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Economic Study of the Connection Safety Factor

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ABSTRACT

The safety factors used in connections (ranging from about 2.2 to 3.0 in the AISI specifications) are higher than in member design. This makes sense, because the strength calculations are less certain, and because the cost of providing more reliability by increasing the safety of connections is less than for members. In a limit states design format (Load and resistance factor design) the problem arises how to select the appropriate safety level for the design of connections. In the widely accepted second moment formats, the reliability is expressed by a safety index, $B$. Procedures are available to select the optimum value of the safety index for member design. The present paper considers the problem of selecting the safety index, and hence the strength factor or safety factor, for connections, given statistical data and given the optimum safety indices for member design.

The analysis is made on the basis of the economic principle of equal marginal returns: When the reliability of members and connections are both optimal, it should be impossible to increase the reliability through reducing the investment in connections by a small amount and reinvesting it in the members, or vice versa.

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It is assumed that the consequences of failure (including loss of life and limb etc.) are the same for member and connection failure. The analysis permits the elimination of these uncertain, subjective and controversial quantities, and reduces to a comparison of the curves of reliability vs. initial cost for members and connections. These curves are generated by comparative designs in coldformed steel deck, cladding and structural members. The results tend to confirm the safety factors employed in the AISI specification and may be used to derive appropriate safety factors for similar specifications, for example for design of temporary buildings.

SUMMARY

Second moment reliability theory and the economic principle of equal marginal returns are used to derive the optimal connection safety factor, given the basic safety factor for member design in cold formed steel. Results tend to confirm the factors in current use.

INTRODUCTION

The use of high safety factors in connection design seems appropriate for a variety of reasons. Some which immediately come to mind are: susceptibility to design and construction errors and the sometimes more serious consequences of failure. Connection failure may also be more likely than member failure.

From an economic viewpoint, it costs less to provide more overall safety by increasing connection reliability. Hence, a higher factor of safety on connections would seem reasonable on the basis of economics as well.
Having established that higher safety factors appear desirable for connection design criteria, the problem arises where to stop: Intuitively a designer (or, rather code committee) will decide after a certain point that it is no longer worthwhile to spend extra for safer connections. What he is describing is, in fact, an economic equilibrium point. He is allocating resources (money) at his disposal so as to receive the optimum return (structural reliability, interest on other investments, etc.) for his client’s investment.

As a resource allocation problem or an optimization problem, it may be possible to select an appropriate safety level for connections by many methods. This paper considers one of the simplest: the economic principle of equal marginal returns. However, before applying this simple idea to a structural problem, it is wise to consider why its application to such a problem is reasonable.

THE PRINCIPLE OF EQUAL MARGINAL RETURNS

This principle has been described by Baumol (2) and others as the rule of relative levels of economic activity. It states that optimal results occur when activity levels (or resource allocations) all yield the same "marginal returns". For the present purpose the principle can be explained with reference to Fig 1.

The surface represents the expected failure cost $F = F(M, C)$, where $M$ is the member cost and $C$ is the cost of connections. $F$ is a convex function, as shown, by the principle of diminishing returns. Plane ABCDE is a plane of constant initial cost $M + C = \text{const.}$ or, equivalently, $dM = -dC$. At the minimum point E,
\[ dF = \frac{\partial F}{\partial M} \, dM + \frac{\partial F}{\partial C} \, dC = \left( \frac{\partial F}{\partial M} - \frac{\partial F}{\partial C} \right) dM = 0 \]  

Hence, a necessary condition for minimum failure cost is

\[ \frac{\partial F}{\partial M} = \frac{\partial F}{\partial C} \]  

The partial derivatives \( \frac{\partial F}{\partial M} \) and \( \frac{\partial F}{\partial C} \) are called the marginal returns

(of \( F \) on \( M \) and \( C \), respectively). Fig. 1 shows the locus of all relative equilibrium points \( E \).

If it is assumed that code committees give considerable thought to the appropriate values of safety factors for member design, it is reasonable to consider such a safety factor as indicative of the optimum point with respect to member design. By designing for a few safety levels above and below this optimum point, the equilibrium marginal return \( \frac{\partial F}{\partial M} \) can be determined as the slope in Fig 2.

Similarly, failure costs can be determined for several connection designs and a graph constructed. Using this graph and the condition, Eq. 2, the optimum connection design can be located. Working backwards, the corresponding reliability and hence safety factor can be determined.

In many designs the expected cost of failure, defined as the product of the failure probability and the failure cost, is almost directly proportional to the probability of failure. This occurs when damage to non-structural goods outweighs the damage to the structure itself. Similarly, in many cases the consequences of member and connection failure are about the same. Again, this is likely to occur when non-structural damage is considerably greater than structural damage. The selection of an appropriate safety index can then be made directly from graphs.
of cost versus the safety index.

EVALUATION OF NOMINAL RELIABILITY

Based on the widely accepted second moment formats, reliability is expressed in terms of a safety index $B$. This $B$ is a measure of the number of standard deviations away from the mean of the formulation variable (e.g. ratio of strength to loading) that the failure condition lies. By assuming an appropriate distribution for the formulation variable, the safety index can be transformed into a nominal probability of failure.

Allen (1) describes in detail the calculation of the safety index and the nominal reliability, and gives statistical data, used in the following example in this paper.

EVALUATION OF COSTS:

The expected cost of failure can be rewritten as follows:

$$EC_F = P_F C_F = (P_C + P_M - P_{MC}) C_F \approx (P_C + P_M) C_F \approx [P(B_C) + P(B_M)] C_F$$

where the approximation has been used that $P_{MC}$, the probability of simultaneous member and connection failure, is negligible; in Eq. 3, $P_C$ and $P_M$ are the probabilities of connection and member failure, respectively, while $P$ is the failure probability function, assumed lognormal; $B_C$ and $B_M$ are the safety indexes for connections and members, respectively (see Fig 3).

The cost of failure $C_F$ is assumed constant and is the same for member and connection failure. It is important to notice that $C_F$ is then a constant factor
which can be ignored, eliminating the need to calculate this nebulous and somewhat controversial factor.

**EXAMPLE: CONNECTION SAFETY LEVEL IN COLD FORMED STEEL DESIGN**

The following example, while as realistic as possible for the short period in which it was undertaken, is meant primarily as an illustration of the method outlined in this paper. To draw firmer conclusions would require a carefully selected set of such examples representative of current practice and intimate knowledge of the cost estimating procedures and bidding policies current in the industry.

The cladding of a 45 ft x 100 ft warehouse (Fig 4) was designed for "average" loading conditions according to climatic data from six urban centers across Canada. The designs were carried out for three member factors of safety and three connection factors of safety.

All designs were based on CSA Standard S136-1974 otherwise, and were governed by flexure in the case of member designs. The designs considered were a typical roof deck, wall panel, roof purlin, wall stringer, and the corresponding connections to each other and the hot rolled frame.

For each of these designs a corresponding safety index was calculated. This, in turn, was related to the nominal probability of failure using the graph in Fig 3. From the nominal reliability the expected cost of failure was calculated by Eq. 3. A cost analysis was made to determine the initial cost of each design. The results are shown in Figs 5 and 6.
Connection costs were heavily constrained by available connector sizes. In order to make the comparison more reasonable, fictitious intermediate connector sizes were also considered. A differentiable fictitious cost was then calculated and plotted as the dashed line in Fig. 6.

Member and connection cost graphs were compared, and the slope criterion, Eq. 2, was used to determine the connector safety factor corresponding to the member safety factor 1.60. According to the analysis, the connection safety factor should be about 2.30.

CONCLUSIONS

1. For any value of the basic safety factor in a structural code there is a corresponding optimum value of the safety factor on connections, dictated by the economic principle of equal marginal returns.

2. If the consequences of member and connection failure are the same, the value loss attached to failure does not influence the optimum connection safety level for a given member safety level.

3. The example shows that it is possible to calculate the optimum connection safety level in practice. To reach firm conclusions about the exact numerical value of the connection safety factor for a structural code, a number of such calculations would have to be made. The calculations can be based, at the present level of information, only on nominal reliabilities according to second moment rationales; however, this is not believed to have a major influence on the resulting safety factors.
4. The example suggests that current connection safety factors in the CSA S136-74, AISI-1968, and similar specifications (2.2 to 3.0) are not far from the economic optimum.

5. The example further suggests that the optimum reliability of connections is higher than for members, with a failure probability ratio of about 1:50.

ACKNOWLEDGMENT

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REFERENCES


APPENDIX 1 - Notation

\( B \) = Safety index according to the second moment rationale (1).

\( C \) = connection cost

\( C_F \) = failure cost

\( F \) = expected failure cost

\( M \) = member cost
APPENDIX II: WAREHOUSE DESIGN (EXAMPLE)

The cladding for the warehouse in Fig. 4 is designed according to CSA Standard S136-1974. Design loads follow the National Building Code of Canada 1975, for an "average", fictitious, Canadian location when the design values of the 10 year wind and ground snow load are 8 and 36 psf. respectively. In addition, the specified live load on the roof of 300 lbs over a 30 in x 30 in area was superimposed on the snow load.

The following constraints were accepted. (a) The thickness of cladding sections at least 0.0239 in. (old gage No. 24), an industrial standard. (b) Material limited to cold formed steel in yield strengths commonly used (about 50 ksi). (c) Connections limited to available types, notably those used for quick semi-automatic installation. (d) All other criteria of the CSA Standard S136 -1974 except the specified safety factors. However, deflection constraints were deliberately omitted, because the objective of the design, and the basis for comparison, is the protection of goods against damage, not loss of serviceability.

Designs for typical roof deck, wall panel, roof purlin and wall beam members were done for three factors of safety: 1.35, 1.60, 1.90. The corresponding connections were designed for the factors of safety: 1.95, 2.30, 2.70. Some solutions, particularly in wall panel design, were constrained by minimum thickness requirements. The designs aim to represent practice.
Except for the thicknesses, the roof deck and wall panels were similar to the examples in the AISI manual.

The roof purlins and wall stringers were 10 in. x 3.5 in. and 7 in. x 2.75 in. Zee. The connections were of two types: (1) Self-tapping hexagonally headed and slotted screw bolts with washers, 33 ksi yield strength and (2) A325 bolts with washers (33ksi). Table 1 and 2 show the results of the design.

The safety index, calculated as in Ref.(1), and the nominal failure probability from Fig. 3 are given in Table 3. The failure costs indicated in Figs 5 and 6 were based on an assumed failure cost value of $ 10/ft$^3$, but this value does not influence the conclusions. Building costs were estimated from available construction cost data in a conventional manner.
TABLE 1: WAREHOUSE DESIGN EXAMPLE
MEMBER THICKNESSES IN INCHES

<table>
<thead>
<tr>
<th>F.S.</th>
<th>Deck</th>
<th>Wall</th>
<th>Purlin</th>
<th>Stringer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.35</td>
<td>0.0239</td>
<td>0.0239</td>
<td>0.105</td>
<td>0.075</td>
</tr>
<tr>
<td>1.60</td>
<td>0.0269</td>
<td>0.0239</td>
<td>0.120</td>
<td>0.090</td>
</tr>
<tr>
<td>1.90</td>
<td>0.0269</td>
<td>0.0239</td>
<td>0.135</td>
<td>0.105</td>
</tr>
</tbody>
</table>

TABLE 2: WAREHOUSE DESIGN EXAMPLE CONNECTIONS STANDARD AND (CONTINUOUS) SIZES IN INCHES

<table>
<thead>
<tr>
<th>Connection Type</th>
<th>Spacing</th>
<th>F.S. = 1.95</th>
<th>F.S. = 2.30</th>
<th>F.S. = 2.70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deck Screw</td>
<td>7.5</td>
<td>1/8 (0.108)</td>
<td>1/8 (0.117)</td>
<td>3/16 (0.127)</td>
</tr>
<tr>
<td>Wall Screw</td>
<td>12</td>
<td>1/8 (0.108)</td>
<td>1/8 (0.118)</td>
<td>3/16 (0.128)</td>
</tr>
<tr>
<td>Purlin A325</td>
<td>-</td>
<td>1/2 (0.400)</td>
<td>1/2 (0.436)</td>
<td>1/2 (0.471)</td>
</tr>
<tr>
<td>Beam A325</td>
<td>-</td>
<td>3/8 (0.300)</td>
<td>3/8 (0.326)</td>
<td>3/8 (0.353)</td>
</tr>
</tbody>
</table>

TABLE 3: WAREHOUSE DESIGN EXAMPLE
SAFETY INDICES AND FAILURE PROBABILITIES

<table>
<thead>
<tr>
<th>MEMBERS</th>
<th>F.S. = 1.35</th>
<th>F.S. = 1.60</th>
<th>F.S. = 1.90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safety index B_M</td>
<td>2.43</td>
<td>3.05</td>
<td>3.67</td>
</tr>
<tr>
<td>Failure Probability P_M</td>
<td>7.08 \times 10^{-3}</td>
<td>1.26 \times 10^{-3}</td>
<td>0.166 \times 10^{-3}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CONNECTIONS</th>
<th>F.S. = 1.95</th>
<th>F.S. = 2.30</th>
<th>F.S. = 2.70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safety Index B_C</td>
<td>3.31</td>
<td>4.04</td>
<td>4.63</td>
</tr>
<tr>
<td>Failure probability P_C</td>
<td>5.62 \times 10^{-4}</td>
<td>2.81 \times 10^{-5}</td>
<td>1.26 \times 10^{-6}</td>
</tr>
</tbody>
</table>
Fig 1. Principle of Equal Marginal Returns.

Fig 2. Cost Tradeoff Characteristic for Member Design
Fig 3. Nominal Reliability vs. Safety
Fig 4. Example: 45' x 100' Warehouse, Structural geometry. (1) Roof Deck continuous over 3 spans of 8' each; (2) Wall Panel, continuous over 2 spans of 7.5' each; (3) Roof Purlin, simply supported, 20' span. (4) Wall Beam, simply supported, 20' span.
CONNECTION SAFETY FACTOR

Fig 5 Example: Member Cost Tradeoff Characteristic
Fig 6. Example: Connection Cost Tradeoff Characteristic