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ABSTRACT

A number of investigators have attributed the reduced turbulent flow drag exhibited by polymer solutions to their high elongational viscosity. The results of a recent theoretical study of drag reduction are summarized, which show that it may instead be a consequence of the non-Newtonian behavior of polymer solutions in time-varying shear fields. Experiments are proposed to ascertain the relative importance of these transient shear effects and elongational effects in reducing drag.

INTRODUCTION

The turbulent flow of dilute polymer solutions has been the subject of numerous experimental and theoretical studies during the past decade. The explanation for the substantial reduction in turbulent flow drag exerted by such solutions, compared to Newtonian liquids of equal density and steady-shear viscosity, is still not known with certainty, however. In the present work two continuum explanations for the phenomenon are reviewed. Experiments are suggested to ascertain the relative importance of the two proposed mechanisms of drag reduction.

Of primary importance in the reduction of turbulent flow drag is the effect of the dissolved polymer on the wall region of the flow[1] (i.e., the viscous sublayer and transition regions). The characteristics of the flow of a Newtonian fluid in this region are known from the recent visual studies of Corino and Brodkey[2] and the earlier investigations of Kline and coworkers[3]. Volumes of liquid moving at less than the local mean-axial velocity of the flow periodically develop adjacent to the wall. The decelerated liquid is overtaken by higher velocity liquid from upstream, giving rise to a large, transient, shear gradient. Shortly thereafter, the low-velocity liquid is ejected outward from the wall. It mixes with the higher velocity liquid from upstream, resulting in the formation of the small-scale turbulent eddies responsible for the majority of the turbulent energy dissipation in the wall region.

The ejection of the low velocity liquid away from the wall is principally a transient, elongational flow, or one where the velocity gradient is along the streamlines. This has led a number of investigators[4,5] to suggest that drag reduction is caused by a high elongational viscosity (μ_E) of the polymer solution, which impedes the ejection process and thereby lessens the energy dissipated in the subsequent mixing of high and low velocity liquids. Large values of μ_E have been measured[6] for dilute polymer solutions in quasi-steady elongational flows. Recent theoretical work[7] indicates, however, that the duration of transient, elongational flows must in general be at least comparable to the solution relaxation time λ for μ_E to be significantly larger than the Newtonian value. Whether or not this condition is satisfied by the ejection processes (transient elongational flows) in turbulent flows with reduced drag is an unanswered question at this time.

Another possibility is that drag reduction results from the non-Newtonian behavior of polymer solutions in the large, transient shear gradients in the wall region. The possible importance of transient shear effects was originally suggested by the independent investigations of Meek and Baer[8], the present author[9,10] and Ruckenstein[11]. The laminar flow adjacent to an impulsively started flat plate was examined theoretically for a fluid described by the convected Maxwell constitutive equation[12]. That is to say, the plate motion was described by the following boundary and initial conditions, in terms of the orthogonal xyz coordinate system shown in Fig. 1:

$$t < o,\ u = U = o;$$
$$t \geq o,\ y = o,\ u = U; \tag{1}$$
$$t \geq o,\ y \to \infty,\ u \to o$$

Here u denotes the x component of velocity, U the plate velocity, and t the time from the start of the plate. The behavior of the Maxwell fluid differed significantly in this transient, laminar, shear flow from that originally predicted by Stokes[13] for a Newtonian fluid. Moreover, when these results were combined with the simple Einstein-Li[14] model for the turbulent boundary layer, a reduction in turbulent flow drag was predicted for the Maxwell liquid compared to the Newtonian one.

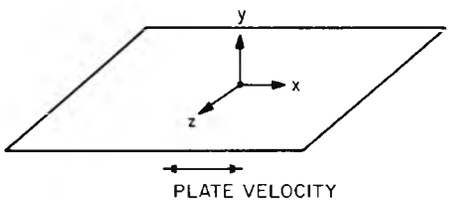

Figure 1: The xyz Coordinate System

SUMMARY OF RECENT THEORETICAL WORK

This theoretical work on the importance of transient shear effects in drag reduction has recently been extended by the present author to the flow of a fluid described by a three-constant constitutive equation proposed by Oldroyd[15]. The work was motivated in part by Darby's[16] success in correlating transient, laminar, shear flow data for polymer solutions in the 100 to 500 parts per million by weight concentration range with this equation. These are the lowest concentrations for which transient shear response measurements have been made. They also fall within the concentration range where most turbulent flow drag reduction experiments have been conducted (approximately 1 to 1000 parts per million by weight). As in the preceeding investigations, the flow adjacent to an impulsively started flat plate has been studied and the results combined with the Einstein-Li model for the turbulent boundary layer.

The most general form of the Oldroyd equation for a stationary coordinate system is[15]:

$$\tau^{ik} + \lambda\left(\frac{\partial \tau^{ik}}{\partial t} - v^{i}{}_{,m}\tau^{mk} - v^{k}{}_{,m}\tau^{im} + v^{m}\tau^{ik}{}_{,m}\right)$$
$$= 2\mu e^{ik} + \frac{2\mu\lambda}{K}\left(\frac{\partial e^{ik}}{\partial t} - v^{i}{}_{,m}e^{mk} - v^{k}{}_{,m}e^{im} + v^{m}e^{ik}{}_{,m}\right) \tag{2}$$

Here τ^{ik} denotes the ik component of the stress tensor, e^{ik} the corresponding strain rate tensor component, v^{m} the m component of the velocity vector and $(\ )_{,m}$ the covariant derivative of $(\ )$. μ denotes the steady-shear viscosity of the polymer solution, λ its relaxation time, and K the ratio of λ to the retardation time. K exceeds unity for the recently tested polymer solutions[16]. For the case of a laminar flow with boundary and initial conditions given by

(1) this relationship simplifies to:

$$\tau + \lambda \frac{\partial \tau}{\partial t} = \mu \frac{\partial u}{\partial y} + \frac{\mu\lambda}{K} \frac{\partial^2 u}{\partial y \partial t} \qquad (3)$$

Here τ is used to denote shear stresses exerted in the x direction on y = constant planes and in the y direction on x = constant planes. This particularly simple form is due to the absence of velocity gradients in the x and z directions and of y and z velocity components.

The equation of fluid motion for a laminar flow with boundary conditions (1) has been shown previously[10] to be:

$$\rho \frac{\partial u}{\partial t} = \frac{\partial \tau}{\partial y} \qquad (4)$$

Simultaneous solution of Eqs. 3 and 4 for the time-averaged wall shear stress, $\bar{\tau}_o$, has been accomplished by Laplace transform techniques. (Details of this solution will be published elsewhere.) $\bar{\tau}_o$ is defined as follows:

$$\bar{\tau}_o = \frac{1}{T} \int_{t=o}^{t=T} \tau_o \, dt, \qquad (5)$$

where τ_o is the instantaneous wall shear stress.

The Einstein-Li model for turbulent boundary layer flow, which has been used rather successfully[17] with Newtonian fluids, is based on the assumptions that (a) localized regions of growth and decay of the wall region exist; (b) the decay occurs in a negligibly small time compared to the growth, which is governed by viscous processes; and (c) the growth stage may be represented by the impulsively started flat plate problem with U replaced by U_δ, the fluid velocity at the outer edge of the periodic region. The governing equation for the turbulent boundary layer flow of a Newtonian fluid derived on this basis is[14]:

$$\left(\frac{U_\delta}{u_*}\right)^2 = \frac{\pi u_*^2 T}{4\nu} = \left(\frac{\pi u_*^2 \lambda}{2\nu}\right)\left(\frac{T}{2\lambda}\right) \qquad (6)$$

Here u_* denotes the friction velocity and T the period of growth of the periodic region. The corresponding result for an Oldroyd liquid is:

$$\left(\frac{U_\delta}{u_*}\right)^2 = \left(\frac{\pi u_*^2 T}{4\nu}\right)\frac{1}{Q^2} = \left(\frac{\pi u_*^2 \lambda}{2\nu}\right)\left(\frac{T/2\lambda}{Q^2}\right). \qquad (7)$$

Here Q is the ratio of $\bar{\tau}_o$ for the Oldroyd liquid to that for a Newtonian liquid with the same value of ν, at a given T and U_δ. The function Q and the quotient $\frac{T}{2\lambda}/Q^2$ [or equivalently, the ratio $\left(\frac{U_\delta}{u_*}\right)^2 / \frac{\pi u_*^2 \lambda}{2\nu}$ from Eq. 7] have been evaluated over a range of $\frac{T}{2\lambda}$ and K. The results are presented in Figs. 2 and 3, along with those for a Newtonian fluid from Eq. 6.

A number of significant predictions relating to the turbulent flow of a polymer solution may be deduced from these figures. First, drag is reduced by the polymer additive (i.e., for a fluid for which $K > 1$ and $\lambda > 0$.) Recent experiments[18] have suggested that the ratio $\frac{u_*^2 T}{\nu}$ appearing in Eq. 6 and Eq. 7 is approximately the same in turbulent flows of Newtonian fluids and those turbulent flows where small to moderate deviations from Newtonian behavior are caused by a polymer additive. It follows from Eq. 7 and Fig. 2 that this deviation will assume the form of reduced drag. That is to say, the ratio $\frac{U_\delta}{u_*}$ will be larger for the polymer solution than for the Newtonian fluid; and U_δ is known to be of the order of the mean velocity V in pipe flows[17,19].

For large deviations from Newtonian behavior in turbulent flows, which might be expected for $\frac{T}{2\lambda} \ll 1$ (large flow rates), the value of $\frac{u_*^2 T}{\nu}$ is unknown.

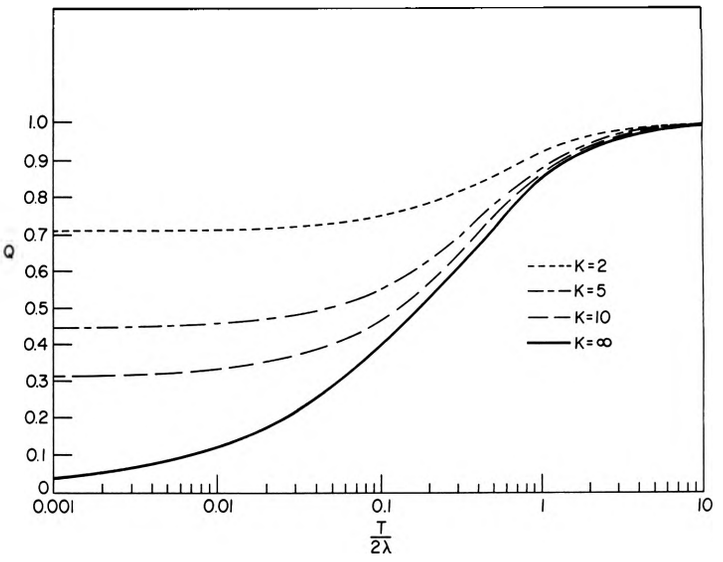

Figure 2: Q as a Function of Time

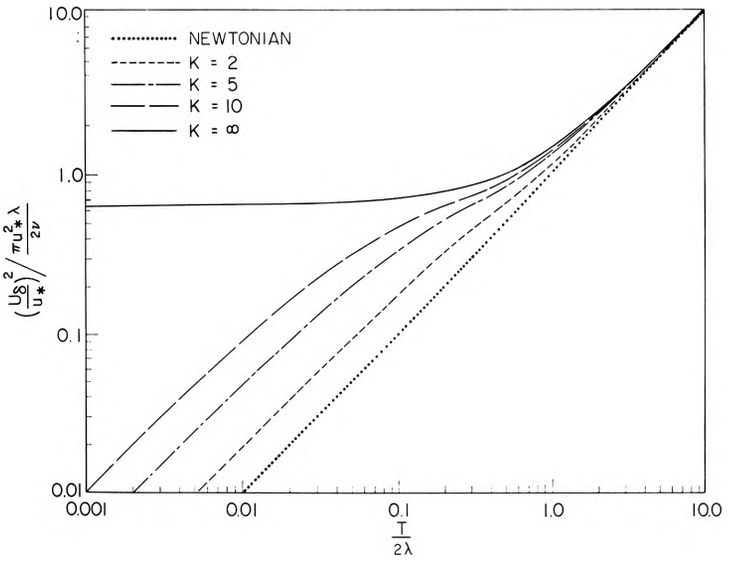

Figure 3: Theoretical Predictions for the Turbulent Boundary Layer

Under these circumstances, however, Eq. 7 asymptotically approaches the form (for finite K):

$$\left(\frac{U_\delta}{u_*}\right)^2 = \frac{\pi u_*^2 T}{4\nu'} \qquad (8)$$

where

$$\nu' = \frac{\nu}{K} \qquad (9)$$

Eq. 8 is identical to that governing a Newtonian fluid with viscosity ν', as is evident from Eq. 6. Thus, for small $\frac{T}{2\lambda}$ the polymer additive is predicted to have the same effect on the flow in the wall region as a reduction in viscosity by a factor $\frac{1}{K}$. Because U_δ and V are of the same order of magnitude, this result indicates that drag is reduced in the limit of small $\frac{T}{2\lambda}$. It further indicates that, in this limit, the turbulent pipe flow of a polymer solution should resemble that of a lower viscosity Newtonian fluid. In other words, a log u_* vs log V plot of the polymer solution data should have the same appearance as that representing a Newtonian fluid of lower viscosity. Virk's data[20] for the most dilute polyethylene oxide solutions tested in a 3.21 cm diameter pipe approximates this behavior, when the wall shear stress exceeds about three times the "onset" value.

A consequence of the analysis which is apparent in Fig. 3 is that an upper bound exists on the amount by which the drag may be reduced. This condition is realized when $K = \infty$, which corresponds to a fluid described by the convected Maxwell constitutive equation. This upper bound has been shown elsewhere[10] to correspond to a variation in U_δ with u_*^2. The experimentally observed[21] maximum drag reduction asymptote may be shown characterized by a variation in V with u_*^n, where n varies monotonically from 1.6 at a Reynolds number of 3400 to 1.3 at 80,000. By virtue of the similarity in magnitude of U_δ and V, qualitative agreement between the experimental and theoretical results may be said to exist.

A final prediction of the theoretical work is that the polymer solution behavior asymptotically approaches that of a Newtonian fluid as $\frac{T}{2\lambda}$ becomes large compared to unity. This behavior is in apparent contrast to the observations of a number of investigators[20,22,23] of a critical onset wall shear stress for drag reduction. The discrepancy between theory and experiments may be attributable to the inadequacy of presently used experimental methods to measure very small deviations from Newtonian behavior. Alternatively, a critical onset condition may be a consequence of some characteristic of the turbulent boundary layer which is not represented by the simple Einstein-Li model.

PROPOSED EXPERIMENTAL WORK

A substantial body of additional experimental work is needed to test the validity of these theoretical results and assess the relative importance of transient shear and elongational effects in drag reduction. First, experiments should be conducted to study further the transient shear response of the very dilute polymer solutions of greatest interest in drag reduction. These are needed to assess the utility of the Oldroyd constitutive equation at concentrations below those tested by Darby (i.e., below 100 parts per million by weight). If this relationship is adequate, values of λ and K are required to make quantitative comparisons of the experimental and theoretical results for turbulent flow. This work will presumably require the development of new techniques to measure the transient shear response of a material. Those presently used are not suitable for materials with relaxation times as small as are thought to characterize very dilute polymer solutions (probably much less than 10^{-3} seconds[24]).

Second, experiments to measure the durations of and strain rates in the elongational and transient shear regions near the wall in turbulent pipe flows are proposed. Visual studies similar to those made by Corino and Brodkey[2] may be suitable for this purpose. The experiments should be conducted in a single test section with a Newtonian fluid and a polymer solution of the same steady-shear viscosity, over the range of mean flow velocity V where the polymer solution exhibits reduced drag. The results should be analyzed together with those of the proposed rheological studies to provide answers to the following questions. (1) Over this range of V does the shear rate in the wall region of the Newtonian flow vary significantly over a time period of the order λ or less? (2) Are the durations of and strain rates in the elongational regions of the Newtonian flow such that addition of the polymer would make μ_E large compared to its Newtonian value of 3μ? (3) How do the durations of, strain rates in, and frequency of occurrence of the elongational and transient shear regions of the polymer solution flow compare with these measured for the Newtonian case at the same values of V? An affirmative answer to the first question would confirm the importance of transient shear effects in reducing drag, since non-Newtonian behavior becomes evident in transient, laminar, shear flows when the time period over which changes in shear rate occur is of the order of λ or less. An affirmative answer to the second question would suggest that elongational effects are important in reducing drag. The observed alterations in the wall region due to the presence of the polymer should also reflect the important mechanisms in drag reduction. A large elongational viscosity for example, might be expected to decrease the average elongational strain rate in the wall region of the polymer solution flow compared to Newtonian flow at the same V. Detailed comparisons of the flows in the wall region might also reveal the importance of mechanisms of drag reduction other than transient shear and elongational effects. It may be, for example, that the hydrodynamic stability of some region of the flow near the wall is important and that the stability of this region is affected by the presence of the polymer additive.

Finally, the theoretical work outlined above relates the characteristic time, T, associated with the wall region to the time average flow parameters, U_δ and u_*. Consequently, measurements of the turbulent burst period in flows with reduced drag would be of some interest. A spectrum of burst periods would be measured at a given value of V, from which a mean burst period could be obtained. U_δ, the flow velocity which appears in the theory, and V, that which would be determined experimentally, are of the same order of magnitude. A comparison which should therefore be illustrative is (mean burst period)/2λ vs. $\left(\frac{V}{u_*}\right)^2 / \frac{\pi u_*^2 \lambda}{2\nu}$ from the experiments and $\frac{T}{2\lambda}$ vs. $\left(\frac{U_\delta}{u_*}\right)^2 / \frac{\pi u_*^2 \lambda}{2\nu}$ from Eq. 7. Similarity in the form of the two functional relationships should be observed if the transient shear explanation of drag reduction is correct (i.e., if the Oldroyd constitutive equation describes the polymer solution being tested and if large elongational effects do not invalidate the assumptions on which the Einstein-Li model for the turbulent boundary layer is based).

CONCLUSIONS

The theoretical work outlined above shows that turbulent flow drag reduction may be due to the unusual transient shear response of polymer solutions. A number of investigators have suggested that the high elongational viscosity of these solutions may cause drag reduction. The experimental work which has been proposed will establish whether or not one of these effects is primarily responsible for the drag reduction phenomenon. If some effect other than these two is of primary importance the proposed work should also serve to identify it.

SYMBOLS

e^{ik}	ik component of the strain rate tensor
K	ratio of relaxation to retardation times for the polymer solution
Q	ratio of $\bar{\tau}$ for an Oldroyd liquid, to that for a Newtonian liquid having the same value of ν, for a given T and U
t	time from the impulsive start of the flat plate
T	growth period of the periodic region of the turbulent boundary layer
u	x component of fluid velocity
u_*	friction velocity
U	plate velocity
U_δ	fluid velocity at the outer edge of the periodic region of the turbulent boundary layer
v^m	m component of fluid velocity
V	mean flow velocity in a pipe
x,y,z	coordinates of the orthogonal system of Fig. 1
λ	polymer solution relaxation time
μ	steady-shear viscosity of the polymer solution

μ_E	elongational viscosity of the polymer solution
ν	kinematic viscosity of the polymer solution
ν'	ν/K
ρ	polymer solution density
τ	shear stress exerted in the x direction on y = constant planes and in the y direction on x = constant planes in the fluid bounded by an impulsively-started, flat plate
τ_o	instantaneous wall shear stress exerted on the impulsively-started, flat plate
$\bar{\tau}_o$	time-average of τ_o
τ^{ik}	ik component of the stress tensor

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DISCUSSION

T. J. HANRATTY (University of Illinois): Gilead Fortuna attempted some calculations like this about four years ago. We did not use an Oldroyd model. We used linear models which involved the time derivative of the rate of strain as well as the time derivative of the stress. We abandoned further work on these models since they predicted that under conditions needed for appreciable drag reduction, the time-averaged shear stress at the wall is not related to the time-averaged velocity gradient at the wall by Newton's law of viscosity. Measurements which we had at hand at that time indicated that this was not the case. Have you examined this in your models? If this was not a problem for you, then maybe we ought to go back and take a look at our calculations to see if we overlooked something.

HANSEN: The present analysis indicates that the time-averaged shear stress will be proportional to the time-averaged shear rate when $\frac{T}{2\lambda} << 1$ and K is finite (as well as for $\frac{T}{2\lambda} >> 1$). This may be inferred directly from Eq. 3 by considering its limiting form for small times, or from Fig. 3 where the polymer solution is seen to behave as a Newtonian fluid with a reduced viscosity for $\frac{T}{2\lambda} << 1$. The proportionality constant will be $\frac{\mu}{K}$ for small $\frac{T}{2\lambda}$ and μ for large $\frac{T}{2\lambda}$. When $\frac{T}{2\lambda}$ is of order unity, a proportionality of time-averaged shear stress to time-averaged shear rate is not predicted by this work.

L. THOMAS (Akron University): In which region are you when you talk about dilute solutions, maybe 5-20 parts per million?

HANSEN: I do not know at present. The best experiments that have been done in looking at the transient shear response of dilute polymer solutions are with high molecular weight polyacrylamides in weight concentration ranges of 100 to 500 parts per million. So, as I said before, tests of the rheological properties of more dilute solutions are needed. If a rheological model such as the Oldroyd model is appropriate, determination of λ and K as a function of concentration is required.

THOMAS: If you use a relaxation time of something like 10^{-5} or 10^{-4} for about 20 parts per million of Polyox or Separan, that is going to put your dimensionless parameter at around 10 to 100. In that region you're saying that the Einstein-Li or Hanratty model would be appropriate. Measurements that I have made recently suggest that this type of model is completely adequate at least in the concentrations that I have been talking about. I'm just wondering if you would ever get down into the region where you have the bifurcation.

HANSEN: Your question is a good one. I think it important to realize, however, that the orders of magnitude of the relaxation times of very dilute polymer solutions are not known at present. An additional consideration may be relevant. Velocity profiles in the turbulent boundary layer of a Newtonian fluid obtained at Stanford show that transient shear gradients with characteristic times much shorter than the mean burst period are present. Moreover, the fact that a distribution of burst periods is present has been well established. It may be that the short time shear gradients revealed by the Stanford work or the shortest burst periods are more pertinent to drag reduction than the mean burst period.

W. G. TIEDERMAN (Oklahoma State University): Just recently we have done some of the experiments that you have suggested. For his Ph.D. thesis George Donohue is measuring the spanwise spacing and the bursting rates of the turbulent wall flow structure by seeping dye into the wall layers of a drag reducing solution of polyethylene oxide, about a hundred parts per million. The flow field is a fully developed two-dimensional channel. We haven't reduced this data in terms of non-dimensional characteristics yet. However, there is a very marked effect on the wall-layer flow structure and it is pretty much as one might suspect, that is at the same flow rate the streaks are elongated and the bursting frequencies for the drag reducing solutions are decreased approximately an order of magnitude.

G. L. DONOHUE (Oklahoma State University): I question your relaxation times here. I seem to recall that the relaxation times in the polymers that I was looking at were something like a millisecond. That is, it is 2 orders of magnitude smaller than the time between bursts and it just did not look like a very promising non-dimensional grouping.

HANSEN: Perhaps you are correct. I would once again point out, however, that the relaxation times you quote are calculated from the Rouse or Zimm theories; and it is not clear at all that these times in fact characterize the continuum behavior of the polymer solutions of interest. The experimental work of Professor Darby on laminar, transient, shear flows was conducted with polyacrylamide solutions in the same concentration ranges you have used. The

relaxation and retardation times he measured were in the 0.10 to 1.0 second range.

A. FABULA (Naval Undersea Research and Development Center): My first comment is on the proposal that "start-shear" flow is more promising than "start-elongation" flow for investigating and understanding those polymer solution properties that are important in polymeric friction reduction. I would suggest that they are equally promising, since both can cause the polymer coil to be substantially elongated if the strain rate is suitably large compared with the inverse terminal relaxation time. With respect to the steady-state limits of these two deformation histories, we already know that simple-shear viscosity tells us little about the mechanism of friction reduction or about the relative effectiveness of various polymers, and it remains to be seen if steady state elongational viscosity is more relevant.

My second comment is on the use of an Einstein-Li type model of turbulent flow. They chose the duration of the flow for an impulsively started wall in a fluid at rest so as to make the resultant temporal mean velocity profile fit the observed turbulent mean profile. Thus, to extend the Einstein-Li model to cases of friction reduction, it seems that the author ought to proceed as follows. Assuming some particular non-Newtonian constitutive relation, he would also choose the time duration so the resultant temporal mean velocity profile fits the experimental profile for a selected case of friction reduction. Then he would compare the corresponding predicted temporal mean wall shear stress with the corresponding experimental value, and so test the model and the constitutive relation.

HANSEN: Concerning your first point, it is not yet clear in my opinion that a high elongational viscosity always implies the kind of transient shear response which results in reduced drag, and <u>vice versa</u>. This being the case, I think there is motivation for asking which mechanism (if either) dominates. As to the second, the characteristic time which you mention may be determined in a relatively straightforward manner as you say. Whether or not it is of the order of magnitude of those time scales in the turbulent boundary layer which are important in drag reduction remains to be demonstrated, however.