Output feedback controller for operation of spark ignition engines at lean conditions using neural networks

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Output Feedback Controller for Operation of Spark Ignition Engines at Lean Conditions Using Neural Networks

Jonathan Blake Vance, Member, IEEE, Brian C. Kaul, Sarangapani Jagannathan, Senior Member, IEEE, and James A. Drallmeier

Abstract—Spark ignition (SI) engines operating at very lean conditions demonstrate significant nonlinear behavior by exhibiting cycle-to-cycle bifurcation of heat release. Past literature suggests that operating an engine under such lean conditions can significantly reduce NOx emissions by as much as 30% and improve fuel efficiency by as much as 5%–10%. At lean conditions, the heat release per engine cycle is not close to constant, as it is when these engines operate under stoichiometric conditions where the equivalence ratio is 1.0. A neural network controller employing output feedback has shown ability in simulation to reduce the nonlinear cyclic dispersion observed under lean operating conditions. This neural network (NN) output controller consists of three NNs: a) an NN observer to estimate the states of the engine such as total fuel and air; b) a second NN for generating virtual input; and c) a third NN for generating actual control input. The uniform ultimate boundedness of all closed-loop signals is demonstrated by using the Lyapunov analysis without using the separation principle. Persistency of the excitation condition, the certainty equivalence principle, and the linearity in the unknown parameter assumptions are also relaxed.

The controller is implemented for a research engine as a program running on an embeddable PC that communicates with the engine through a custom hardware interface, and the results are similar to those observed in simulation. Experimental results at an equivalence ratio of 0.77 show a drop in NOx emissions by around 98% from stoichiometric levels with an improvement of fuel efficiency by 5%. A 30% drop in unburned hydrocarbons from uncontrolled case is observed at this equivalence ratio of 0.77. Similar performance was observed with the controller on a different engine.

Index Terms—Adaptive control, neural network (NN) hardware, neural networks (NNs), neurocontrollers, observers, output feedback.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>CFR</td>
<td>Cooperative fuel research.</td>
</tr>
<tr>
<td>COV</td>
<td>Coefficient of variation.</td>
</tr>
<tr>
<td>IMEP</td>
<td>Indicated mean effective pressure, Work/Disp. Volume.</td>
</tr>
<tr>
<td>uHC</td>
<td>Unburned hydrocarbons.</td>
</tr>
<tr>
<td>CE(k)</td>
<td>Combustion efficiency.</td>
</tr>
<tr>
<td>d1(k)</td>
<td>Unknown disturbance in air.</td>
</tr>
<tr>
<td>d2(k)</td>
<td>Unknown disturbance in fuel.</td>
</tr>
<tr>
<td>F(k)</td>
<td>Fraction of unreacted gas and fuel remaining from previous cycle.</td>
</tr>
<tr>
<td>R</td>
<td>Stoichiometric air-fuel mass ratio.</td>
</tr>
<tr>
<td>u(k)</td>
<td>Mass change fuel input.</td>
</tr>
<tr>
<td>x1(k)</td>
<td>Mass of air.</td>
</tr>
<tr>
<td>x2(k)</td>
<td>Mass of fuel.</td>
</tr>
<tr>
<td>φ(k)</td>
<td>Equivalence ratio.</td>
</tr>
<tr>
<td>φl, φu</td>
<td>Lower 10% and upper 90% locations of the combustion efficiency function.</td>
</tr>
<tr>
<td>φm</td>
<td>Midpoint between φl and φu.</td>
</tr>
</tbody>
</table>

I. INTRODUCTION

MODERN automobiles utilize microprocessor-based engine control systems to meet stringent federal regulations governing fuel economy and the emissions of CO, NOx, and uHC. Current efforts aim to decrease emissions and minimize the fuel consumption. To address these requirements, lean combustion control technology has received increasing attention [1]. Unfortunately, significant cyclic dispersion is exhibited when operating spark ignition engines at extreme lean conditions [2], [3], causing engine instability and poor performance.

Several control schemes have been proposed to stabilize engine operation at lean conditions. Inoue et al. [1] designed a lean combustion engine control system using a combustion pressure sensor. With the measurement of engine torsional acceleration, Davis et al. [4] developed a feedback control approach, which uses fuel as the control variable to reduce the cyclic dispersion. However, system stability is not guaranteed in either [1] or [4] since analysis of stability for nonlinear unknown engine dynamics during combustion is difficult. On the other hand, several control schemes [5]–[7] using state feedback are available.
to maintain air to fuel ratio near stoichiometric levels. Maintaining air to fuel ratio near a target value is different than reducing cyclic dispersion at lean engine operating conditions. Cyclic variability at lean engine operation causes instability and degraded performance levels.

Therefore, He et al. [8] proposed an adaptive neural network (NN) backstepping controller to maintain stable operation of the spark ignition (SI) engine at lean conditions by altering the fuel intake as the control variable. The NN is used to model the complex unknown engine dynamics. Lyapunov analysis is applied to ensure the uniformly ultimate boundedness (UUB) of the internal system signals. However, to implement the controller, total mass of air and fuel (system states) are required for each engine cycle. These are extremely difficult if not impossible to measure and, therefore, this controller cannot be implemented. In [9], another control scheme is presented using state feedback for air to fuel ratio control at stoichiometric conditions in order to maximize the benefits of the catalytic converter. As mentioned before, controlling air to fuel ratio at stoichiometric conditions is a totally different problem from reducing cyclic dispersion using heat release as the feedback parameter at lean engine operation. Additionally, cyclic variability exhibits very nonlinear, but to some level deterministic, behavior under lean conditions while being stochastic near stoichiometric operation.

Conventional control schemes [8] have been found incapable of reducing the cyclic dispersion to the levels needed to implement these concepts since the engine dynamics are not taken into consideration. Moreover, the total amount of fuel and air in a given cylinder is normally not measurable on a per-cycle basis which necessitates the development of output feedback control schemes.

Several output feedback controller designs in discrete time are proposed for the single-input–single-out (SISO) nonlinear systems [10]–[16]. However, no output feedback control scheme currently exists for the proposed class of nonstrict feedback nonlinear discrete-time systems. No controller design is available for nonstrict feedback nonlinear systems even with state feedback.

The separation principle [10], [12] does not hold for nonlinear systems, since an exponentially decaying state estimation error can lead to instability at finite time [10]. Consequently, the output feedback control design is in general quite difficult for nonlinear discrete-time systems even though it is highly necessary.

To make the controller implementation more practical, a heat release-based neuro-output feedback controller is proposed in discrete-time to reach stable operation of a single-cylinder SI engine at lean conditions. Noncatalytic SI engine designs (e.g., generator sets and other industrial applications) could make use of lean operation to reduce engine-out NOx as well as improve fuel efficiency. The proposed output feedback controller has an observer and a controller. The NN observer is designed to estimate the total mass of air and fuel in the cylinder by using a measured value of heat release. The estimated values are used by an adaptive NN controller. Consequently, the cyclic dispersion is reduced and the engine is stable even when an exact knowledge of engine dynamics is not known to the controller making the NN controller model-free.

The proposed controller is designed for a class of nonlinear discrete-time systems in nonstrict feedback form. Both simulation and experimental results show satisfactory performance of the controller. It is important to note that in this work, the output is an unknown function of system states unlike in the existing literature [10]–[16] where the system output is a known linear function of system states.

The stability analysis of the closed-loop control system is given and the boundedness of the closed-loop signals is shown since a stable open-loop system can still become unstable with a controller. This stability permits higher levels of diluents to be considered for a specific engine, further enhancing NOx reduction and fuel efficiency than would be realized on an uncontrolled engine. The NN weights are tuned online, with no offline learning phase required. Moreover, separation principle, persistency of excitation condition, certainty equivalence, and linearity in the unknown parameters assumptions are relaxed. Performance of the NN controller is evaluated on different engines and results show satisfactory performance of the controller.

II. CONTROLLER DESIGN
A. Background

1) Engine Dynamics: According to the Daw model [2], [3], SI engine dynamics can be expressed as a class of nonlinear systems in nonstrict feedback form

\[
\begin{align*}
    x_1(k+1) &= A F(k) + F(k)x_1(k) - R \\
    & \cdot F(k)CE(k)x_2(k) + d_1(k) \\
    x_2(k+1) &= (1 - CE(k))F(k)x_2(k) \\
    & + (MF(k) + u(k)) + d_2(k) \\
    y(k) &= x_2(k)CE(k) \\
    \varphi(k) &= R \frac{x_2(k)}{x_1(k)} \\
    CE(k) &= \frac{CE_{\text{max}}}{1 + 100\% (\varphi(k) \varphi_m)} \\
    \varphi_m &= \frac{\varphi_u - \varphi_l}{2}
\end{align*}
\]

where \( x_1(k) \) and \( x_2(k) \) are total mass of air and fuel, respectively, in the cylinder before kth burn, \( y(k) \) is the heat release at kth instant, \( CE(k) \) is combustion efficiency for \( 0 < CE_{\text{min}} < CE(k) < CE_{\text{max}} \), \( CE_{\text{max}} \) is the maximum combustion efficiency, \( F(k) \) is residual gas fraction for \( 0 < F_{\text{min}} < F(k) < F_{\text{max}} \), \( AF(k) \) is mass of fresh air per cycle, \( R \) is stoichiometric air-fuel ratio, \( MF(k) \) is mass of fresh fuel per cycle, \( u(k) \) is change in mass of fresh fuel per cycle, \( \varphi(k) \) is input equivalence ratio, \( \varphi_m, \varphi_u, \varphi_l \) are constant system parameters, and \( d_1(k) \) and \( d_2(k) \) are unknown but bounded disturbances. Since \( y(k) \) varies each cycle, the engine is unstable. In the previously described engine dynamics, both \( F(k) \) and \( CE(k) \) are unknown nonlinear functions of \( x_1(k) \) and \( x_2(k) \).

Remark 1: For the system represented by (1)–(3), states of \( x_1(k) \) and \( x_2(k) \) are typically not measurable [17] and output \( y(k) \) can be made available. The control objective is to stably operate the engine at lean conditions \( (0 < \varphi(k) < 1) \) with only heat release information available—to stabilize \( y(k) \) around \( y_{\text{d}} \), where \( y_{\text{d}} \) is the target heat release value.

Remark 2: We notice that in (3) the available system output \( y(k) \) is an unknown nonlinear function of both unmeasurable...
states of $x_1(k)$ and $x_2(k)$, unlike that in all past literatures [10]–[16], where $y(k) = x_1(k)$ or $y(k)$ is a known linear combination of system states. This issue makes the observer design more challenging.

2) Engine Dynamics in a Different Form: Substituting (3) into both (1) and (2), we get

$$
x_1(k+1) = AF(k) + F(k)x_1(k) - R \cdot F(k)y(k) + d_1(k) 
$$

(7)

$$
x_2(k+1) = F(k)(x_2(k) - y(k)) + (MF(k) + u(k)) + d_2(k).
$$

(8)

For actual engine operation, fresh air $AF(k)$, fresh fuel $MF(k)$, and residual gas fraction $F(k)$, can all be viewed as nominal values plus some small and bounded disturbances

$$
AF(k) = AF_0 + \Delta AF(k)
$$

(9)

$$
MF(k) = MF_0 + \Delta MF(k)
$$

(10)

$$
F(k) = F_0 + \Delta F(k)
$$

(11)

where $AF_0$, $MF_0$, and $F_0$ are known nominal fresh air, fresh fuel, and residual gas fraction values, respectively. $\Delta AF(k)$, $\Delta MF(k)$, and $\Delta F(k)$ are small, unknown but bounded disturbances for fresh air, fresh fuel, and residual gas fraction, respectively. Their bounds are given by

$$
0 \leq |\Delta AF(k)| \leq \Delta AF_m
$$

(12)

$$
0 \leq |\Delta MF(k)| \leq \Delta MF_m
$$

(13)

$$
0 \leq |\Delta F(k)| \leq \Delta F_m
$$

(14)

where $\Delta AF_m$, $\Delta MF_m$, and $\Delta F_m$ are the respective upper bounds for $\Delta AF(k)$, $\Delta MF(k)$, and $\Delta F(k)$.

Combine (9)–(11) with (7) and (8), and rewrite (7) and (8) to get

$$
x_1(k+1) = AF_0 + F_0x_1(k) - R \cdot F_0 \cdot y(k) + \Delta AF(k)
$$

$$
+ \Delta F(k)x_1(k) - R\Delta F(k)y(k) + d_1(k)
$$

(15)

$$
x_2(k+1) = F_0(x_2(k) - y(k)) + (MF_0 + u(k))
$$

$$
+ \Delta F(k)(x_2(k) - y(k)) + \Delta MF(k) + d_2(k).
$$

(16)

Now, at the $k$th step and based on (3), future heat release, $y(k+1)$ can be predicted as

$$
y(k+1) = x_2(k+1)CE(k+1)
$$

$$
= f_3(x_1(k), x_2(k), y(k), u(k))
$$

(17)

where $f_3(x_1(k), x_2(k), y(k), u(k))$ is an unknown nonlinear function.

It is important to note that the closed-loop stability analysis has to be performed with the proposed NN controller even though many of the engine terms are considered bounded above since a stable open-loop system can still become unstable with a controller unless the NN weight update laws are properly selected. Moreover, a Lyapunov-based stability analysis is needed in order to see the relaxation of the separation principle for the observer and certainty equivalence principle for the controller. Next, the NN observer design is introduced.

B. NN Observer Design

A two-layer NN predicts the heat release in the subsequent time interval. The heat release prediction error is utilized to design the system observer. From (17) $y(k+1)$ can be approximated by using a one layer NN as

$$
y(k+1) = w_1^T \phi_1 (v_1^T z_1(k)) + \epsilon_1 (z_1(k))
$$

(18)

where $z_1(k) = [x_1(k), x_2(k), y(k), u(k)]^T \in R^4$ is the network input, matrices $w_1 \in R^{m_1}$ and $v_1 \in R^{1 \times m_1}$ represent target output and hidden layer weights, $\phi_1(\cdot)$ represents the hidden layer activation function, $v_1$ denotes the number of the hidden layer nodes, and $\epsilon_1(z_1(k)) \in R$ is the functional approximation error. As demonstrated in [18], if the hidden layer weight, $v_1$, is chosen initially at random and held constant and the number of hidden layer nodes is sufficiently large, the approximation error $\epsilon_1(z_1(k))$ can be made arbitrarily small over the compact set since the activation function forms a basis.

For simplicity define

$$
\phi_1 (z_1(k)) \equiv \phi_1 (v_1^T z_1(k)) = \epsilon_1 (z_1(k)).
$$

(19)

Given (19) and (20), (18) is rewritten as

$$
y(k+1) = w_1^T \phi_1 (z_1(k)) + \epsilon_1 (z_1(k)).
$$

(21)

1) Observer Structure: Since states $x_1(k)$ and $x_2(k)$ are not measurable, $z_1(k)$ is not available either. Using the estimated values $\hat{x}_1(k), \hat{x}_2(k)$, and $\hat{y}(k)$ instead of $x_1(k), x_2(k)$, and $y(k)$, the proposed heat release observer is given as

$$
\hat{y}(k+1) = \hat{w}_1^T(k)\phi_1 (v_1^T \hat{z}_1(k)) + l_{1}\hat{y}(k)
$$

$$
= \hat{w}_1^T(k)\phi_1 (\tilde{z}_1(k)) + l_{1}\hat{y}(k).
$$

(22)

where $\hat{y}(k+1)$ is the predicted heat release, $\hat{w}_1(k) \in R^{m_1}$ are output layer weights, $\tilde{z}_1(k) = [\tilde{x}_1(k), \tilde{x}_2(k), \tilde{y}(k), u(k)]^T \in R^4$ is the network input, $l_{1} \in R$ is the observer gain, $\hat{y}(k)$ is the heat release estimation error, where

$$
\hat{y}(k) = \hat{z}_1(k) - y(k)
$$

(23)

and $\phi_1 (\tilde{z}_1(k))$ represents $\phi_1 (v_1^T \tilde{z}_1(k))$, for simplicity.

Using the heat release estimation error, the proposed system observer is given as

$$
\hat{x}_1(k+1) = AF_0 + F_0 \hat{x}_1(k) - R \cdot F_0 \cdot \hat{y}(k) + l_{2}\hat{y}(k)
$$

(24)

$$
\hat{x}_2(k+1) = F_0 (\hat{x}_2(k) - \hat{y}(k)) + (MF_0 + u(k)) + l_3\hat{y}(k)
$$

(25)

where $l_2 \in R$ and $l_3 \in R$ are observer gains. Here, the initial value of $u(0)$ is assumed to be bounded. Equations (22)–(25) represent the proposed system observer to estimate the states of $x_1(k)$ and $x_2(k)$. 

(10)
2) Observer Error Dynamics: Let us define the state estimation errors as
\[
\tilde{x}_i(k) = \hat{x}_i(k) - x_i(k) \quad (i = 1, 2).
\]
Combining (21)–(26), we obtain the estimation error dynamics as
\[
\begin{align*}
\dot{\tilde{x}}_1(k+1) &= F_0 \tilde{x}_1(k) + (I_2 - R \cdot F_0) \tilde{y}(k) - \Delta F(k) x_1(k) + R \Delta F(k) \tilde{y}(k) - d_1(k) \\
\dot{\tilde{x}}_2(k+1) &= F_0 \tilde{x}_2(k) + (I_2 - F_0) \tilde{y}(k) - \Delta F(k) (x_2(k) - y(k)) - \Delta M(k) - d_2(k)
\end{align*}
\]
(27)
\[
\begin{align*}
\tilde{y}(k+1) &= \tilde{w}_1^T(k) \tilde{\varphi}_1 (\tilde{z}_1(k)) + l_1 \tilde{y}(k) - w_1 \varphi_1 (z_1(k)) - \varepsilon_1(k) \\
&= \tilde{w}_1^T(k) \varphi_1 (\tilde{z}_1(k)) + l_1 \tilde{y}(k) - w_1 \varphi_1 (z_1(k)) - \varepsilon_1(k) \\
&= \zeta_1(k) + l_1 \tilde{y}(k) - \varepsilon_1(k) + w_1 \varphi_1 (\tilde{z}_1(k))
\end{align*}
\]
(28)
where
\[
\begin{align*}
\tilde{w}(k) &= w(k) - w_1 \\
\zeta_1(k) &= \tilde{w}_1^T(k) \varphi_1 (\tilde{z}_1(k))
\end{align*}
\]
(30)
and, for simplicity, \( \tilde{\varphi}_1 (\tilde{z}_1(k)) = (\varphi_1 (\tilde{z}_1(k)) - \varphi_1 (z_1(k))) \).
These substitutions are made to simplify the analysis and to show the boundedness of the closed-loop signals.

C. Adaptive NN Output Feedback Controller

Heat release cyclic dispersion is observed at lean conditions, and, thus, engine operation is unsatisfactory. To stabilize the engine at lean conditions, our control objective is to reduce the heat release cyclic dispersion—drive the heat release toward the target operating point of \( y_d \). Given \( y_d \) and the engine dynamics (1)–(5), we could obtain the operating point of total mass of air and fuel in the cylinder, \( x_{1d} \) and \( x_{2d} \), respectively. By driving states \( x_1(k) \) and \( x_2(k) \) to approach their respective operating points \( x_{1d} \) and \( x_{2d} \), \( y(k) \) will approach the desired value \( y_d \). Then the control objective is realized. With the estimated states \( \hat{x}_1(k) \) and \( \hat{x}_2(k) \), the controller design follows the backstepping technique [19] detailed in Sections III-C1 and III-C2.

1) Adaptive NN Output Feedback Controller Design:

Step 1) Virtual controller design. Define system error as
\[
e_1(k) = x_1(k) - x_{1d}.
\]
(32)
Combining with (1), (32) can be rewritten as
\[
e_1(k+1) = x_1(k+1) - x_{1d} = AF(k) + F(k) x_1(k) - x_{1d} - R \\
\cdot F(k)CE(k) x_2(k) + d_1(k)
\]
(33)
For simplicity, let us denote
\[
\begin{align*}
f_1(k) &= AF(k) + F(k) x_1(k) - x_{1d} \\
g_1(k) &= R \cdot F(k)CE(k)
\end{align*}
\]
(34)
(35)
Then the system error equation can be expressed as
\[
e_1(k+1) = f_1(k) - g_1(k) x_2(k) + d_1(k).
\]
(36)
By viewing \( x_2(k) \) as a virtual control input, a desired feedback control signal can be designed as
\[
x_2(k) = \frac{f_1(k)}{g_1(k)}.
\]
(37)
The term \( x_2(k) \) can be approximated by the second NN as
\[
x_2(k) = u_2^T \varphi_2 (v_2^T x(k)) + \varepsilon_2 (x(k))
\]
\[
= u_2^T \varphi_2 (x(k)) + \varepsilon_2 (x(k))
\]
(38)
where the input is the state \( x(k) = [x_1(k), x_2(k)]^T \), \( u_2 \in \mathbb{R}^{n_2} \), and \( v_2 \in \mathbb{R}^{2 \times m_2} \) denote the constant ideal output and hidden layer weights, \( n_2 \) is the number of hidden layer nodes, the hidden layer activation function of the input and hidden layer weights, \( \varphi_2 (v_2^T x(k)) \), is abbreviated as \( \varphi_2 (x(k)) \), and \( \varepsilon_2 (x(k)) \) is the approximation error.
Since both \( x_1(k) \) and \( x_2(k) \) are unavailable, the estimated state \( \hat{x}(k) \) is selected as the NN input. Consequently, the virtual control input is taken as
\[
\dot{x}_2(k) = u_2^T \varphi_2 (v_2^T \hat{x}(k)) = u_2^T \varphi_2 (\hat{x}(k))
\]
(39)
where \( u_2^T \varphi_2(\hat{x}(k)) \in \mathbb{R}^{n_2} \) is the actual weight matrix for the second NN. Define the weight estimation error by
\[
\dot{w}_2(k) = \tilde{w}(k) - w_2.
\]
(40)
Define the error between \( x_2(k) \) and \( \dot{x}_2(k) \) as
\[
e_2(k) = x_2(k) - \dot{x}_2(k).
\]
(41)
Equation (36) can be expressed using (41) for \( x_2(k) \) as
\[
e_1(k+1) = f_1(k) - g_1(k) (e_2(k) + \dot{x}_2(k)) + d_1(k)
\]
(42)
or, equivalently
\[
e_1(k+1) = f_1(k) - g_1(k) \cdot (e_2(k) + x_2(k) - x_{2d}(k) + \dot{x}_2(k)) + d_1(k)
\]
\[
= -g_1(k) (e_2(k) + x_2(k) - x_{2d}(k) + \dot{x}_2(k)) + d_1(k)
\]
\[
= -g_1(k) \left( e_2(k) + w_2^T \varphi_2 (\hat{x}(k)) - w_2^T \varphi_2 (\hat{x}(k)) \right) + d_1(k).
\]
(43)
Similar to the calculation of (29), (43) can be further expressed as
\[
e_1(k+1) = -g_1(k) \left( e_2(k) + \zeta_2(k) + w_2^T \varphi_2 (\hat{x}(k)) - \varepsilon_2 (x(k)) \right) + d_1(k)
\]
(44)
where
\[
\begin{align*}
\omega_2(k) &= \hat{u}_2^T(k)\phi_2(\hat{x}(k)) \\
\omega_2^T \phi_2(\hat{x}(k)) &= \omega_2^T \phi_2(\hat{x}(k)) - \phi_2(x(k)))
\end{align*}
\]  
(45)
(46)

Step 2) Design of Control Input $u(k)$. Rewriting the error $e_2(k)$ from (41) as

\[
e_2(k + 1) = x_2(k + 1) - \hat{x}_2(k + 1) = (1 - CE(k)) F(k)x_2(k) + (MF(k) + u(k)) - \hat{x}_2(k + 1) + d_2(k)
\]
(47)

for simplicity, let us denote

\[
f_2(k) = (1 - CE(k)) F(k)x_2(k) + MF(k).
\]
(48)

Equation (47) can be written as

\[
e_2(k + 1) = f_2(k) + u(k) - \hat{x}_2(k + 1) + d_2(k).
\]
(49)

Here, the future value $\hat{x}_2(k + 1)$ is not available in the current time step. However, from (37) and (39), observe that $\hat{x}_2(k + 1)$ is a smooth nonlinear function of the state $x(k)$ and the virtual control input $\hat{x}_2(k)$. Consequently, $\hat{x}_2(k + 1)$ is assumed to be approximated by using another NN with semirecurrent architecture since a first-order predictor generated by this NN is sufficient to obtain this value. Alternatively, a first-order filter can be used to obtain the value as given in [20].

Using the third NN, we can now select the desired control input as

\[
u_2(k) = \left(-f_2(k) + \hat{x}_2(k + 1)\right).
\]

\[
u_3^T \phi_3(\hat{x}_3^T z_3(k)) + \varepsilon_3(z_3(k))
\]

\[
u_3^T \phi_3(z_3(k)) + \varepsilon_3(z_3(k))
\]
(50)

where $u_3 \in R^{n_3}$ and $v_3 \in R^{3 \times n_3}$ denote the constant ideal output and hidden layer weights, $n_3$ is the number of hidden layer nodes, the activation function $\phi_3\left(v_3^T z_3(k)\right)$ is abbreviated by $\phi_3(z_3(k))$, $\varepsilon_3(z_3(k))$ is the approximation error, and $z_3(k) \in R^3$ is the NN input, which is given by (51). Considering that both $x_1(k)$ and $x_2(k)$ cannot be measured, $z_3(k)$ is substituted with $\hat{z}_3(k) \in R^3$, where

\[
z_3(k) = [x(k), \hat{x}_2(k)]^T \in R^3
\]

\[
\hat{z}_3(k) = [\hat{x}(k), \hat{x}_2(k)]^T \in R^3.
\]
(51)

(52)

Define
\[
\hat{e}_1(k) = \hat{x}_1(k) - x_{1d}
\]
\[
\hat{e}_2(k) = \hat{x}_2(k) - x_{2d}
\]
(53)

(54)

The actual control input is now selected as
\[
u(k) = \hat{u}_3^T(k)\phi_3(v_3^T \hat{z}_3(k)) + L_4 \hat{e}_2(k)
\]

\[
= \hat{u}_3^T(k)\phi_3(\hat{z}_3(k)) + L_4 \hat{e}_2(k)
\]
(55)

where $\hat{u}_3(k) \in R^{n_3}$ is the actual output layer weights and $l_4 \in R$ is the controller gain selected to stabilize the system.

Similar to the derivation of (29), combine (49), (50), and (55) yielding

\[
e_2(k + 1) = I_4 \hat{e}_2(k) + \xi_3(k) + u_3^T \phi_3(\hat{z}_3(k)) - \varepsilon_3(z_3(k)) + d_2(k)
\]
(56)

where

\[
\hat{u}_3(k) = \hat{w}_3(k) - u_3
\]

\[
\xi_3(k) = \hat{w}_3^T(k)\phi_3(\hat{z}_3(k))
\]

\[
u_3^T \phi_3(\hat{z}_3(k)) = u_3^T \phi_3(\hat{z}_3(k)) - \phi_3(z_3(k))
\]
(57)

(58)

(59)

Equation (44) and (56) represent the closed-loop error dynamics. It is necessary to show that the estimation errors (23) and (26), the system errors (44) and (56), and the NN weight matrices $\hat{w}_1(k), \hat{w}_2(k), \hat{w}_3(k)$ are bounded.

2) Weight Updates for Guaranteed Performance:

Assumption 1 (Bounded Ideal Weights): Let $w_1, w_2,$ and $w_3$ be the unknown output layer target weights for the observer and two action NNs and assume that they are bounded above so that

\[
\|w_1\| \leq w_{1m} \quad \|w_2\| \leq w_{2m} \quad \|w_3\| \leq w_{3m}
\]
(60)

where $w_{1m}, w_{2m},$ and $w_{3m} \in R^+$ represent the bounds on the unknown target weights where the Frobenius norm is used.

3) Fact 1: The activation functions are bounded above by known positive values so that

\[
\|\phi_i(\cdot)\| \leq \phi_{im}, \quad i = 1, 2, 3
\]
(61)

where $\phi_{im}, i = 1, 2, 3$ are the upper bounds.

Assumption 2 (Bounded NN Approximation Error): The NN approximation errors $e_1(z_1(k)), e_2(x(k)),$ and $e_3(z_3(k))$ are bounded over the compact set by $e_{1m}, e_{2m},$ and $e_{3m},$ respectively.

Theorem 1: Consider the system given in (1)–(3) and let the Assumptions 1 and 2 hold. Let the unknown disturbances be bounded by $|d_1(k)| \leq d_{1m}$ and $|d_2(k)| \leq d_{2m}$, respectively. Let the observer NN weight tuning be given by

\[
\hat{w}_1(k + 1) = \hat{w}_1(k) - \alpha_1 \phi_1(\hat{z}_1(k)) \cdot (\hat{u}_1^T(k)\hat{\phi}_1(\hat{z}_1(k) + L_3 \hat{y}(k))
\]
(62)

with the virtual control NN weight tuning provided by

\[
\hat{w}_2(k + 1) = \hat{w}_2(k) - \alpha_2 \phi_2(\hat{x}(k)) \cdot (\hat{w}_2^T(k)\phi_2(\hat{x}(k) + L_4 \hat{e}_2(k))
\]
(63)

and the control input weight be tuned by

\[
\hat{w}_3(k + 1) = \hat{w}_3(k) - \alpha_3 \phi_3(\hat{z}_3(k)) \cdot (\hat{w}_3^T(k)\phi_3(\hat{z}_3(k)) + L_5 \hat{e}_2(k)
\]
(64)

where $\alpha_1 \in R, \alpha_2 \in R, \alpha_3 \in R, \text{and } l_5 \in R, l_6 \in R, \text{and } l_7 \in R$. 

\[\]
$R$ are design parameters. Let the system observer be given by (22), (24), and (25), virtual and actual control inputs be defined as (39) and (55), respectively. The estimation errors (27)–(29), the tracking errors (44) and (56), and the NN weights $\hat{w}_1(k)$, $\hat{w}_2(k)$, and $\hat{w}_3(k)$ are UUB with the bounds specifically given by (A.17)–(A.24) provided the design parameters are selected as

\begin{align}
    (a) & \quad 0 < \alpha_i |\hat{\psi}_i(k)|^2 < 1, \quad i = 1, 2, 3 \quad (65) \\
    (b) & \quad \bar{E}_3 < 1 - \frac{(l_1 - R \cdot F_0)^2}{6\tilde{P}^2 \cdot \Delta F_m^2} - \frac{(l_2 - F_0)^2}{6\Delta F_m^2} - \gamma_0^2 \quad (66) \\
    (c) & \quad \bar{E}_0 < \min \left( \frac{1 - F_0^2}{18\tilde{P}^2 \cdot \Delta F_m^2}, \frac{1}{18\tilde{P}^2} \right) \quad (67) \\
    (d) & \quad \bar{E}_0 + 6\bar{E}_3 < \min \left( \frac{1 - F_0^2}{6\Delta F_m^2}, \frac{1}{3} \right). \quad (68)
\end{align}

**Proof:** See Appendix A.

**Remark 3:** Given specific values of $R$, $F_0$, and $\Delta F_m$, we can derive the design parameters of $l_i$, $i = 1, \ldots, 7$. For instance, given $R = 14.6$, $F_0 = 0.14$, and $\Delta F_m = 0.02$, we can select $l_1 = 1.99$, $l_2 = 0.13$, $l_3 = 0.4$, $l_4 = 0.14$, $l_5 = 0.25$, $l_6 = 0.016$, and $l_7 = 0.1067$ to satisfy (66)–(68).

**Remark 4:** Given the hypotheses, this proposed neuro-output control scheme and the weight updating rules in Theorem 1 with the parameter selection based on (65)–(68), the state $x_2(k)$ approaches the operating point $x_{2d}$.

**Remark 5:** A well-defined controller is developed in this paper since a single NN is utilized to approximate two nonlinear functions thereby avoiding division by zero.

**Remark 6:** It is important to note that in this theorem there is no persistency of excitation (PE) condition for the NN observer and NN controller in contrast with standard work in the discrete-time adaptive control [21] since the first difference of the Lyapunov function in Appendix A does not require the PE condition on input signals to prove the boundedness of the weights. Even though the input to the hidden-layer weight matrix is not updated and only the hidden to the output-layer weight matrix alone is tuned, the NN method relaxes the linearity in the unknown parameter assumption. Additionally, certainty equivalence principle is not used in the proof.

**Remark 7:** Generally, the separation principle used for linear systems does not hold for nonlinear systems and hence it is relaxed in this paper for the controller design since the Lyapunov function is a quadratic function of system errors and weight estimation errors of the observer and controller NNs.

**Remark 8:** It is important to notice that the NN outputs are not fed as delayed inputs to the network whereas the outputs of each layer are fed as delayed inputs to the same layer. Thus, the NN weight tuning proposed in (62)–(74) renders a semi-recurrent architecture due to the proposed weight tuning law even though feed forward NNs are utilized in the observer and controller. This semi-recurrent NN architecture creates a dynamic NN which is capable of predicting the state one step-ahead overcoming the non causal controller design.

**Remark 9:** It is only possible to show boundedness of all the closed-loop signals by using an extension of Lyapunov stability [21], [22] due to the presence of approximation errors and bounded disturbances consistent with the literature.

The block diagram representation of the controller with observer, controller and engine are shown in Fig. 1. The SI engine block represents the model during simulations and, during experimentation, the research engine itself.

### III. Simulation

System parameters are selected as: $R = 15.13$, $F = 0.09$, $\varphi_u = 0.725$, $\varphi_t = 0.095$ (by prior analysis to match the simulation output with the experimental data), $CF_{\text{max}} = 1.0$. The controller gains are $l_1 = 1.99$, $l_2 = 0.25$, $l_3 = 0.59$, $l_4 = 0.12$, $l_5 = 0.5$, $l_6 = 0.1$, and $l_7 = 0.7$. Adaptation gains for weight updating are selected as $\alpha_1 = 0.005$, $\alpha_2 = 0.03$, and $\alpha_3 = 0.03$. All of the neural networks have 35 hidden layer nodes. The neuron activation functions are hyperbolic tangent sigmoid in order to ensure the NN approximation capability.

Parameters are chosen to correlate with the research engine used for implementation. Uncontrolled simulation of the engine model is performed for 5000 cycles whereupon model heat release is stored for analysis. Controlled simulation for 5000 cycles follows on the engine model using the same parameters. The entire time series of heat release values is plotted in Fig. 2. The 5000 cycles recorded during control exhibit less instability than the first 5000 cycles where the engine model was run without control. Observe in Fig. 2 that the average controlled heat release is slightly higher than for uncontrolled, a result of a slight increase in the operating equivalence ratio.

---

**Fig. 1.** Structure of system and controller shows the relationship between the observer and controller neural networks as well as the connection to the engine.

**Fig. 2.** Discrete time series of heat release shows control beginning at cycle 5001. Cyclic dispersion decreases since less misfires occur after control is applied. Mean heat release also increases.
Fig. 3. Uncontrolled and controlled heat release return maps in normalized units of joules generated from the engine model. Current heat release $HR(k)$ is plotted against next heat release $HR(k+1)$, where $k$ represents cycle number.

Fig. 4. Simulation heat release output in normalized units of joules from the engine model. Current heat release to the next is detected, which is essentially a misfire. The controller modifies the fuel control input when such a misfire is detected. Increased fuel intake during control drives the equivalence ratio $\phi$ slightly higher than 0.74.

The scale of heat release shown in Fig. 3 is different from that shown in Fig. 4. The heat release values of the return maps in Fig. 3 are those from the engine model, but the heat release values plotted in Fig. 4 are the internal, controller-scaled, normalized heat release values used in calculations. Also, in Fig. 4 one can see that the observer-estimated heat release is less than the engine model heat release, but there is an observer heat release decrease that indicates engine model misfire detection.

IV. CONTROLLER HARDWARE DESIGN

Implementation of the controller is carried out on a CFR engine. Additional results are obtained on a Ricardo Hydra research engine with a Ford Zetec head. The controller itself is implemented in software, and the algorithm is processed by an embeddable PC running a Linux-based operating system. A special hardware board had to be designed in order to interface the engine and PC signals. Both engines are port-fuel-injected, with the fuel injector being driven by an injector driver that receives a TTL signal from this interface board.

The research engines, shown in Fig. 5(a) and (b), are connected to an electric dynamometer which maintains a constant engine speed of 1000 r/min. The use of a single cylinder engine eliminates the dynamics that would be introduced from interactions between multiple cylinders. A shaft encoder is mounted on the crank shaft to provide a crank angle signal and a hall effect sensor on the cam shaft provides a start of cycle signal. There are 720° of crankshaft rotation per engine cycle, so a crank angle degree is detected approximately every 167 $\mu$s at 1000 r/min.

In-cylinder pressure measurements are obtained using a Kistler model 6061B water-cooled pressure transducer, coupled to a charge amplifier, which converts the pC charge from the transducer to a 0–10 V signal. The laboratory-grade pressure transducers used in collecting experimental data are too expensive and fragile for production use. However, low-cost, in-cylinder pressure measurement devices are being developed including lower-cost piezo-resistive sensors [23], spark plug boss mounted sensors [24], and fiber-optic sensors [25], so that in-cylinder pressure measurements will be feasible in production automotive engines in the near future. Production quality in-cylinder pressure sensors are currently under development by various companies including Siemens, Kistler, and Delphi.

Heat release for a given engine cycle is calculated by integrating in-cylinder pressure and volume over time. In-cylinder pressure is measured from the engine every half crank angle degree during combustion, over a cycle window from 345° to 490° for the CFR engine [see Fig. 5(a)], and every crank angle from 330° to 490° for the Ricardo engine [see Fig. 5(b)], for a total of 290, and 130 pressure measurements, respectively. At 1000 r/min pressure measurements must be made approximately every 83.3 $\mu$s.

Fig. 6 shows the timing events in terms of degrees and again in seconds. Start of cycle is labeled SOC, and top dead center is labeled TDC. The pressure window is shown in milliseconds on the second plot as well as the calculation window and the fuel injection window.

Notice the timing constraints that are present when an engine is running at 1000 r/min. The pressure measurement window from 345° to 490° corresponds to 24.167 ms. Also, observe the fuel for the next cycle is injected at the end of the current cycle.
The measurement of pressure data and the injection of fuel leave about 17.67 ms for the PC to collect the pressure measurements, calculate heat release, run the controller algorithm, and return the new fuel pulse width to the fuel injector.

The control input is an adjustment to the nominal fuel required at a given equivalence ratio. Fuel injection is controlled by a TTL signal to a fuel injector driver circuit developed for the engine. Pressure measurements come from a charge amplifier which receives a signal from a water-cooled piezoelectric pressure transducer inside the cylinder.

An engine-to-PC interface board was designed to manage the shaft encoder signals, pressure measurements, and fuel injector signal since timing is crucial to correct engine operation. The board uses a microcontroller to buffer the engine hardware signals. A high speed 8-bit analog-to-digital (A/D) converts the pressure measurements. Pressure measurements are sent to the PC where heat release is calculated and then passed to the controller algorithm. A change to the fuel control input, $u(k)$, is returned by the controller algorithm and used to calculate the fuel pulse width for the next engine cycle. This pulse width is a function of mass of fuel to be injected.

The controller algorithm and neural network data structures are implemented in C and compiled to run on an x86 PC. The controller was compiled using the same structure and parameters as for simulation. Configuration files allow the controller parameters to be modified without recompiling. In Fig. 7, a plot of the controller runtime to calculate heat release and the new fuel control input is shown for varying neural network hidden layer size.

Since the number of nodes required in a multilayer NN for a given approximation error is not clear in the literature, the plot in Fig. 7 illustrates that even with a large number of hidden-layer NNs the proposed controller can be implemented on the embedded hardware. However, it was found from offline analysis that the improvement in approximation accuracy is not significant beyond 35 hidden-layer nodes and, therefore, the hidden-layer NN nodes in the observer and controller are limited to 35. From Fig. 7, one can see that the time to compute the controller calculations is less than 100 $\mu$s.

V. EXPERIMENTAL RESULTS

During experimentation, the controller was tested at a variety of steady-state operating conditions (determined by a combination of engine speed and load) on the engines. The speed was maintained at a constant 1000 r/min for all tests, and the pressure in the intake manifold [manifold absolute pressure (MAP)]
was maintained at around 80 kPa for the CFR engine which is roughly a mid-load operating condition, and at around 90 kPa for the Ricardo engine. MAP at full load would be nearly atmospheric pressure and at low load is typically around 40 kPa.

Since the work output from the engine varies with equivalence ratio because reduction in fuel will reduce the engine output, each operating condition is a unique speed/load case. The operation on two different engines also yields more varied test conditions for the controller.

Before activating the controller, air flow is measured and nominal fuel is calculated for the desired equivalence ratio by

\[ \varphi = R \left( \frac{MF}{AF} \right) \]

(69)

where \( MF \) is nominal mass of fuel and \( AF \) is nominal mass of air. The nominal fuel and air are loaded into the controller configuration. During data acquisition, ambient pressure is measured when the exhaust valve is fully open at 600º and used to calibrate the combustion pressure measurements. This is necessary to remove any bias generated by charge accumulation on the pressure transducer from which pressure measurements are obtained.

Uncontrolled and controlled heat release data were collected at lean equivalence ratios from 0.79 down to 0.72. NO\textsubscript{x} and uHC emissions data were also collected for both uncontrolled and controlled engine operations.

NO\textsubscript{x} data were measured using a Rosemount Analytical Model 951A NO\textsubscript{x} analyzer, and uHC data were measured using a Rosemount Analytical Model 400A flame ionization detector. All emissions data are dry gas measurements, averaged over 2 min through a data acquisition system.

Uncontrolled engine data means the controller algorithm was not used to modify the fuel injected for each cycle, but the amount of fuel to be injected was set to a nominal value. Controlled engine data comes from the controller modifying the fuel injector pulse width for every cycle. The engine ran for 3000 cycles uncontrolled, and then 5000 cycles with the control. Before collecting data the engine was allowed to reach a steady state for each set point according to stable exhaust temperature.

Heat release data is shown in time series and return maps. Time series show the heat release data for the last 500 cycles without control and for the first 500 cycles with control. This illustrates the change in heat release when control is activated. Return maps of heat release are the current cycle of heat release plotted against the next cycle of heat release. This shows the heat release on a per-cycle-basis as well as the general cyclic dispersion. For fair comparison of cyclic dispersion, 3000 cycles are used to create the uncontrolled return map and 3000 cycles for the controlled return map.

On each return map of controlled data, there is a percentage that the equivalence ratio increased during control. This percentage increase of the set-point is due to the mean value of fuel during control increasing from the nominal value injected for the cycles without controller operation.

Fig. 8 shows the time series of heat release for an equivalence ratio of 0.79. At index \( k = 0 \) the controller is activated, and mean heat release increases. Note that heat release increases when control is activated, and there are fewer misfires. In Fig. 9, return maps of the uncontrolled and controlled heat release are plotted next to each other. Both the return maps exhibit cyclic dispersion, however, with control the dispersion has decreased. This fact is emphasized by the lower COV of heat release per cycle calculated for each return map.

The COV metric is used to quantify cyclic dispersion in heat release, and is often used as a measure of variability in engine output. It is calculated as the standard deviation of a set of heat release data divided by the mean heat release for that set. A larger COV indicates that heat release values were more dispersed on the return map. With regard to COV, a goal for
this controller implementation is to observe a reduction in COV when the control loop is closed on the engine.

Note that heat release appears to be much higher than average after a misfire or partial burn. This stronger-than-average burn can be explained by residual fuel left over in the cylinder from the previous cycle that experienced the weak burn. This results in more fuel to burn for the next cycle causing a higher heat release since the engine is operating lean.

Next, in Fig. 10, the time series of heat release for equivalence ratio 0.77 is plotted. Without control, there is more instability seen at this leaner equivalence ratio than at 0.79. From the plot, one can see abundant misfires for the uncontrolled portion of the time series where control begins at index \( k = 0 \). With control applied, the instabilities in the heat release time series reduce substantially. Coefficient of variation decreases from 38.7% to 13.6% when control has been applied.

Looking at Fig. 11, one can see the return maps for the data collected at equivalence ratio 0.77. A decrease in cyclic dispersion is shown by the drop in COV from the uncontrolled return map to the controlled return map.

In Figs. 12 and 13, the time series and return maps of heat release for equivalence ratio 0.75 are plotted. Again, with control applied, instabilities in the heat release time series are reduced substantially. Comparison of the uncontrolled and controlled return maps at equivalence ratio 0.75 in Fig. 13 shows significant decrease in cyclic dispersion. Coefficient of variation decreases from 46.3% to 20.7% when control has been applied.

The COV for all of the uncontrolled and controlled heat release return maps is shown in Table I. For each equivalence ratio, the uncontrolled COV is greater than the controlled COV, since cyclic dispersion reduced when control was applied. The most significant decrease in cyclic dispersion was observed at equivalence ratio 0.77, where COV fell from 38.6% to 13.6%. This reduction in dispersion translated into a drop of 30% in measured unburned hydrocarbons compared to the uncontrolled case at an equivalence ratio of 0.77. Measured NO\(_x\) values decreased by around 98% from levels at stoichiometric conditions.

Emissions data are given in Table II. The \((u)\) and \((c)\) prefixes in the column headings stand for uncontrolled and controlled, respectively. The exhaust gas analyzers were used to measure parts-per-million of NO\(_x\) and parts-per-million C\(_3\) uHC. Looking at the uncontrolled and controlled data independently, uHC increases as equivalence ratio decreases due to more abundant partial fuel burns. To reduce uHC at lower equivalence ratios, cyclic dispersion must be decreased. The controller is able to reduce the cyclic dispersion which in turn minimizes the uHC. NO\(_x\) is decreased at lower equivalence ratios because of lower combustion temperatures.

Additional results from the Ricardo research engine also show the controller’s effectiveness at reducing cyclic dispersion. The Ricardo engine was operated at 1000 r/min like the CFR. The same emissions analyzers were used, and the in-cylinder pressure measurement is similar. In Figs. 14 and
TABLE II
EMISSIONS DATA FOR LEAN SET-POINTS OF THE CFR ENGINE

<table>
<thead>
<tr>
<th>( \Phi_{\text{set-point}} )</th>
<th>(c) NO(_x) (PPM)</th>
<th>(u) uHC (PPM)</th>
<th>(c) uHC (PPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.79</td>
<td>351.7</td>
<td>81.4</td>
<td>77.7</td>
</tr>
<tr>
<td>0.77</td>
<td>48.2</td>
<td>387.3</td>
<td>283.7</td>
</tr>
<tr>
<td>0.75</td>
<td>54.5</td>
<td>913.3</td>
<td>386.1</td>
</tr>
</tbody>
</table>

Fig. 14. Ricardo engine—Time series of heat release at equivalence ratio 0.72.

Fig. 15. Ricardo engine—Return maps of heat release at equivalence ratio 0.72.

15, time series and return maps are shown for lean equivalence ratio 0.72.

Figs. 16 and 17 contain the heat release information recorded at equivalence ratio 0.75.

The COV for the uncontrolled and controlled heat release return maps of the Ricardo engine is shown in Table III. For each equivalence ratio, the uncontrolled COV is greater than the controlled COV. This is an expected result, since the controller should be reducing the cyclic dispersion.

The indicated fuel conversion efficiency, \( \eta_f \), is a measure of the efficiency of the engine in converting the chemical potential energy present in the fuel to actual work. This metric was also calculated for both the uncontrolled and controlled cases. To determine \( \eta_f \), the net IMEP is calculated by integrating the pressure measured in the cylinder with respect to the cylinder volume, then normalizing by the displacement volume of the engine. The net IMEP, which is a measure of the work output of the engine, is combined with the engine speed to determine an indicated power. Dividing the fuel consumed by the power produced will yield a specific fuel consumption rate, which is then used along with the lower heating value of the fuel, which quantifies its chemical potential energy content, to determine the indicated fuel conversion efficiency.

Due to reduced cyclic dispersion and fewer misfires and low energy cycles, a gain of approximately 5% in indicated fuel conversion efficiency was observed for controlled engine operation.

In Table IV one can see that NO\(_x\) levels are lower at reduced equivalence ratios. Since cyclic dispersion has been reduced and the engine can operate in a more stable fashion, the amount of partial burns and misfires are reduced. This leads to a reduction of unburned hydrocarbons in the exhaust.

Results from the controller implementation on two different engines exemplify the controller’s flexibility. Only engine parameters such as fuel injector information and cylinder geometry had to be changed to extend the controller from the CFR engine to the Ricardo engine. No offline NN training is required and the controller is model-free. Finally, the task of identifying stabilizing initial weights for the observer and controller NNs, a well known problem in the literature [21] and [22], is overcome by initializing the NN weights to zero.
VI. CONCLUSION

The SI engine controller aims to decrease emissions by reducing cyclic dispersion encountered during lean operation. Both in model simulation and engine experimentation the controller minimizes estimated heat release error given by (23) returning a noticeable decrease in cyclic dispersion. Although model heat release output does not exhibit all the nonlinearities of actual engine heat release, the controller is still able to reduce heat release error. Correlating the reduction in cyclic dispersion to the measured values of NOx and unburned hydrocarbons, it is clear that a modest drop in emission products is observed between controlled and uncontrolled scenarios and a significant drop in NOx from stoichiometric levels while the fuel conversion efficiency shows a 5% improvement. Persistency of excitation condition is not needed, separation principle and certainty equivalence principle are relaxed and linearity in the unknown parameter assumption is not used.

While transient conditions are also encountered in actual engine operations, it is necessary to first develop the ability to control the engine dynamics under steady state conditions. Also, the avoidance of speed and load transients eliminates the need for additional controllers in the system to control equivalence ratio, spark timing, and other parameters, leaving the controller being tested as the only controller in the system so that there are no conflicts or impacts due to other control systems. Once control of lean engine dynamics under steady state speed and load conditions is perfected, transient control will be a logical next step.

Experimental results indicate that the controller can improve engine stability and reduce unburned hydrocarbons at lean engine operation where significant reductions in NOx can be realized. Furthermore, the controller is flexible enough to be implemented on two spark ignition research engines.

APPENDIX A

PROOF OF THEOREM 1: Define the Lyapunov function

\[ J(k) = \sum_{i=1}^{3} \frac{\gamma_i}{\alpha_i} \tilde{u}_i^T(k) \tilde{u}_i(k) + \frac{\gamma_4}{\alpha_4} \tilde{r}_1^2(k) + \frac{\gamma_5}{5} \tilde{z}_2^2(k) + \frac{\gamma_6}{3} \tilde{y}_2^2(k) + \frac{\gamma_7}{3} \tilde{e}_1^2(k) + \frac{\gamma_8}{4} \tilde{e}_3^2(k) \]

(A.1)

where \( \gamma_i \), \( i = 1, 5, 8 \) are auxiliary constants; the NN weights estimation errors \( \tilde{u}_1 \), \( \tilde{u}_2 \), and \( \tilde{u}_3 \) are defined in (30), (40), and (57), respectively; the observation errors \( \tilde{r}_1(k) \), \( \tilde{r}_2(k) \), and \( \tilde{y}(k) \) are defined in (26) and (23), respectively; the system errors \( \tilde{e}_1(k) \) and \( \tilde{e}_2(k) \) are defined in (32) and (41), respectively; and \( \alpha_i \), \( i = 1, 2, 3 \) are NN adaptation gains. The Lyapunov function (A.1) consisting of the system errors, observation errors, and the weights estimation errors obviates the need for CE condition.

The first difference of the Lyapunov function is given by

\[ \Delta J(k) = \sum_{i=1}^{8} \Delta J_i(k). \]

(A.2)

The first item of \( \Delta J_1(k) \) is obtained using (62) as

\[ \Delta J_1(k) = \frac{\gamma_1}{\alpha_1} \left[ \tilde{u}_1^T(k+1) \tilde{u}_1(k+1) - \tilde{u}_1^T(k) \tilde{u}_1(k) \right] \]

\[ \leq -\gamma_1 \left[ 1 - \alpha_1 ||\phi_1(*)||^2 \right] \tilde{u}_1^T(k) \phi_1(*) + l_5 \tilde{z}(k) \]

\[ -\gamma_1 \dot{\zeta}_1^2(k) + 2\gamma_1 \dot{\beta}_2^2(k) + 2\gamma_1 \left( \tilde{u}_1^T \phi_1(*) \right)^2. \]

(A.3)

where \( \zeta_1(k) \) is defined in (31).

Now taking the second term in the first difference (A.1) and substituting (63) into (A.2), obtain

\[ \Delta J_2(k) = \frac{\gamma_2}{\alpha_2} \left[ \tilde{u}_2^T(k+1) \tilde{u}_2(k+1) - \tilde{u}_2^T(k) \tilde{u}_2(k) \right] \]

\[ \leq -\gamma_2 \left[ 1 - \alpha_2 ||\phi_2(*)||^2 \right] \]

\[ \cdot \left( \tilde{u}_1^T(k) \phi_2(*) \right) + l_6 \dot{z}_1(k) + l_6 \dot{e}_2(k) \]

\[ -\gamma_2 \dot{\zeta}_2^2(k) + 3\gamma_2 \dot{\beta}_1^2(k) \]

\[ + 3\gamma_2 \dot{\beta}_2^2(k) + 3\gamma_2 \left( \tilde{u}_2^T \phi_2(*) \right)^2. \]

(A.4)

Taking the third term in the first difference (A.1) and substituting (64) into (A.2), then

\[ \Delta J_3(k) = \frac{\gamma_3}{\alpha_3} \left[ \tilde{u}_3^T(k+1) \tilde{u}_3(k+1) - \tilde{u}_3^T(k) \tilde{u}_3(k) \right] \]

\[ \leq -\gamma_3 \left[ 1 - \alpha_3 ||\phi_3(*)||^2 \right] \]

\[ \cdot \left( \tilde{u}_1^T(k) \phi_3(*) \right) + l_7 \dot{x}_2(k) + l_7 \dot{e}_2(k) \]

\[ -\gamma_3 \dot{\zeta}_3^2(k) + 3\gamma_3 \dot{\beta}_1^2(k) + 3\gamma_3 \dot{\beta}_2^2(k) + 3\gamma_3 \left( \tilde{u}_3^T \phi_3(*) \right)^2. \]

(A.5)

Similarly

\[ \Delta J_4(k) = \gamma_4 \left[ F_0^2 \dot{z}_2^2(k) + (l_1 - R \cdot F_0)^2 \right] \]

\[ + \gamma_4 \left[ \left( \tilde{u}_1^T(k) \right)^2 e_2^2(k) + \left( \tilde{u}_1^T(k) \right)^2 \zeta_2^2(k) \right] \]

\[ + d_{11}(k) - \tilde{z}_2^2(k) \]

(A.6)

where

\[ \tilde{u}_1^T(k) = R \cdot \Delta F(k) \cdot CE(k) \]

(A.7)

\[ d_{11}(k) = R \cdot \Delta F(k) \cdot CE(k) \cdot \tilde{u}_2^T \phi_2(*) - \Delta AF(k) - d_1(k) \]

(A.8)
and $\zeta_2(k)$ is defined in (45)

$$
\Delta J_5(k) = \gamma_5 \left[ F_0^2 \delta_2^2(k) + (l_2 - F_0)^2 \delta^2(k) + d_{21}(k) - \delta_2^2(k) \right]
+ \gamma_5 \left[ ((1 - CE(k)) \Delta F(k))^2 (\delta_2^2(k) + \zeta_2^2(k)) \right]
$$

(A.9)

where

$$
d_{21} = - d_2(k) - \Delta F(k) (1 - CE(k)) \cdot w_T^T \phi_2(k)
$$

(A.10)

$$\Delta J_6(k) \leq \gamma_6 \left( \delta_3^2(k) + \delta_3^2(k) + d_3^2(k) - \delta_3^2(k) \right)
$$

(A.11)

$$\Delta J_7(k) \leq \gamma_7 \left( \delta_3^2(k) + \delta_3^2(k) + d_3^2(k) - \delta_3^2(k) \right)
$$

(A.12)

$$\Delta J_8(k) \leq \gamma_8 \left( \delta_3^2(k) + \delta_3^2(k) + \delta_3^2(k) + d_3^2(k) - \delta_3^2(k) \right).
$$

(A.13)

Combining (A.3)–(A.13) to get the first difference of the Lyapunov function and simplifying it, get

$$
\Delta J(k) \leq - \gamma_1 \left( 1 - \alpha_1 \| \phi_1(\cdot) \|^2 \right) \left( \tilde{w}_T^T(k) \phi_1(\cdot) + l_3 \tilde{y}(k) \right)^2
- \gamma_2 \left( 1 - \alpha_2 \| \phi_2(\cdot) \|^2 \right) \left( \tilde{w}_T^T(k) \phi_2(\cdot) + l_6 \tilde{x}_1(k) + l_6 \epsilon_1(k) \right)^2
- \gamma_3 \left( 1 - \alpha_3 \| \phi_3(\cdot) \|^2 \right) \left( \tilde{w}_T^T(k) \phi_3(\cdot) + l_7 \tilde{x}_2(k) + l_7 \epsilon_2(k) \right)^2
- \gamma_7 \left( 1 - CE(k) \right) \Delta F(k)^2
$$

$$
- \gamma_7 \gamma_7 \left( \delta_3^2(k) + \delta_3^2(k) - \delta_3^2(k) \right)
- \left( 1 - F_0^2 \right) \gamma_4 - 3 \gamma_2 \gamma_3 \delta_3^2(k)
- \left( 1 - F_0^2 \right) \gamma_5 - 3 \gamma_3 \gamma_3 \delta_3^2(k)
- \left( 1 - F_0^2 \right) \gamma_6 - (l_1 - R \cdot F_0)^2 \gamma_4
- \left( 1 - F_0^2 \right) \gamma_7 - 2 \gamma_1 \gamma_5
- \left( 1 - F_0^2 \right) \gamma_8 - \gamma_3 \gamma_3 \delta_3^2(k)
- \gamma_4 \Delta F^2(k) \delta_3^2(k) + D_M \delta_3^2(k)
- \gamma_5 \Delta F^2(k) \delta_3^2(k) + D_M \delta_3^2(k)
$$

(A.14)

Choose $\gamma_1 = 2$, $\gamma_2 = 1$, $\gamma_3 = 2$, $\gamma_4 = (1/6 R^2 \Delta F_m^2)$, $\gamma_5 = (1/6 \Delta F_m^2)$, $\gamma_6 = 1$, $\gamma_7 = (1/3 R^2)$, and $\gamma_8 = 1$, then, (A.14) is simplified as

$$
\Delta J(k) \leq - 2 \left( 1 - \alpha_1 \| \phi_1(\cdot) \|^2 \right) \left( \tilde{w}_T^T(k) \phi_1(\cdot) + l_3 \tilde{y}(k) \right)^2
- \left( 1 - \alpha_2 \| \phi_2(\cdot) \|^2 \right) \left( \tilde{w}_T^T(k) \phi_2(\cdot) + l_6 \tilde{x}_1(k) + l_6 \epsilon_1(k) \right)^2
- 2 \left( 1 - \alpha_3 \| \phi_3(\cdot) \|^2 \right) \left( \tilde{w}_T^T(k) \phi_3(\cdot) + l_7 \tilde{x}_2(k) + l_7 \epsilon_2(k) \right)^2
- \gamma_7 \left( \delta_3^2(k) + \delta_3^2(k) - \delta_3^2(k) \right)
- \frac{1}{3} \gamma_2 \delta_3^2(k) - \delta_3^2(k)
- \frac{1}{6 R^2 \Delta F_m^2 - 3 \delta_3^2(k) \cdot \delta_3^2(k) + D_M \delta_3^2(k)
- \left( 1 - F_0^2 \right) \gamma_4 - 3 \gamma_2 \gamma_3 \delta_3^2(k)
- \left( 1 - F_0^2 \right) \gamma_5 - 3 \gamma_3 \gamma_3 \delta_3^2(k)
- \left( 1 - F_0^2 \right) \gamma_6 - (l_1 - R \cdot F_0)^2 \gamma_4
- \left( 1 - F_0^2 \right) \gamma_7 - 2 \gamma_1 \gamma_5
- \gamma_4 \Delta F^2(k) \delta_3^2(k) + D_M \delta_3^2(k)
- \gamma_5 \Delta F^2(k) \delta_3^2(k) + D_M \delta_3^2(k)
$$

(A.15)

This implies $\Delta J(k) < 0$ as long as (66)–(68) hold and

$$
|\xi_1(k)| > D_M
$$

(A.17)

or

$$
|\xi_2(k)| > \sqrt{3} D_M
$$

(A.18)

or

$$
|\xi_3(k)| > D_M
$$

(A.19)

or

$$
|\xi_4(k)| > \frac{D_M}{\sqrt{(1 - F_0^2) \Delta F_m^2 - 3 \delta_3^2}}
$$

(A.20)

or

$$
|\xi_5(k)| > \frac{D_M}{\sqrt{(1 - F_0^2) \Delta F_m^2 - 6 \delta_2^2 - \delta_4^2}}
$$

(A.21)

or

$$
|\xi_6(k)| > \frac{D_M}{\sqrt{(1 - F_0^2) \Delta F_m^2 - 6 \delta_2^2 - \delta_4^2}}
$$

(A.22)

or

$$
|\xi_7(k)| > \frac{D_M}{\sqrt{(1 - F_0^2) \Delta F_m^2 - 6 \delta_2^2 - \delta_4^2}}
$$

(A.23)

where

$$
D_M^2 = 2 \gamma_1 u_{1m}^2 \theta_{1m}^2 + 3 \gamma_2 u_{2m} \theta_{2m}^2 + 3 \gamma_3 u_{3m} \theta_{3m}^2 + \gamma_4 \theta_{1m}^2
+ \gamma_5 \theta_{2m}^2 + \gamma_6 \theta_{3m}^2 + \gamma_7 \theta_{4m}^2 + \gamma_8 \theta_{5m}^2.
$$
According to a standard Lyapunov extension theorem [22], this demonstrates that the system tracking error and the weight estimation errors are $Ub$. The boundedness of $\|u_1(k)\|$, $\|u_2(k)\|$, and $\|\theta_0(k)\|$ implies that $\|v_1(k)\|$, $\|v_2(k)\|$, and $\|\delta_0(k)\|$ are bounded, and, further, that the weights estimates $\hat{\theta}_1(k)$, $\hat{\theta}_2(k)$, and $\hat{\delta}_0(k)$ are bounded. Therefore, signals in the closed-loop system are bounded.

REFERENCES


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