information.

The method's independence of distribution assumptions is both beneficial and detrimental. It reflects the essential statistical information that is available and avoids the problem of extrapolating a distribution curve fitted to given data outside the range of the data into a region characterized by such rare occurrences that sufficient data may never be available for proper definition. It fails to reflect, however, the possible "shape effect" of a distribution.
Some of the approximations involved are quite crude, notably the first order approximation of the mean of the logarithmic function. The method is also not invariant with respect to changes in the definition of resistance that are consistent with the laws of mechanics.

5.2 Load and Resistance Factor Design Criteria

Load and resistance factor design\(^{(33)}\) is an extension and formalization of the first order probabilistic method described in the previous section. It consists of load factors and strength reduction factors applied to load effects and strengths, but differs from deterministic design in that the factors reflect the inherent uncertainties of the design variables. Although variations in form and interpretation are possible, the criteria proposed in Reference 33 is presented here.

5.2.1 Development of the Criteria

The design criterion is defined by

\[
\varnothing R_n > \gamma_A (\gamma_D c_D m_D + \gamma_L c_L m_L + \cdots ) \quad (5.15)
\]

where \(\varnothing\) is the "resistance factor", reflecting the uncertainties of the resistance; \(R_n\) is the "nominal resistance"; \(\gamma_A\) is a factor accounting for the uncertainties of structural analysis; \(\gamma_D, \gamma_L\) are the dead and live load factors; \(m_D, m_L\) are the mean dead and live load intensities; and \(c_D, c_L\) are deterministic influence coefficients translating load intensities into load effects.
In accordance with the basic first order method, the member resistance $R$ is expressed by

$$ R = R_n M F E $$

(5.16)

where $M$, $F$ and $E$ are the same variables as in Eq. 5.3. The coefficient of variation of the resistance $V_R$ is provided by Eq. 5.7.

The load effect $Q$ for combined dead and live load, $D$ and $L$, is assumed to be of the form

$$ Q = A (c_D X D + c_L Y L) $$

(5.17)

where $X$ and $Y$ are random variables reflecting the uncertainties in the transformation of loads into load effects and $A$ is a random variable representing the uncertainties in analysis. Assuming the transformations and analysis to be unbiased, the mean load effect is

$$ m_Q = c_D m_D + c_L m_L = c (m_D + m_L) $$

(5.18)

and the coefficient of variation is

$$ V_Q = \sqrt{\frac{V_A^2 + m_D^2 (V_X^2 + V_D^2) + m_L^2 (V_Y^2 + V_L^2)}{(m_D + m_L)^2}} $$

(5.19)

The central safety factor $\theta$, defined by Eq. 5.13, combines the uncertainties of the resistance and the load effects. Lind$^{(34)}$ suggested a linear approximation to the square root term to permit the independent determination of the resistance
and load factors. The approximation

$$\sqrt{V_R^2 + V_Q^2} \approx \alpha (V_R + V_Q)$$  \hspace{1cm} (5.20)

allows the first order probabilistic design criterion (Eq. 5.14) to be written as

$$\exp(-\alpha \beta V_R) m_R \geq \exp(\alpha \beta V_Q) m_Q$$  \hspace{1cm} (5.21)

The idea of a linear approximation can be further developed, allowing an expansion of the load term into dead and live load effects. The design criterion then becomes

$$\exp(-\alpha \beta V_R) m_R \geq \exp(\alpha \beta V_A) \left[ (1 + \alpha \beta \sqrt{V_X^2 + V_D^2}) c_D m_D ight. \\
+ \left. (1 + \alpha \beta \sqrt{V_Y^2 + V_L^2}) c_L m_L \right]$$  \hspace{1cm} (5.22)

where $\alpha = 0.55$ was found to produce a reasonably small error in the central safety factor $\theta$ over the entire range of likely variation of all the parameters. (33) This equation permits the direct determination of the resistance factor $\theta$ and the load factors $\gamma$ as a function of the pertinent statistical parameters of each component.

This discussion applies only to the ultimate limit state for a dead and live load combination. Formulations based on serviceability criteria are possible, as are extensions to combinations of other load effects, e.g. dead plus live plus wind loads.
5.2.2 The Problem of Invariant Safety Measures

The question of the invariance of the previously defined safety index $\beta$ (Eq. 5.12) arises in situations involving ambiguity in the definition of resistance and the corresponding load effect. Different definitions may lead to different designs for the same value of $\beta$. Ditlevsen (31) has shown that no second moment reliability measure based on a comparison of resistance $R$ and load effect $Q$ is both addition invariant and multiplication invariant. Quotient forms are not addition invariant and difference forms are not multiplication invariant. Ditlevsen, and Hasofer and Lind (32) have proposed criteria for an invariant safety index. Both proposals use all of the basic random variables involved in the design in the determination of the safety index.

5.3 Basis for a Partial Coefficient Code

Ditlevsen has proposed the following second moment reliability model for partial safety factor codes. Danish codes presently employ a partial safety factor format and the Nordic Committee for Building Regulations (35) has proposed codes based on this concept.

Let $M_1, \ldots, M_p$ be $p$ random material strengths; $L_1, \ldots, L_q$ be $q$ random load effects (or load intensities); and $X_1, \ldots, X_r$ be $r$ other relevant variables (e.g. geometric variables, some special loads). Assume that the failure criteria is defined by some continuous function $G$ of the variables such that failure occurs if
\[ G(M_1, \ldots, M_p, L_1, \ldots, L_q, X_1, \ldots, X_r) \leq 0 \quad (5.23) \]

For a large range of problems the function \( G \) may be chosen to be non-decreasing in \( M \) and non-increasing in \( L \). Now define coefficients \( \theta_1, \ldots, \theta_{p+q} \) such that

\[ G(M_1, \ldots, M_p, \theta_{p+1} L_1, \ldots, \theta_{p+q} L_q, X_1, \ldots, X_r) = 0 \quad (5.24) \]

that is, a random relation \( C_\theta \) between the \( \theta \) values. The point \((1, \ldots, 1)\) is of special interest since the position of the random surface \( C_\theta \) relative to this point determines whether or not failure occurs. If a continuous curve, monotonically increasing in all coordinates, connecting the origin and \((1, \ldots, 1)\) crosses \( C_\theta \) before \((1, \ldots, 1)\) there is failure. If the crossing is beyond \((1, \ldots, 1)\) there is no failure.

It is desirable that each \( \theta \) coefficient be an increasing function of the coefficient of variation of the variable to which it applies. It is therefore convenient to define the curve connecting the origin and \((1, \ldots, 1)\) with the parametric representation

\[ \begin{align*}
\exp (t V_{M_i}) , & \quad i = 1, \ldots, p \\
\theta_i = \exp (t V_{L_{i-p}}) , & \quad i = p+1, \ldots, p+q
\end{align*} \quad (5.25) \]

The value of the parameter \( t \) for which the curve crosses \( C_\theta \) is a random variable \( \nu \). Failure occurs if \( \nu \leq 0 \).
Denote the mean and standard deviation of \( v \) by \( m_v \) and \( \sigma_v \). Then larger values of \( m_v \), measured in terms of \( \sigma_v \), imply greater structural reliability. Hence a second moment reliability index can be defined as

\[
\beta = \frac{m_v}{\sigma_v} = \frac{1}{V_v} \tag{5.26}
\]

The design criterion is then

\[
G \left( \frac{m_{M_1}}{m_{\theta_1}}, \ldots, \frac{m_{\theta_{p+1}}}{m_{M_1}}, \ldots, \frac{m_{X_1}}{m_{X_1}}, \ldots \right) = 0 \tag{5.27}
\]

where the partial coefficients are given by

\[
m_{\theta_i} = \exp \left( m_v V_{M_i} \right), \quad i = 1, \ldots, p
\]

\[
m_{\theta_i} = \exp \left( m_v V_{L_{i-p}} \right), \quad i = p+1, \ldots, p+q \tag{5.28}
\]

In order to determine \( m_v \), and hence calculate the partial coefficients corresponding to a given \( \beta \), the standard deviation is determined within first order approximation from Eqs. 5.24 and 5.25 after linearizing and solving with respect to \( v \). This calculation involves solution of a nonlinear algebraic equation.

Denoting the factored strengths and loads as the design state, it can be shown that if some arbitrarily selected total safety factor associated with any particular resistance definition is greater than unity, then any and all safety factors following from changes in the definition are also greater than unity. The amount of exceedance has no physical significance since it depends on the choice of definition of resistance. It
merely indicates that the reliability is larger than the required reliability. Stabilizing loads are to be first considered as resistances and divided by coefficients and then as loads and multiplied by coefficients. The smaller of the resulting β's is then defined as the reliability index of the structure.

5.4 Summary and Conclusions

Despite the shortcomings produced by various approximations, the first order probabilistic code format incorporated in the load and resistance factor design criteria is an important step towards fully probabilistic design. Its simplicity and compatibility with existing codes are favorable characteristics, and it permits the independent treatment of resistance and load effects. It also allows the specification writer or designer flexibility in reflecting the uncertainties affecting the safety of a particular design, e.g. location or quality control. It can be extended to various load combinations and can be formulated for serviceability criteria.

The calculation of the coefficients in Ditlevsen's partial coefficient format is not trivial. He has suggested a comprehensive study of the population of structures to which the criteria are to be applied to produce a number of different sets of fixed partial safety factors. Such sets, however, cannot encompass all design situations. The inability to treat resistance and load effects independently and the difficulty of reflecting the particular uncertainties affecting a specific design are considered to be serious restrictions of the method.

The invariance problem could be handled in the load and
resistance factor format by either specifying the definition of resistance in instances where the definition is ambiguous or providing different sets of factors corresponding to different definitions. The Hasofer-Lind criterion for a safety index could be employed for calibration purposes in cases where explicit definition of load and resistance is difficult, e.g. short columns.

One drawback of first order probabilistic formats is the failure to account for possible effects of distribution shapes. Other questions to be faced include system or subsystem reliability versus member reliability, the consequences of different types of failure and the influence of time on loads and resistances. Possible extensions might involve improvements in the determination of member reliability and the use of expected costs as the basis for decisions.
CHAPTER 6
RESISTANCE FACTORS AND TEST EVALUATION METHOD

The proposed test procedures presented as Appendix A were designed to measure the strength or resistance of a connection subjected to forces producing a single shear, pull-over or pull-out failure. The load and resistance factor design (LRFD) format, as represented by Eq. 5.15, separates resistance and load effects and allows their independent treatment. It is therefore a convenient framework for a test evaluation method. The purpose of this chapter is to determine the resistance factors $\phi$ which will be used in conjunction with the strength values obtained with the proposed procedures to form the basis of a corresponding evaluation method.

The underlying philosophy to be followed is that the connection should be stronger than the parts it connects. A second criterion of fundamental importance in connection design is ductility. This criterion is typically met by proper design and the nature of cold-formed steel, which usually possesses sufficient ductility for structural applications.

6.1 Resistance Factors

A major advantage of LRFD over deterministic design is the ability of the former to account for varying degrees of uncertainty. Hence the resistance factors should reflect uncertainties in local conditions, e.g. workmanship and inspection. They should also account for the uncertainty in estimating the true mean strength from a limited number of tests. LRFD permits different levels of safety for different types of
structures or assemblies, and the resistance factors should distinguish relatively unimportant from important structures.

Finally, there is the question of consequences of failure, or in broader terms system reliability versus member reliability. For example, failure of a single connection in an assembly subject to single shear does not produce the same consequences as failure of a connection subject to pull-over. In the former case, provided the material is sufficiently ductile, some additional load is distributed among adjacent connections. In the latter case the result is immediate failure of the assembly due to an "unbuttoning" effect.

6.1.1 Coefficient of Variation

A comparison of Eqs. 5.15 and 5.22 reveals that

\[ \phi R_n = m_R \exp(-\alpha \beta V_R) \]  

(6.1)

The coefficient of variation of the resistance \( V_R \) is given by Eq. 5.7 and depends on the coefficients of variation of the material properties (M), the fabrication (F) and the theory or prediction equation (E).

Prediction equations do not exist and the proposed test procedures specify that the following features of the test be identical, or as nearly similar as possible, to those of the actual application: materials; fastener and fastener head assembly or fastener accessories; fastener hole diameter, tolerance and location with respect to any corrugations; tightening torque; fastener hole creation and fastener driving and/or tightening techniques and equipment; and the presence of filler
material such as insulation. Consequently the coefficients of variation resulting from the Bayesian distributions on ultimate strength determined in Chapter 4 should include the influences of M, F and E. These coefficients of variation shall be denoted as $V_{R'}$.

It is recognized that conditions in the field are not controlled to the degree that they are in the laboratory. For this reason another fabrication term, representing the effects of the additional variation over laboratory conditions produced by field conditions, should be included. Furthermore this factor should reflect the amount of discrepancy between the two circumstances. In a broader sense, this factor may be considered to reflect the degree of simulation of actual conditions by the proposed laboratory tests. This factor shall be denoted by $V_{F'}$, and it is proposed that it assume the following values for the indicated conditions:

- 0.05 good workmanship and inspection, slight variation between laboratory and field conditions
- 0.10 average workmanship and inspection, moderate variation between laboratory and field conditions
- 0.15 below average workmanship and inspection, substantial variation between laboratory and field conditions

If the effects of field conditions are assumed to be normally distributed about a mean value that represents laboratory conditions, a coefficient of variation of 0.05 implies a 95 percent probability that field conditions are within $\pm$ 10 percent of laboratory conditions. This probability is reduced to 50 percent for a coefficient of variation of 0.15. Conversely,
there is a 95 percent probability that field conditions are within $\pm 10$, $\pm 20$ and $\pm 29$ percent of laboratory conditions for respective coefficients of variation of 0.05, 0.10 and 0.15.

An exception to the proposed values may be necessary for pull-out strength. Pull-out is more sensitive to tightening torque than are the other modes of failure. It has what may be termed a relatively well-defined optimum, and tightening torques below or above this optimum result in reduced strength. This fact is evident in the negative skew of its nearly triangular probability density function, determined in Chapter 4. To account for this additional sensitivity, or to reflect the shape of the underlying distribution, it proposed that for pull-out strength $V_F$, equal 0.075, 0.125 and 0.175 for good, average and below average workmanship and inspection.

Another source of uncertainty stems from estimating the true mean ultimate strength from a limited number of tests. If the nominal resistance $R_n$ is taken as the mean value of the test results, the variation of this mean about the true mean represents the error in estimation. This situation is completely analogous to the error in the prediction equation in LRFD. The coefficients of variation of the sample mean as a function of sample size were determined in Chapter 4 from the Bayesian distributions, and shall be denoted by $V_F$.

The coefficient of variation of the resistance, when the mean value is measured in the laboratory from a limited number of tests, is then given approximately by
6.1.2 Safety Index

The safety index is presently determined by calibrating the LRFD criteria to existing design standards. Such a calibration to the 1969 AISC Specification for beams and columns revealed that \( \beta = 3.0 \) provided a good estimate of the reliability inherent in current design, and this value was suggested\(^{33}\) as the basis for LRFD criteria for all other types of structural elements. The requirement that connections be more reliable than the connected members necessitates a safety index greater than 3.0 for such elements.

Private communications from T. V. Galambos indicate that \( \beta = 4.5 \) is an appropriate choice of safety index for connections at the present time. This value stems from calibration to the 1969 AISC Specification for the ultimate limit state of connections formed with fillet welds and high strength bolts. Consistent connection design implies that connections in cold-formed steel have a similar reliability, and it is proposed that \( \beta = 4.5 \) be adopted for such connections.

In the strict sense, the choice of safety index should reflect the consequences of failure, or system reliability versus member reliability. As previously mentioned, failure of a connection in single shear does not typically constitute collapse of the assembly, whereas failure in pull-over or pull-out generally does. Thus the assembly has greater reliability in
single shear than in pull-over or pull-out. The assembly can be made to have consistent reliability by increasing the safety index for element pull-over and pull-out failures. This action will not be taken for the following reasons: (1) Present LRFD criteria deal only with element reliability, with failure of the system defined as failure of any element. This definition will be adopted for connections. (2) Connection strengths are not uncorrelated because the same load effect applies to a number of connections or connectors. Hence the probability that another connection will fail, given that one has already failed, is quite high. In the single shear example it may be said that upon failure of a connection, failure of the entire assembly is imminent. (3) The proposed value of the safety index is sufficiently large that even when decreased to reflect connection system reliability it is substantially greater than the safety index of the structural members.

A major asset of LRFD is the capability to distinguish between different types and utilizations of structures. Present design procedures imply that temporary structures should have the same reliability as very important structures. Such restrictions produce results that are sometimes clearly inefficient and unnecessary. The versatility of cold-formed steel permits its use in a very wide range of applications. This fact is particularly true with the fastener types under consideration. Hence the choice of safety index should reflect the important of the structure.

The choice of safety index to reflect structural importance
is somewhat arbitrary. It has been suggested\(^{(33)}\) that for structural members the following values might be used: \(\beta = 2.5\) for temporary structures, \(\beta = 3.0\) for routine design and \(\beta = 4.5\) for vital structures. Direct scaling for connections results in the respective values of 3.75, 4.5 and 6.75. A safety index of 6.75 is considered to be much too large, even for vital structures. Assuming the ratio of resistance to load effects to be lognormally distributed, \(\beta = 6.75\) implies a probability of failure on the order of \(10^{-11}\). It is proposed that the following values be used for the safety index of connections: \(\beta = 3.5\) for temporary structures, \(\beta = 4.5\) for standard designs and \(\beta = 5.5\) for very important structures. Under the same distribution assumption, the corresponding probabilities of failure are \(2.3 \times 10^{-4}\), \(3.4 \times 10^{-6}\) and \(1.9 \times 10^{-8}\).

6.1.3 Modified Resistance Factors

The use of a safety index for connections that differs from the one used for members introduces some operational difficulties. The LRFD design criterion is defined by Eq. 5.15 as

\[
\phi R_n \geq \gamma_A (\gamma_D c_D m_D + \gamma_L c_L m_L)
\]

A comparison with Eq. 5.22 reveals that

\[
\gamma_A = \exp(\alpha \beta V_A)
\]

(6.3)

\[
\gamma_D = 1 + \alpha \beta \sqrt{V_X^2 + V_D^2}
\]

(6.4)
\[ \gamma_L = 1 + \alpha \beta \sqrt{V_Y^2 + V_L^2} \quad (6.5) \]

It can be seen that the load factors depend on the choice of safety index. Hence the load factors for connection design will differ from those for member design. It would be desirable to maintain only one set of load factors, and thus avoid unnecessary confusion and greater chances of error in the design calculations. This goal can be achieved by placing all of the penalty for the greater safety index on the resistance factor. The result is a modified resistance factor \( \overline{\gamma} \) given by

\[
\overline{\gamma} = \gamma_A \frac{[\gamma_D c_D m_D + \gamma_L c_L m_L]}{[\gamma_A (\gamma_D c_D m_D + \gamma_L c_L m_L)]} \quad (6.6)
\]

The design criterion for connections is then

\[
\overline{\gamma} R_n \geq \gamma_A (\gamma_D c_D m_D + \gamma_L c_L m_L + \ldots) \quad (6.7)
\]

where the load factors \( \gamma \) are determined from the safety index for members and \( \overline{\gamma} \) is determined from the safety index for connections.

The following values were used for the coefficients of variation in the calibration of LRFD for beams and columns: \( V_A = 0.05, V_X = 0.04, V_D = 0.04, V_Y = 0.2 \) and \( V_L = 0.13 \). With \( \alpha = 0.55 \), the load factors corresponding to various values of the safety index become
For beams, columns and other main members $\gamma_A = 1.1$ and $\gamma_D = 1.1$ may be used for all types of structures, while $\gamma_L$ may be taken as 1.3, 1.4 and 1.6 for temporary, standard and very important structures.

The modified resistance factor for connections in temporary structures is given by Eq. 6.6 as

$$\bar{\phi} = \frac{1.1 (1.1 c_D m_D + 1.3 c_L m_L)}{1.1 (1.1 c_D m_D + 1.5 c_L m_L)}$$

The reduction term varies from 0.92 to 0.88 as the ratio $c_L m_L/c_D m_D$ varies from one to ten, and is asymptotic to 0.867 as the ratio of live to dead load effects approaches infinity. For standard structures the corresponding variation is 0.93 to 0.88 with an asymptote of 0.875 and for very important structures it is 0.85 to 0.86 with an asymptote of 0.863. The variation in all cases is not large, and in cold-formed steel construction the live load is typically substantially greater than the dead load. This fact is particularly true for the fastener types and load effects under consideration. Thus it is proposed that for connections in cold-formed steel
6.1.4 Extensions to Situations Beyond the Scope of the Proposed Test Procedures

The proposed test procedures possess several limitations, as indicated in Chapter 3. They were designed to determine the static strength of a connection and thus are not applicable to repeated loads. The pull-over test does not adequately simulate the behavior of connections formed with members exhibiting torsional behavior under load because the amount of torsional stiffness, and hence the degree of prying action on the thin connected material by the fastener head assembly, depends on the member properties and the support conditions and these factors are difficult to simulate in a small scale test. The proposed procedures also do not specify the treatment of designs based on deflection constraints. The following suggestions are intended to assist in overcoming some of these limitations.

6.1.4.1 Deflection Considerations

Designs may be governed by deflection considerations rather than ultimate strength. Deflection considerations are particularly important in those instances where relatively thin members are connected with fasteners which exhibit rotation under single shear load effects. Such connections are quite flexible and may result in relatively large deflections at
low load levels. For this reason the proposed test procedures specify the acquisition of some form of load-deformation curve with single shear tests. Although it is possible to obtain an LRFD formulation based on serviceability criteria, such a formulation depends on expressions for deflection which are not available. Hence a simple criterion based on the load-deformation curve is proposed.

The design is initially based on the required resistance against the factored loads. The deflection $\delta$ corresponding to this design at service loads ($c_D m_D + c_L m_L$) can be determined from the load-deformation curve obtained from the tests. If $\delta$ is less than the allowable deflection $\delta_a$, the design is strength dependent and deflection limitations need not be further considered. If $\delta$ is greater than $\delta_a$, the stiffness should be increased in proportion to the ratio $\delta/\delta_a$. This action is equivalent to increasing the resistance by the ratio $\delta/\delta_a$ if the load-deformation curve is linear between $\delta_a$ and $\delta$.

6.1.4.2 Repeated Loads

The effect of repeated loads on connection strength proper belongs to the resistance factor rather than the load factors. Very little information is presently available on the effects of repeated loads on connections with the fastener types under consideration, and standard procedures for measuring such effects are not yet in existence. When such procedures do exist the effects can be reflected in $m_R$ and $V_R$ to obtain resistance factors for repeated loads. In the interim it is recommended that a general reduction term be applied to $\beta$, that is
\[ \theta_{RL} = \rho \bar{\theta} \]  

(6.8)

where \( \rho \) may depend on the load level and the number of cycles. \( \rho \) may be estimated by comparing prototype or other tests of connections subject to repetitive loading with corresponding static tests performed in accordance with the proposed procedures. The proposed pull-out/pull-over test fixture and possibly the single shear lap joint specimen also might be used for \( P - 0 \) cyclic loading.

6.1.4.3 **Pull-over Strength with Members Exhibiting Torsional Behavior**

The determination of pull-over strength in situations involving main members that exhibit torsional behavior under load may be treated in a manner similar to that for repeated loads. One or more full scale tests of the type described in Chapter 3 might be performed to determine pull-over strength. The ratio of the means of the full scale results to the results of the proposed small scale tests could then be used to estimate the reduction term \( \rho \).

6.1.5 **Calculation of Resistance Factors**

The tests to determine the distributions on ultimate strength were conducted in accordance with the proposed test procedures. If the test procedures are assumed to produce consistent results, the expected value of the mean \( R_n \) of any given number of tests is equal to the true mean \( m_R \). With \( R_n \) equal to \( m_R \), Eq. 6.1 reduces to
\[ \phi = \exp(-\alpha \beta V_R) \]  \hspace{1cm} (6.9)

The term \( \alpha \) in this expression is a constant equal to 0.55. The value of the safety index \( \beta \) is selected as 3.5, 4.5 and 5.5 for temporary, standard and vital structures, respectively.

The coefficient of variation of the resistance \( V_R \) is provided by Eq. 6.2 and depends on the coefficients of variation reflecting the inherent variability of the resistance as measured in the laboratory \( V_{R}' \), the influence of fabrication or workmanship variability \( V_{F}' \), and the error in estimating the mean resistance from a limited number of tests \( V_{E}' \).

The modified resistance factor \( \bar{\phi} \), which permits the use of the same load factors for both connections and main members was found in Section 6.1.3 to equal 0.88 \( \phi \) for temporary and standard structures and 0.85 \( \phi \) for very important structures.

6.1.5.1 Single Shear (Lap Joint) Test

The value of \( V_R \) is given in Table 4.4 as 0.045, and Table 4.17 provides the values of \( V_E \) as a function of the sample size. \( V_F \) is chosen as 0.05, 0.10 and 0.15 to represent good, average and below average workmanship and inspection. The resulting values of \( V_R \) are presented in Table 6.1. It can be seen that the variation is slight for samples of size two or more, reflecting the small amount of scatter in the test results and hence the dominance of the \( V_F \), term.

These characteristics are also evident in the corresponding values of \( \bar{\phi} \), given in Table 6.2, which exhibit only slight
variation for samples whose size is greater than about two. The practical resistance factors for such samples may effectively be taken as the following:

<table>
<thead>
<tr>
<th>Workmanship and Inspection</th>
<th>Good</th>
<th>Average</th>
<th>Below Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.77</td>
<td>0.71</td>
<td>0.65</td>
</tr>
<tr>
<td>Temporary Structures</td>
<td>0.74</td>
<td>0.67</td>
<td>0.60</td>
</tr>
<tr>
<td>Standard Structures</td>
<td>0.69</td>
<td>0.61</td>
<td>0.53</td>
</tr>
</tbody>
</table>

6.1.5.2 Single Shear (Simulated Diaphragm Action) Test

Table 4.6 gives the value of $V_R'$ as 0.063, and the values of $V_E'$ may be obtained from Table 4.18. Use of the same values of $V_F'$, as in the lap joint test produces the values of $V_R$ shown in Table 6.3. The variation is slight for samples of size two or more because the effect of $V_E'$ is overshadowed by the two other terms.

The corresponding values of $\bar{\theta}$, presented in Table 6.4, also display very little variation. The following practical resistance factors could apply to typical samples of three to five specimens:

<table>
<thead>
<tr>
<th>Workmanship and Inspection</th>
<th>Good</th>
<th>Average</th>
<th>Below Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.74</td>
<td>0.69</td>
<td>0.64</td>
</tr>
<tr>
<td>Temporary Structures</td>
<td>0.71</td>
<td>0.65</td>
<td>0.58</td>
</tr>
<tr>
<td>Standard Structures</td>
<td>0.65</td>
<td>0.58</td>
<td>0.51</td>
</tr>
</tbody>
</table>
These values are somewhat smaller than those for the lap joint test because of the greater test scatter, i.e. the greater value of $V_{R'}$ and hence of $V_{E'}$, with this procedure.

### 6.1.5.3 Pull-over Test

The value of $V_{R'}$ is given in Table 4.8 as 0.073, and Table 4.19 indicates the values of $V_{E'}$, as a function of sample size. $V_{E'}$ is again selected as 0.05, 0.10 and 0.15 to represent good, average and below average workmanship and inspection. The resulting values of $V_{R'}$ are given in Table 6.5. The variation is small for samples of size three or more because $V_{E'}$ decreases with sample size.

The corresponding values of $\bar{V}$ are shown in Table 6.6. The following practical resistance factors could be used for typical samples of three to five specimens:

<table>
<thead>
<tr>
<th>Workmanship and Inspection</th>
<th>Workmanship and Inspection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>Average</td>
</tr>
<tr>
<td>0.73</td>
<td>0.69</td>
</tr>
<tr>
<td>0.69</td>
<td>0.64</td>
</tr>
<tr>
<td>0.63</td>
<td>0.57</td>
</tr>
</tbody>
</table>

These values are smaller than the single shear values because of the greater scatter in test results for the pull-over test.

### 6.1.5.4 Pull-out Test

Table 4.10 provides the value of $V_{R'}$, as 0.039, and the values of $V_{E'}$, may be obtained from Table 4.20. $V_{E'}$ is chosen as 0.075, 0.125 and 0.175 for the various qualities of
workmanship because of the sensitivity of pull-out strength to variations in fabrication, notably tightening torque. Table 6.7 presents the resulting values of \( V_R \). The variation in these values as a function of sample size is very slight because \( V_R \) and \( V_E \) are small relative to \( V_F \).

Table 6.8 provides the corresponding values of \( \bar{V} \), which are nearly independent of sample size. Practical resistance factors for pull-out strength may be taken as the following:

<table>
<thead>
<tr>
<th>Workmanship and Inspection</th>
<th>Below Average</th>
<th>Good</th>
<th>Average</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temporary Structures</td>
<td>0.62</td>
<td>0.74</td>
<td>0.68</td>
<td>0.62</td>
</tr>
<tr>
<td>Standard Structures</td>
<td>0.56</td>
<td>0.71</td>
<td>0.63</td>
<td>0.63</td>
</tr>
<tr>
<td>Vital Structures</td>
<td>0.49</td>
<td>0.65</td>
<td>0.57</td>
<td>0.57</td>
</tr>
</tbody>
</table>

These values are similar to those for the pull-over test.

6.1.6 Relative Probabilities of Failure

It has been previously mentioned that extrapolation of distributions beyond the range of the data is hazardous because results largely depend on the distribution assumptions. Such assumptions were minimized in the present work through the use of a generalized beta distribution and the method of inverse inference. It is still felt, however, that a discussion of the absolute probabilities of failure corresponding to the modified resistance factors and factored loads would serve little useful purpose. An examination of the variation in the probability of failure with type of structure and quality of
workmanship, on the other hand, may be of interest. The probabilities of failure corresponding to the modified resistance factors can be obtained directly from the cumulative distribution functions determined in Chapter 4.

It is found that in general the probability of failure at design loads decreases by approximately one order of magnitude with each step of increase in the importance of the structure. Thus for a given quality of workmanship and type of connection failure, the probability of occurrence in a standard structure is about one tenth that in a temporary structure and ten times that in a very important structure. This result is consistent with the choice of safety index for temporary, standard and vital structures.

6.1.7 Summary and Conclusions

The coefficient of variation of the resistance, when the mean value is measured in the laboratory from a limited number of tests, may be taken as

$$V_R = \sqrt{V_R'^2 + V_F'^2 + V_E'^2}$$

where $V_R'$ is the coefficient of variation of the underlying distribution determined from a relatively large number of laboratory tests, $V_F'$ reflects the effects of the additional variation in workmanship and inspection produced by field conditions and $V_E'$ represents the error in estimating the mean value of the resistance from a limited number of tests.

The safety index $\beta$ may be chosen as 3.5, 4.5 and 5.5 for connections in temporary, standard and very important structures.
These values are based on the principle that the connection be stronger than the members connected, and correspond to $\beta$ values of 2.5, 3.0 and 4.5 for main members. To enable the use of the same load factors for both connections and main members, a modified resistance factor $\bar{\beta}$ equal to 0.88 $\bar{\beta}$ for temporary and standard structures and 0.85 $\bar{\beta}$ for vital structures should be used for connections.

The variation in $\bar{\beta}$ as a function of sample size is slight for any given test, and may be taken as constant for typical tests of three to five specimens. The variation in $\bar{\beta}$ between the different tests is also not substantial. Hence it is proposed that a single set of modified resistance factors be employed for all connections in cold-formed steel formed with small diameter fasteners. These modified resistance factors would be used in conjunction with the proposed test procedures and would require a minimum of three test specimens. The recommended values are as follows:

<table>
<thead>
<tr>
<th>Workmanship and Inspection</th>
<th>Below Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>Average</td>
</tr>
<tr>
<td>0.75</td>
<td>0.70</td>
</tr>
<tr>
<td>0.70</td>
<td>0.625</td>
</tr>
<tr>
<td>0.65</td>
<td>0.55</td>
</tr>
<tr>
<td>Standard Structures</td>
<td></td>
</tr>
<tr>
<td>Vital Structures</td>
<td></td>
</tr>
</tbody>
</table>

These values are typically slightly conservative for standard and very important structures, and not conservative for temporary structures.\(^1\)

\(^1\) A discussion of the "safety factors" corresponding to these modified resistance factors is presented in Appendix C.
In the determination of the resistance factors, it was found that in all cases the governing parameter is not the number of tests performed in the laboratory but the quality of workmanship and degree of inspection in the field. Hence the decision on the level of a workmanship that exists, or the degree of simulation provided by the laboratory tests, should be made with care. "Good" workmanship typically implies a 95 percent confidence that the strengths of connections made in the field are within ±10 percent of the strengths of connections made and tested in the laboratory. These limits are ±20 and ±29 percent for "average" and "below average" workmanship, respectively.

The probability of failure corresponding to the modified resistance factors generally decreases by an order of magnitude with each step of increase in the importance of the structure. This result is consistent with the selected values for the safety index.

6.2 Commentary on the Test Evaluation Method

The test evaluation method presented as Appendix B is intended to complement the proposed single shear, pull-over and pull-out test procedures by providing a standard interpretation of the test results. The method is based on the concepts of load and resistance factor design as developed in Reference 33, and uses the test results to define the resistance portion of the design criterion. The method evolved directly from the resistance factors determined in Section 6.1.5, and employs the single set of resistance factors
recommended in the previous section for all connections in
cold-formed steel formed with small diameter fasteners and
tested in accordance with the proposed test procedures.

6.2.1 Sample Size

The scatter in test results is characteristically rather
small, and this should be the case with all connections in
cold-formed steel tested in accordance with the proposed test
procedures. This slight scatter produces resistance factors
that are nearly independent of sample size for sizes greater
than about two. The resistance factors determined herein in­
creased an average of 0.8 percent as the sample increased from
three to ten specimens, and the maximum increase was only 2.1
percent.

Three is considered the minimum number of specimens re­
quired for a reasonable estimate of the mean, and the slight
increase in the resistance factor is regarded as insufficient
to warrant additional tests.

6.2.2 Scatter Limitations

The small scatter permits a reasonable estimate of the
mean from three test specimens. Limits are placed on the
extreme values of the sample to assure that the scatter is
indeed small. These limits are convenient percentages of the
sample mean which correspond to a compromise between the 95
and 99 percent two-sided confidence limits of the underlying
probability distributions. The purpose of these limits is to:

a) assure a reasonable estimate of the true mean, b) indicate
possible violations of the test procedures which produce inconsistencies in results and c) indicate possible improprieties in the test specimens, e.g. mixing specimens with different components.

Exceedance of the limits should be uncommon, for the probability that any test result will fall within the limits is between 0.95 and 0.99. Should the limits be exceeded, the sample size is to be increased to six specimens to assure a good estimate of the true mean strength.

6.2.3 Load Factors

The load factors were determined in Section 6.1.3 with the values of loading uncertainties suggested in Reference 33 and a safety index of 2.5, 3.0 and 4.5 for temporary, standard and vital structures, respectively.

6.2.4 Deflection Considerations

The proposed deterministic design criterion for deflection limitations is simply to design for service loads with the resistance corresponding to the allowable deflection. Although this criterion provides no visible safety margin, it is considered adequate for the following reasons: (1) Service loads as defined by building authorities are rarely exceeded under normal conditions. (2) The variation in the stiffness of the connection is slight at the deflection levels of concern. (3) The consequences of exceeding the allowable deflection are typically not severe.
6.2.5 Possible Extensions

The procedures developed herein for the determination of resistance factors and an evaluation method can be extended to any standard test procedure. Such procedures might be developed for repeated loads, or the proposed procedures might be modified for that purpose. A similar circumstance exists in the determination of the pull-over strength of connections to a main member which is subject to torsional behavior under load.

In the interim, methods such as large scale or prototype tests might be used to relate the strength under repeated loads or with main members that exhibit torsional behavior to the static strength as measured by the proposed procedures. An estimate of the reduction in static strength could then be applied to the resistance factors prescribed herein, and the same evaluation method could be employed.
CHAPTER 7
SUMMARY AND CONCLUSIONS

This investigation naturally divides itself into two independent phases. The first phase is the development of test fixtures and procedures suitable for tests of mechanical connections employed by the cold-formed steel industry, and the second is the determination of an evaluation method to be used in conjunction with the proposed procedures. The independent treatment of these two phases is continued in this chapter.

7.1 Test Fixtures and Procedures

A review of the literature reveals that established and nationally recognized standard test procedures exist for most mechanical fasteners. Standard tests to evaluate the performance of mechanical connections, on the other hand, are limited and appear to be infrequently used. Connection tests are typically devised to satisfy a particular demand without regard to standardization.

The performance of connections formed with mechanical fasteners other than bolts is typically determined from tests. The lack of standardized test fixtures and procedures impedes the acceptance of connection designs and the exchange of information vital to understanding the behavior of connections formed with a large number of fasteners with diverse properties.

7.1.1 Summary

Test fixtures and procedures are developed which are
considered to provide consistent and unbiased estimates of the single shear, pull-over and pull-out strength of connections in cold-formed steel. These tests are designed for use with most mechanical fasteners employed by the industry, but are expected to be used primarily with small diameter fasteners. Because these tests are intended to determine the strength of connections in the actual application, it is important that the connection in the test specimen be as similar to that in the actual application as practicable.

7.1.1.1 *Single Shear Test Procedures*

Single shear tests of joints formed with two overlapping straps are perhaps the most common connection test. For this reason a single shear lap joint test is developed as a proposed standard. A specimen configuration with two fasteners located parallel to the direction of force, as shown in Figure 3.1b, is selected because it provides more consistent results and is more representative of in-line connections than a single fastener connection. The frequency of shear connections formed with unequal thickness components and the use of fasteners exhibiting distinct head side and point side behavior necessitates employing a test specimen with the thickness combination used in the actual application and both fastener heads on the same side of the joint as in actual practice. The requirement that the specimen represent twice the strength of a single fastener connection results in a specimen designed to fail by yield in bearing.

Bolted connection formulas are used to establish the
specified edge distance and strap width of the proposed specimen because these connections represent the most severe case for tension and edge failures. The fastener spacing is chosen to assure the independent behavior of the fasteners, and the strap length is specified to establish a common base for the joint eccentricity. Test results indicate the need for shims to reduce joint eccentricity in specimens formed with substantially thick components.

Connections formed with mechanical fasteners often display a varying stiffness when loaded to failure, and for this reason the speed of testing is expressed both in terms of a displacement rate and a load rate. These rates are both set sufficiently low to allow the initiation and propagation of yielding to occur gradually. Some fasteners also characteristically exhibit relatively large displacements at ultimate load, necessitating designs based on displacement rather than strength. Consequently, the taking of load-displacement measurements is recommended and the measurement of the displacement at ultimate load is required.

A single shear test fixture that is thought to provide a good simulation of the behavior of connections in shear diaphragms is included as an alternative to the lap joint test. The fixture, shown in Figures A15 and A20, consists of a set of two loading arms between which the specimen is bolted in a friction type connection. The arms are constrained by guide tracks to move only in the plane of the specimen, and their form is such as to produce a set of co-linear forces which
induce shear in the test fasteners. The design of the test specimen and the determination of the test procedure are based on the same principles as the lap joint specimen and procedure. A comparison of results obtained with the two procedures indicates that the simpler lap joint test will prove adequate in almost all circumstances.

7.1.1.2 Pull-out/Pull-over Test Procedures

Pull-out or pull-over failures are normally associated with connections in relatively thin panel sections connected to more substantial supporting members and loaded in uplift or similar loading which produces tension in the connecting fasteners.

The proposed test fixture basically consists of a base plate to which the specimen is bolted and a loading channel through which the load is applied to the test fastener. These components are shown in Figures A3 and A4. The complete fixture with a specimen installed is shown in Figure A1.

The bolting provides the primary source of membrane stresses in the test specimen, and the proper allowances for these stresses is very important in the pull-over test. Corrugated specimens are orthotropic in stiffness, and this orthotropy may produce large deformations and much greater membrane stresses in one direction than the other, resulting in splitting or tearing failures in pull-over tests. The membrane stresses are made more comparable through the use of angle sections as shown in Figure A9.

The speed of testing is chosen to allow the gradual
initiation and propagation of yielding. Furthermore, high ductility materials experience an extrusion type pull-over failure which at or near the ultimate load involves plastic flow in the vicinity of the fastener. This process is not instantaneous, and results in the requirement that the load be applied in increments and maintained for a minimum period of one minute when it approaches the ultimate strength of the connection.

The test fixture is reasonably accurate in its prediction of the failure load when the prescribed procedures are followed. This accuracy extends to situations involving eccentric loading of the fastener by the panel. The major limitation of the test is the accurate prediction of the pull-over load in situations where the actual application involves supporting members which exhibit torsional behavior under load. The amount of rotation, and hence the degree of prying action by the fastener head assembly, depends on the member properties and support conditions, and these factors are difficult to simulate in a small scale test.

This limitation may be overcome by conducting full scale tests with the simple and inexpensive loading system shown in Figure 3.6. The system essentially consists of a frame which serves as a vacuum chamber and a polyethylene covering capable of sealing it. The test specimen is a prototype section composed of the panels and supporting members to be used in the actual application. The specimen is inverted and support on top of the frame, and the entire assembly is sealed airtight
with the polyethylene. The air is removed by any suitable means, including a vacuum cleaner, and the load determined by measurement of air pressure differential. This loading system is not constrained by specimen size or supporting member configuration, span length or number of spans, or supporting conditions, and is believed to offer the best indication of connection strength and performance in uplift.

7.1.2 Conclusions

Double shear test results do not correlate well with single shear results for fasteners which exhibit rotation under single shear loadings because double shear constrains the fastener to remain normal to the shear plane. Since double shear connections are relatively uncommon in thin sheet construction, tests of such connections should essentially be prototype tests.

Driving the fastener into the thicker material in single shear connections formed with materials of different thickness produces increased anchorage for fasteners which exhibit distinct head side and point side behavior, resulting in greater fastener rigidity and clamping force. Thus, while the ultimate strength of single shear connections with fasteners displaying symmetric behavior, e.g. torqued bolted connections, is largely governed by the thickness of the thinner sheet, that of connections using fasteners with non-symmetric behavior is determined by the thickness ratio of the two sheets.

The design, size and thickness of washers and washer assemblies have an important effect on the ultimate strength of shear connections. Washers can affect the clamping force
of the fastener, the effective clamping area and bearing area, and the resistance of the fastener to rotation. These effects are more pronounced with a thin sheet immediately under the washer or washer assembly and the presence of a substantial clamping force. Washers and washer assemblies should consequently be carefully selected.

In shear connections formed with two fasteners in line parallel to the direction of force, any fastener spacing that is sufficiently large to eliminate the effects of one fastener on the other should produce identical results if the mode of failure is yield in bearing or shearing of the fasteners. The proposed test specimen configuration is designed to fail by yield in bearing, and results with the fastener spacing specified are valid for the larger fastener spacings that might actually be employed.

The use of oversize holes in all but the bottom sheet for fasteners which form or cut their own mating threads appears to have slight effect on the strength of the joint in shear. Although this practice eliminates the stripping of the threads formed in the top sheet necessary to achieve a tight joint, which may be difficult with thicker materials, the best approach is to employ test hole sizes identical to those used in the actual application.

The primary consideration in the design of any test fixture for conducting general pull-out/pull-over tests is the ability to provide for the membrane stresses which are present in the actual application. This requirement necessitates
restraining the test specimen along its boundary in a manner that severely inhibits motion in the plane of the specimen. Test fixtures which employ a simply supported test specimen perform satisfactorily only when the specimen possesses sufficient stiffness to resist flexure failures. Otherwise the specimen acts primarily as a simply supported beam under point load, and the resulting distortion alters the geometry of the section in the vicinity of the fastener to a state totally remote from that of the actual application. This situation is further aggravated if the fastener location and specimen configuration are such as to load the fastener eccentrically.

Both pull-out and pull-over failure are very localized in their effect and behavior, and are dependent on the mechanical properties of the material in the immediate vicinity of the fastener. Thus the location of the fastener can affect connection strength, and placing the fastener in regions of the panel that have been strain hardened in the forming process increases the connection strength if the resulting ductility is adequate and the increase is not offset by eccentricities that may be introduced. The localized nature of pull-out and pull-over failures also indicates that specimen size is unimportant to the test procedure provided that substantial membrane stresses can be developed without excessive deformations.

Pull-out strength is highly dependent on fastener hole size and tolerance, with even slight variations having a significant effect. In general, the smallest fastener hole
that still allows the fastener to be driven should be used. Tightening torque also has a great effect on pull-out strength. Pull-over strength is affected by the type and size of the washer or washer assembly used under the fastener head, as well as the tightening torque.

7.1.3 Proposed Future Research

Research of the performance and behavior of connections in cold-formed steel formed with mechanical fasteners other than bolts is still essentially in its initial stages, and consequently a large amount of work still needs to be completed before the behavior of such fasteners is thoroughly understood.

The test fixtures and procedures developed herein are intended to apply to single shear, pull-out and pull-over tests of steel-to-steel connections formed with mechanical fasteners. Other fixtures and procedures might be developed to test combined loadings, e.g. the combination of shear and tension, or other types of connections, such as that formed with a button punch. The suitability of the proposed fixtures and procedures to connections formed with other materials common to the cold-formed steel industry (e.g. wood, fiberglass, wallboard) should be checked, and alternate fixtures might be developed if the proposed fixtures are deemed inadequate.

The performance of connections subject to repetitive loadings is very important in the evaluation of mechanical fasteners. Although this investigation excludes such loadings, it is felt that the pull-out/pull-over fixture and possibly
the single shear lap joint specimen might be used in P - 0 cyclic loading. Should these fixtures prove unsuitable, alternate fixtures and procedures could be developed for such loadings.

A systematic examination of connection parameters would prove very useful in understanding mechanical fastener performance. Some of the connection parameters that might be studied are the size and thread pitch of the fastener, the fastener hole size, the type and size of the washer or washer assembly, the tightening torque, the material types, and the material thickness combination. The effects of repetitive loadings also should be examined.

The number of variables in connections formed with some mechanical fasteners will defy the establishment of "exact" design rules and formulas, but general formulas could be established with the idea that they would be refined with the use of the test procedures prescribed herein and additional experience.

7.2 Test Evaluation Method

The purpose of this phase of the investigation is the development of a rational procedure for the determination of connection strength when the sole basis of information is the results of tests performed in the laboratory in accordance with a standard test procedure. The method of load and resistance factor design (LRFD) serves as a convenient basis for this development because it accounts for the various uncertainties associated with connection strength and treats
strength or resistance and load effects independently. The procedure developed herein for connection strength may be generally applied in situations where the design is based on the outcomes of laboratory experiments.

7.2.1 Summary

The design criteria for LRFD may be expressed as

\[ \Phi R_n \geq Y_A (\gamma_D c_D m_D + \gamma_L c_L m_L + \ldots ) \]

where \( \Phi \) is a resistance factor; \( R_n \) is the nominal resistance; \( Y_A, \gamma_D \) and \( \gamma_L \) are load factors; \( m_D \) and \( m_L \) are the mean dead and live load intensities; and \( c_D \) and \( c_L \) are influence coefficients translating load intensities into load effects. The proposed evaluation method defines the resistance portion of this design criterion.

The factored resistance is determined from the expression

\[ \Phi R_n = m_R \exp(-\alpha \beta V_R) \]

where \( m_R \) and \( V_R \) are the respective mean value and coefficient of variation of the resistance, \( \beta \) is the safety index and \( \alpha \) is a constant equal to 0.55 which permits the separation of the resistance and load effects. The nominal resistance \( R_n \) is taken as the mean value of the test results. Its expected value is the true mean of the resistance \( m_R \) for test procedures that provide consistent and unbiased results. The procedure thus consists of the evaluation of the resistance factor \( \Phi \).
7.2.1.1 Coefficient of Variation

The coefficient of variation of the resistance $V_R$, when the mean value is determined from a limited number of laboratory tests, may be taken as

$$V_R = \sqrt{\frac{V_R'^2}{1} + \frac{V_F'^2}{1} + \frac{V_E'^2}{1}},$$

where $V_R'$ is the coefficient of variation of the underlying distribution of laboratory results, $V_F'$ reflects the effects on strength of the additional variation in fabrication produced by field conditions and $V_E'$ represents the error in estimating the mean value of the resistance from a limited number of tests.

An estimate of the coefficient of variation of the underlying distribution $V_R'$, can be obtained by performing a relatively large number of identical tests and calculating the coefficient of variation of the results. This procedure, however, does not utilize all of the information contained in the results. Such information includes the shape of the underlying distribution and the direction of the possible skew. The underlying distributions on connection strength are determined by the method of inverse inference and the method of maximum likelihood, using a generalized beta distribution as the mathematical model.

The method of inverse inference provides distributions on the model parameters, which are treated as random variables. Lack of sufficient information to assign prior probabilities to particular parameter combinations suggests the use of diffuse priors on the parameters. The resulting Bayesian distributions
on strength prove relatively insensitive to the parameter ranges used in determining the joint posterior density functions on the parameters. Hence the only requirement for the selection of parameter ranges is that they be reasonably broad. The resulting distributions are very nearly symmetrical for single shear and pull-over strength, and a normal or lognormal probability model would be acceptable. The advantage of using a beta model is evident in the nearly triangular distribution that results for pull-out strength, indicating a definite upper bound.

Maximum likelihood estimates of the beta parameters provide good estimates of the characteristics of the distributions and give a general indication of their shapes. A computer program for the automated determination of these parameters might thus be employed in a preliminary investigation to select the form of the model to be used. For small sample sizes maximum likelihood estimators generally ignore the regions of the tails, however, and consequently should not be used for decisions sensitive to those regions.

The selection of $V_F$, should be based on the degree of simulation of actual conditions by laboratory tests. For single shear and pull-over failures, $V_F$, is chosen as 0.05, 0.10 and 0.15 to represent good, average and below average workmanship and inspection, respectively. For pull-out strength the greater sensitivity to tightening torque results in the increased values of 0.075, 0.125 and 0.175 for $V_F$.

The value of $V_E$, is determined from the distribution of
the sample mean. Samples of various size are drawn from the underlying distributions by simulation, the sample means are calculated and their distributions determined by a method of moments fit to a beta distribution. It is found that this purpose could be served by invoking the central limit theorem with $V_{R'}$, even for samples of size two and a triangular underlying distribution.

7.2.1.2 Safety Index

The selection of the safety index $\beta$ is based on the principle that the connection be stronger than the members connected. $\beta$ values of 3.5, 4.5 and 5.5 are used for connections in temporary, standard and very important structures, respectively. For main members the corresponding values have been suggested as 2.5, 3.0 and 4.5.

The load factors $\gamma$ also depend on the choice of safety index. To enable the use of the same load factors for both connections and main members, a modified resistance factor $\Theta$ equal to 0.88 $\Theta$ for temporary and standard structures and 0.85 $\Theta$ for very important structures should be used for connections.

7.2.1.3 Resistance Factors

The variation in the modified resistance factor $\Theta$ as a function of sample size is slight for any given test. The implication is that the amount of scatter characteristic of connection strengths in cold-formed steel is such that three tests are sufficient to determine a reasonable estimate of
the true mean strength. The information gained from additional tests does not warrant their expense.

The variation in $\bar{J}$ between different test procedures is also not substantial. Hence it is proposed that a single set of modified resistance factors be employed for all connections in cold-formed steel formed with small diameter fasteners and tested in accordance with the proposed procedures. Use of these modified resistance factors would require a minimum of three test specimens. The recommended values of $\bar{J}$ are:

<table>
<thead>
<tr>
<th>Good</th>
<th>Average</th>
<th>Below Average</th>
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<tbody>
<tr>
<td>0.75</td>
<td>0.70</td>
<td>0.65</td>
</tr>
<tr>
<td>0.70</td>
<td>0.625</td>
<td>0.55</td>
</tr>
<tr>
<td>0.65</td>
<td>0.55</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Temporary Structures
Standard Structures
Vital Structures

The probability of failure corresponding to the modified resistance factors generally decreases by an order of magnitude with each step of increase in the importance of the structure.

7.2.2 Conclusions

The method of LRFD is applicable to situations where the resistance of a structural element or assembly is determined from tests. The method's independent treatment of resistance and load effects makes it well suited to form the basis of test evaluation methods. It can produce results which reflect not only the degree of variation in the measured resistance,
but also the degree of simulation of actual conditions pro-
vided by the tests, the bias in test results, the effects of
a limited number of tests and the nature of the application.

Flexibility in shape combined with adjustable upper and
lower limits make the beta distribution a valuable probability
model. The model can be used with the method of maximum like-
lihood to obtain estimates of the underlying distribution
characteristics which compare well with those obtained by
the method of posterior likelihoods, at a fraction of the com-
putational effort. Although the shapes of the distributions
resulting from the method of maximum likelihood were rather
approximate in this investigation, they can be considered to
improve with larger sample sizes.

The procedure used herein to establish connection strength
can be modified somewhat to take advantage of LRFD's inde-
pendence of distribution assumptions and still produce good
results. Maximum likelihood estimates with a beta law can be
used in the determination of \( V_R \), and the central limit theorem
invoked to determine \( V_E \), even for very small sample sizes.
An estimation of the degree of simulation provided by the
laboratory tests as well as their possible bias would then
provide the required information for implementation of the
procedure. These modifications would result in substantial
computational savings and produce a convenient method for
establishing design strengths.

Three tests appear sufficient to estimate the mean
strength of most connections in cold-formed or hot-rolled
steels because the coefficients of variations of such elements are characteristically small, and hence additional tests provide little additional information.

The method of inverse inference can be used to update the results of this and other investigations as additional information becomes available. Although such information is not expected to greatly alter the characteristics, it would establish better estimates of the upper and lower bounds and eventually lead to well defined priors which could be examined and possibly extended to other situations.

Finally, a number of computational savings appear possible in using the method of posterior likelihoods. Lognormal or shifted lognormal distributions might be used with crude approximations of the prior distributions for future tests on cold-formed steel connections subject to single shear or pull-over load effects. The method of posterior likelihoods also might be used to obtain point estimates of the model parameters, producing a distribution which should be similar in all respects to the Bayesian distribution if reasonably broad ranges are selected for the parameters. Normalization of the data with respect to the mean and the direct determination of one of the Beta parameters from the constraint that the mean of the resulting distribution equal unity appears to have a negligible effect on the resulting Bayesian distribution and reduces the computations by an order of magnitude. This technique might also prove useful for three or even two parameter distributions if the sample size is reasonably large.
7.2.3 Proposed Future Research

The first order probabilistic code format incorporated in LRFD can be refined and improved. The possible effects of the approximations used in the establishment of the code format might be examined and refinements made where necessary. The invariance problem needs to be resolved to permit the treatment of situations where certain variables may have either a resistance or a load effect, e.g. the axial load on short concrete columns. The question of system or subsystem reliability versus member reliability must eventually be considered. This question is related to definitions of "series" and "parallel" systems for various types of failure and the concept of "progressive failure" or "unbuttoning". The treatment of possible "shape effects" of distributions, which are not reflected in a first order probabilistic format, needs to be studied. One possible solution might be the use of a function of the third central moment as a modification factor on the coefficient of variation. The consequences of different types of failure and their treatment need to be determined, as does the influence of time on loads and resistance. Possible extensions might involve the treatment of various load combinations and formulations based on serviceability criteria. The use of expected costs as the basis for decisions with the code format should also receive study.

A need for the rational treatment of deflection considerations is evident in this investigation. Such a treatment might be based on progressive levels of unserviceability. A
REFERENCES


APPENDIX A

RECOMMENDED PROCEDURES FOR CONDUCTING PULL-OVER, PULL-OUT AND SINGLE SHEAR TESTS OF MECHANICAL CONNECTIONS

A1. SCOPE

A1.1 These methods cover procedures and definitions for the mechanical testing of joints formed by connecting a cold-formed steel member to a cold-formed or hot-rolled steel member with one or more mechanical fasteners. The purpose of these procedures is the determination of the load capacities of the joints so formed.

A1.2 The term "mechanical fastener" or "fastener" shall be defined as any mechanical device used in the connection of two or more members, inclusive of such devices as bolts, tapping screws and rivets but exclusive of welds and adhesives.

A1.3 The following mechanical tests are described:

<table>
<thead>
<tr>
<th>Sections</th>
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<tbody>
<tr>
<td>Pull-over</td>
</tr>
<tr>
<td>Pull-out</td>
</tr>
<tr>
<td>Single Shear (Lap Joint)</td>
</tr>
<tr>
<td>Single Shear (Simulated Diaphragm Action)</td>
</tr>
</tbody>
</table>

A2. GENERAL PROCEDURES AND PRECAUTIONS

A2.1 All fastener holes in test specimens shall be of an identical diameter to those employed in the actual application. Fastener holes shall be created with equipment and techniques
that are identical, or as nearly similar as possible, to those used in the actual application.\footnote{Recommended hole diameters are available from some fastener manufacturers for specific products. Also refer to the complete ANSI B18.6.4 standard available from ASME for thread-forming and thread-cutting screws.}

A2.2 All fasteners in test specimens shall be tightened to an amount of torque equal to that employed in the actual application. Fastener driving and/or torqueing devices shall be identical, or as nearly similar as possible, to those used in the actual application. The same procedures shall be employed in both instances.

A2.3 Improper machining or preparation of test specimens may give erroneous results. Care should be taken to assure good workmanship in specimen preparation. Improperly prepared specimens should be discarded and other specimens substituted.

A2.4 Should a test specimen fail improperly due to faulty arrangements, such as failure of the testing equipment or improper specimen installation, it may be discarded and another specimen taken.

A2.5 All fasteners in test specimens shall be tested with those accessories employed in the actual application. Such accessories are to include gaskets, washers and other items used in conjunction with mechanical fasteners.

A2.6 References made herein to self-tapping fasteners
apply to all types of such fasteners: Self-Tapping fasteners and Self-Drilling, Self-Tapping fasteners, including both thread-cutting and thread-forming varieties.

A3. DESIRABILITY OF PROTOTYPE TESTING

A3.1 The large inventory of mechanical fasteners of various types, sizes and mechanical properties in existence, together with the innovations possible in the design of connections, may create situations where the tests described herein are less than adequate to fully evaluate the connection under consideration. Thus it is recommended that whenever practicable a full-scale prototype of the proposed connection be tested. Such testing is especially important when a unique design or special material is proposed.

PULL-OVER TEST

A4. DEFINITION AND DESCRIPTION

A4.1 A pull-over failure shall be defined as a connection failure caused by the fastener head assembly pulling through the material immediately beneath it, or conversely, the connected material pulling over the fastener head assembly. This type of failure is normally associated with relatively thin panel sections connected to heavier members and subjected to uplift or other loading which produces tension in the fastener.

A4.2 The Pull-over Test is designed to determine the ultimate load capacity of a connection subjected to uplift where the mode of failure is the extraction of the material from
under the head assembly of the fastener. A fastener is driven through an 8 in. square test specimen into a loading channel, the specimen is attached to a support with four 1/2 in. bolts and the loading channel is connected to a loading arm. The entire assembly is then placed in a tension testing machine, as shown in Figure A1, and the force required to pull the fastener head assembly through the specimen is measured.

A5. TEST LIMITATIONS

A5.1 This test is believed to be a reasonable simulation of the behavior of most connections subjected to uplift where pull-over is the mode of failure. It can be readily modified to accommodate many specimen shapes and connections and in some instances can utilize all components of the actual application.

A5.2 This test shall not be used to determine the pull-over strength of connections to main members subject to torsional behavior under load.

A5.3 Use of the test fixture described herein shall be limited to test specimens whose thickness is less than or equal to 0.10 in. Specimens with a thickness greater than this limit may be tested in a similar fixture with suitably increased dimensions.

A6. TEST SPECIMEN PARAMETERS

A6.1 Flat-Shaped Specimens. The test specimen parameters to be used for flat-shaped specimens tested in pull-over are given in Figure A2. The governing parameter is that the 9/16
in. diameter holes be centered relative to the fastener location on a 6 in. square.

A6.2 Corrugated (Ribbed) Specimens. The test specimen parameters to be used for corrugated specimens are identical to those for flat-shaped specimens specified in Paragraph A6.1. The fastener location relative to the corrugations shall be identical to the location used in the actual application.

A6.3 The material employed in this test shall be identical to that used in the actual application. Surface treatment, if any, shall remain undisturbed.

A7. PREPARATION OF TEST SPECIMEN

A7.1 Test specimens may be removed from the parent stock by any suitable means, including cutting and shearing, that does not affect the material properties in the vicinity of the fastener location.

A7.2 The four 9/16 in. diameter holes shall be produced in the test specimen by drilling, punching or any other technique that results in a properly located smooth round hole. If desired, test specimens may be stacked, clamped and drilled altogether. Rough edges incurred in this process shall be removed by a suitable means to leave a smooth, flat surface.

A7.3 The fastener hole shall be created in the test specimen in a location relative to any ribs or corrugations that is identical to that used in the actual application, employing the same hole diameter and creation techniques used in the actual application.
A8. TESTING APPARATUS AND EQUIPMENT

A8.1 Test Fixture. The basic test fixture consists of an 8 in. square base plate with two 1 in. square bars welded onto it. A rod is attached to the center of the base plate to permit placement in a tension testing machine and the square bars are tapped for 1/2 in. bolts to enable attachment of the test specimen. The test fixture is completed with a loading channel and a loading arm to transmit the load from the testing machine to the channel. Figure A1 presents an elevation view of the test fixture with a test specimen in place.

A8.2 Test Fixture Components. The following are the descriptions of the various component parts of the test fixture.

A8.2.1 Base Plate. A steel plate 8 in. square and a minimum of 3/8 in. thick, drilled and tapped at its center to accommodate a threaded rod with a minimum diameter of 1/2 in. To this plate are welded two steel rectangular bars 8 in. long and 1 in. square in cross-section with their longitudinal axes 6 in. apart. Care shall be taken in welding to assure that the components remain plane. Each bar shall have two drilled and threaded holes for 1/2 in. bolts, located 6 in. apart and centered with respect to both axes. A drawing of the base plate assembly is presented as Figure A3.

A8.2.2 Loading Channel. A channel formed by welding together three steel plates as shown in Figure A4. The plates are a minimum of 3/8 in. thick and are welded in a manner that leaves the sections plane. The thickness requirement may be waived if the grip length of the test fastener necessitates a
thinner channel base plate. A hole is drilled and tapped at the center of the base of the channel to mate with the test fastener being employed. A piece of the actual member to which the connection is to be made may be used if desired, provided that a piece of roughly similar dimensions to the loading channel can be obtained from it.

A8.2.3 Loading Arm. A steel rectangular bar 8 in. long and 1 in. by 2 in. in cross-section, drilled and tapped at one end to accommodate a threaded rod with a minimum diameter of 1/2 in. and drilled at the other for a 1/2 in. diameter pin. A drawing of the loading arm is presented as Figure A5.

A8.2.4 Miscellaneous Items. In addition to the 1/2 in. diameter pin and 1/2 in. bolts and washers, two angle sections and as many as four spacer sleeves may be required. The angle sections are 8 in. long and a minimum of 1/8 in. thick, slotted with two 9/16 in. slots as shown in Figure A6. The spacer sleeves are tubular sections with an inside diameter slightly larger than 1/2 in. and a length determined by the configuration of the test specimen employed.

A8.3 Loading System. The loading system is a tension testing machine. This machine shall be maintained in good operating condition, used only in the proper loading range, and calibrated periodically in accordance with the latest revision of ASTM Methods E4, Verification of Testing Machines. Both upper and lower gripping or holding devices of the testing machine shall be suitable for round sections.

A8.4 Displacement Measurement. A displacement measurement,
if desired, can be obtained with a dial gage and such magnetic or mechanical holding devices as may be deemed necessary for its attachment to the base plate or loading arm.

A9. ASSEMBLY OF TEST COMPONENTS

A9.1 Fastener Installation. The fastener shall be installed through the test specimen into the loading channel employing the same driving technique and equipment as used in the actual application. If filler material such as insulation is used between the section and main members in the actual application, the same material of equal thickness shall be used between the test specimen and the loading channel. Tightening torque affects the pull-over strength of a connection and thus it is important that the tightening torque used on the test specimen be identical, or as nearly similar as practicable, to that employed in the actual application. If necessary or appropriate, properly calibrated devices shall be used to assure compliance with this requirement.

A9.2 Specimen Installation. The test specimen shall be installed on the base plate with four 1/2 in. diameter high strength bolts. In all cases the bolts shall be of such a length to assure a minimum of 1/2 in. thread engagement in the base plate, shall employ a washer immediately under the head of the bolt, and shall be tightened to approximately 20 ft.-lbs. of torque.

A9.2.1 Flat-Shaped Specimens. Flat-shaped specimens, connected to the loading channel in accordance with the provisions
of Paragraph A9.1, shall be installed directly onto the base plate with the four 1/2 in. bolts. Care shall be exercised to assure proper alignment of the base plate and test specimen.

A9.2.2 Corrugated (Ribbed) Specimens. If the test specimen has sufficient stiffness in the direction perpendicular to the corrugations to resist excessive deformation, or deformation under load which would alter the fastener hole geometry, it shall be installed in the manner given for flat-shaped specimens in the preceding paragraph provided that the four 9/16 in. holes lie in the same plane. This situation is depicted in Figure A7. If the 9/16 in. holes lie in different planes appropriate length spacer sleeves shall be employed as shown in Figure A8. If sleeves are employed, a washer shall be used between the sleeve and the test specimen. Some distortion of the corrugations is permitted to assure proper tightening of the bolts.

A9.2.2.1 If the corrugated test specimen has insufficient stiffness to resist excessive deformations it shall be installed on the base plate with the two slotted angle sections and appropriate length spacer sleeves in a manner similar to that shown in Figure A9. The angle sections shall be positioned such that 45 degree lines may be drawn from the fastener location to the points defined by the intersection of the angle section and the rib most distant from the fastener, as shown in Figure A10. If spacer sleeves are employed, a washer shall be used between the sleeve and the test specimen.
A10. TEST PROCEDURE

A10.1 Loading. The base plate assembly, with the test specimen and loading channel attached in accordance with the provisions of Sectin A9, shall be placed in the upper gripping or holding device of the testing machine. The loading arm shall then be attached to the loading channel with a 1/2 in. diameter pin and secured in the lower gripping or holding device of the testing machine. The final configuration of the complete assemblage is shown in Figure A1. Care shall be taken to assure that the centers of the grips are in alignment, insofar as practicable, with the axis of the fixture at the beginning and during the test.

A10.2 Measurement of Relative Displacement. Relative displacement measurements, if desired, may be obtained by suitably attaching a dial gage of a desired accuracy between some point on the loading arm or channel and the base plate of the test fixture with the complete assembly in place in the testing machine.

A10.3 Speed of Testing. The speed of testing shall not be greater than that at which load and relative displacement readings can be made accurately. In addition, up to the vicinity of the ultimate load the speed of testing shall not exceed either a 0.02 in. per min. rate of separation of the two heads of the testing machine under load or a 100 lb. per min. rate of load, whichever produces the greater rate of separation of the two heads of the testing machine under load. In the vicinity of the ultimate load the loading shall be conducted
in increments, with the size of each increment determined by the accuracy desired. The use of very small increments is strongly recommended. The load shall be maintained at each increment for a minimum period of one minute before proceeding to the next increment.

A11. EVALUATION OF TEST RESULTS

A11.1 The ultimate strength of the connection in pull-over shall be taken as the value of the ultimate load attained in this test.

A11.2 The load-deformation curve of the connection in pull-over shall be the load-deformation curve obtained from this test. Such a curve, if obtained, should be used with caution since actual deformation will be governed by support and boundary conditions.

PULL-OUT TEST

A12. DEFINITION AND DESCRIPTION

A12.1 A pull-out failure shall be defined as a connection failure caused by the threaded portion of the fastener stripping out of the material in which thread engagement existed. This type of failure is normally associated with fasteners which form their own mating threads in the material into which they are driven (self-tapping fasteners).

A12.2 The Pull-out Test is designed to determine the ultimate load capacity of a connection subjected to uplift where the mode of failure is the extraction of the fastener from the
material into which it was driven. A fastener is driven through a loading channel into a test specimen, the specimen is attached to a support with four 1/2 in. bolts and the loading channel is connected to a loading arm. The entire assembly is then placed in a tension testing machine, as shown in Figure A11, and the force required to extract the fastener from the specimen is measured.

A13. TEST LIMITATIONS
A13.1 This test is a good simulation of the behavior of a connection subjected to uplift where pull-out is the mode of failure. It can be readily modified to accommodate most connections and specimen configurations, and with additional modification may be used to approximate eccentric pull-out behavior.

A13.2 Use of the test fixture described herein shall be limited to test specimens whose thickness is less than or equal to 0.20 in. Specimens with a thickness greater than this limit may be tested in a similar fixture with suitably increased dimensions.

A14. TEST SPECIMEN PARAMETERS
A14.1 The test specimen parameters for flat-shaped and corrugated (ribbed) specimens are identical to those for the Pull-over Test and are specified in Paragraphs A6.1 and A6.2, respectively.

A14.1.1 Formed Specimens. Formed specimens, e.g. channel, zee or rectangular sections, are to be a minimum of 8 in. in length and identical to that used in the actual application.
A14.2 The material employed in this test shall be identical to that used in the actual application. Surface treatment, if any, shall remain undisturbed.

A15. PREPARATION OF TEST SPECIMEN
A15.1 The test specimen shall be prepared in accordance with the provisions of Section A7.

A16. TESTING APPARATUS AND EQUIPMENT
A16.1 Test Fixture. The Pull-out Test employs the same test fixture used in the Pull-over Test, except that the loading channel components shall have a minimum thickness of 1/4 in. and the hole in the loading channel shall be slightly larger than the diameter of the fastener used. A description of the test fixture and its assembly is presented in Sections A8.1 and A8.2.

A16.2 Loading System. The loading system is a tension testing machine that complies with the provisions of Paragraph A8.3.

A16.3 Displacement Measurement. The provisions of Paragraph A8.4 shall apply if a displacement measurement is desired.

A17. ASSEMBLY OF TEST COMPONENTS
A17.1 Fastener Installation. The fastener shall be installed through the loading channel into the test specimen employing the same driving technique and equipment as used in the actual application. Tightening torque materially affects the pull-out strength of a connection and thus it is imperative
that the tightening torque used on the test specimen be identical, or as nearly similar as practicable, to that employed in the actual application. If necessary or appropriate, properly calibrated devices shall be used to assure compliance with this requirement.

A17.2 Specimen Installation. The test specimen shall be installed on the base plate in accordance with the provisions of Paragraph A9.2.

A17.2.1 Flat-Shaped Specimens. Flat-shaped specimens, connected to the loading channel in accordance with the provisions of Paragraph A17.1, shall be installed in accordance with the provisions of Paragraph A9.2.1.

A17.2.2 Corrugated (Ribbed) Specimens. The test specimen shall be installed in accordance with the provisions of Section A9.2.2, although the proper positioning of the angle sections is not critical.

A17.2.3 Formed Specimens. Formed specimens shall be installed on the base plate by using two suitable angle sections as shown in Figure A12. Care shall be exercised to assure that the test fastener is centered with respect to both axes of the base plate. The bolts shall be sufficiently tightened to assure that the test specimen is securely clamped without being distorted.

A18. TEST PROCEDURE

A18.1 Loading. The base plate assembly, with the test specimen and loading channel attached in accordance with the provisions of Section A17, shall be placed in the testing
machine in accordance with the provisions of Paragraph A10.1.

A18.2 Measurement of Relative Displacement. Relative displacement measurements, if desired, may be obtained in accordance with the provisions of Paragraph A10.2.

A18.3 Speed of Testing. The speed of testing shall not be greater than that at which load and relative displacement readings can be made accurately. In addition, the speed of testing shall not exceed either a 0.02 in per min. rate of separation of the two heads of the testing machine under load or a 100 lb. per min. rate of loading, whichever produces the greater rate of separation of the two heads of the testing machine under load.

A19. EVALUATION OF TEST RESULTS

A19.1 The ultimate strength of the connection in pull-out shall be taken as the value of the ultimate load attained in this test.

A19.2 The load-deformation curve of the connection in pull-out shall be the load-deformation curve obtained from the test. Such a curve, if obtained, should be used with caution since actual deformation will be governed by support and boundary conditions.

SINGLE SHEAR (LAP JOINT) TEST

A20. DEFINITION AND DESCRIPTION

A20.1 The Single Shear (Lap Joint) Test is a common and simple test designed to determine the shear capacity of a
simple overlap joint. Two straps, each of a desired thickness, are joined together with two fasteners located parallel to the direction of force. The assemblage is then placed in a tension testing machine and the ultimate strength of the connection is measured.

A21. TEST LIMITATIONS

A21.1 Some eccentricity in the transfer of force across the joint together with some out-of-plane distortions combine to make this test less than ideal for some shear transfer simulations, notably some simulations of diaphragm action. This test is considered adequate for most situations, however. If increased accuracy in simulating diaphragm action is desired or deemed necessary the more complex Single Shear (Simulated Diaphragm Action) Test should be performed.

A22. TEST SPECIMEN PARAMETERS

A22.1 Figure A13 shows the typical test specimen configuration. The following are the values to be used for the parameters designated in the figure, where d is the nominal fastener diameter:

A22.1.1 Edge distance e. If edge failures are not to be considered the edge distance e shall be taken as the greater of 1 in. or 4d. If edge failures are to be considered, the test specimen shall employ a single test fastener and the edge distance to be used in the actual application.

A22.1.2 Fastener spacing s. The fastener spacing s shall be taken as the greater of 2 in. or 8d.
A22.1.3 Specimen width $w$. The specimen width $w$ shall be taken as the greater of 2 in. or 10d.

A22.1.4 Specimen strap length $L$. The length $L$ of each component strap of the specimen shall be at least the greater of 15 in. or 60d. Strap lengths longer than this minimum are desirable as they tend to decrease the eccentricity of the joint tested.

A22.1.5 Specimen thicknesses $t_1$ and $t_2$. The specimen thicknesses $t_1$ and $t_2$ shall be identical to, and in the same position relative to the head of the fastener as, the actual application.

A22.2 If either one or both of the component straps of the test specimen are not flat-shaped, or have a reasonably flat area at least as wide as the specimen width $w$, the specimen width specification in Paragraph A22.1.3 does not apply and the actual section configuration, together with flanges, if any, shall be used. In this case approximately 4 in. at the end of such a strap shall be appropriately deformed, by either cutting or bending or both, to form a surface suitable for gripping.

A22.3 The material employed in this test shall be identical to that used in the actual application. Surface treatment, if any, shall remain undisturbed except for an approximate 4 in. length at the end of each component strap which may be altered by any suitable means to provide for a more slip resistant grip.
A23. PREPARATION OF TEST SPECIMEN

A23.1 Component straps for the test may be removed from the parent stock by any suitable means, including mechanical cutting and shearing, that does not measurably effect the material properties at a distance of one fastener diameter from the edge. Edge roughness incurred in this process that might prevent the component straps from mating completely shall be removed by a suitable means, preferably filing.

A23.2 If the actual application is to involve the creation of holes in both component parts of the joint simultaneously, the two component straps of the test specimen shall be well aligned and suitably clamped in the final specimen configuration. Care shall be exercised to assure proper alignment and thus the elimination of bending in the plane of the specimen at the joint. The fastener holes shall be created with the specimen in this clamped position, employing the same hole diameters and creation techniques as used in the actual application.

A23.3 If the actual application is to involve the creation of holes in each component part of the joint individually, such holes shall be created in each component strap of the test specimen employing the same hole diameters and creation techniques used in the actual application. Additional care must be exercised to assure that both component parts of the test specimen will be well aligned when mated and thus avoid bending in the plane of the specimen at the joint.

A23.4 The two fasteners shall be installed from the same
side of the test specimen with both component straps carefully mated and clamped together, employing a technique identical to that used in the actual application. In all circumstances the tightening torque shall be identical, or as nearly similar as practicable, to that employed in the actual application. If necessary or appropriate, suitably calibrated devices shall be used to assure fulfillment of this condition.

A23.5 Approximately the final 4 in. of length at each end of the test specimen shall have the surface on both sides roughened sufficiently to prevent slip in the grips, employing any suitable means that does not significantly reduce the tensile strength across the section. This requirement is especially critical for thin specimens.

A23.6 If a load-displacement recorder for autographic plotting of load-displacement curves is not available and a load-displacement curve for the test specimen is desired, the specimen shall be gage marked with a center punch, scribe marks, multiple device, or drawn with ink. Punch marks, if used, shall be light, sharp and accurately spaced. The gage marks shall be made on the same side of the specimen, in line with the two fasteners, and at a distance of 0.5 in. away from the ends of the overlapped portion of the specimen, for a gage length of \((2e + s + 1)\) in. The purpose of these two gage marks is to determine the relative movement of the two component straps across the joint.

A23.7 If the test specimen is not gage marked in accordance with Paragraph A23.6, it shall be marked in a fashion that
will permit the accurate determination of the total relative movement of the two component straps at the ultimate load. Such a marking may be a single scribe mark across both components of the joint. Markings used shall in no way affect either the strength of the material or the strength of the joint.

A24. TESTING APPARATUS AND EQUIPMENT

A24.1 Loading System. The loading system is a tension testing machine with gripping or holding devices suitable for flat sections. This machine shall comply with the provisions of Paragraph A8.3.

A24.2 It is desirable to use a load-displacement recorder, such as an extensometer, for autographic plotting of load-displacement curves. Such a record, if used, should have a displacement range of approximately 0.50 in. over a gage length of \((2e + s + 1)\) in., although use over a smaller gage length is permitted.

A25. TEST PROCEDURE

A25.1 Loading. The test specimen, prepared in accordance with the provisions of Section A23, shall be placed in the gripping or holding devices of the testing machine. It is essential that the load be transmitted axially to keep bending at a minimum. This implies that the centers of the grips shall be in alignment, insofar as practicable, with the axis of the specimen at the beginning and during the test. The specimen shall be gripped over approximately the final 4 in. of length at each end, although this length may be increased if necessary.
to avoid slippage in the grips. Shims of appropriate thicknesses shall be used as shown in Figure A14 if either $t_1$ or $t_2$ is greater than 0.075 in. to reduce loading eccentricity.

A25.2 Speed ofTesting. The speed of testing shall not be greater than that at which load and relative displacement readings can be made accurately. In addition, the speed of testing shall not exceed either a 0.05 in. per min. rate of separation of the two heads of the testing machine under load or a 100 lb. per min. rate of loading, whichever produces the greater rate of separation of the two heads of the testing machine under load.

A25.3 Measurement of Relative Displacement. The load-displacement recorder, if employed, should be set at a gage length of $(2e + s + 1)$ in., with the ends of the measuring device 0.5 in. away from the edge of each overlap. Should this gage length be unattainable for the particular device used, the gage length for which the device was designed may be used provided that it is centered with reference to the two fasteners, i.e. each gage mark is at an equal distance from the fastener nearest to it.

A25.3.1 If no load-displacement recorder is available and a load-deformation curve for the specimen is desired, displacement measurements between the gage marks made in accordance with Paragraph A23.6 shall be made at appropriate intervals using a set of dividers or similarly suitable instrument. Accuracy should be on the order of $0.01 + 0.005$ in.

A25.4 Measurement of Relative Displacement at Ultimate
Load. Upon attainment of the ultimate load, or as soon thereafter as practicable, the testing machine drive shall be stopped and the specimen held in the strained position. Measurement of the total relative displacement at ultimate load shall be made between the marks made in accordance with Paragraph A23.6 or A23.7 employing a device suitable for such a purpose. Accuracy should be on the order of $0.01 \pm 0.005$ in. After the completion of this measurement the specimen may be further strained to obtain the complete load-deformation curve or unloaded, as desired.

A26. EVALUATION OF TEST RESULTS

A26.1 The ultimate strength of the connection in single shear, per fastener, shall be taken as one-half the value of the ultimate load attained in this test.

A26.2 The load-deformation curve of the connection in single shear, per fastener, shall be the load-deformation curve obtained from this test with the load values reduced by one-half. If no load-deformation curve was obtained from the test, and it is felt that deformations might govern in the design, the load-deformation curve shall be taken as a straight line from the origin to the point defined by one-half the ultimate load attained in this test and the relative displacement at ultimate load as measured per Paragraph A25.4.

SINGLE SHEAR (SIMULATED DIAPHRAGM ACTION) TEST

A27. DEFINITION AND DESCRIPTION
A27.1 The Single Shear (Simulated Diaphragm Action) Test is designed to determine the single shear capacity of a connection in a cold-formed steel diaphragm. The two joint components, each of the desired thickness and connected with two fasteners located parallel to the direction of force, are each clamped between two flat, heavy plates using high strength bolts. The flat plates are constrained by guide tracks to movement only in their own plane and in the direction of force. The force is transmitted from the flat plates to the joint components, and across the components by the fasteners. The test fixture is depicted in Figure A15.

A28. TEST LIMITATIONS

A28.1 This test, although considerably more complicated than the Single Shear (Lap Joint) Test, is believed to be a good simulation of the behavior of a connection in a shear diaphragm. It readily lends itself to several specimen shapes and with no modifications can be used to simulate a double shear connection. The primary limitations of this test are its complexity and the requirement for great care in machining and preparation of test specimens.

A29. TEST SPECIMEN PARAMETERS

A29.1 Figure A16 shows the typical test specimen configuration. All constant parameters are given in the figure. The nature of this test and the required tolerances demand that these parameters be closely met. The following values are to be used for the variable parameters:
A29.1.1 Fastener spacing $s$. The fastener spacing $s$ shall be taken as the greater of 2 in. or $8d$, where $d$ is the fastener diameter.

A29.1.2 Specimen thicknesses $t_1$ and $t_2$. The specimen thicknesses $t_1$ and $t_2$ shall be identical to, and in the same position relative to the head of the fastener as, the actual application.

A29.2 In addition to the test specimen two spacer plates, one of thickness $t_1$ and the other of thickness $t_2$, where $t_1$ and $t_2$ are the specimen thicknesses, are required. The typical spacer plate configuration is shown in Figure A17.

A29.3 If either one or both components of the test specimen are not flat-shaped appropriate measures, including bending and cutting, may be taken to produce a configuration suitable for the test fixture. Such measures shall in no way affect the strength of the connection nor significantly alter the geometry of the section in the vicinity of the fasteners.

A29.4 The material employed in this test shall be identical to that used in the actual application. Surface treatment, if any, shall remain undisturbed.

A30. PREPARATION OF TEST SPECIMEN

A30.1 Specimen components for the test may be removed from the parent stock by any suitable means, including mechanical cutting and shearing, that produces a clean, straight cut and does not measurably affect the material properties at a distance of 1/16 in. from the edge on the side nearest the fastener location. Spacer plates may be removed from the parent stock
by any suitable means, provided that the final dimensions are met. Edge roughness incurred in this process that might prevent the component sections and spacer plates from mating completely shall be removed by a suitable means, preferably filing.

A30.2 The four 9/16 in. holes shall be produced in accordance with the provisions of Paragraph A7.2.

A30.3 If the actual application is to involve the creation of holes in both component parts of the connection simultaneously the provisions of Paragraph A23.2 shall apply.

A30.4 If the actual application is to involve the creation of holes in each component of the connection individually the provisions of Paragraph A23.3 shall apply.

A30.5 The two fasteners shall be installed in accordance with the provisions of Paragraph A23.4.

A31. TESTING APPARATUS AND EQUIPMENT

A31.1 Test Fixture. The basic test fixture consists primarily of a base plate with three tracked supports, a center support and two outside supports, as shown in Figure A18. The test specimen is bolted between two sets of shear plates and placed on the supports in the manner indicated in Figures A19 and A20. The shearing force, produced by a hydraulic ram, is transmitted to the shear plates and hence the test specimen through the arrangement depicted in Figure A21.

A31.2 Test Fixture Components. The following are the descriptions of the various component parts of the test fixture. Mechanical connections are used rather than welding to eliminate the possibility of distortion due to heat.
A3l.2.1 Base Plate. A steel plate 10 in. wide, 16 in. long and 5/8 in. thick drilled with 3 rows of countersunk holes to accommodate 1/4 in. cap screws. A drawing of the base plate is presented as Figure A22.

A3l.2.2 Center Support. A steel tee section 12 in. long, 1 1/2 in. wide and 1 in. high drilled and tapped to be attached to the base plate with 1/4 in. cap screws. Teflon pads, 1/8 in. thick, are bonded to the inside surfaces to reduce friction. A drawing of the center support is presented as Figure A23.

A3l.2.3 Outside Supports. Two steel angles 15 in. long and 5/8 in. thick, with legs of 2 1/4 in. and 1 in., drilled and tapped to be attached to the base plate with 1/4 in. cap screws. A teflon pad 1/8 in. thick is bonded to the bottom inside surface to reduce friction. A drawing of an outside support is presented as Figure A24.

A3l.2.4 Vertical Guides. Two steel plates 1 1/4 in. wide, 15 in. long and 1/2 in. thick drilled to fit the outside supports and designed to restrain the test specimen from movement in the vertical plane. A drawing of a vertical guide is presented as Figure A25.

A3l.2.5 Shear Plates. Four steel plates (two to be positioned above and two below the test specimen) 2 7/8 in. wide, 18 in. long and 5/8 in. thick drilled with 9/16 in. diameter holes to accommodate the specimen between the two sets of plates. These plates are designed to transmit a shearing force in line with the fasteners across the two components of the test specimen. Drawings of a bottom and top shear plate
are given as Figures A26 and A27, respectively.

A31.2.6 Miscellaneous Items. In addition to such common items as dowels, cap screws, bolts and clamps, a number of miscellaneous items are required. These items include: a rigid support for the test fixture, such as an I-beam, with appropriate end restraints for the assembly; two yokes of a convenient size, complete with pins, to transmit the shearing force to the shear plates; a loading rod and restraining rod to transmit the shearing force to the yokes; a hydraulic ram to generate the force; a load measurement device, such as a load cell or calibrated rod; and two dial gages with appropriate supports accurate to 0.001 in. to measure the relative displacement of the two sets of shear plates.

A32. ASSEMBLY OF TEST COMPONENTS

A32.1 Placement of Test Specimen Between Shear Plates. The two bottom shear plates shall be placed on two supports approximately 11 in. apart, enabling access to the 9/16 in. holes from below, and separated by two 1 in. spacers as shown in Figure A28. The spacer plate of thickness $t_1$ shall be placed on one of the shear plates, taking care to assure that the 9/16 in. holes are in alignment and that the wider portion of the spacer plate is to the outside. The test specimen, prepared in accordance with Section A30, shall then be placed on the shear plates with the fastener heads directed downwards and the test specimen component of thickness $t_2$ on the shear plate containing the spacer plate. Care shall be taken to assure the proper alignment of the 9/16 in. diameter holes in
the test specimen, spacer plate of thickness $t_1$ and shear plates. The top shear plate shall then be placed directly over that bottom shear plate which contains the spacer of thickness $t_1$ and test specimen component of thickness $t_2$. It is recommended that 17/32 in. dowels be used to aid in the alignment of the 9/16 in. holes. After proper alignment is attained, 1/4 in. dowels shall be placed through the dowel holes in the top and bottom shear plates and both plates shall be clamped to the supports. The spacer plate of thickness $t_2$ shall then be placed over the test specimen component of thickness $t_1$, again assuring the alignment of the 9/16 in. holes. The remaining top shear plate shall then be placed directly over the other bottom shear plate. After properly aligning the 9/16 in. holes, 1/4 in. dowels shall be placed through the dowel holes in the top and bottom shear plates and both plates shall be clamped to the supports.

A32.2 Bolting of Test Specimen Between Shear Plates. The test specimen shall be bolted between the shear plates with eight 1/2 in. high strength bolts 2 1/2 in. long. With the shear plates clamped to the supports as described in the previous paragraph, the bolts shall be placed through the 9/16 in. holes in the plates from below, with a washer under both the head of the bolt and the nut. After all eight bolts are installed and fingertight they shall all be torqued to 40 ft.-lbs. using an accurately calibrated torque wrench, and then torqued again to 80 ft.-lbs. This tightening produces a friction joint and avoids stress concentrations and distortions due to bearing
of the bolts on thin steel sheeting. After the final tightening the clamps holding the plates to the supports and the 1/4 in. dowels shall be removed.

A32.3 Assembly of Basic Test Fixture. The center support and two outside supports are attached to the base plate with 1/4 in. hexagon socket head cap screws. The screws are placed through the bottom of the base plate into the three supports in such a manner that the final configuration is as shown in Figure A18. This assemblage is then clamped to a rigid support in such a manner that the longitudinal axis of the assemblage coincides with the longitudinal axis of the support.

A32.3.1 The shear plates, with the test specimen bolted between them in accordance with Paragraph A32.2, shall be lifted from the supports and placed into the assembly described in the preceding paragraph. Extreme care shall be exercised to assure that no bending or twisting of the specimen is incurred during this process. The test specimen shall be centered in the fixture by carefully sliding both sets by shear plates simultaneously into the desired position. With the specimen centered, 1/8 in. thick teflon pads shall be placed between the edges of the shear plates and the outside supports. The two vertical guides shall then be attached to the tops of the outside supports with 1/4 in. hexagon socket head cap screws. Teflon pads, 1/8 in. thick, shall be placed between the shear plates and vertical guides in such a manner that they are centered under the No. 5 cap screw holes, and No. 5 hexagon
socket head cap screws shall be positioned and finger-tightened to assure a positive restraint in the vertical direction. Figures A19 and A20 show the configuration of the basic test fixture in plan view and vertical section, respectively.

A32.4 Completion of Fixture Assembly. With the specimen installed in the basic test fixture and the test fixture clamped to a rigid support as prescribed in the two preceding paragraphs, the following steps will complete assembly of the test fixture. A yoke shall be connected to each set of shear plates with a 5/8 in. diameter pin. A restraining rod shall be attached to one of the yokes and supported at its opposite end in such a manner that it provides an immovable support and its axis coincides with the longitudinal axis of the test specimen. The restraining rod may be instrumented with strain gages or similar devices to measure the force transmitted across the test specimen with sufficient accuracy and it should be threaded at one or both ends to enable sufficient tightening to prevent substantial rigid body motion of the test specimen. A loading rod shall be attached to the other yoke and passed through a hydraulic ram with a hollow core, supported such that its axis coincides with the longitudinal axis of the test specimen. If the restraining rod is not instrumented, a load cell shall be placed between the hydraulic ram and the end support to measure the force generated. A drawing of the completely assembled test fixture is shown as Figure A21.

A32.4.1 Two dial gages, accurate to 0.001 in. and supported in a suitable manner, shall be positioned as shown in Figure
A29 to measure the relative displacement of the two sets of shear plates.

A33. TEST PROCEDURE

A33.1 Loading. The test specimen, prepared and installed in the test fixture in accordance with the provisions of Sections A30 and A32, shall be loaded by producing a tensile force in the loading rod by means of the hydraulic ram. There shall be a sufficient number of load increments to assure the production of a proper load-displacement curve, with a minimum of ten increments used. It is recommended that the magnitude of the load increment be reduced as the ultimate load is approached to improve the accuracy in determining the ultimate load.

A33.2 Measurement of Relative Displacement. Both dial gages, positioned in accordance with Paragraph A32.4.1, shall be read and the readings recorded at the beginning of the test (zero load) and at each load increment thereafter. The relative displacement of the two component parts of the test specimen shall be the difference in the displacements of the two sets of shear plates.

A34. EVALUATION OF TEST RESULTS

A34.1 The ultimate strength of the connection in single shear, per fastener, shall be taken as one-half the value of the ultimate load attained in the test.

A34.2 The load-deformation curve of the connection in single shear shall be the load-deformation curve obtained from the test, with the values of the load taken as one-half those obtained from the test.
APPENDIX B

TEST EVALUATION METHOD FOR SINGLE SHEAR, PULL-OVER AND PULL-OUT TESTS OF MECHANICAL CONNECTIONS

B1. SCOPE

B1.1 This method applies only to single shear, pull-over and pull-out tests conducted strictly in accordance with the "Recommended Procedures for Conducting Pull-over, Pull-out and Single Shear Tests of Mechanical Connections," hereafter referred to as the Recommended Procedures. All restrictions and limitations cited in the Recommended Procedures are applicable herein.

B1.2 All definitions given in the Recommended Procedures are valid herein. Particular attention should be paid to the definition of the ultimate strength of the connection.

B2. SAMPLE SELECTION

B2.1 Each sample shall consist of no less than three test specimens.

B2.1.1 The sample mean shall be defined as the arithmetic average of the ultimate strengths of the specimens constituting the sample.

B2.1.2 No values shall be excluded in the determination of the sample mean except under the provisions of Paragraphs A2.3 and A2.4 of the Recommended Procedures.

B2.2 Each sample shall represent the connection strength resulting from a particular set of conditions. Changes in
conditions which may affect connection strength shall require a separate sample to represent the altered state. Such changes shall include, but not be limited to, the following: use of material with different material or geometrical properties, alterations in the fastener head assembly or fastener accessories, changes in the fastener hole creation techniques or equipment, variations in the fastener driving and/or tightening techniques or equipment.

B2.3 The extreme values of the sample shall be within \( \pm 10 \) percent of the sample mean for pull-out and single shear (lap joint) tests and \( \pm 15 \) percent of the sample mean for pull-over and single shear (simulated diaphragm action) tests. Failure to meet this criterion shall require an increase in the sample size to a minimum of six specimens, and all values shall be included in the determination of the sample mean.

B2.3.1 Scatter greater than the allowable may be an indication that the test procedures are not being properly followed. The procedures, test set-up and equipment should therefore be reviewed prior to conducting the addition tests.

B2.3.2 Excessive scatter may also be in indication that one of the specimens represents a different set of conditions than the others. This possibility should be checked, and if found to exist a separate sample shall be taken from each set of conditions.

B3. DETERMINATION OF THE DESIGN STRENGTH

B3.1 The design strength shall be defined as the product of the sample mean \( R_n \) and a resistance factor \( \phi \) selected from the following table:
Workmanship and Inspection

<table>
<thead>
<tr>
<th>Good</th>
<th>Average</th>
<th>Below Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>0.70</td>
<td>0.65</td>
</tr>
<tr>
<td>0.70</td>
<td>0.625</td>
<td>0.55</td>
</tr>
<tr>
<td>0.65</td>
<td>0.55</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Temporary Structures
Standard Structures
Vital Structures

B3.2 The appropriate resistance factor $\phi$ shall be determined by the nature of the actual application and the quality of workmanship and degree of inspection at the construction site. The classification of workmanship and inspection as "good", "average" or "below average" shall be based on the following:

B3.2.1 The term "good workmanship and inspection" shall imply at least a 95 percent confidence that the strengths of connections made in the actual application are within $\pm 10$ percent of the strengths of the connections made and tested in the laboratory.

B3.2.2 The term "average workmanship and inspection" shall imply at least a 95 percent confidence that the strengths of connections made in the actual application are within $\pm 20$ percent of the strengths of the connections made and tested in the laboratory.

B3.2.3 The term "below average workmanship and inspection" shall imply at least a 95 percent confidence that the strengths of connections made in the actual application are within $\pm 30$ percent of the strengths of the connections made and tested in the laboratory.
B4. APPLICATION OF THE DESIGN STRENGTH

B4.1 The determination of the design strength $\varnothing R_n$ was based on the philosophy that the connection should be stronger than the parts it connects.

B4.2 The design strength only accounts for uncertainties that may affect the strength of the connection, and does not account for any uncertainties in loading. The design loads should therefore be selected to include the uncertainties associated with the loadings.

B4.3 In the absence of additional guidance, the following design criterion may be used for the combination of dead and live loads:

$$ \text{Design Strength } \varnothing R_n \geq 1.1 \left(1.1 \ c_D \ m_D + \gamma \ c_L \ m_L\right) $$

where $m_D$ and $m_L$ are the mean values of the dead and live load intensities, $c_D$ and $c_L$ are influence coefficients translating load intensities into load effects (e.g. single shear per fastener), and $\gamma$ is a live load factor equal to 1.3, 1.4 and 1.6 for temporary, standard and vital structures, respectively.\(^1\) The loads specified by building authorities may be used for the values of $m_D$ and $m_L$ in many instances.

B4.3.1 This design criterion shall not be used for designs involving repeated loads. The Recommended Procedures define only the static strength of a connection.

---

1. This design criterion and load factors derive from:
B3.2 The above criterion shall not be used for connection design against pull-over with main members that exhibit torsional behavior under load.

B5. DEFLECTION CONSIDERATIONS

B5.1 Single shear designs may be governed by the deflection at service loads rather than ultimate strength. Service load deflections may be determined from the load-deformation curves obtained from the tests performed in accordance with the Recommended Procedures. The following design criterion may be used if the deflection limitation is not satisfied:

Allowable Deflection Strength \( \delta_{R_n} \geq c_D m_D + c_L m_L \)

where \( \delta_{R_n} \) is the strength on the load-deformation curve corresponding to the allowable deflection and \( c_D, c_L, m_D \) and \( m_L \) are as defined in Section B4.3 of this Evaluation Method.

B6. REQUIRED DOCUMENTATION

B6.1 The test results of each sample shall be recorded. The record shall contain at least the following information:

a) Date of tests
b) Name(s) of person(s) performing tests
c) Type of test
d) Brief description of the fasteners, fastener accessories and materials used in the connections tested
e) Ultimate strength of each specimen
f) Mean value of the sample

B6.2 Samples for designs that may be governed by deflection considerations rather than strength shall have the load-deformation curves of the specimens attached to the record.
APPENDIX C

SAFETY FACTORS

The nominal "safety factors" that are associated with the modified resistance factors $\bar{f}$ proposed in Section 6.1.7 and employed in the test evaluation method can be determined from the load and resistance factor design criterion. This criterion is expressed by Eq. 6.7 and is suggested in paragraph B4.3 of the test evaluation method.

Dividing Eq. 6.7 by $\bar{f}$ results in

$$R_n \geq \frac{\gamma_A}{\bar{f}} (\gamma_D c_D m_D + \gamma_L c_L m_L)$$

with the variables as defined in Section 5.2.1. The load factors $\gamma$ may be determined from Eqs. 6.3-6.5. Use of the coefficients of variation employed in Reference 33 and cited in Section 6.1.3, and a safety index $\beta$ of 2.5, 3.0 and 4.5 for temporary, standard and very important structures, results in $\gamma_A = \gamma_D = 1.1$ and $\gamma_L = 1.3, 1.4$ and 1.6 for temporary, standard and vital structures. The proposed values of $\bar{f}$ are then sufficient to determine the safety factors as a function of the ratio of live to dead load effects $c_L m_L / c_D m_D$.

A plot of the safety factor versus $c_L m_L / c_D m_D$ for connections made with average workmanship and inspection is presented as Figure C1. The safety factor increases with increasing ratios of live to dead load effects because of the greater uncertainty associated with the live load effects. For typical ratios of
live to dead load the safety factor is nearly constant and may be taken as 2.0, 2.4 and 3.1 for connections in temporary, standard and vital structures made with average workmanship and inspection.

The safety factors associated with good and below average workmanship and inspection may be obtained by direct scaling. Hence typical safety factors associated with connections made with good workmanship and inspection in temporary, standard and very important structures are 1.9, 2.1 and 2.6, respectively. These values are below those for average workmanship because they reflect the reduced uncertainty produced by good workmanship and inspection. The corresponding values for below average workmanship and inspection are 2.2, 2.7 and 3.8.
APPENDIX D

PROBABILISTIC CONCEPTS

This appendix is taken as is in its entirety from Appendix A of Ref. 33. The author wishes to acknowledge the permission of Prof. T. V. Galambos to include this material in this report.
APPENDIX 4

PROBABILISTIC CONCEPTS

INTRODUCTION

In the following some definitions and explanations of probabilistic concepts introduced in the Report are given. Though there are many excellent texts available to fulfill this purpose just the same, it is thought that these concepts, however elementary they may seem to those trained in probability theory, may be so unfamiliar to others as to turn them off from this fascinating subject. Hence is the need for the following.

For more detailed discussion with illustrations from civil engineering practice, the book by Benjamin and Cornell is highly recommended. Attention is drawn particularly to Chap. 1, Secs. 2.1, 2.2.1, 2.4.1, 2.4.2, 2.4.4, 3.1.1, 3.1.2, 3.2.1 and 3.3.

PROBABILITY

If we can predetermine the outcome (e.g., saying that the yield stress will be 40 ksi) of an experiment (in a most general sense) when it is planned (e.g., testing a tension coupon) then the experiment is deterministic. In engineering practice, we cannot make such an absolute statement; we are uncertain about the outcome because of natural variations or of our incomplete professional knowledge. When the element of uncertainty is to be considered explicitly, the engineering problem is probabilistic and subject to analysis by the rules of a branch of mathematics known as theory of probability.

The qualitative or quantitative outcome of an experiment conducted under completely defined conditions is called an event. An event which may or may not occur under a given set of conditions is known as a random event. The yield stress of the tension coupon may lie either between 36 ksi and 38 ksi or between 40 ksi and 42 ksi. Both events are random; however, their
possibilities of occurrence are not identical. The mathematical estimation (assignment of "weight") of the possibility of occurrence of a random event is its probability.

Two most important of the several interpretations of probability are: the relative frequency and subjective probability.

1. **Relative Frequency**

When the experiment under consideration is repeated \( N \) times and if the event \( A \) occurs \( n \) times, then the probability of the event \( A \), denoted \( P(A) \), is defined as the limit of the relative frequency \( n/N \) of the occurrence of \( A \)

\[
P(A) = \lim_{N \to \infty} \frac{n}{N}
\]

(D1)

The classical reference to this interpretation of probability is the book by Richard von Mises D2.

2. **Subjective Probability**

The probability of an event is a subjective measure of the degree of belief one has in a proposition. For an excellent discussion of this interpretation, see Tribus D3.

Whatever be the interpretation given to the concept of probability (note: it is a concept and not a physical property), the assignment of "weights" or probabilities to the events should satisfy the following axioms:

**Axiom I.** The probability of an event is a number greater than or equal to zero but less than or equal to unity.

\[
0 \leq P(A) \leq 1
\]

(D2)

**Axiom II.** The probability of the certain event \( S \) is unity

\[
P(S) = 1
\]

(D3)

where \( S \) is the event associated with all outcomes of an experiment.
Axiom III. The probability of an event which is the union of two mutually exclusive events (events which cannot occur simultaneously as a result of an experiment) is the sum of the probabilities of these two events:

\[ P[A \cup B] = P[A] + P[B], \]

where the symbol \( \cup \), read union of, means, in general, the occurrence of either the event A or the event B or both.

Probabilities are also expressed as percentages.

**RANDOM VARIABLE**

A numerical variable associated with random events is called a random variable, and as such its specific value cannot be determined before an experiment.

The behavior of a random variable is usually described by its probability density function which is defined as follows.

If \( X \) is a random variable, \( x \) is a specific value it takes. \( X \) is called a continuous random variable if it can take any value \( x \) on the real axis (\( X \) could also take values only in a finite interval \( (a,b) \)). The probability density function of a continuous random variable \( f_X(x) \) is defined such that the probability that \( X \) is in interval \( x \) to \( x + dx \) is \( f_X(x)dx \) (see Fig. A.1)

The probability density function must satisfy the following two conditions:

\[ f_X(x) > 0 \] \hspace{1cm} (D5)

\[ \int_{-\infty}^{\infty} f_X(x)dx = 1 \] \hspace{1cm} (D6)

Alternatively, a random variable can be characterized by its cumulative distribution function \( F_X(x) \)
\[ F_X(x) = P[X \leq x] = P[-\infty \leq X \leq x] = \int_{-\infty}^{x} f_X(x_0) \, dx_0 \quad (D7) \]

(where the dummy variable of integration \( x_0 \) is used to avoid confusion with the limit of integration \( x \)).

\[
\frac{dF_X(x)}{dx} = f_X(x) \quad (D8)
\]

**Jointly distributed random variable**

When two or more random variables are being considered simultaneously, their joint behavior is described a joint cumulative distribution function or equivalently by a joint probability density function.

Consider two random variables \( X \) and \( Y \). The joint probability density function \( f_{X,Y}(x,y) \) is defined such that the probability \( X \) lies in the interval \( \{x, x + dx\} \) and \( Y \) lies in the interval \( \{y, y + dy\} \) is \( f_{X,Y}(x,y) \, dx \, dy \) (see Fig.A2).

With this definition,

\[
P[(x_1 \leq X \leq x_2) \text{ and } (y_1 \leq Y \leq y_2)] = \int_{x_1}^{x_2} \int_{y_1}^{y_2} f_{X,Y}(x,y) \, dx \, dy \quad (D9)
\]

Similar to the case of a single random variable,

\[
f_{X,Y}(x,y) \geq 0 \quad (D10)
\]

\[
\int \int f_{X,Y}(x,y) = 1 \quad (D11)
\]

The joint cumulative distribution function \( F_{X,Y}(x,y) \) is

\[
F_{X,Y}(x,y) = P[(X \leq x) \text{ and } (Y \leq y)] = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{X,Y}(x_0,y_0) \, dx_0 \, dy_0 \quad (D12)
\]

(Nota the use of dummy variables \( x_0 \) and \( y_0 \))
While studying the joint behavior of random variables (say, \( X \) and \( Y \)), two special types of probability density functions are important.

**Marginal Probability Density Function**

The *marginal probability density function* \( f_X(x) \) describes the behavior of \( X \) only, when one ignores the random variable \( Y \) and is obtained by integrating the joint density function over all values of \( Y \).

\[
f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) \, dy \tag{D13}
\]

Similarly, \( f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) \, dx \tag{D14} \)

**Conditional Probability Density Function**

The *conditional probability density function* of \( X \) given \( Y \), \( f_{X|Y}(x,y) \) is defined as

\[
f_{X|Y}(x,y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} \tag{D15}
\]

**Independent Random Variables**

Two random variables \( X \) and \( Y \) are said to be *stochastically independent* if the conditional density function \( f_{X|Y}(x,y) \) is identical to the marginal density function \( f_X(x) \)

i.e. \( f_{X|Y}(x,y) = f_X(x) \tag{D16} \)

Stochastic independence is an important concept. Eq. A14 says that the knowledge about \( Y \) (e.g., \( Y = y \) and \( Y \leq y \)) does not provide any additional information to describe the behavior of \( X \) (other than its marginal density function \( f_X(x) \)). Alternatively,

\[
f_{X,Y}(x,y) = f_X(x) f_Y(y) \tag{D17}
\]

Assumption of independence helps to simplify probabilistic analysis; but should be made based on experience, judgment and with caution.
Moments of a Random Variable

In many engineering problems, knowledge of the probability density function (or the cumulative distribution function) over the range of the variable may not be possible or sometimes be essential; it is sufficient to find some numerical descriptors which summarize the dominant features of the behavior of a random variable. Such descriptors are called moments of a random variable.

The mean $X_m$ of a random variable $X$ is defined as

$$ X_m = \int_{-\infty}^{\infty} x f_x(x) \, dx $$

(D18)

In the mean, we are condensing the information in the probability density function $f_x(x)$ into a single number ($X_m$) by summing over all possible values of $X$ the product of the value of $x$ and its probability of occurrence $f_x(x) \, dx$ (see Fig. D.3). In practical problems, the arithmetical average of a large number of observations can be used to approximate closely the mean of the underlying random variable. Thus, the mean describes the central tendency of a random variable.

The variance $\sigma_x^2$ is the most common and useful measure of the scatter or dispersion of a random variable. It is defined as the weighted average of the squared deviations from the mean (Fig. D.3).

$$ \sigma_x^2 = \text{Var}[X] = \int_{-\infty}^{\infty} (x - X_m)^2 f_x(x) \, dx $$

(D19)

The variance $\sigma_x^2$ is the second central moment of the area of the probability density function with respect to its center of gravity $X_m$. Smaller variances indicate that the variable is less widely spread about the mean.

The positive square root of the variance is called the standard deviation $\sigma_x$.

$$ \sigma_x = (\int_{-\infty}^{\infty} (x - X_m)^2 f_x(x) \, dx)^{\frac{1}{2}} $$

(D20)
It should be noted that the mean $X_m$ and the standard deviation $\sigma_X$ have the same units as the variable $X$ itself, e.g. if $X$ is the random variable yield stress of steel measured in kips per square inch, the mean $X_m$ and the standard deviation $\sigma_X$ are also expressed in kips per square inch.

A non-dimensional characteristic is of special importance: coefficient of variation $V_X$. It is defined as

$$V_X = \frac{\sigma_X}{X_m} \quad (D21)$$

For example, we say that the coefficient of variation of the yield stress of steel is 0.11 or 11 percent implying that a steel (A 36 data) of mean yield stress 44.0 ksi has a standard deviation in yield stress of about 5 ksi. The coefficient of variation of a random variable is easily understood in engineering practice (e.g. in quality control). It is also useful in comparing variables of different units.

A coefficient that characterizes the asymmetry of the probability density function of a random variables is the coefficient of skewness $g_1$.

$$g_1 = \frac{\int_{-\infty}^{\infty} (x-X_m)^3f_X(x)dx}{\sigma_X^3} \quad (D22)$$

If a distribution is symmetrical, this coefficient is zero (converse is not necessarily true). Positive values of $g_1$ correspond to the probability density functions with dominant tails on the right; negative values to long tails on the left (see Fig. D4).

**Nominal Value**

A nominal value $X_n$ of a random variable $X$ is defined with reference to a probability level. A nominal maximum value $X_{n,\text{max}}$ is defined such that the probability that the random variable $X$ exceeds this value $X_{n,\text{max}}$ is $p$ (see Fig. D5). A nominal minimum value $X_{n,\text{min}}$ is defined such that the
probability that the random variable $X$ falls below this value $(X_{n,\text{min}})$ is $p_1$.

The difference between $X_n$ and $X_m$ is usually expressed as a number $K$ of standard deviations of $X$. This number $K$ relates to the probability $p$ (or $p_1$.)

$$X_n = X_m + K\sigma_X$$  \hspace{1cm} (D23)

The nominal value is also referred to in literature as characteristic value, (Comité Europeen du Beton) percentile or quantile.

**Covariance**

The joint behavior of two random variables is usually summarized by the covariance $\sigma_{X,Y}$:

$$\sigma_{X,Y} = \int_{-\infty}^{\infty} (x - X_m)(y - Y_m) f_{X,Y}(x,y)dx dy$$  \hspace{1cm} (D24)

The covariance corresponds to the product moment of inertia with respect to the axes in the $x$ and $y$ direction passing through the centroid of a thin plate of variable density.

**Correlation Coefficient**

The correlation coefficient $\rho_{X,Y}$ is a dimensionless characteristic obtained by dividing the covariance of $X$ and $Y$ by the product of their standard deviations.

$$\rho_{X,Y} = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y}$$  \hspace{1cm} (D25)

Usually $\rho_{X,Y}$ lies between -1 and 1. Fig. D6 shows the joint density function contours of correlated random variables. It should be noted that $\rho_{X,Y}$ is only a measure of linear dependence between $X$ and $Y$. 

NORMAL PROBABILITY DISTRIBUTION

Normal (or Gaussian) probability distribution is the most widely used model in applied probability theory. The normal density function \( f_X(x) \) is defined as (see Fig. A7).

\[
f_X(x) = \frac{1}{\sigma_X \sqrt{2\pi}} \exp \left[ -\frac{(x - \mu)^2}{2\sigma^2} \right]
\]

where \( \mu \) and \( \sigma \), the mean and the standard deviation of the variable \( X \), are the two parameters of the distribution, in that they completely define the distribution.

The cumulative distribution function \( F_X(x) \) is

\[
F_X(x) = \int_{-\infty}^{x} f_X(u) \, du
\]

In most text books (for example, Benjamin and Cornell\(^1\), Table D1) the density function and the cumulative distribution function of a standardized normal random variable are tabulated. The standardized normal random variable \( U \) is defined as

\[
U = \frac{X - \mu}{\sigma_X}
\]

and has a mean 0 and standard deviation 1. The density function \( f_U(u) \) and the cumulative distribution function \( F_U(u) \) are given by

\[
f_U(u) = \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{1}{2} u^2 \right] = \sigma_X f_X(x)
\]

and

\[
F_U(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} \exp \left[ -\frac{1}{2} v^2 \right] dv = F_X(x)
\]

for \(-\infty \leq u \leq \infty\).
From the tabulated standardized variable, it is seen that the probability that the variable lies within ±1σ about mean is 67 percent, ±2σ about mean is 95 percent and ±3σ about mean is 99.5 percent. In this context, the importance of the standard deviation as a measure of dispersion is realized. Greater the standard deviation, greater are these intervals about the mean for specified probability levels (such as 67, 95 and 99.5 percent).

The nominal values are also understood easily with reference to a normal distribution. The nominal value \( X_{n,\text{max}} \) which is exceeded only with a 5 percent probability is \( K (=1.64) \) number of standard deviations from the mean i.e.,

\[
X_n = X_m + 1.64\sigma_X
\]

**EXTREME VALUE DISTRIBUTIONS**

In structural design, we are often interested in the largest or smallest of a number of random variables; for example, our concern may lie in the performance of a structure under the maximum load. Resistance of a structure could also be modelled as the strength of the weakest of many elementary components.

**Type I:** Distribution of largest value (Gumbel distribution)

\[
F_X(x) = \exp [-e^{-\alpha(x-q_1)}] \quad -\infty \leq x \leq \infty
\]

\[
X_m = q_1 + \frac{0.577}{\alpha} \quad, \quad \sigma_X = \frac{1.282}{\alpha}
\]

**Type II:** Distribution of smallest value

\[
F_X(x) = 1 - \exp [-e^{-\alpha(x-q_2)}] \quad -\infty \leq x \leq \infty
\]
Type II: Distribution of largest value

\[ F_X(x) = \exp \left(- \frac{q_3}{x} \right) \]  

(D36)

The parameters \( q_3 \) and \( k \) are expressed in terms of the mean \( X_m \) and standard deviation \( \sigma_X \).

Type III: Distribution of smallest value

\[ F_X(x) = 1 - \exp \left(- \frac{x - \varepsilon}{w - \varepsilon} \right)^{k_1} \quad \text{for} \quad x \geq \varepsilon \]  

(D37)

The parameters \( w \) and \( k_1 \) are expressed in terms of \( X_m \) and \( \sigma_X \).

**SECOND MOMENT THEORY**

In traditional engineering practice, all variables and processes were characterized by typical values such as best estimates or some conservative estimates. In an ideal probabilistic analysis, the complete probability law (density or function) is needed to describe a random variable. The traditional approach does not explicitly recognize the "variability" of the variable; whereas the ideal probabilistic approach in offering the complete description of the variable makes the analysis highly complicated; in addition, we do not have enough data to describe any variable completely. As a first order probabilistic approach, the first two moments are used to characterize random variables. Means, standard deviations and correlation coefficients concisely describe the best predictions, the uncertainty and the joint behavior of variables. Simple first order relationships between these characteristics
(i.e., means, standard deviations and correlation coefficients) can be developed when the variables themselves are related.

MOMENT ALGEBRA

Some simple relationships for the means, standard deviations and correlation coefficients of functions of random variables in terms of the first two moments of the component variables are presented here. It is useful to define a mathematical symbol; the expectation operator.

The expected value (mean) of a function $h(X)$ of a random variable $X$ is obtained by summing over all possible values of $X$, the product of $h(x)$ and its probability of occurrence $f_X(x)dx$

$$E[h(x)] = \int_{-\infty}^{\infty} h(x) f_X(x)dx$$  \hspace{1cm} (D38)

$E[\cdot]$ is known as the expectation operator. When $h(X) = [X - X_m]^2$, the resulting expectation is called $\text{Var}[X]$ and $\text{Var}[\cdot]$ is also used as variance operator.

**Properties of Expectation**

$$E[c] = c$$

$$E[cX] = cX_m$$

$$E[a + bX] = a + bX_m$$  \hspace{1cm} (D39)

$$E[h_1(X) + h_2(X)] = E[h_1(X)] + E[h_2(X)]$$

where $a$, $b$ and $c$ are deterministic constants

$$\text{Var}[X] = E[X^2] - (X_m)^2$$

$$\text{Var}[c] = 0$$

$$\text{Var}[cX] = c^2 \sigma_X^2$$

$$\text{Var}[a + bX] = b^2 \sigma_X^2$$  \hspace{1cm} (D40)
Sum of Random Variables

Let \( Z = X + Y \) \hspace{1cm} (D41)

then \( Z_m = X_m + Y_m \) \hspace{1cm} (D42)

and \( \sigma_Z^2 = \sigma_X^2 + \sigma_Y^2 + 2\rho_{XY}\sigma_X\sigma_Y \) \hspace{1cm} (D43)

If \( X \) and \( Y \) are uncorrelated (\( \rho_{XY} = 0 \))

\[ \sigma_Z^2 = \sigma_X^2 + \sigma_Y^2 \] \hspace{1cm} (D44)

Difference of Two Random Variables

Let \( Z = X - Y \) \hspace{1cm} (D45)

then \( Z_m = X_m - Y_m \) \hspace{1cm} (D46)

and \( \sigma_Z^2 = \sigma_X^2 + \sigma_Y^2 - 2\rho_{XY}\sigma_X\sigma_Y \) \hspace{1cm} (D47)

If \( X \) and \( Y \) are uncorrelated,

\[ \sigma_Z^2 = \sigma_X^2 + \sigma_Y^2 \] \hspace{1cm} (D48)

Notice that if \( X \) and \( Y \) are uncorrelated, whether \( Z \) is the sum or difference of \( X \) and \( Y \), the variances \( \sigma_X^2 \) and \( \sigma_Y^2 \) always add to give \( \sigma_Z^2 \).

In general, if the variable \( Y \) is a linear function of a number of random variables \( X_i \),

\[ Y = \sum_{i=1}^{n} a_i X_i \] \hspace{1cm} (D49)

\[ E[Y] = Y_m = \sum_{i=1}^{n} a_i E[X_i] \] \hspace{1cm} (D50)

and \[ \text{Var}[Y] = \sum_{i=1}^{n} a_i^2 \text{Var}[X_i] + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} a_i a_j \rho_{X_i X_j} \sigma_{X_i} \sigma_{X_j} \] \hspace{1cm} (D51)

If the variables \( X_i \) are uncorrelated,

\[ \text{Var}[Y] = \sum_{i=1}^{n} a_i^2 \text{Var}[X_i] \] \hspace{1cm} (D52)
**Product of Two Random Variables**

Let \( Z = XY \) \( (D53) \)

\[
Z_m = X_m Y_m + \rho_{X,Y} \sigma_X \sigma_Y \quad (D54)
\]

If \( X \) and \( Y \) are uncorrelated,

\[
Z_m = X_m Y_m \quad (D55)
\]

and \( \text{Var}(XY) = X_m^2 \sigma_Y^2 + Y_m^2 \sigma_X^2 + \sigma_X^2 \sigma_Y^2 \) \( (D56) \)

which is simplified to

\[
\nu_Z^2 = \nu_X^2 + \nu_Y^2 + \nu_X^2 \nu_Y^2 \quad (D57)
\]

**Approximations**

If \( Y = g(X_1, X_2, \ldots, X_k) \),

\[
Y_m = g(X_{1m}, X_{2m}, \ldots, X_{km}) \quad (D58)
\]

and

\[
\sigma_Y^2 = \sum_{i=1}^{k} \left( \frac{\partial g}{\partial X_i} \right)^2 \sigma_{X_i}^2 \quad (D60)
\]

where the variables \( X_i \) are uncorrelated and \( \frac{\partial g}{\partial X_i} \) is the partial derivative of \( g(X_1, X_2, \ldots, X_k) \) with respect to \( X_i \) evaluated at \( X_{1m}, X_{2m}, \ldots, X_{km} \). These approximations are used extensively in the Report.

**STOCHASTIC PROCESS**

A random variable \( X(t) \) that is a function of time is called a random (stochastic) process; i.e. the value \( x \) assumed by the random variable \( X \) at a particular time cannot be predetermined but in addition this value changes with time.

For example \( X(t) \) could be the floor live load present at time \( t \) or the force on a building in a wind storm. Fig. B9 shows the variation of wind
force with time. The record indicates a realization or sample function. The collection or ensemble of all such realizations is a stochastic process. The objective of mathematical studies of stochastic processes is to describe the probabilistic properties of the process (the recommended reference is Parzen, E., "Stochastic Processes").
APPENDIX D

REFERENCES

D1. Benjamin, J. R. and Cornell, C. A.
    *Probability, Statistics and Decision for Civil Engineers*

D2. Von Mises, R.
    *Probability, Statistics and Truth*

D3. Tribus, M.
    *Rational Descriptions, Decisions and Designs*
    Pergamon Press, New York, 1969

D4. Comité Européen du Beton
    "International Recommendations for the Design and Construction of
    Concrete Structures"
    *Principles and Recommendations, FIP Sixth Congress, Prague, June 1970.*

D5. Parzen, E.
    *Stochastic Processes*
### APPENDIX D

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, B</td>
<td>random events</td>
</tr>
<tr>
<td>E[·]</td>
<td>expectation operation</td>
</tr>
<tr>
<td>$F_X(x)$</td>
<td>cumulative distribution function of the random variable $X$</td>
</tr>
<tr>
<td>$f_X(x)$</td>
<td>probability density function of the random variable $X$; marginal density function</td>
</tr>
<tr>
<td>$F_{X,Y}(x,y)$</td>
<td>joint cumulative distribution function of the random variables $X$ and $Y$</td>
</tr>
<tr>
<td>$f_{X,Y}(x,y)$</td>
<td>joint probability density function of the random variables $X$ and $Y</td>
</tr>
<tr>
<td>$f_{X/Y}(x,y)$</td>
<td>conditional probability density function of $X$ given $Y</td>
</tr>
<tr>
<td>$g_1$</td>
<td>skewness of a random variable</td>
</tr>
<tr>
<td>$h(X)$</td>
<td>a function of a random variable $X$</td>
</tr>
<tr>
<td>$k$</td>
<td>a number defining the nominal value of a random variable</td>
</tr>
<tr>
<td>$k_1$</td>
<td>parameters of extreme value distributions</td>
</tr>
<tr>
<td>$N$</td>
<td>total number of repetitions</td>
</tr>
<tr>
<td>$n$</td>
<td>number of times the event $A$ occurs</td>
</tr>
<tr>
<td>$P[A]$</td>
<td>probability of event $A$</td>
</tr>
<tr>
<td>$q_1, q_2, q_3$</td>
<td>parameters of extreme value distributions</td>
</tr>
<tr>
<td>$S$</td>
<td>event associated with all outcomes of an experiment (certain event)</td>
</tr>
<tr>
<td>$U$</td>
<td>standardized variable (zero mean, unit variance)</td>
</tr>
<tr>
<td>Var[·]</td>
<td>variance operation</td>
</tr>
<tr>
<td>$V_X$</td>
<td>coefficient of variation of the random variable $X$</td>
</tr>
<tr>
<td>$X, Y$</td>
<td>random variables</td>
</tr>
<tr>
<td>$x$</td>
<td>a specific value that $X$ takes</td>
</tr>
<tr>
<td>$X_m$</td>
<td>mean of the random variable $X$</td>
</tr>
<tr>
<td>$X_n$</td>
<td>nominal value of $X$</td>
</tr>
</tbody>
</table>
\( w, a, \varepsilon \) parameter in extreme value distributions

\( \sigma_X \) standard deviation of \( X \) (\( \sigma_X^2 \) is the variance of \( X \))

\( \sigma_{X,Y} \) covariance of \( X \) and \( Y \)

\( \rho_{X,Y} \) correlation coefficient of \( X \) and \( Y \)

\( U \) "union" of two events
Table 2.1

Average Material Properties for Preliminary Tests

<table>
<thead>
<tr>
<th>Sheet Designation</th>
<th>Thickness (in.)</th>
<th>Yield Stress (ksi)</th>
<th>Ultimate Stress (ksi)</th>
<th>% Elongation in 2 Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Test Material</strong></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>16 Gage</td>
<td>0.060</td>
<td>42.8</td>
<td>51.9</td>
<td>33</td>
</tr>
<tr>
<td><strong>Additional Test Materials</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 Gage</td>
<td>0.123</td>
<td>38.6</td>
<td>48.8</td>
<td>41</td>
</tr>
<tr>
<td>16 Gage</td>
<td>0.061</td>
<td>41.5</td>
<td>48.0</td>
<td>32</td>
</tr>
<tr>
<td>22 Gage</td>
<td>0.030</td>
<td>35.1</td>
<td>45.6</td>
<td>30</td>
</tr>
<tr>
<td>26 Gage</td>
<td>0.021</td>
<td>37.4</td>
<td>45.5</td>
<td>32</td>
</tr>
</tbody>
</table>
Table 2.2

Summary of Initial Test Results

<table>
<thead>
<tr>
<th>Specimen Designation</th>
<th>Single Shear Lap Joint - 1 Fastener</th>
<th>Single Shear Lap Joint - 2 Fasteners</th>
<th>Simulated Diaphragm Action Test</th>
<th>Sheet-to-Sheet Pull-out</th>
<th>Sheet-to-Structural Pull-out</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 Fastener</td>
<td>2 Fasteners</td>
<td>2 Fasteners</td>
</tr>
<tr>
<td>Specimen Designation</td>
<td>Ultimate Load per Fastener (lbs.)</td>
<td>1 Fastener</td>
<td>2 Fasteners</td>
<td>2 Fasteners</td>
<td>2 Fasteners</td>
</tr>
<tr>
<td>SS1A (e = 1/2&quot;)</td>
<td>1250</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS2A (e = 1 1/4&quot;)</td>
<td>1625</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS3A (e = 2 1/2&quot;)</td>
<td>1960</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS1B (e = 1/2&quot;)</td>
<td></td>
<td>1565</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS2B (e = 1 1/4&quot;)</td>
<td></td>
<td>1515</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS3B (e = 2 1/2&quot;)</td>
<td></td>
<td>1625</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T1 (4.25&quot; square)</td>
<td>640</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T2 (6.25&quot; square)</td>
<td>780</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T3 (8.25&quot; square)</td>
<td>835</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sheet Perpendicular</td>
<td>400</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sheet Parallel</td>
<td>990</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2.3

Single Shear Test Results - Equal Thickness Sheets

(Ultimate load in pounds)
(Failure by yield in bearing)

<table>
<thead>
<tr>
<th>Oversize Hole Employed</th>
<th>10 Gage</th>
<th>16 Gage</th>
<th>22 Gage</th>
<th>26 Gage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material Combination</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Group I Lap Joint (Heads on Opposite Sides)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>5840*</td>
<td>3410</td>
<td>1160</td>
<td>630</td>
</tr>
<tr>
<td></td>
<td>6210*</td>
<td>3345</td>
<td>990</td>
<td>615</td>
</tr>
<tr>
<td>No</td>
<td>6040*</td>
<td>3805</td>
<td>1240</td>
<td>735</td>
</tr>
<tr>
<td></td>
<td>5580*</td>
<td>3495</td>
<td>1220</td>
<td>645</td>
</tr>
<tr>
<td></td>
<td>Group II Lap Joint (Heads on Same Side)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>3535</td>
<td>970</td>
<td>725</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3425</td>
<td>925</td>
<td>520</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Simulated Diaphragm Action (Heads on Same Side)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>7000*</td>
<td>3290</td>
<td>1200</td>
<td>710</td>
</tr>
<tr>
<td></td>
<td>6800*</td>
<td>3240</td>
<td>1115</td>
<td>705</td>
</tr>
</tbody>
</table>

*Failure by shearing of the fastener.
<table>
<thead>
<tr>
<th>Oversize Hole Employed</th>
<th>16 Gage</th>
<th>22 Gage</th>
<th>26 Gage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-10 Gage</td>
<td>-10 Gage</td>
<td>-16 Gage</td>
</tr>
<tr>
<td>Group I Lap Joint (Heads on Opposite Sides)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>4670</td>
<td>1925</td>
<td>1765</td>
</tr>
<tr>
<td></td>
<td>4560</td>
<td>1620</td>
<td>1755</td>
</tr>
<tr>
<td>No</td>
<td>5070</td>
<td>1730</td>
<td>1720</td>
</tr>
<tr>
<td></td>
<td>4480</td>
<td>1725</td>
<td>1640</td>
</tr>
<tr>
<td>Group II Lap Joint (Heads on Same Side)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>5340</td>
<td>2100</td>
<td>2395</td>
</tr>
<tr>
<td></td>
<td>5280</td>
<td>1935</td>
<td>2290</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2195</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2130</td>
<td></td>
</tr>
<tr>
<td>Simulated Diaphragm Action (Heads on Same Side)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>6780</td>
<td>2760</td>
<td>1880</td>
</tr>
<tr>
<td></td>
<td>6660</td>
<td>2720</td>
<td>1780</td>
</tr>
</tbody>
</table>
Table 2.5

Double Shear Test Results
(Ultimate load in pounds)
(Failure by yield in bearing)

<table>
<thead>
<tr>
<th>Cover Plate Material</th>
<th>16 Gage</th>
<th>22 Gage</th>
<th>26 Gage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fastener Heads on Same Side</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4120</td>
<td>1490</td>
<td>850</td>
</tr>
<tr>
<td></td>
<td>3950</td>
<td>1470</td>
<td>775</td>
</tr>
<tr>
<td></td>
<td>4120</td>
<td>1505</td>
<td>830</td>
</tr>
<tr>
<td></td>
<td>4015</td>
<td>1465</td>
<td>825</td>
</tr>
<tr>
<td></td>
<td>Fastener Heads on Opposite Sides</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4150</td>
<td>1090</td>
<td>695</td>
</tr>
<tr>
<td></td>
<td>3840</td>
<td>1025</td>
<td>650</td>
</tr>
<tr>
<td></td>
<td>3940</td>
<td>1120</td>
<td>675</td>
</tr>
<tr>
<td></td>
<td>3620</td>
<td>1100</td>
<td>655</td>
</tr>
</tbody>
</table>
Table 2.6

Pull-out Test Results
(Sheet clamped on 6 inch square)
(Ultimate load in pounds)

<table>
<thead>
<tr>
<th>Test Material</th>
<th>10 Gage</th>
<th>16 Gage</th>
<th>22 Gage</th>
<th>26 Gage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pull-out from Formed Channel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1240</td>
<td>766</td>
<td>376</td>
<td>285</td>
<td></td>
</tr>
<tr>
<td>1220</td>
<td>734</td>
<td>356</td>
<td>267</td>
<td></td>
</tr>
<tr>
<td>1400</td>
<td>745</td>
<td>338</td>
<td>227</td>
<td></td>
</tr>
<tr>
<td>1365</td>
<td>712</td>
<td>336</td>
<td>184</td>
<td></td>
</tr>
<tr>
<td></td>
<td>752</td>
<td>371</td>
<td>310</td>
<td></td>
</tr>
<tr>
<td></td>
<td>651</td>
<td>306</td>
<td>263</td>
<td></td>
</tr>
<tr>
<td></td>
<td>735</td>
<td>326</td>
<td>249</td>
<td></td>
</tr>
<tr>
<td></td>
<td>661</td>
<td>102*</td>
<td>216</td>
<td></td>
</tr>
<tr>
<td>Mean Values</td>
<td>1306</td>
<td>720</td>
<td>344</td>
<td>250</td>
</tr>
<tr>
<td>Pull-out from Clamped Sheet</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1325</td>
<td>517</td>
<td>247</td>
<td>177</td>
<td></td>
</tr>
<tr>
<td>1295</td>
<td>483</td>
<td>219</td>
<td>159</td>
<td></td>
</tr>
<tr>
<td>1270</td>
<td>585</td>
<td>310</td>
<td>71*</td>
<td></td>
</tr>
<tr>
<td>1240</td>
<td>496</td>
<td>302</td>
<td>243</td>
<td></td>
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<tr>
<td>1270</td>
<td>536</td>
<td>297</td>
<td>193</td>
<td></td>
</tr>
<tr>
<td>1210</td>
<td>527</td>
<td>281</td>
<td>218</td>
<td></td>
</tr>
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<td></td>
<td>519</td>
<td>272</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>492</td>
<td>267</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Values</td>
<td>1268</td>
<td>519</td>
<td>274</td>
<td>192</td>
</tr>
</tbody>
</table>

*Connected member broke loose prior to full distortion, probably due to partial stripping of formed threads during fastener driving and tightening.
Table 2.7

Comparison of Pull-out Test Results
(Sheet clamped on 6 inch vs. 3 inch square)
(Ultimate load in pounds)

<table>
<thead>
<tr>
<th>Test Material</th>
<th>10 Gage</th>
<th>16 Gage</th>
<th>22 Gage</th>
<th>26 Gage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Mean</strong> Values</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 Gage</td>
<td><strong>1268</strong></td>
<td>519</td>
<td>274</td>
<td>192</td>
</tr>
<tr>
<td>16 Gage</td>
<td><strong>1268</strong></td>
<td>519</td>
<td>274</td>
<td>192</td>
</tr>
<tr>
<td>22 Gage</td>
<td><strong>1283</strong></td>
<td>583</td>
<td>256</td>
<td>166</td>
</tr>
<tr>
<td>26 Gage</td>
<td><strong>1283</strong></td>
<td>583</td>
<td>256</td>
<td>166</td>
</tr>
</tbody>
</table>

*Threads partially stripped during fastener tightening.

**Excluded from mean value. If included, mean = 1249.
Table 2.8

Pull-over Test Results
(Ultimate load in pounds)

<table>
<thead>
<tr>
<th>Test Material</th>
<th>22 Gage</th>
<th>26 Gage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pull-over of Formed Channel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Value</td>
<td>926</td>
<td>833</td>
</tr>
<tr>
<td></td>
<td>1003</td>
<td>953</td>
</tr>
<tr>
<td></td>
<td></td>
<td>928</td>
</tr>
<tr>
<td>Pull-over of Sheet</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Values</td>
<td>1130</td>
<td>798</td>
</tr>
<tr>
<td></td>
<td>1127</td>
<td>767</td>
</tr>
<tr>
<td></td>
<td>1129</td>
<td>783</td>
</tr>
<tr>
<td>Pull-over of Sheet Clamped on 6 Inch Square</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Values</td>
<td>1148</td>
<td>826</td>
</tr>
<tr>
<td></td>
<td>1120</td>
<td>697</td>
</tr>
<tr>
<td></td>
<td>1134</td>
<td>762</td>
</tr>
</tbody>
</table>
Table 3.1
A Comparison of Single Shear Lap Joint and Simulated Diaphragm Action Test Results
(Load in pounds per fastener)

<table>
<thead>
<tr>
<th>Type of Test</th>
<th>Material Combination</th>
<th>Ultimate Load</th>
<th>10 Gage</th>
<th>16 Gage</th>
<th>22 Gage</th>
<th>26 Gage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>-16 Gage</td>
<td>-22 Gage</td>
<td>-26 Gage</td>
<td>-16 Gage</td>
</tr>
<tr>
<td>Lap</td>
<td></td>
<td></td>
<td>2670</td>
<td>1050</td>
<td>770</td>
<td>1765</td>
</tr>
<tr>
<td>Joint</td>
<td></td>
<td></td>
<td>2640</td>
<td>970</td>
<td>685</td>
<td>1710</td>
</tr>
<tr>
<td>Diaphragm</td>
<td></td>
<td></td>
<td>3390</td>
<td>1380</td>
<td>790</td>
<td>1645</td>
</tr>
<tr>
<td>Action</td>
<td></td>
<td></td>
<td>3330</td>
<td>1360</td>
<td>750</td>
<td>1620</td>
</tr>
<tr>
<td></td>
<td>&quot;Yield&quot; Load</td>
<td></td>
<td>2540</td>
<td>970</td>
<td>685</td>
<td>1645</td>
</tr>
<tr>
<td>Lap</td>
<td></td>
<td></td>
<td>2500</td>
<td>940</td>
<td>680</td>
<td>1630</td>
</tr>
<tr>
<td>Joint</td>
<td></td>
<td></td>
<td>970</td>
<td>965</td>
<td></td>
<td>445</td>
</tr>
<tr>
<td>Diaphragm</td>
<td></td>
<td></td>
<td>3120</td>
<td>1170</td>
<td>750</td>
<td>1645</td>
</tr>
<tr>
<td>Action</td>
<td></td>
<td></td>
<td>2970</td>
<td>1125</td>
<td>725</td>
<td>1620</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>460</td>
</tr>
</tbody>
</table>
Table 3.2

Average Material Properties for Tests to Estimate Ultimate Load Distributions and Verify Fastener Spacing

<table>
<thead>
<tr>
<th>Specimen Designation</th>
<th>Thickness (in.)</th>
<th>Yield Stress (ksi)</th>
<th>Ultimate Stress (ksi)</th>
<th>% Elongation in 2 Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 Gage</td>
<td>0.063</td>
<td>35.1</td>
<td>43.3</td>
<td>33</td>
</tr>
<tr>
<td>18 Gage</td>
<td>0.048</td>
<td>39.9</td>
<td>46.5</td>
<td>31</td>
</tr>
<tr>
<td>26 Gage</td>
<td>0.020</td>
<td>33.2</td>
<td>42.4</td>
<td>32</td>
</tr>
</tbody>
</table>
Table 3.3
Results of Single Shear Tests to Estimate Ultimate Load Distributions
(Load in pounds per fastener)
(Failure by yield in bearing)

<table>
<thead>
<tr>
<th>Sample Characteristics</th>
<th>Lap Joint Tests</th>
<th>Simulated Diaphragm Action Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&quot;Yield&quot;</td>
<td>Ultimate</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>900</td>
<td>1155</td>
<td>1010</td>
</tr>
<tr>
<td>1025</td>
<td>1205</td>
<td>810</td>
</tr>
<tr>
<td>1030</td>
<td>1235</td>
<td>970</td>
</tr>
<tr>
<td>940</td>
<td>1265</td>
<td>975</td>
</tr>
<tr>
<td>1110</td>
<td>1370</td>
<td>960</td>
</tr>
<tr>
<td>1080</td>
<td>1270</td>
<td>950</td>
</tr>
<tr>
<td>1065</td>
<td>1325</td>
<td>970</td>
</tr>
<tr>
<td>1035</td>
<td>1250</td>
<td>940</td>
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<tr>
<td>1060</td>
<td>1350</td>
<td>1010</td>
</tr>
<tr>
<td>1050</td>
<td>1300</td>
<td>1020</td>
</tr>
<tr>
<td>1035</td>
<td>1200</td>
<td>950</td>
</tr>
<tr>
<td>1085</td>
<td>1300</td>
<td>1010</td>
</tr>
<tr>
<td>1050</td>
<td>1290</td>
<td>1040</td>
</tr>
<tr>
<td>1025</td>
<td>1230</td>
<td>890</td>
</tr>
<tr>
<td>1050</td>
<td>1290</td>
<td>870</td>
</tr>
<tr>
<td>1050</td>
<td>1265</td>
<td>1000</td>
</tr>
<tr>
<td>1055</td>
<td>1305</td>
<td>1000</td>
</tr>
<tr>
<td>1040</td>
<td>1320</td>
<td>950</td>
</tr>
<tr>
<td>1000</td>
<td>1380</td>
<td>900</td>
</tr>
<tr>
<td>1000</td>
<td>1230</td>
<td>930</td>
</tr>
<tr>
<td>1010</td>
<td>1250</td>
<td>940</td>
</tr>
<tr>
<td>1000</td>
<td>1210</td>
<td>860</td>
</tr>
<tr>
<td>1000</td>
<td>1280</td>
<td>1050</td>
</tr>
<tr>
<td>1045</td>
<td>1330</td>
<td>940</td>
</tr>
<tr>
<td>915</td>
<td>1220</td>
<td>1040</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Value</td>
<td>1025</td>
<td>960</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>0.047</td>
<td>0.062</td>
</tr>
<tr>
<td>Median</td>
<td>1035</td>
<td>960</td>
</tr>
<tr>
<td>Range</td>
<td>210</td>
<td>240</td>
</tr>
</tbody>
</table>
Table 3.4

Results of Single Shear Tests with a Ten Inch Fastener Spacing

(Ultimate load in pounds per fastener) (Failure by yield in bearing)

<table>
<thead>
<tr>
<th>Lap Joint</th>
<th>Simulated Diaphragm Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>1185</td>
<td>1170</td>
</tr>
<tr>
<td>1345</td>
<td>1025</td>
</tr>
<tr>
<td>1250</td>
<td>1150</td>
</tr>
<tr>
<td>1225</td>
<td>1245</td>
</tr>
<tr>
<td>1300</td>
<td>1185</td>
</tr>
</tbody>
</table>

Mean Values

| 1260      | 1155                        |

Mean of 25 Tests with a 2" Fastener Spacing

| 1275      | 1145                        |

Percent Difference

| -1.2      | +0.9                        |
Table 3.5

Results of Single Shear Tests with Various Specimen Configurations

(Ultimate load in pounds per fastener)
(Failure by yield in bearing)

<table>
<thead>
<tr>
<th>Configuration A</th>
<th>Configuration B</th>
<th>Configuration C</th>
<th>Lap Joint</th>
<th>Simulated Diaphragm Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>1005</td>
<td>1125</td>
<td>1385</td>
<td>1185</td>
<td>1195</td>
</tr>
<tr>
<td>1145</td>
<td>1060</td>
<td>1305</td>
<td>1090</td>
<td>1050</td>
</tr>
<tr>
<td>1025</td>
<td>1095</td>
<td>1430</td>
<td>1135</td>
<td>1010</td>
</tr>
<tr>
<td>1090</td>
<td>1150</td>
<td>1345</td>
<td>1115</td>
<td>1170</td>
</tr>
<tr>
<td>1115</td>
<td>1040</td>
<td>1400</td>
<td>1170</td>
<td>1095</td>
</tr>
</tbody>
</table>

Mean Values

| 1075 | 1095 | 1375 | 1140 | 1105 |
Table 3.4

Results of Single Shear Tests with a Ten Inch Fastener Spacing
(Ultimate load in pounds per fastener) (Failure by yield in bearing)

<table>
<thead>
<tr>
<th>Lap Joint</th>
<th>Simulated Diaphragm Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>1185</td>
<td>1170</td>
</tr>
<tr>
<td>1345</td>
<td>1025</td>
</tr>
<tr>
<td>1250</td>
<td>1150</td>
</tr>
<tr>
<td>1225</td>
<td>1245</td>
</tr>
<tr>
<td>1300</td>
<td>1185</td>
</tr>
</tbody>
</table>

Mean Values

1260       1155

Mean of 25 Tests with a 2" Fastener Spacing

1275       1145

Percent Difference

-1.2       +0.9
Table 3.5
Results of Single Shear Tests with Various Specimen Configurations
(Ultimate load in pounds per fastener)
(Failure by yield in bearing)

<table>
<thead>
<tr>
<th>Configuration A</th>
<th>Configuration B</th>
<th>Configuration C</th>
<th>Lap Joint</th>
<th>Simulated Diaphragm Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>1005</td>
<td>1125</td>
<td>1385</td>
<td>1185</td>
<td>1195</td>
</tr>
<tr>
<td>1145</td>
<td>1060</td>
<td>1305</td>
<td>1090</td>
<td>1050</td>
</tr>
<tr>
<td>1025</td>
<td>1095</td>
<td>1430</td>
<td>1135</td>
<td>1010</td>
</tr>
<tr>
<td>1090</td>
<td>1150</td>
<td>1345</td>
<td>1115</td>
<td>1170</td>
</tr>
<tr>
<td>1115</td>
<td>1040</td>
<td>1400</td>
<td>1170</td>
<td>1095</td>
</tr>
</tbody>
</table>

Mean Values

| 1075 | 1095 | 1375 | 1140 | 1105 |
Table 3.6

Average Material Properties of Panel Sections Used in Large Scale Uplift Tests

<table>
<thead>
<tr>
<th>Panel Configuration</th>
<th>Panel Designation</th>
<th>Thickness (in.)</th>
<th>Yield Stress (ksi)</th>
<th>Ultimate Stress (ksi)</th>
<th>% Elongation in 2 Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>22 Gage</td>
<td>0.028</td>
<td>100.6*</td>
<td>103.4</td>
<td>4</td>
</tr>
<tr>
<td>II</td>
<td>22 Gage</td>
<td>0.029</td>
<td>48.2</td>
<td>56.7</td>
<td>28</td>
</tr>
<tr>
<td>III</td>
<td>24 Gage</td>
<td>0.022</td>
<td>81.9*</td>
<td>81.9</td>
<td>2.5</td>
</tr>
<tr>
<td>IV</td>
<td>24 Gage</td>
<td>0.022</td>
<td>42.8</td>
<td>48.1</td>
<td>30</td>
</tr>
</tbody>
</table>

* Determined from 0.2 percent offset.
Table 3.7
Comparison of Large and Small Scale Uplift Test Results
(Ultimate load in pounds per fastener)

<table>
<thead>
<tr>
<th>Panel Configuration</th>
<th>Large Scale Uplift Test Result</th>
<th>Small Scale Pull-over Test Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22 Gage Low Ductility</td>
<td>975 (Film over entire assembly)</td>
<td>745</td>
</tr>
<tr>
<td></td>
<td></td>
<td>770</td>
</tr>
<tr>
<td></td>
<td><strong>Mean</strong> 910</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>1700 (Film over entire assembly)</td>
<td>1565</td>
</tr>
<tr>
<td>22 Gage High Ductility</td>
<td>1650 (Film between panel and member)</td>
<td>1465</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1430</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1540</td>
</tr>
<tr>
<td></td>
<td><strong>Mean 1480</strong></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>900* (Film between panel and member)</td>
<td>1020</td>
</tr>
<tr>
<td>24 Gage Low Ductility</td>
<td></td>
<td>1005</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1080</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1150</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1075</td>
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<tr>
<td></td>
<td></td>
<td>1075</td>
</tr>
<tr>
<td></td>
<td></td>
<td>920</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1160</td>
</tr>
<tr>
<td></td>
<td><strong>Mean 1060</strong></td>
<td></td>
</tr>
</tbody>
</table>

*This test result is in doubt. The panel edge failed prematurely, was braced, and the section was re-tested.
Table 3.8
Additional Large and Small Scale Uplift Test Results
(Ultimate load in pounds per fastener)

<table>
<thead>
<tr>
<th>Panel Configuration</th>
<th>Large Scale Uplift Test Result</th>
<th>Proposed Pull-over Fixture Result (Without Angles)</th>
<th>Reduced Pull-over Fixture Result (Without Angles)</th>
<th>Proposed Pull-over Fixture Result (With Angles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test A</td>
<td>1060</td>
<td>1000</td>
<td>975</td>
<td>1095</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1095</td>
<td>1065</td>
<td></td>
</tr>
<tr>
<td>Test B</td>
<td>1140</td>
<td>1105</td>
<td>1140</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean 1065</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>740</td>
<td>820</td>
</tr>
<tr>
<td></td>
<td></td>
<td>780</td>
<td>795</td>
<td>950</td>
</tr>
<tr>
<td></td>
<td></td>
<td>940</td>
<td>880</td>
<td>950</td>
</tr>
<tr>
<td></td>
<td></td>
<td>920</td>
<td>950</td>
<td>Mean 930</td>
</tr>
<tr>
<td>Test C</td>
<td>780</td>
<td>915</td>
<td>1020</td>
<td>Mean 785</td>
</tr>
<tr>
<td>Test D</td>
<td>930</td>
<td>820</td>
<td>675</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean 930</td>
</tr>
</tbody>
</table>
Table 3.9

Results of Pull-out and Pull-over Tests to Estimate Ultimate Load Distributions

(Ultimate load in pounds)

<table>
<thead>
<tr>
<th>Pull-out from 16 Gage Sheet</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>574</td>
<td>562</td>
<td>547</td>
<td>569</td>
<td>559</td>
</tr>
<tr>
<td>597</td>
<td>554</td>
<td>591</td>
<td>594</td>
<td>573</td>
</tr>
<tr>
<td>596</td>
<td>564</td>
<td>583</td>
<td>567</td>
<td>586</td>
</tr>
<tr>
<td>548</td>
<td>575</td>
<td>579</td>
<td>584</td>
<td>554</td>
</tr>
<tr>
<td>594</td>
<td>539</td>
<td>582</td>
<td>593</td>
<td>526</td>
</tr>
</tbody>
</table>

Mean Value = 572
Median = 574
Coefficient of Variation = 0.033
Range = 71

<table>
<thead>
<tr>
<th>Pull-over of 26 Gage Sheet</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>818</td>
<td>707</td>
<td>680</td>
<td>805</td>
<td>696</td>
</tr>
<tr>
<td>698</td>
<td>722</td>
<td>757</td>
<td>683</td>
<td>682</td>
</tr>
<tr>
<td>852</td>
<td>763</td>
<td>700</td>
<td>674</td>
<td>749</td>
</tr>
<tr>
<td>712</td>
<td>708</td>
<td>716</td>
<td>608</td>
<td>777</td>
</tr>
<tr>
<td>709</td>
<td>702</td>
<td>703</td>
<td>672</td>
<td>676</td>
</tr>
</tbody>
</table>

Mean Value = 719
Median = 707
Coefficient of Variation = 0.072
Range = 244
Table 4.1
Single Shear Test Results
to Estimate Ultimate Load Distributions

(Load in pounds per fastener)
(Failure by yield in bearing)

<table>
<thead>
<tr>
<th>Sample Characteristics</th>
<th>Lap Joint Tests</th>
<th>Simulated Diaphragm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original</td>
<td>Normalized</td>
</tr>
<tr>
<td>Simulated Diaphragm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint Tests</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Action</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1155</td>
<td>0.9073</td>
<td>1250</td>
</tr>
<tr>
<td>1205</td>
<td>0.9466</td>
<td>1000</td>
</tr>
<tr>
<td>1235</td>
<td>0.9701</td>
<td>1160</td>
</tr>
<tr>
<td>1265</td>
<td>0.9937</td>
<td>1200</td>
</tr>
<tr>
<td>1370</td>
<td>1.0762</td>
<td>1115</td>
</tr>
<tr>
<td>1270</td>
<td>0.9976</td>
<td>1120</td>
</tr>
<tr>
<td>1325</td>
<td>1.0408</td>
<td>1120</td>
</tr>
<tr>
<td>1250</td>
<td>0.9819</td>
<td>1085</td>
</tr>
<tr>
<td>1350</td>
<td>1.0605</td>
<td>1145</td>
</tr>
<tr>
<td>1300</td>
<td>1.0212</td>
<td>1060</td>
</tr>
<tr>
<td>1200</td>
<td>0.9427</td>
<td>1250</td>
</tr>
<tr>
<td>1300</td>
<td>1.0212</td>
<td>1190</td>
</tr>
<tr>
<td>1290</td>
<td>1.0134</td>
<td>1155</td>
</tr>
<tr>
<td>1230</td>
<td>0.9662</td>
<td>1020</td>
</tr>
<tr>
<td>1290</td>
<td>1.0134</td>
<td>1100</td>
</tr>
<tr>
<td>1265</td>
<td>0.9937</td>
<td>1170</td>
</tr>
<tr>
<td>1305</td>
<td>1.0251</td>
<td>1095</td>
</tr>
<tr>
<td>1320</td>
<td>1.0369</td>
<td>1085</td>
</tr>
<tr>
<td>1380</td>
<td>1.0841</td>
<td>1125</td>
</tr>
<tr>
<td>1230</td>
<td>0.9662</td>
<td>1195</td>
</tr>
<tr>
<td>1250</td>
<td>0.9819</td>
<td>1210</td>
</tr>
<tr>
<td>1210</td>
<td>0.9505</td>
<td>1075</td>
</tr>
<tr>
<td>1280</td>
<td>1.0055</td>
<td>1285</td>
</tr>
<tr>
<td>1330</td>
<td>1.0448</td>
<td>1210</td>
</tr>
<tr>
<td>1220</td>
<td>0.9584</td>
<td>1165</td>
</tr>
</tbody>
</table>

Mean Value            1275       1.0000   1145       1.0000
Coefficient of Variation 0.043   0.0430   0.061   0.0611
Median                  1270       0.9976   1145       1.0014
Range                   225       0.1767   285       0.2493
Table 4.2
Ranges and Increments on Beta Parameters for Single Shear (Lap Joint) Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum Value</th>
<th>Increment</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter Set 1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>0.72</td>
<td>0.02</td>
<td>0.90</td>
</tr>
<tr>
<td>b</td>
<td>1.085</td>
<td>0.025</td>
<td>1.310</td>
</tr>
<tr>
<td>r</td>
<td>2.0</td>
<td>1.0</td>
<td>11.0</td>
</tr>
<tr>
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<td></td>
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</tr>
<tr>
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<td>b</td>
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<tr>
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<td>2.0</td>
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Table 4.3

Characteristics of the Marginal Posterior Density Functions on the Beta Parameters for Single Shear (Lap Joint) Results

<table>
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<th>Standard Deviation</th>
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<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
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<td>0.833</td>
<td>0.822</td>
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</tr>
<tr>
<td>b</td>
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<td>1.129</td>
<td>1.184</td>
<td>0.0604</td>
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<td>r</td>
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</tr>
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<td>0.760</td>
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</tr>
<tr>
<td>b</td>
<td>1.208</td>
<td>1.210</td>
<td>1.210</td>
<td>0.0594</td>
</tr>
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<td>r</td>
<td>11.32</td>
<td>10.00</td>
<td>11.26</td>
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<td></td>
<td></td>
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<td>0.280</td>
<td>0.281</td>
<td>0.1614</td>
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<td>b</td>
<td>2.127</td>
<td>1.800</td>
<td>2.121</td>
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<td>r</td>
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<td>150.0</td>
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Table 4.4

Characteristics of the Bayesian Distributions on Single Shear (Lap Joint) Connection Strength

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<th>Median</th>
<th>Standard Deviation</th>
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<td>1.0000</td>
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<td>1.0000</td>
<td>1.0001</td>
<td>0.04397</td>
</tr>
<tr>
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<td>1.0000</td>
<td>0.9998</td>
<td>0.04466</td>
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Table 4.5

Beta Parameters Used for Single Shear (Simulated Diaphragm Action) Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum Value</th>
<th>Increment</th>
<th>Maximum Value</th>
</tr>
</thead>
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<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
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<td>0.04</td>
<td>0.86</td>
</tr>
<tr>
<td>b</td>
<td>1.14</td>
<td>0.04</td>
<td>1.50</td>
</tr>
<tr>
<td>r</td>
<td>2.0</td>
<td>2.5</td>
<td>24.5</td>
</tr>
<tr>
<td>Parameter Set 2</td>
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<td></td>
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<td>0.09</td>
<td>0.81</td>
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<tr>
<td>b</td>
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<tr>
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<td>100.0</td>
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Characteristics of the Marginal Posterior Probability Density Functions

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<th>Mode</th>
<th>Median</th>
<th>Standard Deviation</th>
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</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
</tr>
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<td>a</td>
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<td>0.645</td>
<td>0.0888</td>
</tr>
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<td>b</td>
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<td>1.300</td>
<td>1.324</td>
<td>0.0978</td>
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<tr>
<td>r</td>
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<td>14.50</td>
<td>14.69</td>
<td>5.736</td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
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<td>0.270</td>
<td>0.299</td>
<td>0.1936</td>
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<tr>
<td>b</td>
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<td>1.550</td>
<td>1.635</td>
<td>0.2389</td>
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<tr>
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<td>60.00</td>
<td>57.94</td>
<td>23.90</td>
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</table>
Table 4.6

Characteristics of the Bayesian Distributions on Single Shear (Simulated Diaphragm Action) Connection Strength

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Parameter Set 1</th>
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</thead>
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<tr>
<td>Mean</td>
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<td>1.0000</td>
</tr>
<tr>
<td>Mode</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>Median</td>
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<td>1.0004</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.06329</td>
<td>0.06314</td>
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</table>
Table 4.7

Pull-over and Pull-out Test Results to Estimate Ultimate Load Distributions

(Ultimate load in pounds)

<table>
<thead>
<tr>
<th>Sample Characteristics</th>
<th>Pull-over Tests</th>
<th>Pull-out Tests</th>
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<tr>
<td></td>
<td>Original</td>
<td>Normalized</td>
</tr>
<tr>
<td>818</td>
<td>1.1381</td>
<td>574</td>
</tr>
<tr>
<td>707</td>
<td>0.9836</td>
<td>562</td>
</tr>
<tr>
<td>680</td>
<td>0.9461</td>
<td>547</td>
</tr>
<tr>
<td>805</td>
<td>1.1200</td>
<td>569</td>
</tr>
<tr>
<td>696</td>
<td>0.9683</td>
<td>559</td>
</tr>
<tr>
<td>698</td>
<td>0.9711</td>
<td>597</td>
</tr>
<tr>
<td>722</td>
<td>1.0045</td>
<td>554</td>
</tr>
<tr>
<td>757</td>
<td>1.0532</td>
<td>591</td>
</tr>
<tr>
<td>683</td>
<td>0.9502</td>
<td>594</td>
</tr>
<tr>
<td>682</td>
<td>0.9489</td>
<td>573</td>
</tr>
<tr>
<td>852</td>
<td>1.1854</td>
<td>595</td>
</tr>
<tr>
<td>763</td>
<td>1.0616</td>
<td>564</td>
</tr>
<tr>
<td>700</td>
<td>0.9739</td>
<td>583</td>
</tr>
<tr>
<td>674</td>
<td>0.9377</td>
<td>567</td>
</tr>
<tr>
<td>749</td>
<td>1.0421</td>
<td>586</td>
</tr>
<tr>
<td>712</td>
<td>0.9906</td>
<td>548</td>
</tr>
<tr>
<td>708</td>
<td>0.9850</td>
<td>575</td>
</tr>
<tr>
<td>716</td>
<td>0.9962</td>
<td>579</td>
</tr>
<tr>
<td>608</td>
<td>0.8459</td>
<td>584</td>
</tr>
<tr>
<td>777</td>
<td>1.0810</td>
<td>554</td>
</tr>
<tr>
<td>709</td>
<td>0.9864</td>
<td>594</td>
</tr>
<tr>
<td>702</td>
<td>0.9767</td>
<td>539</td>
</tr>
<tr>
<td>703</td>
<td>0.9781</td>
<td>582</td>
</tr>
<tr>
<td>672</td>
<td>0.9349</td>
<td>593</td>
</tr>
<tr>
<td>676</td>
<td>0.9405</td>
<td>526</td>
</tr>
</tbody>
</table>

Mean Value | 719 | 1.0000 | 572 | 1.0000 |
Coefficient of Variation | 0.072 | 0.0723 | 0.033 | 0.0332 |
Median | 707 | 0.9836 | 574 | 1.0042 |
Range | 244 | 0.3395 | 71 | 0.1242 |
Table 4.8
Details of the Distribution on Pull-over Strength

<table>
<thead>
<tr>
<th>Parameter Ranges and Increments</th>
<th>Minimum Value</th>
<th>Increment</th>
<th>Maximum Value</th>
</tr>
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<tbody>
<tr>
<td>a</td>
<td>0.00</td>
<td>0.08</td>
<td>0.72</td>
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<tr>
<td>b</td>
<td>2.00</td>
<td>2.00</td>
<td>20.00</td>
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<tr>
<td>r</td>
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<td>20.00</td>
<td>190.00</td>
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</tbody>
</table>

Characteristics of the Marginal Posterior Probability Density Functions

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<th>Mode</th>
<th>Median</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
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<td>0.253</td>
<td>0.1830</td>
</tr>
<tr>
<td>b</td>
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<td>20.00</td>
<td>11.46</td>
<td>5.044</td>
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<tr>
<td>r</td>
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<td>90.00</td>
<td>99.08</td>
<td>46.72</td>
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Characteristics of the Posterior Probability Density Function on Strength

<table>
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<th>Mean</th>
<th>Mode</th>
<th>Median</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0000</td>
<td>0.9900</td>
<td>0.9977</td>
<td>0.07266</td>
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</tbody>
</table>
### Table 4.9
**Beta Parameters Used for Pull-out Results**

<table>
<thead>
<tr>
<th>Parameters Used for Pull-out Results</th>
<th>Ranges and Increments</th>
<th>Characteristics of the Marginal Posterior Probability Density Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Minimum Value</td>
<td>Increment</td>
</tr>
<tr>
<td>--------------------------------------</td>
<td>---------------</td>
<td>-----------</td>
</tr>
<tr>
<td><strong>Parameter Set 1</strong></td>
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<td></td>
</tr>
<tr>
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<td>0.04</td>
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<tr>
<td>b</td>
<td>1.045</td>
<td>0.025</td>
</tr>
<tr>
<td>r</td>
<td>1.00</td>
<td>1.50</td>
</tr>
<tr>
<td><strong>Parameter Set 2</strong></td>
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<td></td>
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<tr>
<td>a</td>
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<td>0.10</td>
</tr>
<tr>
<td>b</td>
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<tr>
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Table 4.10

Characteristics of the Bayesian Distributions on Pull-out Strength

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<tr>
<td>Median</td>
<td>1.0092</td>
<td>1.0102</td>
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<tr>
<td>Standard Deviation</td>
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<td>0.03930</td>
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Table 4.11
Single Shear (Lap Joint) Results with Maximum Likelihoods

<table>
<thead>
<tr>
<th>Maximum Likelihood Estimators of Beta Parameters</th>
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</thead>
<tbody>
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<td>Parameter</td>
<td>Estimated Value</td>
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<tr>
<td>b</td>
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<tr>
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<tr>
<td>t</td>
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</tbody>
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<table>
<thead>
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<th>Characteristics of Beta Distribution</th>
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<tr>
<td>Mode</td>
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</tr>
<tr>
<td>Median</td>
<td>1.0014</td>
</tr>
<tr>
<td>Standard Deviation</td>
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</tr>
<tr>
<td>Coefficient of Variation</td>
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</table>
Table 4.12
Single Shear (Simulated Diaphragm Action)
Results with Maximum Likelihoods

<table>
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<th>Estimated Value</th>
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<td>t</td>
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Characteristics of Beta Distribution

<table>
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<th>Value</th>
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<td>Mode</td>
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<tr>
<td>Median</td>
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<td>Coefficient of Variation</td>
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Table 4.13
Pull-over Results with Maximum Likelihoods

<p>| Maximum Likelihood Estimates of Beta Parameters |</p>
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</thead>
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<tr>
<td>t</td>
<td>129.1</td>
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</table>

<table>
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<th>Characteristics of Beta Distribution</th>
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<tr>
<td>Median</td>
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<tr>
<td>Standard Deviation</td>
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<tr>
<td>Coefficient of Variation</td>
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Table 4.14
Pull-out Results with Maximum Likelihoods

<table>
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</thead>
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<tr>
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<tr>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
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</tbody>
</table>
Table 4.15

Comparison of Bayesian and Maximum Likelihood Distributions on Single Shear Strength

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<th>Maximum Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Single Shear (Lap Joint) Strength</strong></td>
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<td></td>
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<td>1.0004</td>
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<tr>
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<tr>
<td>Standard Deviation</td>
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<td>0.04409</td>
</tr>
<tr>
<td><strong>Single Shear (Simulated Diaphragm Action) Strength</strong></td>
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<td></td>
</tr>
<tr>
<td>Mean</td>
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<td>0.9993</td>
</tr>
<tr>
<td>Mode</td>
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<td>0.9991</td>
</tr>
<tr>
<td>Median</td>
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<td>0.9980</td>
</tr>
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<td>Standard Deviation</td>
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</table>
Table 4.16
Comparison of Bayesian and Maximum Likelihood Distributions on Pull-over and Pull-out Strength

<table>
<thead>
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<th>Maximum Likelihood</th>
</tr>
</thead>
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<tr>
<td><strong>Pull-over Strength</strong></td>
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<td></td>
</tr>
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<td>0.07108</td>
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<tr>
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<td></td>
</tr>
<tr>
<td>Mean</td>
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<td>0.9994</td>
</tr>
<tr>
<td>Mode</td>
<td>1.0400</td>
<td>1.0431</td>
</tr>
<tr>
<td>Median</td>
<td>1.0102</td>
<td>1.0051</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.03930</td>
<td>0.03214</td>
</tr>
</tbody>
</table>
Table 4.17

Distributions of the Sample Mean for Single Shear (Lap Joint) Strength with a Beta Model

Limits of the distributions: \( a = 0.68 \)
\( b = 1.33 \)

Mean of the distributions: 0.9967

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>( r )</th>
<th>( t )</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>49.94</td>
<td>102.5</td>
<td>0.03194</td>
</tr>
<tr>
<td>3</td>
<td>75.77</td>
<td>155.5</td>
<td>0.02597</td>
</tr>
<tr>
<td>4</td>
<td>102.9</td>
<td>211.1</td>
<td>0.02231</td>
</tr>
<tr>
<td>5</td>
<td>121.5</td>
<td>249.3</td>
<td>0.02053</td>
</tr>
<tr>
<td>6</td>
<td>149.1</td>
<td>306.1</td>
<td>0.01854</td>
</tr>
<tr>
<td>7</td>
<td>164.7</td>
<td>338.1</td>
<td>0.01764</td>
</tr>
<tr>
<td>8</td>
<td>187.3</td>
<td>384.4</td>
<td>0.01655</td>
</tr>
<tr>
<td>9</td>
<td>187.3</td>
<td>384.4</td>
<td>0.01655</td>
</tr>
<tr>
<td>10</td>
<td>231.2</td>
<td>474.4</td>
<td>0.01490</td>
</tr>
</tbody>
</table>
Table 4.18

Distributions of the Sample Mean for Single Shear (Simulated Diaphragm Action) Strength with a Beta Model

Limits of the distributions:  
\[ a = 0.51 \]
\[ b = 1.43 \]

Mean of the distributions:  
1.0005

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>r</th>
<th>t</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>49.29</td>
<td>92.44</td>
<td>0.04748</td>
</tr>
<tr>
<td>3</td>
<td>74.91</td>
<td>140.5</td>
<td>0.03859</td>
</tr>
<tr>
<td>4</td>
<td>93.14</td>
<td>174.7</td>
<td>0.03095</td>
</tr>
<tr>
<td>5</td>
<td>116.8</td>
<td>219.0</td>
<td>0.03463</td>
</tr>
<tr>
<td>6</td>
<td>152.5</td>
<td>286.1</td>
<td>0.02709</td>
</tr>
<tr>
<td>7</td>
<td>179.9</td>
<td>337.6</td>
<td>0.02495</td>
</tr>
<tr>
<td>8</td>
<td>174.8</td>
<td>327.7</td>
<td>0.02531</td>
</tr>
<tr>
<td>9</td>
<td>233.8</td>
<td>438.5</td>
<td>0.02189</td>
</tr>
<tr>
<td>10</td>
<td>198.7</td>
<td>372.6</td>
<td>0.02374</td>
</tr>
</tbody>
</table>
Table 4.19

Distributions of the Sample Mean for Pull-over Strength with a Beta Model

Limits of the distributions:  
<table>
<thead>
<tr>
<th>Size</th>
<th>$r$</th>
<th>$t$</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>41.03</td>
<td>100.9</td>
<td>0.05158</td>
</tr>
<tr>
<td>3</td>
<td>63.68</td>
<td>156.6</td>
<td>0.04147</td>
</tr>
<tr>
<td>4</td>
<td>84.13</td>
<td>206.9</td>
<td>0.03611</td>
</tr>
<tr>
<td>5</td>
<td>112.6</td>
<td>277.0</td>
<td>0.03123</td>
</tr>
<tr>
<td>6</td>
<td>136.8</td>
<td>336.5</td>
<td>0.02834</td>
</tr>
<tr>
<td>7</td>
<td>144.6</td>
<td>355.7</td>
<td>0.02757</td>
</tr>
<tr>
<td>8</td>
<td>154.8</td>
<td>380.7</td>
<td>0.02665</td>
</tr>
<tr>
<td>9</td>
<td>217.1</td>
<td>533.9</td>
<td>0.02251</td>
</tr>
<tr>
<td>10</td>
<td>219.4</td>
<td>539.5</td>
<td>0.02239</td>
</tr>
</tbody>
</table>

Mean of the distributions: 1.0010

Limits: $a = 0.57$, $b = 1.63$
Table 4.20

Distributions of the Sample Mean for Pull-out Strength with a Beta Model

Limits of the distributions: \( a = 0.46 \)
\( b = 1.18 \)

Mean of the distributions: \( 1.0000 \)

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>( r )</th>
<th>( t )</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>95.17</td>
<td>126.9</td>
<td>0.02757</td>
</tr>
<tr>
<td>3</td>
<td>150.1</td>
<td>200.2</td>
<td>0.02198</td>
</tr>
<tr>
<td>4</td>
<td>189.2</td>
<td>252.3</td>
<td>0.01959</td>
</tr>
<tr>
<td>5</td>
<td>276.7</td>
<td>368.9</td>
<td>0.01621</td>
</tr>
<tr>
<td>6</td>
<td>339.6</td>
<td>452.8</td>
<td>0.01463</td>
</tr>
<tr>
<td>7</td>
<td>383.0</td>
<td>510.7</td>
<td>0.01378</td>
</tr>
<tr>
<td>8</td>
<td>453.8</td>
<td>605.1</td>
<td>0.01266</td>
</tr>
<tr>
<td>9</td>
<td>543.8</td>
<td>725.0</td>
<td>0.01157</td>
</tr>
<tr>
<td>10</td>
<td>481.3</td>
<td>641.7</td>
<td>0.01230</td>
</tr>
</tbody>
</table>
Table 6.1
Coefficients of Variation of the Resistance $V_R$
for Single Shear (Lap Joint) Tests

<table>
<thead>
<tr>
<th>Number of Samples</th>
<th>Quality of Workmanship and Degree of Inspection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Good ($V_F = 0.05$)</td>
</tr>
<tr>
<td>1</td>
<td>0.081</td>
</tr>
<tr>
<td>2</td>
<td>0.074</td>
</tr>
<tr>
<td>3</td>
<td>0.072</td>
</tr>
<tr>
<td>4</td>
<td>0.071</td>
</tr>
<tr>
<td>5</td>
<td>0.070</td>
</tr>
<tr>
<td>6</td>
<td>0.070</td>
</tr>
<tr>
<td>7</td>
<td>0.070</td>
</tr>
<tr>
<td>8</td>
<td>0.069</td>
</tr>
<tr>
<td>9</td>
<td>0.069</td>
</tr>
<tr>
<td>10</td>
<td>0.069</td>
</tr>
</tbody>
</table>
Table 6.2
Modified Resistance Factors $\bar{f}$ for Single Shear (Lap Joint) Tests

<table>
<thead>
<tr>
<th>Number of Samples</th>
<th>Temporary Structures ($\beta = 3.5$)</th>
<th>Standard Structures ($\beta = 4.5$)</th>
<th>Vital Structures ($\beta = 5.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Workmanship and Inspection</td>
<td>Workmanship and Inspection</td>
<td>Workmanship and Inspection</td>
</tr>
<tr>
<td></td>
<td>Good</td>
<td>Average</td>
<td>Below Average</td>
</tr>
<tr>
<td>1</td>
<td>0.753</td>
<td>0.700</td>
<td>0.643</td>
</tr>
<tr>
<td>2</td>
<td>0.763</td>
<td>0.707</td>
<td>0.647</td>
</tr>
<tr>
<td>3</td>
<td>0.766</td>
<td>0.708</td>
<td>0.648</td>
</tr>
<tr>
<td>4</td>
<td>0.768</td>
<td>0.709</td>
<td>0.649</td>
</tr>
<tr>
<td>5</td>
<td>0.769</td>
<td>0.709</td>
<td>0.649</td>
</tr>
<tr>
<td>6</td>
<td>0.769</td>
<td>0.711</td>
<td>0.649</td>
</tr>
<tr>
<td>7</td>
<td>0.769</td>
<td>0.711</td>
<td>0.649</td>
</tr>
<tr>
<td>8</td>
<td>0.771</td>
<td>0.711</td>
<td>0.649</td>
</tr>
<tr>
<td>9</td>
<td>0.771</td>
<td>0.711</td>
<td>0.649</td>
</tr>
<tr>
<td>10</td>
<td>0.771</td>
<td>0.711</td>
<td>0.650</td>
</tr>
</tbody>
</table>
Table 6.3

Coefficients of Variation of the Resistance $V_R$ for Single Shear (Simulated Diaphragm Action) Tests

<table>
<thead>
<tr>
<th>Number of Samples</th>
<th>Quality of Workmanship and Degree of Inspection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Good $(V_F, = 0.05)$</td>
</tr>
<tr>
<td>1</td>
<td>0.102</td>
</tr>
<tr>
<td>2</td>
<td>0.093</td>
</tr>
<tr>
<td>3</td>
<td>0.089</td>
</tr>
<tr>
<td>4</td>
<td>0.088</td>
</tr>
<tr>
<td>5</td>
<td>0.086</td>
</tr>
<tr>
<td>6</td>
<td>0.085</td>
</tr>
<tr>
<td>7</td>
<td>0.084</td>
</tr>
<tr>
<td>8</td>
<td>0.084</td>
</tr>
<tr>
<td>9</td>
<td>0.083</td>
</tr>
<tr>
<td>10</td>
<td>0.084</td>
</tr>
</tbody>
</table>
Table 6.4

Modified Resistance Factors $\bar{\phi}$ for Single Shear (Simulated Diaphragm Action) Tests

<table>
<thead>
<tr>
<th>Number of Samples</th>
<th>Temporary Structures ($\bar{\phi} = 3.5$)</th>
<th>Standard Structures ($\bar{\phi} = 4.5$)</th>
<th>Vital Structures ($\bar{\phi} = 5.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Workmanship and Inspection</td>
<td>Workmanship and Inspection</td>
<td>Workmanship and Inspection</td>
</tr>
<tr>
<td></td>
<td>Good Average Below Average</td>
<td>Good Average Below Average</td>
<td>Good Average Below Average</td>
</tr>
<tr>
<td>1</td>
<td>0.723 0.680 0.630</td>
<td>0.684 0.632 0.572</td>
<td>0.624 0.567 0.502</td>
</tr>
<tr>
<td>2</td>
<td>0.736 0.689 0.636</td>
<td>0.699 0.643 0.579</td>
<td>0.642 0.579 0.510</td>
</tr>
<tr>
<td>3</td>
<td>0.741 0.693 0.638</td>
<td>0.706 0.647 0.582</td>
<td>0.649 0.584 0.513</td>
</tr>
<tr>
<td>4</td>
<td>0.743 0.694 0.639</td>
<td>0.708 0.649 0.584</td>
<td>0.651 0.586 0.514</td>
</tr>
<tr>
<td>5</td>
<td>0.746 0.696 0.639</td>
<td>0.711 0.651 0.584</td>
<td>0.655 0.588 0.514</td>
</tr>
<tr>
<td>6</td>
<td>0.747 0.697 0.641</td>
<td>0.713 0.652 0.585</td>
<td>0.657 0.589 0.516</td>
</tr>
<tr>
<td>7</td>
<td>0.749 0.697 0.641</td>
<td>0.715 0.652 0.585</td>
<td>0.659 0.589 0.516</td>
</tr>
<tr>
<td>8</td>
<td>0.749 0.697 0.641</td>
<td>0.715 0.652 0.585</td>
<td>0.659 0.589 0.516</td>
</tr>
<tr>
<td>9</td>
<td>0.750 0.698 0.642</td>
<td>0.717 0.654 0.586</td>
<td>0.661 0.591 0.518</td>
</tr>
<tr>
<td>10</td>
<td>0.749 0.697 0.642</td>
<td>0.715 0.652 0.586</td>
<td>0.659 0.589 0.518</td>
</tr>
</tbody>
</table>
Table 6.5
Coefficients of Variation of the Resistance $V_R$ for Pull-over Tests

<table>
<thead>
<tr>
<th>Number of Samples</th>
<th>Quality of Workmanship and Degree of Inspection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Good $(V_F, = 0.05)$</td>
</tr>
<tr>
<td>1</td>
<td>0.115</td>
</tr>
<tr>
<td>2</td>
<td>0.103</td>
</tr>
<tr>
<td>3</td>
<td>0.098</td>
</tr>
<tr>
<td>4</td>
<td>0.096</td>
</tr>
<tr>
<td>5</td>
<td>0.094</td>
</tr>
<tr>
<td>6</td>
<td>0.093</td>
</tr>
<tr>
<td>7</td>
<td>0.093</td>
</tr>
<tr>
<td>8</td>
<td>0.093</td>
</tr>
<tr>
<td>9</td>
<td>0.091</td>
</tr>
<tr>
<td>10</td>
<td>0.091</td>
</tr>
</tbody>
</table>
Table 6.6
Modified Resistance Factors $\bar{\beta}$ for Pull-over Tests

<table>
<thead>
<tr>
<th>Number of Samples</th>
<th>Temporary Structures ($\beta = 3.5$)</th>
<th>Standard Structures ($\beta = 4.5$)</th>
<th>Vital Structures ($\beta = 5.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Workmanship and Inspection</td>
<td>Workmanship and Inspection</td>
<td>Workmanship and Inspection</td>
</tr>
<tr>
<td></td>
<td>Good  Average Below Average</td>
<td>Good  Average Below Average</td>
<td>Good  Average Below Average</td>
</tr>
<tr>
<td>1</td>
<td>0.705  0.667  0.620</td>
<td>0.662  0.616  0.561</td>
<td>0.600  0.550  0.490</td>
</tr>
<tr>
<td>2</td>
<td>0.722  0.680  0.628</td>
<td>0.682  0.632  0.571</td>
<td>0.622  0.567  0.501</td>
</tr>
<tr>
<td>3</td>
<td>0.729  0.685  0.632</td>
<td>0.690  0.638  0.575</td>
<td>0.632  0.574  0.505</td>
</tr>
<tr>
<td>4</td>
<td>0.732  0.686  0.633</td>
<td>0.694  0.639  0.576</td>
<td>0.636  0.575  0.507</td>
</tr>
<tr>
<td>5</td>
<td>0.734  0.688  0.634</td>
<td>0.697  0.641  0.578</td>
<td>0.640  0.577  0.508</td>
</tr>
<tr>
<td>6</td>
<td>0.736  0.689  0.636</td>
<td>0.699  0.643  0.579</td>
<td>0.642  0.579  0.510</td>
</tr>
<tr>
<td>7</td>
<td>0.736  0.689  0.636</td>
<td>0.699  0.643  0.579</td>
<td>0.642  0.579  0.510</td>
</tr>
<tr>
<td>8</td>
<td>0.736  0.689  0.636</td>
<td>0.699  0.643  0.579</td>
<td>0.642  0.579  0.510</td>
</tr>
<tr>
<td>9</td>
<td>0.739  0.690  0.637</td>
<td>0.703  0.644  0.581</td>
<td>0.645  0.581  0.511</td>
</tr>
<tr>
<td>10</td>
<td>0.739  0.690  0.637</td>
<td>0.703  0.644  0.581</td>
<td>0.645  0.581  0.511</td>
</tr>
</tbody>
</table>
Table 6.7
Coefficients of Variation of the Resistance $V_R$
for Pull-out Tests

<table>
<thead>
<tr>
<th>Number of Samples</th>
<th>Quality of Workmanship and Degree of Inspection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Good $(V_F, = 0.075)$</td>
</tr>
<tr>
<td>1</td>
<td>0.093</td>
</tr>
<tr>
<td>2</td>
<td>0.089</td>
</tr>
<tr>
<td>3</td>
<td>0.087</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
</tr>
<tr>
<td>5</td>
<td>0.086</td>
</tr>
<tr>
<td>6</td>
<td>0.086</td>
</tr>
<tr>
<td>7</td>
<td>0.086</td>
</tr>
<tr>
<td>8</td>
<td>0.086</td>
</tr>
<tr>
<td>9</td>
<td>0.085</td>
</tr>
<tr>
<td>10</td>
<td>0.085</td>
</tr>
</tbody>
</table>
Table 6.8
Modified Resistance Factors $\bar{f}$ for Pull-out Tests

<table>
<thead>
<tr>
<th>Number of Samples</th>
<th>Temporary Structures ($\beta = 3.5$)</th>
<th>Standard Structures ($\beta = 4.5$)</th>
<th>Vital Structures ($\beta = 5.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Workmanship and Inspection</td>
<td>Workmanship and Inspection</td>
<td>Workmanship and Inspection</td>
</tr>
<tr>
<td></td>
<td>Good</td>
<td>Average</td>
<td>Below Average</td>
</tr>
<tr>
<td>1</td>
<td>0.736</td>
<td>0.676</td>
<td>0.619</td>
</tr>
<tr>
<td>2</td>
<td>0.741</td>
<td>0.680</td>
<td>0.621</td>
</tr>
<tr>
<td>3</td>
<td>0.744</td>
<td>0.681</td>
<td>0.621</td>
</tr>
<tr>
<td>4</td>
<td>0.744</td>
<td>0.683</td>
<td>0.622</td>
</tr>
<tr>
<td>5</td>
<td>0.746</td>
<td>0.683</td>
<td>0.622</td>
</tr>
<tr>
<td>6</td>
<td>0.746</td>
<td>0.683</td>
<td>0.622</td>
</tr>
<tr>
<td>7</td>
<td>0.746</td>
<td>0.683</td>
<td>0.622</td>
</tr>
<tr>
<td>8</td>
<td>0.746</td>
<td>0.683</td>
<td>0.622</td>
</tr>
<tr>
<td>9</td>
<td>0.747</td>
<td>0.684</td>
<td>0.622</td>
</tr>
<tr>
<td>10</td>
<td>0.747</td>
<td>0.684</td>
<td>0.622</td>
</tr>
</tbody>
</table>
D = Nominal Fastener Diameter.  
When edge margin is to be investigated as a variable, it is permissible to change the 2D dimension.

H = Hole Diameter.  
Use hole size specified in governing specification. For Blind, Taper Shank and other special fasteners use hole size required for the product application.

* ARTC Report No. 33 specifies a width of 6D; MIL-STD-1312 specifies 8D.

Fig. 2.1 - Single shear lap joint test specimen configuration
Fig. 2.2 - Plan view of single shear (simulated diaphragm action) test fixture
Fig. 2.3 - Initial single shear test specimen configurations
Fig. 2.4 - Typical load-deformation curves for two fastener lap joint and simulated diaphragm action test specimens.
Dimensions B, C, D, T depend on fastener size. Chamfer all hole edges to provide head fillet clearance. Material: 4140 Alloy Steel heat treated to 200,000 psi or equal.

Fig. 2.5 - Plates for tension plate type fixture
Fig. 2.6 - Configuration of tension plate type test fixture
Gusset Plate (Both Sides)

Material: 2 x 4 Lumber

Fig. 2.7 - Sheet-to-structural uplift test specimen configuration
Fig. 2.8 - Sheet-to-sheet uplift test specimen configuration
Group I: Fasteners on Opposite Sides

Group II: Fasteners on Same Side

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b. Pull-out from formed channel
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a. Standard single shear joint test specimen

b. General specimen configuration for single shear (lap joint) test

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36" Coverage
Panel I

30" Coverage
Panel II

36" Coverage
Panel III

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Fig. A9 - Test specimen installation for pull-out/pull-over tests with flexible corrugated specimens
Angle Section

Fig. A10 - Angle section location for pull-over tests
Fig. A11 - Pull-out/pull-over test fixture with specimen installed for pull-out test
Fig. A12 - Test specimen installation for pull-out tests with formed sections
Fig. A13 - Single shear (lap joint) test specimen parameters

\[ e = 4 \, d \geq 1 \, \text{in.} \]
\[ s = 8 \, d \geq 2 \, \text{in.} \]
\[ w = 10 \, d \geq 2 \, \text{in.} \]
\[ L = 60 \, d \geq 15 \, \text{in.} \]
Fig. A14 - Test specimen configuration using shims for single shear (lap joint) test
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(14) 9/32" Dia. Hole
(14) 13/32" x 5/16" Countersink
Drill & Tap for 1/4" Hexagon Socket Head Cap Screw (Typical)

Bonded Teflon Pads, 1/8" Thick

Fig. A23 - Center support for single shear (simulated diaphragm action) test fixture
Fig. A24 - Outside support for single shear (simulated diaphragm action) test fixture
Fig. A25 - Vertical guide for single shear (simulated diaphragm action) test fixture
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Fig. A28 - Placement of bottom shear plates for single shear (simulated diaphragm action) test
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Fig. C1 - Safety factor vs. the ratio of live to dead load effects for connections made with average workmanship and inspection
Probability Density Function

Cumulative Distribution Function of a Random Variable

Fig. D-1. Random Variable and its Probability Distribution
Fig. D-2. Joint Probability Density Function
Frequency

\[ \text{Area} = f_X(x) \, dx \]

Mean \( \mu_X = \frac{\int_{-\infty}^{\infty} xf_X(x) \, dx}{\int_{-\infty}^{\infty} f_X(x) \, dx} \)

Variance \( \sigma_X^2 = \int_{-\infty}^{\infty} (x - \mu_X)^2 f_X(x) \, dx \)

Center of Mass = \( \frac{\text{First Moment}}{\text{Total mass}} \)

\[ \bar{x} = \frac{\int_{-\infty}^{\infty} xg(x) \, dx}{\int_{-\infty}^{\infty} g(x) \, dx} \]

Central Moment of Inertia

\[ I = \frac{\int_{-\infty}^{\infty} (x-\bar{x})^2 g(x) \, dx}{\int_{-\infty}^{\infty} g(x) \, dx} \]

If the mass is unity

\[ I = \int_{-\infty}^{\infty} (x-\bar{x})^2 g(x) \, dx \]

**Probability Density Function**

**Non Uniform Mass Function**

Fig. D-3. Physical Interpretation of the Moments of a Random Variable
Fig. D-4. Variation of Shape of Probability Density Functions with the Coefficient of Skewness $g_1$

Fig. D-5. Nominal Values
Fig. D-6. Correlation Coefficient
Normal Density Function

$\text{Mean } = 0$

Standardized Normal Variable

$\sigma = 1$

Fig. D-7. Normal Probability Distribution
Fig. 8. Extreme Value Distributions
Fig. 3-9. Stochastic Process $X(t)$