Comparison of AISI Specification Methods for Members with Single Intermediate Longitudinal Stiffeners

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Comparison of AISI Specification Methods for Members With Single Intermediate Longitudinal Stiffeners

RESEARCH REPORT RP06-3

July 2006

Committee on Specifications for the Design of Cold-Formed Steel Structural Members

American Iron and Steel Institute
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Abstract

In the current AISI specification, there are two different methods for calculating the effective widths of longitudinally stiffened elements. The first method “B4.1” applies solely to the element with only one intermediate stiffener, whereas the second method “B5.1” works only for those elements with more than one intermediate stiffener. Both methods are accurate in finding the design moments if used appropriately. Although two methods are developed to handle different cases, for some sections with one intermediate stiffener, the second method also works accurately, giving the same result as the first method does for calculating design moments. For some other sections with one intermediate stiffener, the multiple stiffener method when applied to the case of a single stiffened becomes different. These different sections can be distinguished with sets of parameters discussed herein. By comparing the design moments obtained with the same sets of parameters, one can predict when the two methods would have the largest difference and no difference in calculating the design moments and effective widths.
1. Introduction

The addition of longitudinal intermediate stiffeners to the compression flange can increase the flexural strength of the member significantly. The use of stiffeners is, however, unpractical without knowing the actual strength increase. Although sophisticated computer programs can accurately predict the strength increase, such methods are impractical for everyday use. In contrast, the AISI specification methods are useful shortcuts in obtaining the effect of stiffeners with a high degree of accuracy. The procedure to find the flexural strength of a member involves calculating the effective width of the compression flange. Because the elements under the compression loading do not experience the uniformly distributed stress due to the buckling of the member, the method of effective width is necessary. Finding the effective width of a compression flange without any stiffener is relatively easy. However, with the addition of intermediate stiffeners in compression elements, the stress distribution and effective width become more complicated. The procedure of finding the effective width of the compression flange with one or more intermediate stiffeners becomes more complicated, but with the help of studies and research, there are now two methods in AISI Specification which calculate the effective width of the compression elements with longitudinal stiffeners. These AISI Specification methods are “B4.1 Uniformly Compressed Elements with One Intermediate Stiffener” and “B5.1 Uniformly Compressed Elements with Multiple Intermediate Stiffeners.” Although not originally intended for the elements with only one intermediate stiffener, this study focuses on the applicability of B5.1 for the one stiffener case, comparing it with the B4.1 method and also with previously conducted nonlinear finite element analysis results. The ultimate goal of this study is to contribute to future or current studies that focus on finding one method that will eventually take both places of AISI methods, B4.1 and B5.1.
2. Brief Presentations of AISI Specification B4.1 and B5.1

I. AISI Specification B4.1 Uniformly Compressed Elements with One Intermediate Stiffener

\[ S = 1.28 \sqrt{\frac{E}{f}} \]  

(B4.1-1)

For \( \frac{b_o}{t} \leq S \)

\[ I_a = 0 \]

\[ b = w \]  

(B4.1-1)

\[ A_s = A_s' \]  

(B4.1-2)

For \( \frac{b_o}{t} > S \)

\[ A_s = A_s'(R_i) \]  

(B4.1-3)

\[ n = \left[ 0.583 - \frac{b_o}{t} \right] \geq \frac{1}{3} \]  

(B4.1-4)

\[ k = 3(R_i)^{\frac{n}{3}} + 1 \]  

(B4.1-5)

\[ R_i = I_o/I_a \leq 1 \]  

(B4.1-6)

where

i) For \( S < \frac{b_o}{t} < 3S \)

\[ I_a = t^4 \left[ 50 \frac{b_o}{t} - 50 \right] \]  

(B4.1-7)

ii) For \( \frac{b_o}{t} \geq 3S \)

\[ I_a = t^4 \left[ 128 \frac{b_o}{t} - 285 \right] \]  

(B4.1-8)

The effective width, \( b \), is calculated in accordance with Section B2.1(a).
II. **AISI Specification B5.1 Uniformly Compressed Elements with Multiple Intermediate Stiffeners**

The effective width, \( b_e = \rho \left( \frac{A_g}{t} \right) \)  

(B5.1-1)

The effective element is placed at the centroid of the entire element so that the neutral axis location is unaffected by eqn. B5.1-1.

\[ \rho = 1 \quad \text{when } \lambda \leq 0.673 \]  

(B5.1-2)

\[ \rho = \frac{1 - 0.22 / \lambda}{\lambda} \quad \text{when } \lambda > 0.673 \]  

(B5.1-3)

\[ \lambda = \frac{f}{\sqrt{F_{cr}}} \]  

(B5.1-4)

\[ F_{cr} = k \frac{\pi^2 E}{12(1-\mu^2)} \left( \frac{t}{b_o} \right)^2 \]  

(B5.1-5)

\[ k = \text{the minimum of } R_k d \text{ and } k_{loc} \]  

(B5.1-6)

\[ R = 2 \quad \text{when } b_o/h < 1 \]  

(B5.1-7)

\[ R = \frac{11 - b_o/h}{5} \geq \frac{1}{2} \quad \text{when } b_o/h \geq 1 \]  

(B5.1-8)

**B5.1.2 General Case: Arbitrary Stiffener Size, Location and Number**

\[ k_{loc} = 4 \left( \frac{b_o}{b_p} \right)^2 \]  

(B5.1.2-1)

\[ k_d = \frac{(1 + \beta^2)^2 + 2 \sum_{i=1}^{n} \gamma_i \omega_i}{\beta^2 \left( 1 + 2 \sum_{i=1}^{n} \delta_i \omega_i \right)} \]  

(B5.1.2-2)

\[ \beta = \left( \sum_{i=1}^{n} \gamma_i \omega_i + 1 \right)^{1/4} \]  

(B5.1.2-3)

\[ \gamma_i = \frac{10.92 (I_{yp})_i}{b_o t^3} \]  

(B5.1.2-4)

\[ \omega_i = \sin^2 \left( \pi \frac{c_i}{b_o} \right) \]  

(B5.1.2-5)

\[ \delta_i = \frac{(A_s)_i}{b_o t} \]  

(B5.1.2-6)
3. Summary of Cross-Sections

The focus of this study is a hat shape section made of cold-formed steel. Although the AISI codes B4.1 and B5.1 apply to any shape with longitudinal stiffener(s), the hat section is the most commonly studied for this case. The compression flange of a hat section is a stiffened element, having webs on both sides. Its tension flanges also help eliminate factors such as torsion and tension failure since it increases the moment of inertia about y-axis and moves the neutral axis closer to the tension flange, causing the first yield to occur in the compression flange.

![Fig 3.1 Summary of Cross-Section](image)

The cross-sections used in this study are distinguished by three different parameters, the height of the web ($h_w$), the width of the subelement of the flange ($w$), and the size of the stiffener ($w_s$). The geometry of the stiffeners is defined as $ds = \frac{w_s}{2}$. Although the length of the tension flanges could play a part as one of the parameters, it was kept constant to concentrate the study into the compression part of the section since the two codes are dealing with the effective widths of the flanges. In this study thirty sets of parameters were selected from earlier research (Schafer, 1994), in order to compare the AISI values with the nonlinear FEA values. The following table summarizes the selected parameters.
Table 3.1 Selected Sets of Parameters (dimensions in mm)

<table>
<thead>
<tr>
<th>w/t</th>
<th>Ws</th>
<th>ds</th>
<th>hw</th>
<th>wtf</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>14.92</td>
<td>7.46</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>30</td>
<td>17.48</td>
<td>8.74</td>
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<td>150</td>
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<tr>
<td>50</td>
<td>20.94</td>
<td>10.47</td>
<td>100</td>
<td>150</td>
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<tr>
<td>70</td>
<td>23.48</td>
<td>11.74</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>20</td>
<td>11.86</td>
<td>5.93</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>30</td>
<td>13.88</td>
<td>6.94</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>35</td>
<td>14.68</td>
<td>7.34</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>45</td>
<td>16.02</td>
<td>8.01</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>50</td>
<td>16.62</td>
<td>8.31</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>70</td>
<td>18.64</td>
<td>9.32</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>20</td>
<td>9.42</td>
<td>4.71</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>30</td>
<td>11.02</td>
<td>5.51</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>35</td>
<td>11.64</td>
<td>5.82</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>50</td>
<td>13.2</td>
<td>6.6</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>70</td>
<td>14.8</td>
<td>7.4</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>30</td>
<td>17.48</td>
<td>8.74</td>
<td>50</td>
<td>150</td>
</tr>
<tr>
<td>40</td>
<td>19.38</td>
<td>9.69</td>
<td>50</td>
<td>150</td>
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</tr>
<tr>
<td>60</td>
<td>22.28</td>
<td>11.14</td>
<td>50</td>
<td>150</td>
</tr>
<tr>
<td>70</td>
<td>23.48</td>
<td>11.74</td>
<td>50</td>
<td>150</td>
</tr>
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<td>30</td>
<td>11.02</td>
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<td>150</td>
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<td>40</td>
<td>12.2</td>
<td>6.1</td>
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<td>13.2</td>
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<td>150</td>
</tr>
<tr>
<td>60</td>
<td>14.04</td>
<td>7.02</td>
<td>50</td>
<td>150</td>
</tr>
<tr>
<td>70</td>
<td>14.8</td>
<td>7.4</td>
<td>50</td>
<td>150</td>
</tr>
</tbody>
</table>

For the comparison between B4.1 and B5.1 690 trials were performed. These sets of parameter are the combinations of 6 flange width to thickness ratios, 4 web heights, and 30 stiffener widths/depths. The following summarizes the dimension of those parameters.

Table 3.2. All Parameters

<table>
<thead>
<tr>
<th>w/t, width to thickness ratios</th>
<th>20, 30, 35, 45, 50, 70</th>
</tr>
</thead>
<tbody>
<tr>
<td>hw, web heights (mm)</td>
<td>25, 50, 100, 150</td>
</tr>
<tr>
<td>ws, stiffener widths (mm)</td>
<td>9.42, 11.02, 11.64, 11.86, 13.2, 13.88, 14.68, 14.8, 14.92, 16.02, 16.62, 18.64, 17.48, 20.94, 23.48</td>
</tr>
</tbody>
</table>
4. Comparison with Nonlinear FEA predictions

Figures 4.1.1 and 4.1.2 given below provide a comparison of the AISI methods B4.1 and B5.1 with the ultimate strength predicted using nonlinear finite element analysis from Schafer (1994). The test-to-predicted ratio statistics, where in this case the “test” value is the nonlinear FEA analysis, are presented in the Table 4.1.1.

<table>
<thead>
<tr>
<th></th>
<th>$M_{FEA}/M_{B4.1}$</th>
<th>$M_{FEA}/M_{B5.1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg</td>
<td>0.96</td>
<td>0.98</td>
</tr>
<tr>
<td>COV</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

The comparison between the values by the AISI codes and the nonlinear FEA predictions shows that both AISI B4.1 and AISI B5.1 are reliable for calculating the bending capacity of the hat shape sections with one intermediate stiffener. Although both codes are less than 5% different from the FEA values, surprisingly, B5.1 the multiple stiffener code has a better result as seen with the difference of only 2%. This means that B5.1 is a more reliable and better code than B4.1 for any number of intermediate stiffeners in hat shape sections. From this limited data, it cannot be concluded whether B5.1 is reliable for all other sections with different shapes. However, it is known from previous studies that the most of the flexural members’ strengths are heavily depended on the effective width of the compression flange; therefore it is likely that the use of method B5.1 will be reliable for other cases as well. The only question left is AISI B4.1; why it is not as reliable as B5.1 and what makes it different from B5.1? The major difference between the two methods is the way in which one treats the area of a stiffener. The next study (parameter study with the comparison of two methods) includes more details on this difference.
Figure 4.1.1 AISI B4.1 & B5.1 compared with nonlinear FEA.

Figure 4.1.2 AISI B4.1 & B5.1 compared with nonlinear FEA.
5. Parameter study comparing B5.1 to B4.1

Though it appears reliable to use either of the two methods to obtain the effective width of an element with one stiffener, the differences between two methods must be studied in order to create a better method. To study the differences, the ratio B5.1 to B4.1 is plotted against two varying parameters per graph. One varying parameter that is always present in following graphs is the web height, hw. Table 3.2 summarizes the parameter varied in this study.

Although B4.1 and B5.1 are for finding the effective width of the flange part of the member, unlike B4.1 method, B5.1 method involves the height of web when determining the effective width of the flange as the following equations show,

\[
R = \frac{11-b_{o}/h}{5} \geq \frac{1}{2} \text{ when } b_{o}/h \geq 1 \quad (B5.1-8)
\]
\[
R = 2 \quad \text{ when } b_{o}/h < 1 \quad (B5.1-7)
\]
Figure 5.1 shows that when \( \frac{b_o}{h} \geq 1 \) or \( \frac{h}{b_o} \leq 1 \), there are large number of disagreements between \( M_{B5.1} \) and \( M_{B4.1} \), as they do not coincide with each other. It seems \( R \) is one of the main driving factors that are responsible for the difference of two methods. In other words, both methods will produce similar values for the flexural strength for the sections that are “tall” and “narrow.”

As mention before, one major difference between \( B4.1 \) and \( B5.1 \) is the way they treat the area of stiffener. In \( B4.1 \) the area of stiffener is kept out of the calculation of the effective width, but used throughout the rest of flexural strength calculation, locating the neutral axis and calculating the effective moment of inertia. Unlike \( B4.1 \), in \( B5.1 \) the area of stiffener is absorbed into the effective width of the flange as its area is reduced by \( \rho \), reduction factor, together with the flat
subelements. Although B5.1 places the effective element at the centroid of the original element to be consistent in calculating the neutral axis, the absorption of the area of stiffener can have large effect on the moment of inertia of the whole section and therefore on the flexural strength. This effect is apparent when the size of stiffener is very small or very large. In Figure 5.2, each group with different web height forms a trend which shows that the difference between $M_{B5.1}$ and $M_{B4.1}$ increases as the size of stiffener increases or decreases away from the medium size, except for the group with $hw=25$, because in this case the stiffener width is small relative to the whole section. B5.1 produces higher flexural strength values when the relative size of the stiffener is small and lower strength values for relatively large stiffeners.

Figure 5.3 $M_{B5.1}/M_{B4.1}$ against stiffener $ws/b_0$.  

Figure 5.3 $M_{B5.1}/M_{B4.1}$ against stiffener $ws/b_0$. 
A large stiffener in a small flange will give similar values for the two methods and a small stiffener will give higher values for B5.1 except for the small web height because of the R factor.

It is also apparent in Figure 5.3 that smaller web height groups slowly stray away from B5.1/B4.1=1 as the stiffener becomes too large.

![AISI Spec B4.1 vs B5.1](image)

**Figure 5.4 Combination of Figure 5.1 and Figure 53**

This figure is the combination of the first graph (Figure 5.1) and third graph (Figure 5.3). The figure indicates that B4.1 and B5.1 will produce almost identical values for the “tall” and “narrow” sections with large stiffeners, and very different values for the “short” and “wide” sections with small stiffeners.
6. Conclusions and Recommendations

This study shows that it is unnecessary for AISI Specification to have two different procedures for calculating the effective widths of the elements with one or more stiffeners because AISI B5.1, the current procedure for multiple stiffeners, is reliable for both cases of one stiffener and multiple stiffeners. Replacing AISI B4.1 with B5.1 is possible. The comparison of B4.1 and B5.1 to finite-element analysis shows that B5.1 is slightly more reliable than B4.1 procedure in the overall perspective. However, the parameter study shows that B4.1 and B5.1 can each produce flexural strength values with up to 10% difference, though they both are reliable in the 30 selected parameters cases considered here. To ensure applicability of B5.1 in one-stiffener cases, new studies could focus on the parameters which produce the large differences between B4.1 and B5.1, and determine which procedure is more reliable by comparing to FEA, other numerical analysis values, or experiments.
Appendix I: Design Examples

1. AISI B4.1 Method
Parameters:
n=1, w/t=70, ws=23.48mm, ds=11.74mm, hw=100mm, wtf=150mm, Fy=345MPa
E=203400MPa, w = 63.63mm

![Diagram of actual elements](image)

Figure 1 Actual Elements.

1. Calculating the effective width of the compression flange.
   (B4.1 Uniformly Compressed Elements with One Intermediate Stiffener)

\[ b_o = (n+1)(w) + (n)ws = 150.74 \text{mm} \]
\[ b_o/t = 165.8306 \]

Assuming that \( f = f_y \)
\[ S = 1.28 \sqrt{\frac{E}{f}} = 31.0796 \]
\[ b_o/t > (3S = 93.1472) \]

\[ I_a = t^4 \left[ 128 \frac{b_o/t}{S} - 285 \right] = 272.1656 \text{mm}^4 \quad \text{(B4.1-8)} \]

\[ I_s = 2 \left[ \frac{1}{12} \sqrt{t^2 + \left( \frac{w_s/2}{ds} \right)^2} \right] \left( ds^3 \right) = 346.6825 \text{mm}^4 \]

\[ n = \left[ 0.583 - \frac{(b_o/t)}{12S} \right] = 0.178426 \geq \frac{1}{3} \quad n = \frac{1}{3} \quad \text{(B4.1-4)} \]

\[ R_i = I_s/I_a = 1.2738 \leq 1 \quad R_i = 1 \quad \text{(B4.1-6)} \]
2. Locating Neutral Axis with an assumption that the web is fully effective

\[
k = 3(R_i)^n + 1 = 4.25 \leq 4 \quad k=4 \quad \text{(B4.1-5)}
\]

\[
F_{cr} = k \frac{\pi^2 E}{12(1-\mu^2)} \left( \frac{t}{w} \right)^2 = 150.0731 \text{ MPa} \quad \text{(B2.1-5)}
\]

\[
\lambda = \sqrt{\frac{f}{F_{cr}}} = 1.5162 \quad \text{(B2.1-4)}
\]

\[
\rho = \frac{(1-0.22/\lambda)}{\lambda} = 0.56384 \quad \text{when} \quad \lambda > 0.673 \quad \text{(B2.1-3)}
\]

\[
b = \rho w = 35.8773 \text{ mm} \quad \text{(B2.1-7)}
\]

![Figure 2 Effective lengths with fully effective web.](image)

<table>
<thead>
<tr>
<th>Element</th>
<th>Effective Length L (mm)</th>
<th>Distance from Top Fiber y (mm)</th>
<th>( L_y ) (mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>299.091</td>
<td>100.4545</td>
<td>30045.04</td>
</tr>
<tr>
<td>2</td>
<td>1.818</td>
<td>100.4545</td>
<td>182.6263</td>
</tr>
<tr>
<td>3</td>
<td>198.182</td>
<td>50.4545</td>
<td>9999.174</td>
</tr>
<tr>
<td>4</td>
<td>1.818</td>
<td>0.4545</td>
<td>0.826281</td>
</tr>
<tr>
<td>5</td>
<td>71.7546</td>
<td>0.4545</td>
<td>32.61247</td>
</tr>
<tr>
<td>6</td>
<td>33.20572</td>
<td>6.3245</td>
<td>210.0096</td>
</tr>
<tr>
<td>Total</td>
<td>605.8693</td>
<td>40470.29</td>
<td></td>
</tr>
</tbody>
</table>

\[
y_{cg} = \frac{\sum (L_y)}{\sum L} = 66.79705 \text{ mm} > \frac{hw}{2} = 50 \text{ mm}
\]

Since the distance \( y_{cg} \) is more than the half depth of h, 100 mm, the neutral axis is closer to the tension flange and, therefore, the maximum stress occurs in the compression flange. Therefore the assumption \( f=f_y \) is valid in this case.
3. Check the effectiveness of the web.
(B2.3 of AISI Spec)
\[
f_1 = 345 \left( \frac{65.88805}{66.79705} \right) = 340.3051 MPa
\]
\[
f_2 = -345 \left( \frac{33.20295}{66.79705} \right) = -171.49 MPa
\]
\[
\psi = \left| \frac{f_2}{f_1} \right| = 0.50393
\]
\[
k = 4 + 2(1 + \psi)^3 + 2(1 + \psi) = 13.81105
\]
\[
F_{cr} = k \frac{\pi^2 E (t/h)^2}{12(1 - \mu^2)} = 216.604 MPa
\]
\[
\lambda = \sqrt{\frac{f}{F_{cr}}} = 1.26205 \geq 0.673
\]
\[
\rho = (1 - \frac{22}{\lambda})/\lambda = 0.65424
\]
\[
b_c = \rho w = 64.829 \text{ mm} \quad \text{when } \lambda > 0.673
\]
\[
h_o/b_o = 0.66339 \leq 4
\]
\[
b_1 = b_c(3 + \psi) = 18.5018 \text{ mm}
\]
\[
b_2 = b_c/2 = 32.4145 \text{ mm} \quad \text{when } \psi > 0.236
\]
\[
b_1 + b_2 = 50.9163 \text{ mm}
\]
Since the \( b_1 + b_2 \) is less than 65.88805, the compression part of the web, the web element is not fully effective as assumed. Additional iterations are required to relocate the neutral axis.

Figure 3 Effective lengths with partially effective web.
<table>
<thead>
<tr>
<th>Element</th>
<th>Effective Length L (mm)</th>
<th>Distance from Top Fiber y (mm)</th>
<th>Ly (mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>299.091</td>
<td>100.4545</td>
<td>30045.04</td>
</tr>
<tr>
<td>2</td>
<td>1.818</td>
<td>100.4545</td>
<td>182.6263</td>
</tr>
<tr>
<td>3</td>
<td>131.235</td>
<td>67.1913</td>
<td>8817.84</td>
</tr>
<tr>
<td>4</td>
<td>37.0036</td>
<td>10.1599</td>
<td>375.953</td>
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<td>5</td>
<td>1.818</td>
<td>0.4545</td>
<td>0.826281</td>
</tr>
<tr>
<td>6</td>
<td>71.7546</td>
<td>0.4545</td>
<td>32.6125</td>
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<td>7</td>
<td>33.20572</td>
<td>6.3245</td>
<td>210.0096</td>
</tr>
<tr>
<td>Total</td>
<td>575.926</td>
<td>39664.9</td>
<td></td>
</tr>
</tbody>
</table>

\[ y_{cg} = \frac{\sum (Ly)}{\sum L} = 68.8716 \text{ mm} \]

\[ f_1 = 340.447 \text{MPa} \]
\[ f_2 = -155.93 \text{MPa} \]
\[ \psi = \left| \frac{f_2}{f_1} \right| = 0.458 \quad (B2.3-1) \]
\[ k = 4 + 2(1 + \psi)^3 + 2(1 + \psi) = 13.1151 \quad (B2.3-2) \]
\[ F_{cr} = k \frac{\pi^2 E}{12(1 - \mu^2)} \left( \frac{t}{h} \right)^2 = 205.6039 \quad (B2.1-5) \]

\[ \lambda = \sqrt{\frac{f}{F_{cr}}} = 1.29537 > .673 \quad (B2.1-4) \]
\[ \rho = (1 - .22/\lambda)/\lambda = 0.64087 \quad (B2.1-3) \]
\[ b_c = \rho w = 63.5043 \quad \text{when } \lambda > 0.673 \quad (B2.1-7) \]

\[ b_1 = b_\psi/(3 + \psi) = 18.3643 \text{mm} \quad (B2.3-3) \]
\[ b_2 = b_\psi/2 = 31.7522 \text{mm} \quad \text{when } \psi > 0.236 \quad (B2.3-4) \]
\[ b_1 + b_2 = 50.1165 \text{mm} \]

Because the new \( b_1 + b_2 \) is less than the previous \( b_1 + b_2 \) by 1.6%, additional iterations are required.

After few more iterations,
\[ f = 340.473 \text{MPa} \]
\[ \lambda = 1.30174 \]
\[ b_1 = 18.3387 \text{mm} \]
\[ b_2 = 31.6285 \text{mm} \]
\[ y_{cg} = 69.2771 \text{mm} \]

4. Moment of inertia and section modulus.
i. about its own center axis

\[ I_1 = \frac{1}{12} (149.5455)(.909)^3 = 9.36017 \text{ mm}^4 \]
\[ I_2 = \frac{1}{12} (.909)(.909)^3 = 0.053895 \text{ mm}^4 \]
\[ I_3 = \frac{1}{12} (.909)(62.30735)^3 = 18362.049 \text{ mm}^4 \]
\[ I_4 = \frac{1}{12} (.909)(18.32338)^3 = 467.183 \text{ mm}^4 \]
\[ I_5 = \frac{1}{12} (.909)(.909)^3 = 0.0056895 \text{ mm}^4 \]
\[ I_6 = \frac{1}{12} (35.8476)(.909)^3 = 2.2456 \text{ mm}^4 \]
\[ I_7 = I_s = 346.6825 \text{ mm}^4 \]
\[ \sum I = I_Z = 38028.6 \text{ mm}^4 \]

ii. the actual moment of inertia

\[ I_z + \sum (L_y^2)(t) - (\sum L)(y_{cg})^2(t) = I_x = 857155.17 \text{ mm}^4 \]

\[ S_x = \frac{I_x}{y_{cg}} = 12372.851 \text{ mm}^3 \]


\[ M_n = S_x F_y = 4.268634 \text{ kN-m} \]
2. AISI B5.1 Method

Parameters:
n=1, w/t=70, ws=23.48mm, ds=11.74mm, hw=100mm, wtf=150mm, Fy=345MPa
E=203400MPa, w = 63.63mm

Figure 1 Actual Elements.

1. Calculating the effective width of the compression flange.

(B5.1 Uniformly Compressed Elements with One Intermediate Stiffener)

\[ b_o = (n+1)(w) + (n)ws = 150.74 \text{mm} \]
\[ b_p = w = 63.63 \text{mm} \]
\[ k_{loc} = 4 \left( \frac{b_o}{b_p} \right)^2 = 22.449 \]
\[ I_s = 2 \left[ \frac{1}{12} \left( t^2 + t \frac{w_s^2}{ds} \right) \right] = 346.6825 \text{ mm}^4 \]
\[ I_{sp} = I_s + A_s d^2 = 346.6825 + 30.1840(5.87)^2 = 1386.73 \text{ mm}^4 \]
\[ \gamma_i = \frac{10.92(I_{sp})}{b_o t^3} = \gamma_i = \frac{10.92(1387.767)}{(150.74)(.909)^3} = 133.7502 \]
\[ \omega_i = \sin^2 \left( \pi \frac{c_i}{b_o} \right) = \omega_i = 1, c_i = w + ws/2 = 75.37 \]
\[ \delta_i = \left( A_i \right) \frac{b_o t}{b_o t} = 0.22028 \]
\[
\beta = \left( \frac{2}{n} \sum_{i=1}^{n} \gamma_i \omega_i + 1 \right)^{\frac{1}{4}} = \left( \frac{1}{n} \sum_{i=1}^{n} \gamma_i \omega_i + 1 \right)^{\frac{1}{4}} = 4.04796 \quad \text{(B5.1.2-3)}
\]

\[
k_d = \frac{(1 + \beta^2)^2 + 2 \sum_{i=1}^{n} \gamma_i \omega_i}{\beta^2(1 + 2 \sum_{i=1}^{n} \delta_i \omega_i)} = 24.1376 \quad \text{(B5.1.2-2)}
\]

\[
R = \frac{11 - b_o / h}{5} \geq \frac{1}{2} \quad \text{when } b_o / h \geq 1 \quad R = 1.89575 \quad \text{(B5.1-8)}
\]

\[
k = \text{the minimum of } R_kd \text{ and } k_{loc} \quad k = \min(45.75905, 22.449) = 22.449 \quad \text{(B5.1-6)}
\]

\[
F_{cr} = k \frac{\pi^2 E}{12(1 - \mu^2)} \left( \frac{t}{b_o} \right)^2 = 150.069 \text{MPa} \quad \text{(B5.1-5)}
\]

\[
\lambda = \sqrt{\frac{\mu}{F_{cr}}} = 1.51622 \quad \text{(B5.1-4)}
\]

\[
\rho = (1 - 0.22 / \lambda) / \lambda = 0.5638 \quad \text{when } \lambda > 0.673
\]

\[
b_c = \rho \left( \frac{A_c}{t} \right) = 90.4765 \text{mm} \quad \text{(B5.1-1)}
\]

2. Locating Neutral Axis with an assumption that the web is fully effective

* calculating the location of the flange (the centroid of the original flange element)

\[
y_f = \frac{n(A_s)(\frac{t}{2} + \frac{ds}{2}) + (n + 1)(w)(t)\frac{t}{2}}{n(A_s) + n(1)(w)(t)} = 1.65280 \text{ mm (from top fiber)}
\]

\[b = 90.4765 \text{ mm}\]

\[t = 0.909 \text{ mm}\]

\[h = 99.0811 \text{ mm}\]

\[wl = 149.5455 \text{ mm}\]

\[\lambda = \frac{h}{b} = 0.27\]

\[\rho = 0.5638\]

\[k = 22.449\]

\[F_{cr} = 150.069 \text{ MPa}\]

\[\lambda_{cr} = 1.51622\]

\[n = 2\]

\[A_s = 90.4765 \text{ mm}^2\]

\[w = 10 \text{ mm}\]

\[t = 0.909 \text{ mm}\]

\[h = 99.0811 \text{ mm}\]

\[wl = 149.5455 \text{ mm}\]

\[\chi_{cg} = 68.27561\]

\[\chi_f = 1.65280\]

Figure 2 Effective lengths with fully effective Web.
<table>
<thead>
<tr>
<th>Element</th>
<th>Effective Length L (mm)</th>
<th>Distance from Top Fiber y (mm)</th>
<th>Ly (mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>299.091</td>
<td>100.4545</td>
<td>30045.04</td>
</tr>
<tr>
<td>2</td>
<td>1.818</td>
<td>100.4545</td>
<td>182.6263</td>
</tr>
<tr>
<td>3</td>
<td>198.182</td>
<td>50.4545</td>
<td>9999.174</td>
</tr>
<tr>
<td>4</td>
<td>1.818</td>
<td>0.4545</td>
<td>0.826281</td>
</tr>
<tr>
<td>5</td>
<td>90.4765</td>
<td>1.65280</td>
<td>149.4174</td>
</tr>
<tr>
<td>Total</td>
<td>591.3855</td>
<td>40377.2</td>
<td></td>
</tr>
</tbody>
</table>

\[ y_{cg} = \frac{\sum (L_y)}{\sum L} = 68.27561 \text{mm} > \frac{h_w}{2} = 50 \text{ mm} \]

Since the distance \( y_{cg} \) is more than the half depth of \( h, 100 \text{ mm} \), the neutral axis is closer to the tension flange and, therefore, the maximum stress occurs in the compression flange. Therefore the assumption \( f = f_y \) is valid in this case.

3. Check the effectiveness of the web.

(B2.3 of AISI Spec)

\[ f_1 = 345 \left( \frac{67.36661}{68.27561} \right) = 340.4068 \text{ MPa} \]

\[ f_2 = -345 \left( \frac{31.72439}{68.27561} \right) = -160.3056 \text{ MPa} \]

\[ \psi = \left| \frac{f_2}{f_1} \right| = 0.47092 \quad \text{(B2.3-1)} \]

\[ k = 4 + 2(1 + \psi)^3 + 2(1 + \psi) = 13.30685 \quad \text{(B2.3-2)} \]

\[ F_{cr} = k \frac{\pi^2 E}{12(1 - \mu^2)} \left( \frac{t}{h} \right)^2 = 208.633 \text{ MPa} \quad \text{(B2.1-5)} \]

\[ \lambda = \sqrt{\frac{f}{F_{cr}}} = 1.285931 > 0.673 \quad \text{(B2.1-4)} \]

\[ \rho = \left( 1 - \frac{0.22}{\lambda} \right) / \lambda = 0.644605 \quad \text{(B2.1-3)} \]

\[ b_e = \rho w = 63.87457 \text{ mm} \quad \text{when} \quad \lambda > 0.673 \quad \text{(B2.1-3)} \]

\[ b_1 = b_e / (3 + \psi) = 18.40277 \text{ mm} \quad \text{(B2.3-3)} \]

\[ b_2 = b_e / 2 = 31.93729 \text{ mm} \quad \text{when} \quad \psi > 0.236 \quad \text{(B2.3-4)} \]

\[ b_1 + b_2 = 50.34006 \text{ mm} \]

Since the \((b_1 + b_2)\) is less than \(67.36661 \text{ mm}\), the compression part of the web, the web element is not fully effective as assumed. Additional iterations are required to relocate the neutral axis.
Figure 3 Effective lengths with partially effective web.

<table>
<thead>
<tr>
<th>Element</th>
<th>Effective Length ( L ) (mm)</th>
<th>Distance from Top Fiber ( y ) (mm)</th>
<th>( L_y ) (mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>299.091</td>
<td>100.4545</td>
<td>30045.04</td>
</tr>
<tr>
<td>2</td>
<td>1.818</td>
<td>100.4545</td>
<td>182.6263</td>
</tr>
<tr>
<td>3</td>
<td>127.3234</td>
<td>68.16916</td>
<td>8679.526</td>
</tr>
<tr>
<td>4</td>
<td>36.80554</td>
<td>10.11038</td>
<td>372.1182</td>
</tr>
<tr>
<td>5</td>
<td>1.818</td>
<td>0.4545</td>
<td>0.826281</td>
</tr>
<tr>
<td>6</td>
<td>90.4765</td>
<td>1.65280</td>
<td>149.4174</td>
</tr>
<tr>
<td>Total</td>
<td>557.3324</td>
<td></td>
<td>39429.67</td>
</tr>
</tbody>
</table>

\[ \bar{y}_{cg} = \frac{\sum (L_y)}{\sum L} = 70.74714 \text{mm} \]

\[ f_1 = 340.5672 \text{MPa} \]
\[ f_2 = -142.652 \text{MPa} \]
\[ \psi = \left| \frac{f_2}{f_1} \right| = 0.41887 \quad \text{(B2.3-1)} \]
\[ k = 4+2(1+\psi)^3+2(1+\psi) = 12.55061 \quad \text{(B2.3-2)} \]
\[ F_{cr} = k \sqrt{\frac{\pi E}{12(1-\mu^2)}} \left( \frac{t}{h} \right)^2 = 196.684 \text{MPa} \quad \text{(B2.1-5)} \]

\[ \lambda = \sqrt{\frac{f}{F_{cr}}} = 1.324418 > .673 \quad \text{(B2.1-4)} \]
\[ \rho = (1.22/\lambda) / \lambda = 0.629627 \quad \text{(B2.1-3)} \]
\[ b_e = \rho w = 62.39035 \text{mm} \quad \text{when } \lambda > 0.673 \quad \text{(B2.1-7)} \]
\[ b_1 = b_e/(3+\psi) = 18.24884 \text{mm} \quad \text{(B2.3-3)} \]
\[ b_2 = \frac{b_v}{2} = 31.19518 \text{mm} \quad \text{when } \psi > 0.236 \quad (B2.3-4) \]

\[ b_1 + b_2 = 49.44402 \text{mm} \]

Because the new \((b_1 + b_2)\) is still significantly less than the previous \((b_1 + b_2)\), additional iterations are required.

After few more iterations,

\[ f = 340.5977 \text{MPa} \]

\[ \lambda = 1.33182 \]

\[ b_1 = 18.2201 \text{mm} \]

\[ b_2 = 31.0562 \text{mm} \]

\[ y_{cg} = 71.2358 \text{mm} \]

4. Moment of inertia and section modulus.
   i. about its own center axis

\[ I_1 = \frac{1}{12} \times (149.5455)(.909)^3 = 9.36017 \text{ mm}^4 \]

\[ I_2 = \frac{1}{12} \times (.909)(.909)^3 = 0.053895 \text{ mm}^4 \]

\[ I_3 = \frac{1}{12} \times (.909)(59.77326)^3 = 16215.5 \text{ mm}^4 \]

\[ I_4 = \frac{1}{12} \times (.909)(18.20466)^3 = 458.176 \text{ mm}^4 \]

\[ I_5 = \frac{1}{12} \times (.909)(.909)^3 = 0.0056895 \text{ mm}^4 \]

\[ I_6 = \frac{1}{12} \times (90.4026)(.909)^3 = 5.66299 \text{ mm}^4 \]

\[ \sum I = I_Z = 33371.8 \text{ mm}^4 \]

ii. the actual moment of inertia

\[ I_z + \sum (L y^2) - \left( \sum L \right) (y_{cg})^2 = I_x = 797649.63 \text{ mm}^4 \]

\[ S_x = \frac{I_x}{y_{cg}} = 11197.3 \text{ mm}^3 \]


\[ M_n = S_x F_y = 3.863067 \text{ kN-m} \]
Appendix II: Verification Studies

Although the numerical outputs of B4.1 and B5.1 don’t exactly match with those of CFS due to the section’s corner properties, the following comparison shows that the output values of B4.1 and B5.1 are in a reasonable range with CFS.

Table 1 Comparison with CFS predictions.

<table>
<thead>
<tr>
<th>Parameters (mm)</th>
<th>B4.1</th>
<th>B5.1</th>
<th>CFS</th>
</tr>
</thead>
<tbody>
<tr>
<td>w/t</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>20</td>
<td>14.92</td>
<td>7.46</td>
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</tr>
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<td>50</td>
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<td>100</td>
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<td>7.4</td>
<td>100</td>
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<tr>
<td>70</td>
<td>14.8</td>
<td>7.4</td>
<td>50</td>
</tr>
</tbody>
</table>

Summary of CFS

- CFS version 5.0.2
- Procedure: NAS B5.1
- Thickness = 0.909 mm, Default inside radius = 0.0mm
- Modulus of Elasticity = 2.034 × 10^5 MPa, Yield Stress = 345 MPa
Appendix III: Terms

\( A_s = \) reduced area of stiffener
\( A_s' = \) actual area of stiffener
\( b = \) effective width
\( b_0 = \) total length of the flange element
\( b_p = \) length of the largest flat subelement
\( d_s = \) depth of stiffener
\( f = \) applied stress
\( F_y = \) minimum steel yield stress
\( h_w = \) height of web
\( h_0 = \) inner length of web
\( I_{a} = \) adequate moment of inertia of stiffener
\( I_s = \) actual moment of inertia of stiffener
\( k = \) buckling coefficient
\( t = \) design thickness
\( w = \) width of subelement
\( w_s = \) width of stiffener
\( wt_f = \) width of tension lip
\( \rho = \) reduction factor