Load and resistance factor design of cold-formed steel comparative study of design methods for cold-formed steel

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This progress report is the revision of the Eighth Progress Report on the Load and Resistance Factor Design of Cold-Formed Steel. Revisions have been made to reflect some recent changes of resistance factors and design formulas in the proposed LRFD Specification (UMR Civil Engineering Study 88-3) dated February 1988.


Special thanks are extended to Professor T. V. Galambos, Consultant of the project, T. N. Rang, B. Supornsilaphachai, B. K. Snyder, L. C. Pan, and M. K. Rarindra for their contributions to the project.
ABSTRACT

Allowable Stress Design is the current method used to design cold-formed steel structural members and connections. In this design approach, factors of safety are used to compute the allowable design stresses which are compared to the actual maximum stresses that will occur in the member during the life of the structure.

In recent years, the Load and Resistance Factor Design (LRFD) method has been developed for the design of hot-rolled steel shapes and the design of cold-formed steel structural members. This method is based on probabilistic and statistical techniques to account for the many uncertainties involved with the actual design. The LRFD criteria use load factors which are applied to the external loads and resistance factors that are applied to the internal resistance capacities of the structure.

The allowable unfactored loads based on each design method for different types of structural members are compared and shown in graphical forms. For structural members with one type of loading, the dead-to-live load ratio contributes to the difference between the two allowable loads. For members with a combination of loads, cross-sectional geometry, loading conditions, material strength, member length, along with dead-to-live load ratio will affect the difference between the allowable loads computed from allowable stress design and LRFD.
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I. INTRODUCTION

A. GENERAL

The 1986 Edition of the Specification for the Design of Cold-Formed Steel Structural Members published by the American Iron and Steel Institute (AISI) applies to steel members cold-formed to shape from carbon or low-alloy steel sheet, strip, plate or bar not more than one inch in thickness and used for load-carrying purposes in buildings. The specification is based upon the allowable stress concept presented in terms of allowable moments and loads. In the design of cold-formed steel members and connections, the actual moments and loads are computed from service loads that include dead, live, snow, wind, and earthquake loads. The allowable moments and loads are determined by dividing the corresponding nominal capacities by appropriate factors of safety recommended by AISI for different types of structural members and connections.

The Load and Resistance Factor Design (LRFD) criteria for steel members and connections have recently been developed by using probabilistic and statistical techniques to account for the uncertainties in design, fabrication, material properties, and applied loads. The LRFD criteria for hot-rolled shapes, built-up members, and connections have been included in the Load and Resistance Factor Design Manual of Steel Construction published by the American Institute of Steel Construction. For cold-formed steel structural members, the Load and Resistance Factor Design Specification was developed from a joint research project entitled "Load and Resistance Factor Design of Cold-
Formed Steel” conducted at the University of Missouri-Rolla, Washington University and the University of Minnesota.

B. PURPOSE OF INVESTIGATION

The primary purpose of this investigation was to study and compare the proposed Load and Resistance Factor Design (LRFD) Criteria for Cold-Formed Steel with the existing Allowable Stress Design (ASD) Criteria included in the 1986 Specification for the Design of Cold-Formed Steel Structural Members. This comparison involved studies of different variables used for the design of various types of structural members and discussions of different load carrying capacities determined by these two methods.

C. SCOPE OF INVESTIGATION

This study compares the existing Allowable Stress Design Method with the proposed Load and Resistance Factor Design Method for cold-formed steel structural members generally used in building construction. These shapes include channels with stiffened or unstiffened flanges, I-sections made from channels, and hat sections with unreinforced webs. The yield points of steel range from 33 to 50 ksi.

The AISI Specification and the proposed LRFD Specification can be used for the design of tension members, flexural members, compression members, members subjected to a combination of bending and axial loads, bolted connections, welded connections, stiffeners, and wall studs. Even though the allowable stress design provisions and the proposed LRFD criteria were prepared for any combinations of dif-
ferent loads, only dead and live loads were used in this comparison for each type of structural members. Ratios of load carrying capacities were computed and evaluated for different shapes of structural members which are used in typical design situations.
II. REVIEW OF LITERATURE

A. GENERAL

Because of the growing need for a unified approach to structural design for all types of construction materials, many studies have been conducted in recent years. In early 1978, the LRFD criteria for hot-rolled steel shapes were proposed by Galambos as alternative design methods. This proposal was a result of a research project conducted at Washington University under the sponsorship of the American Iron and Steel Institute. This subject was subsequently discussed by Galambos, Ravindra, Yura, Bjorhovde, Cooper, Hansell, Viest, Fisher, Kulak, and Cornell in References 15 through 22. In addition, numerous papers were published in the proceedings of the American Society of Civil Engineers (ASCE) Specialty Conference on Probabilistic Mechanics and Structural Reliability held in January 1979. In Reference 23, Grigoriu, Veneziano, and Cornell discuss the importance of decision making in probability distribution modeling. Chalk and Cortis studied a collection of live load data to develop a probabilistic format for the determination of design live loads for building floors.

During the period from 1979 to 1982, Ellingwood studied statistical information in reinforced concrete, wood, and masonry structures for developing a probability-based limit states design criteria. In a recent study sponsored by the National Bureau of Standards, Galambos, Ellingwood, MacGregor, and Cornell developed a set of load factors, load combinations, and methodology for material specification groups. More recently, the ASCE Committee on Fatigue...
and Fracture Reliability published a series of reports on fatigue reliability.

With regard to cold-formed steel design, a study on reliability based criteria for temporary cold-formed steel buildings was conducted by Knab and Lind in 1975. A joint research project entitled "Load and Resistance Factor Design of Cold-Formed Steel" was conducted by Rang, Supornsilaphachai, Snyder, Pan, Galambos, and Yu at the University of Missouri-Rolla and Washington University since 1976. This project was also under the sponsorship of AISI. References 4 through 8 summarized the studies of the LRFD criteria for cold-formed steel tension members, beams, columns, beam-columns, and connections. The research findings have been discussed at various engineering and specialty conferences and published in several conference proceedings. In September 1985, the Tentative Recommendations on the LRFD Criteria for Cold-Formed Steel Structural Members and Commentary were prepared according to the 1980 edition of the AISI Specification for the allowable stress design. These tentative recommendations were updated in 1987 on the basis of the 1986 edition of the AISI Specification.

In Canada, the Canadian Standards Association uses the limit states design principles in their standard for cold-formed steel.

B. LOAD AND RESISTANCE FACTOR DESIGN CRITERIA

The Load and Resistance Factor Design Specification for Cold-Formed Steel is based on the first-order principles of probabilistic theory. The general format for the LRFD criteria is
In the above, 

\[ \phi R_n \geq \sum_{k=1}^{j} \gamma_k Q_{kn} \]  \hspace{1cm} (II.1)

In the above,

- \( \phi \) = resistance factor
- \( R_n \) = nominal resistance
- \( \gamma_k \) = load factors
- \( Q_{kn} \) = nominal load effects

On the left side of Eq. (II.1), the resistance factor, \( \phi \), is a nondimensional factor less than or equal to one that accounts for the uncertainties in calculating the nominal resistance. The nominal resistance of the structure is the predicted ultimate resistance or load determined from design formulas using specified mechanical properties of material and section properties. It could be a bending moment, axial load, shear force, or an interaction formula when load combinations are presented.

On the right side of the equation, factor \( \gamma \) is a nondimensional load factor used to reflect the possibility of overloads and uncertainties in computing the load effect. Each load factor applies to a nominal load effect \( Q_n \) and the subscript \( k \) corresponds to different types of loads. Only dead and live load effects were used to develop the LRFD criteria for cold-formed steel.

Instead of a safety factor, a safety index is used to determine structural reliability. The safety index, \( \beta \), indicates the probability of failure as shown in Figure 1. The distribution of the \( R/Q \) ratio was assumed to be lognormal. The safety index can be determined by using Eq. (II.2):"
\[ \beta = \frac{\ln\left(\frac{R_m}{Q_m}\right)}{\sqrt{V_R^2 + V_Q^2}} \]  

where

- \( R_m \) = mean value of resistances
- \( Q_m \) = mean value of load effects
- \( V_R \) = coefficient of variation of resistances
- \( V_Q \) = coefficient of variation of load effects

**Figure 1. Probability Distribution of \( \ln R/Q \)**
The target values of safety index used in the development of the LRFD Specification for cold-formed structural members and connections are 2.5 and 3.5, respectively. A probability of failure of $9.8 \times 10^{-3}$ is obtained from the cumulative lognormal distribution for the value of safety index equal to 2.5$^2$.

Unlike the traditional design methods, the resistance of the structure is considered to be a random variable because of variations in mechanical properties and fabrication uncertainties involved in calculations of the resistance. The mean value of the resistances was assumed to be a product of several values as given in Eq. (11.3).

$$ R_m = R \cdot M \cdot F \cdot P $$

where $M_m$, $F_m$ and $P_m$ are the mean values of nondimensional variables reflecting the uncertainties in mechanical properties, sectional properties, and calculation of the resistance.

In Eq. (11.3), $M$ is the material factor which is determined by the ratio of the tested mechanical properties to the specified values. Mechanical properties include yield point, modulus of elasticity, and tensile strength values. The fabrication factor, $F$, accounts for variations of geometric dimensions and uncertainties caused by initial imperfections and tolerances. The professional factor, $P$, accounts for uncertainties that results from the use of approximations and simplifications of complex design formulas based on ideal situations. It is obtained from the ratio of the tested failure loads to the predicted failure loads computed from design formulas.
From statistical studies of applied loads and reliability calculations, the following load combinations and load factors were used for cold-formed steel:

1. \(1.4D_n + L_n\)
2. \(1.2D_n + 1.6L_n + 0.5(L_{rn} \text{ or } S_n \text{ or } R_n)\)
3. \(1.2D_n + 1.6L_m + 0.5(L_{rn} \text{ or } S_n \text{ or } R_n) + (0.5L_n \text{ or } 0.8W_n)\)
4. \(1.2D_n + 1.3W_n + 0.5L_n + 0.5(L_{rn} \text{ or } S_n \text{ or } R_n)\)
5. \(1.2D_n + 1.5E_n + (0.5L_n \text{ or } 0.2S_n)\)
6. \(0.9D_n - (1.3W_n \text{ or } 1.5E_n)\)

where

- \(D_n\) = nominal dead load
- \(E_n\) = nominal earthquake load
- \(L_n\) = nominal live load
- \(L_{rn}\) = nominal roof live load
- \(R_n\) = nominal roof rain load
- \(S_n\) = nominal snow load
- \(W_n\) = nominal wind load (Exception: For wind load on individual purlins, girts, wall panels and roof decks, multiply the load factor for \(W_n\) by 0.9)

Exception: The load factor on \(L_n\) in combination (3), (4), and (5) shall be equal to 1.0 for garages, areas occupied as places of public assembly, and all areas where the live load is greater than 100 psf.

For roof and floor construction, the load combination for dead load, weight of wet concrete, and construction load including equipment, workmen and formwork is suggested in Section A5.1.(2)(a) of the Commentary.
When the structural effects of F, H, P, or T are significant, they shall be considered in design as the following factored loads: 1.3F, 1.6H, 1.2P, and 1.2T, where

\[ F = \text{loads due to fluids with well-defined pressures and maximum heights} \]

\[ H = \text{loads due to the weight and lateral pressure of soil and water in soil} \]

\[ P = \text{loads, forces, and effects due to ponding} \]

\[ T = \text{self-straining forces and effects arising from contraction or expansion resulting from temperature changes, shrinkage, moisture changes, creep in component materials, movement due to differential settlement, or combinations thereof} \]

The preceding load combinations are listed in Section A5.1.4 of the LRFD Specification and should be used in the computation of the load effects. The combination of dead and live load with an assumed dead-to-live load ratio of 1/5 were used to develop the LRFD criteria for cold-formed steel.

The coefficient of variation of the resistances, \( V_R \), is related to the coefficient of variation of M, F, and P as follows:

\[
V_R = \sqrt{V_M^2 + V_F^2 + V_P^2}
\]  

(II.4)

The coefficient of variation of the load effects, \( V_Q \), can be computed from the nominal dead-to-live load ratio and the coefficient of variation of the dead and live loads. For a dead-to-live load ratio equal to 1/5, \( V_Q \) is equal to 0.21.
The resistance factor can be obtained from the following equation developed in Reference 10.

$$\phi = \frac{1.521M F P_m}{m m m m} \exp(\beta \sqrt{\frac{V_p}{R^2 V_Q^2}})$$  (II.5)

All statistical data and calculations for material factors, fabrication factors, professional factors, coefficients of variation of resistances, and resistance factors can be found in References 4 through 13.

In the LRFD criteria, the factored nominal resistance for design is $\Phi R_n$. For the purpose of comparison, the unfactored load combination $(D_n + L_n)$ or allowable load can be computed from the nominal resistance $R_n$, the resistance factor $\phi$, and a given $D_n/L_n$ ratio as follows:

$$\begin{align*}
\Phi R_n & \geq c(1.2D_n + 1.6L_n) \\
\Phi R_n & \geq c(1.2D_n/L_n + 1.6)L_n \\
\Phi R_n & \geq c(1.2D_n/L_n + 1.6)(D_n + L_n)/(D_n/L_n + 1)
\end{align*}$$

Therefore,

$$c(D_n + L_n) \leq \frac{R_n}{(1.2D_n/L_n + 1.6)/[\Phi(D_n/L_n + 1)]}$$  (II.6)

where $c$ is the deterministic influence coefficient to transform the load to load effect.

From Eq. (II.6), the factor of safety against the nominal resistance used in the LRFD is:

$$\text{(F.S.) LRFD} = (1.2D_n/L_n + 1.6)/[\Phi(D_n/L_n + 1)]$$  (II.7)

Equation (II.6) was used in this study to compare the AISI Specification for allowable stress design and the Load and Resistance Factor Design Specification. The results are presented and discussed in Chapters III through IX.
III. TENSION MEMBERS

A. ALLOWABLE STRESS DESIGN (ASD)

According to Section C2 of the AISI Specification, cold-formed steel tension members should be designed to satisfy the following requirement:

"For axially loaded tension members, the applied tensile force shall not exceed $T_a$ determined as follows:

$$T_a = \frac{T_n}{\Omega_t}$$  \hspace{2cm} (III.1)

where

$T_n = \text{Strength of member when loaded in tension}$

$$= A_n F_y$$  \hspace{2cm} (III.2)

$\Omega_t = \text{Factor of safety for tension}$

$$= 1.67$$

$A_n = \text{Net area of the cross section}$

$F_y = \text{Design yield stress}.$

B. LOAD AND RESISTANCE FACTOR DESIGN (LRFD)

Based on Section C2 of the proposed LRFD Specification, the following provisions are used for the design of cold-formed steel tension members:

"For axially loaded tension members, the factored nominal tensile strength, $\phi T_n$, shall be determined as follows:

$$\phi = 0.95$$

$$T_n = A_n F_y$$  \hspace{2cm} (III.3)

where
\( \phi \) = Resistance factor for tension
\( T_n \) = Nominal strength of member when loaded in tension
\( A_n \) = Net area of the cross section
\( F_y \) = Design yield stress.

C. COMPARISON

For a comparison between the allowable stress design and the LRFD approach, the unfactored load can be calculated by using the following equation for both design methods:

\[ P_T = P_{DL} + P_{LL} \]  \quad (III.4)

where

- \( P_T \) = total unfactored load applied to the member
- \( P_{DL} \) = axial tension due to the nominal dead load
- \( P_{LL} \) = axial tension due to the nominal live load

This total unfactored load should be less than or equal to the allowable load. For allowable stress design, the allowable load is

\[ (P_a)_{ASD} = A_n F_y / \Omega_t = A_n F_y / 1.67 \]  \quad (III.5)

For LRFD, the allowable load can be calculated by using Eq. (II.6).

\[ (P_a)_{LRFD} = \phi T_n (D/L+1)/(1.2D/L+1.6) \]  \quad (III.6)

Because \( T_n = A_n F_y \), Eq. (III.6) can be rewritten as

\[ (P_a)_{LRFD} = \phi A_n F_y (D/L+1)/(1.2D/L+1.6) \]  \quad (III.7)
Figure 2. Allowable Load Ratio vs. D/L Ratio for Tension
where $D/L$ is the ratio of the nominal dead load to the nominal live load. From Eq. (III.7) it is clear that the allowable load based on LRFD is a function of not only cross-sectional area and yield strength of the steel but also the dead-to-live load ratio. This will be true for all structural members designed by LRFD method.

Therefore, based on Eqs. (III.5) and (III.7), the allowable load ratio for tension members is

$$\left(\frac{P_a}{P_a}\right)_{\text{LRFD}} = \frac{D/L+1}{1.2D/L+1.6}$$  \hspace{1cm} (III.8)

For the value of $\phi = 0.95$

$$\left(\frac{P_a}{P_a}\right)_{\text{LRFD}} = \frac{1.58}{1.2D/L+1.6}$$  \hspace{1cm} (III.9)

Figure 2 shows the allowable load ratio versus dead-to-live load ratio. When $D/L < 1/25$, the allowable load determined by the LRFD method is slightly less than that determined by the allowable stress design. For $D/L = 1/5$, ASD is about 3.2% conservative compared to LRFD.
IV. FLEXURAL MEMBERS

A. GENERAL

Cold-formed steel flexural members have several possible modes of failure. In the design of beams, consideration should first be given to the section strength based on either initiation of yielding in the effective section or the inelastic reserve capacity as applicable. For beams with inadequate lateral bracing, lateral buckling may limit the moment-resisting capacity. Beam webs have to be designed for shear and combined bending and shear. Because of highly localized concentrations of stress resulting from applied concentrated loads or reactions, web crippling and combined bending and web crippling have to be checked. Excessive deflection due to service live load could also be a problem.

B. STRENGTH FOR BENDING ONLY

1. Allowable Stress Design. Based on Section C3.1 of the AISI Specification, the following provisions are used for the design of cold-formed steel flexural members based on bending strength:

   "In flexural members, the applied moment uncoupled from axial load, shear, and local concentrated forces or reactions shall not exceed the allowable $M_a$ calculated as follows:

   $$M_a = \frac{M_n}{\Omega_f} \quad \text{(IV.1)}$$

   where

   $M_n = \text{Smaller of the nominal moment strength calculated according to Sections C3.1.1 and C3.1.2}$
\( \Omega_f = \text{Factor of Safety for bending} \)
\[ \Omega_f = 1.67 \]

C3.1.1 Nominal Section Strength

Section strength shall be calculated either on the basis of initiation of yielding in the effective section (Procedure I) or on the basis of the inelastic reserve capacity (Procedure II) as applicable.

(a) Procedure I - Based on Initiation of Yielding

Effective yield moment based on section strength, \( M_n \), shall be determined as follows:

\[ M_n = S_e F_y \]  

where

\( F_y = \text{Design yield stress as determined in Section A5.2.1} \)

\( S_e = \text{Elastic section modulus of the effective section calculated with the extreme compression or tension fiber at } F_y \)

(b) Procedure II - Based on Inelastic Reserve Capacity

The inelastic flexural reserve capacity may be used when the following conditions are met:

(1) The member is not subject to twisting or to lateral, torsional, or torsional-flexural buckling.

(2) The effect of cold forming is not included in determining the yield point \( F_y \).

(3) The ratio of the depth of the compressed portion of the web to its thickness does not exceed \( \lambda_1 \).

(4) The shear force does not exceed \( 0.35F_y \) times the web area, \( h \times t \).
(5) The angle between any web and the vertical does not exceed 30 degrees.

The nominal moment strength, $M_n$, shall not exceed either $1.25S_{F_y}$ determined according to Procedure I or that causing a maximum compression strain of $C_y e_y$ (no limit is placed on the maximum tensile strain).

where

$e_y = \text{Yield strain} = \frac{F_y}{E}$

$E = \text{Modulus of elasticity}$

$C_y = \text{Compression strain factor determined as follows:}$

(a) Stiffened compression elements without intermediate stiffeners

$C_y = 3 \text{ for } w/t \leq \lambda_1$

$C_y = 3 - 2(\frac{w/t - \lambda_1}{\lambda_2 - \lambda_1}) \text{ for } \lambda_1 < w/t < \lambda_2$

$C_y = 1 \text{ for } w/t \geq \lambda_2$

where

$\lambda_1 = \frac{1.11}{\sqrt{F_y/E}} \quad \text{(IV.3)}$

$\lambda_2 = \frac{1.28}{\sqrt{F_y/E}} \quad \text{(IV.4)}$

(b) Unstiffened compression elements

$C_y = 1$

(c) Multiple-stiffened compression elements and compression elements with edge stiffeners

$C_y = 1$

When applicable, effective design widths defined in Section B3.1 shall be used in calculating section properties. $M_n$ shall be calculated considering equilibrium of stresses, assuming an ideally elastic-
plastic stress-strain curve which is the same in tension as in compression, assuming small deformation and assuming that plane sections remain plane during bending. Combined bending and web crippling shall be checked by provisions of Section C3.5.

C3.1.2 Lateral Buckling Strength

For the laterally unbraced segments of doubly- or singly-symmetric sections subjected to lateral buckling, \( M_n \) shall be determined as follows:

\[
M_n = S_c (M_c / S_f)^n
\]

where

\( S_f = \) Elastic section modulus of the full unreduced section for the extreme compression fiber

\( S_c = \) Elastic section modulus of the effective section calculated at a stress \( M_c / S_f \) in the extreme compression fiber

\( M_c = \) Critical moment calculated according to (a) or (b) below:

(a) For I- or Z-sections bent about the centroidal axis perpendicular to the web (x-axis):

For \( M_e \geq 2.78 M_y \)

\[
M_c = M_y
\]

(IV.6)

For \( 2.78 M_y > M_e > 0.56 M_y \)

\[
M_c = (10/9) M_y (1 - 10 M_y / 36 M_e)
\]

(IV.7)

For \( M_e \leq 0.56 M_y \)

\[
M_c = M_e
\]

(IV.8)

where

\( M_y = \) Moment causing initial yield at the extreme compression
fiber of the full section

\[ S_y F_y \]  

\( M_e \) = Elastic critical moment determined either as defined in (b) below or as follows:

\[ \pi^2 E C_b (\frac{dI_{yc}}{L^2}) \] for doubly-symmetric I-sections (IV.10)

\[ \pi^2 E C_b \frac{dI_{yc}}{(2L^2)} \] for point-symmetric Z-sections (IV.11)

\( L \) = Unbraced length of the member

\( I_{yc} \) = Moment of inertia of the compression portion of a section about the gravity axis of the entire section parallel to the web, using the full unreduced section

Other terms are defined in (b) below.

(b) For singly-symmetric sections (x-axis is assumed to be the axis of symmetry):

For \( M_e > 0.5 M_y \)

\[ M_c = M_y (1 - \frac{M_e}{4M_y}) \]  

(IV.12)

For \( M_e \leq 0.5 M_y \)

\[ M_c = M_e \]  

(IV.13)

where

\( M_y \) is as defined in (a) above

\( M_e \) = Elastic critical moment

\[ M_e = C_b r_0 A \sqrt{e_y e_t} \] for bending about the symmetry axis (x-axis is the axis of symmetry oriented such that the shear center has a negative x-coordinate).

Alternatively, \( M_e \) can be calculated using the formula for doubly-symmetric I-sections given in (a) above.

(IV.14)
\[ M_e = C_s A \sigma_{ex} \left( j + C_s J + r_0^2 (\sigma_x / \sigma_{ex}) \right) / C_T F \]

for bending centroidal axis perpendicular to the symmetry axis \hspace{1cm} (IV.15)

\[ C_s = + 1 \] for moment causing compression on the shear center side of the centroid

\[ C_s = - 1 \] for moment causing tension on the shear center side of the centroid

\[ \sigma_{ex} = n^2E/(K_x L_x / r_x)^2 \] \hspace{1cm} (IV.16)

\[ \sigma_{ey} = n^2E/(K_y L_y / r_y)^2 \] \hspace{1cm} (IV.17)

\[ \sigma_t = 1/(Ar_0^2)[GJ/n^2EC_w/(K_t L_t)^2] \] \hspace{1cm} (IV.18)

\[ A = \] Full cross-sectional area

\[ C_b = \] Bending coefficient which can conservatively be taken as unity, or calculated from

\[ C_b = 1.75 + 1.05(M_1/M_2) + 0.3(M_1/M_2)^2 \leq 2.3 \]

where

\[ M_1 \] is the smaller and \[ M_2 \] the larger bending moment at the ends of the unbraced length, taken about the strong axis of the member, and where \[ M_1/M_2 \], the ratio of end moments, is positive when \[ M_1 \] and \[ M_2 \] have the same sign (reverse curvature bending) and negative when they are of opposite sign (single curvature bending). When the bending moment at any point within an unbraced length is larger than that at both ends of this length, and for members subject to combined axial load and bending moment (Section CS), \[ C_b \] shall be taken as unity.

\[ E = \] Modulus of elasticity
d = Depth of section

\[ C_{TF} = 0.6 - 0.4(M_1/M_2) \]

where

\( M_1 \) is the smaller and \( M_2 \) the larger bending moment at the ends of the unbraced length, and where \( M_1/M_2 \), the ratio of end moments, is positive when \( M_1 \) and \( M_2 \) have the same sign (reverse curvature bending) and negative when they are of opposite sign (single curvature bending). When the bending moment at any point within an unbraced length is larger than that at both ends of this length, and for members subject to combined axial load and bending moment (Section C5), \( C_{TF} \) shall be taken as unity.

\[ r_0 = \text{Polar radius of gyration of the cross section about the shear center} \]

\[ = \sqrt{r_x^2 + r_y^2 + x_0^2} \quad \text{(IV.19)} \]

\( r_x, r_y \) = Radii of gyration of the cross section about the centroidal principal axes

\( G \) = Shear modulus

\( K_x, K_y, K_t \) = Effective length factors for bending about the \( x \)- and \( y \)-axes, and for twisting

\( L_x, L_y, L_t \) = Unbraced length of compression member for bending about the \( x \)- and \( y \)-axes, and for twisting

\( x_0 \) = Distance from the shear center to the centroid along the principal \( x \)-axis, taken as negative

\( J \) = St. Venant torsion constant of the cross section
2. **LRFD Criteria.** Based on Section C3.1 of the LRFD Specification, for flexural members subjected only to bending moment, the factored nominal bending strength, $\Phi M_n$, shall be the smaller of the values calculated according to nominal section strength and lateral buckling strength.

For nominal section strength, the factored nominal bending strength, $\Phi M_n$, shall be determined with $\Phi = 0.95$ and $0.90$ for sections with stiffened compression flanges and unstiffened compression flanges, respectively, and the nominal section strength, $M_n$, calculated exactly the same as that specified in Section C3.1.1 of the AISI Specification.

For lateral buckling strength, the factored nominal strength of the laterally unbraced segments of doubly- or singly-symmetric sections subjected to lateral buckling, $\Phi M_n$, shall be determined with $\Phi = 0.90$ and $M_n$ calculated exactly the same as that specified in Section C3.1.2 of the AISI Specification.

3. **Comparison.** The unfactored moment can be calculated by using Eq. (IV.21) for both methods (ASD and LRFD) for comparison.

$$M_{TL} = M_{DL} + M_{LL}$$  \hspace{1cm} (IV.21)

where

- $M_{TL} = \text{total unfactored moment}$
- $M_{DL} = \text{moment due to the nominal dead load}$
- $M_{LL} = \text{moment due to the nominal live load}$

For allowable stress design, the allowable moment is determined from either nominal section strength or lateral buckling strength with
a factor of safety of 1.67. Therefore, the allowable moment for beams is

\[
(M_a)_{ASD} = \frac{M_n}{Q_f} = \frac{M_n}{1.67} \quad (IV.22)
\]

For LRFD, the allowable moment can be computed by using the following equation developed from Eq. (II.6).

\[
(M_a)_{LRFD} = \frac{\phi M_n (D/L+1)}{(1.2D/L+1.6)} \quad (IV.23)
\]

The ratio of the allowable moments for both nominal section strength and lateral buckling strength is

\[
\frac{(M_a)_{LRFD}}{(M_a)_{ASD}} = \frac{1.67\phi}{1.2D/L+1.6} \quad (IV.24)
\]

For nominal section strength of sections with stiffened compression flanges, \(\phi = 0.95\)

\[
\frac{(M_a)_{LRFD}}{(M_a)_{ASD}} = \frac{1.58}{1.2D/L+1.6} \quad (IV.25)
\]

Figure 3 shows the allowable moment ratio versus dead-to-live load ratio for beams with stiffened compression flanges based on the nominal section strength. For \(D/L = 1/25\) both design methods will give the same value of allowable moment. However, LRFD will be conservative for \(D/L < 1/25\) and unconservative for \(D/L > 1/25\) as compared with the allowable stress design method.

For nominal section strength of sections with unstiffened compression flanges and lateral buckling strength, \(\phi = 0.90\)

\[
\frac{(M_a)_{LRFD}}{(M_a)_{ASD}} = \frac{1.50}{1.2D/L+1.6} \quad (IV.26)
\]

Figure 4 shows the allowable moment ratio versus dead-to-live load ratio for this case. Both design methods will give the same value for
For $D/L = 0.5$, the allowable moment based on LRFD is about 2.3% larger than the value obtained from allowable stress design. When the dead-to-live load ratio for cold-formed steel is less than $1/3$, the LRFD criteria are found to be conservative for nominal section strength of sections with unstiffened compression flanges and lateral buckling as compared with the allowable stress design method.

C. STRENGTH FOR SHEAR ONLY

There are three possible modes of shear failure in beam webs. For a relatively small $h/t$ ratio, shear yielding will be the failure mode. For webs with large $h/t$ ratios, the webs will fail in elastic shear buckling. For moderate values of $h/t$, the shear buckling will be in the inelastic range.

1. Allowable Stress Design. The shear force at any section shall not exceed the allowable shear, $V_a$, specified in Section C3.2 of the AISI Specification as follows:

(a) For $h/t \leq 1.38\sqrt{Fy/Fv}$

$$V_a = 0.38t^2 \sqrt{k_vFy} \leq 0.4Fyht$$

(IV.27)

(b) For $h/t > 1.38\sqrt{Fy/Fv}$

$$V_a = 0.53Ek_v t^3 /h$$

(IV.28)

where

- $t$ = Web thickness
- $h$ = Depth of the flat portion of the web measured along the plane of the web
- $k_v$ = Shear buckling coefficient determined as follows:
Figure 3. Allowable Moment Ratio vs. D/L Ratio for Bending Strength of Beams With Stiffened Compression Flanges
Figure 4. Allowable Moment Ratio vs. D/L Ratio for Section Strength of Beams With Unstiffened Compression Flanges and Lateral Buckling of Beams
1. For unreinforced webs, \( k_v = 5.34 \)

2. For beam webs with transverse stiffeners satisfying the requirements of Section B6

when \( a/h \leq 1.0 \)

\[
k_v = 4.00 + 5.34/(a/h)^2 \quad \text{(IV.29)}
\]

when \( a/h > 1.0 \)

\[
k_v = 5.34 + 4.00/(a/h)^2 \quad \text{(IV.30)}
\]

where

\( a \) = the shear panel length for unreinforced web element

\( = \) distance between transverse stiffeners for web elements.

For a web consisting of two or more sheets, each sheet shall be considered as a separate element carrying its share of the shear force.

2. LRFD Criteria. According to Section C3.2 of the LRFD Specification, the factored nominal shear strength, \( \phi_v V_n \), at any section shall be calculated as follows:

(a) For \( h/t \leq \sqrt{E k_v / F_y} \)

\[
\phi_v = 1.0 \\
V_n = 0.577 F_y h t \quad \text{(IV.31)}
\]

(b) For \( \sqrt{E k_v / F_y} < h/t \leq 1.415 \sqrt{E k_v / F_y} \)

\[
\phi_v = 0.90 \\
V_n = 0.64 t^2 \sqrt{k_v F_y} \quad \text{(IV.32)}
\]

(c) For \( h/t > 1.415 \sqrt{E k_v / F_y} \)

\[
\phi_v = 0.90 \\
V_n = 0.905 E k_v t^3 / h \quad \text{(IV.33)}
\]

where

\( \phi_v \) = Resistance factor for shear
V
\_n = Nominal shear strength of the beam

3. Comparison. The unfactored shear force can be calculated for both ASD and LRFD methods by using the following equation.

V_T = V_{DL} + V_{LL} \tag{IV.34}

where

V_T = total unfactored shear force
V_{DL} = shear force due to the nominal dead load
V_{LL} = shear force due to the nominal live load

This total unfactored shear force should be less than or equal to the allowable shear capacity. For allowable stress design, the allowable shear load for beam webs is

(V_a)_{ASD} = V_n / \Omega \tag{IV.35}

For LRFD, the allowable shear load equation was developed from Eq. (II.6) and is

(V_a)_{LRFD} = \phi V_n (D/L+1)/(1.2D/L+1.6) \tag{IV.36}

The allowable shear force, V_a, for allowable stress design is determined from shear yielding with a factor of safety of 1.44, from the critical stress for elastic shear buckling with a factor of safety of 1.71, and from the critical stress for inelastic shear buckling with a factor of safety of 1.67. The limits of the h/t ratio were obtained by equating the formulas for the three shear failure modes for both allowable stress and LRFD criteria. Because each failure mode has a different factor of safety, the h/t limits are slightly different for both design criteria. For example, for h/t greater than 1.38\sqrt{E_k/F_y} and less than 1.415\sqrt{E_k/F_y}, inelastic shear buckling will govern for LRFD.
Figure 5. Allowable Shear Ratio vs. D/L Ratio for Shear Strength of Beams
Figure 6. Allowable Shear Ratio vs. h/t Ratio for Shear Strength of Beams
The allowable shear ratios are:

For $h/t \leq \sqrt{E_{k}/F_{y}}$ and $\Phi_{v} = 1.0$,

$$\frac{(V_a)_{LRFD}}{(V_a)_{ASD}} = 1.443\Phi_{v} \frac{D/L+1}{1.2D/L+1.6} = 1.443 \frac{D/L+1}{1.2D/L+1.6}$$  \hspace{0.5cm} (IV.37)

For $\sqrt{E_{k}/F_{y}} < h/t \leq 1.38\sqrt{E_{k}/F_{y}}$ and $\Phi_{v} = 0.90$

$$\frac{(V_a)_{LRFD}}{(V_a)_{ASD}} = 1.674\Phi_{v} \frac{D/L+1}{1.2D/L+1.6} = 1.507 \frac{D/L+1}{1.2D/L+1.6}$$  \hspace{0.5cm} (IV.38)

For $h/t > 1.415\sqrt{E_{k}/F_{y}}$ and $\Phi_{v} = 0.90$

$$\frac{(V_a)_{LRFD}}{(V_a)_{ASD}} = 1.712\Phi_{v} \frac{D/L+1}{1.2D/L+1.6} = 1.541 \frac{D/L+1}{1.2D/L+1.6}$$  \hspace{0.5cm} (IV.39)

Figure 5 shows the allowable shear ratio versus dead-to-live load ratio for the three failure modes. For $D/L = 0.5$, the allowable shear determined according to LRFD may be up to 5% higher than the value obtained from allowable stress design. For $D/L < 0.17$, LRFD is generally conservative. When $D/L > 0.65$, LRFD gives larger values of the allowable shear capacity.

In Figure 6, the relationships of allowable shear ratio and $h/t$ ratio are shown graphically for dead-to-live load ratios equal to 1/5, 1/3, and 1/2. The transition zones between $h/t$ limits can be seen clearly in this figure.

D. **STRENGTH FOR COMBINED BENDING AND SHEAR**
For continuous beams and cantilevers, maximum bending stress and shear stress act simultaneously at supports. The beams will fail at a lower stress than if only one stress were present. The interaction between bending and shear must be checked.

1. **Allowable Stress Design.** For beams subjected to both bending and shear, the following equations should be satisfied in accordance with Section C3.3 of the AISI Specification.

For beams with unreinforced webs, the moment, \( M \), and shear, \( V \), shall satisfy the following interaction equation:

\[
\frac{(M/M_a)^2 + (V/V_a)^2}{2} \leq 1.0
\]  \hspace{1cm} (IV.40)

For beams with transverse web stiffeners, the moment, \( M \), and shear, \( V \), shall not exceed \( M_a \) and \( V_a \) respectively. When \( M/M_a > 0.5 \) and \( V/V_a > 0.7 \), then \( M \) and \( V \) shall satisfy the following interaction equation:

\[
0.6(M/M_a) + (V/V_a) \leq 1.3
\]  \hspace{1cm} (IV.41)

In the above equations:

- \( M_a \) = Allowable moment determined according to Section C3.1.1 when bending alone exists excluding lateral buckling
- \( V_a \) = Allowable shear force when shear alone exists

2. **LRFD Criteria.** Section C3.3 of the LRFD Specification specifies that for beams with unreinforced webs, the bending moment, \( M_D \), and the shear force, \( V_D \), computed on the basis of the factored loads shall satisfy the following interaction equation:

\[
(M_D/\Phi M_n)^2 + (V_D/\Phi V_n)^2 \leq 1.0
\]  \hspace{1cm} (IV.42)

For beams with transverse web stiffeners, the bending moment, \( M_D \), and the shear force, \( V_D \), computed on the basis of the factored loads...
shall not exceed $\phi M_n$ and $\phi V_n$, respectively. When $M_D/\phi M_n > 0.5$ and $V_D/\phi V_n > 0.7$, then $M_D$ and $V_D$ shall satisfy the following interaction equation:

$$0.6(M_D/\phi M_n) + (V_D/\phi V_n) \leq 1.3$$

(IV.43)

In the above equations:

$\phi = $ Resistance factor for bending

$\phi_v = $ Resistance factor for shear

$M_n = $ Nominal ultimate bending moment determined according to Section C3.1.1 when bending alone exists

$V_n = $ Nominal ultimate shear strength when shear alone exists

3. **Comparison.** A typical design example was selected for comparison purposes. The example deals with a three-equal-span continuous beam subjected to a uniformly distributed dead and live load. The combination of the following maximum moment and shear would occur at the interior supports.

$$M_{TL} = M_{DL} + M_{LL} = c_m w_T L^2$$

(IV.44)

$$V_T = V_{DL} + V_{LL} = c_v w_T L$$

(IV.45)

where $c_m$ and $c_v$ are the deterministic influence coefficients for applied moment and shear based on support conditions and number of spans and $w_T$ is the unfactored applied uniform load.

The allowable load based on allowable stress design was calculated as follows:

$$\frac{M}{M_a} = \frac{M_{TL}}{0.6M_n} = \frac{1.667c_m w_T L^2}{M_n}$$

(IV.46)
For $h/t \leq 1.38\sqrt{\frac{E}{F_y}/F_y}$,

$$\frac{V}{V_a} = \frac{V_T}{V_n/1.674} = \frac{1.674c_v w_{T L}}{V_n} \quad \text{(IV.47)}$$

By substituting Eqs. (IV.46) and (IV.47) into Eq. (IV.40),

$$\left(\frac{M}{M_a}\right)^2 + \left(\frac{V}{V_a}\right)^2 = w_T^2 \left[\left(\frac{1.667c_m L_m^2}{M_n}\right)^2 + \left(\frac{1.674c_v L_v^2}{V_n}\right)^2\right] = 1$$

Therefore,

$$(w_T)_{ASD} = \frac{1}{\sqrt{\left(\frac{1.667c_m L_m^2}{M_n}\right)^2 + \left(\frac{1.674c_v L_v^2}{V_n}\right)^2}} \quad \text{(IV.48)}$$

For $h/t > 1.415\sqrt{\frac{E}{F_y}/F_y}$,

$$\frac{V}{V_a} = \frac{V_T}{V_n/1.712} = \frac{1.712c_v w_{T L}}{V_n} \quad \text{(IV.49)}$$

By substituting Eqs. (IV.46) and (IV.49) into Eq. (IV.40),

$$\left(\frac{M}{M_a}\right)^2 + \left(\frac{V}{V_a}\right)^2 = w_T^2 \left[\left(\frac{1.667c_m L_m^2}{M_n}\right)^2 + \left(\frac{1.712c_v L_v^2}{V_n}\right)^2\right] = 1$$

Therefore,

$$(w_T)_{ASD} = \frac{1}{\sqrt{\left(\frac{1.667c_m L_m^2}{M_n}\right)^2 + \left(\frac{1.712c_v L_v^2}{V_n}\right)^2}} \quad \text{(IV.50)}$$
The allowable uniform load based on LRFD was calculated as follows:

\[
\frac{M_D}{\Phi M_n} = \frac{1.2D/L + 1.6}{D/L + 1} \left( \frac{M_{TL}}{\Phi M_n} \right) = \frac{1.2D/L + 1.6}{D/L + 1} \left( \frac{c_m w_T L^2}{\Phi M_n} \right) \quad (IV.51)
\]

\[
\frac{V_D}{\Phi V_n} = \frac{1.2D/L + 1.6}{D/L + 1} \left( \frac{V_T}{\Phi V_n} \right) = \frac{1.2D/L + 1.6}{D/L + 1} \left( \frac{c_v w_T L}{\Phi V_n} \right) \quad (IV.52)
\]

By substituting Eqs. (IV.51) and (IV.52) into Eq. (IV.42),

\[
\left( \frac{M_D}{\Phi M_n} \right)^2 + \left( \frac{V_D}{\Phi V_n} \right)^2 = w_T^2 \left( \frac{1.2D/L + 1.6}{D/L + 1} \right)^2 \left( \frac{c_m L^2}{\Phi M_n} \right)^2 + \left( \frac{c_v L}{\Phi V_n} \right)^2 = 1
\]

Therefore,

\[
(w_T)_{LRFD} = \frac{D/L + 1}{1.2D/L + 1.6} \frac{1}{\frac{\left( \frac{c_m L^2}{\Phi M_n} \right)^2}{L} + \left( \frac{c_v L}{\Phi V_n} \right)^2} \quad (IV.53)
\]

For the design example used in this comparison, the coefficients, \(c_m\) and \(c_v\), are equal to 0.10 and 0.60, respectively. Therefore, by using \(\Phi = 0.95\) and 0.90 for sections with stiffened compression flanges and unstiffened compression flanges, respectively, for nominal section strength and \(\Phi_v = 0.90\), the allowable uniform load ratios are as follows:
For $h/t \leq 1.38\sqrt{\frac{E_k}{F_y}}$,$$
abla \frac{(w_T)_{LRFD}}{(w_T)_{ASD}} = \frac{D/L+1}{1.2D/L+1.6} \sqrt{\frac{2.803+0.07716(V_{L/M})_n^2}{1.235+0.02778(V_{L/\Phi M})_n^2}}$$ (IV.54)

For $h/t > 1.415\sqrt{\frac{E_k}{F_y}}$,$$
abla \frac{(w_T)_{LRFD}}{(w_T)_{ASD}} = \frac{D/L+1}{1.2D/L+1.6} \sqrt{\frac{2.929+0.07716(V_{L/M})_n^2}{1.235+0.02778(V_{L/\Phi M})_n^2}}$$ (IV.55)

Equations (IV.54) and (IV.55) can be expressed in the following form:

$$\frac{(w_T)_{LRFD}}{(w_T)_{ASD}} = \frac{D/L+1}{1.2D/L+1.6} (K_w)$$ (IV.56)

where $K_w$ is a variable determined from section properties, material strength, and span length for a particular design example.

For combined bending and shear, the allowable load ratio can be determined by using Eq. (IV.56) as given above. It is not only a function of dead-to-live load ratio but is also a function of $h/t$, sectional geometry, and material strength. Because of the complexity involved in the comparison, several individual beam sections of different depths and thicknesses were studied.

Figure 7 shows the allowable load ratio versus dead-to-live load ratio for 5 in. x 2 in. standard channel sections with stiffened flanges which are listed in Table 1 of Part V of the AISI Design Manual. Different curves represent the relationships for different thicknesses by using the same span length and material. Table IV.1 shows the sectional properties and calculated values used to obtain the curves which
indicate that thinner members result in slightly lower values for the allowable load ratio except t = 0.048 in. which is governed by Eq. (IV.55) because of the higher h/t ratio.

In Figure 8, the span length was varied for a 5 in. x 2 in. x 0.105 in. channel with stiffened flanges for D/L = 1/5 and $F_y = 33$ to 50 ksi. Span lengths and calculated values used to obtain the curves are included in Table IV.2. It can be seen that the material strength has little effect on the allowable uniform load ratio. This figure also shows that for the channel section used in this comparison, the allowable load permitted by LRFD is larger than that determined by ASD for span length larger than 20 in.

Figure 9 shows the allowable uniform load ratio versus h/t ratio for the 5 in. - deep channels used in Figure 7 and Table IV.1 for a dead-to-live load ratio of 1/5 and a span length of 5 ft. Table IV.3 shows the calculated values for $F_y = 50$ ksi. For $F_y = 33$ and 50 ksi, this figure shows that the smallest allowable load ratio occurs at h/t = 75.

Figure 10 shows the relationship of allowable load ratio and dead-to-live load ratio for channels with stiffened flanges. Sectional properties and other related data are included in Table IV.4. Deeper sections with larger h/t ratios give smaller values of the allowable load ratio as indicated in Figure 10.
Table IV.1
Channels With Stiffened Flanges, 5 in. Depths - Case A.

<table>
<thead>
<tr>
<th>Section</th>
<th>h/t</th>
<th>$V_n$ (Kips)</th>
<th>$M_n$ (K-in.)</th>
<th>$\Phi M_n$ (K-in.)</th>
<th>$K_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5x2x0.135</td>
<td>32.26</td>
<td>26.594</td>
<td>61.803</td>
<td>58.712</td>
<td>1.5790</td>
</tr>
<tr>
<td>0.105</td>
<td>42.05</td>
<td>16.088</td>
<td>49.625</td>
<td>47.144</td>
<td>1.5761</td>
</tr>
<tr>
<td>0.075</td>
<td>62.17</td>
<td>8.208</td>
<td>36.917</td>
<td>35.071</td>
<td>1.5694</td>
</tr>
<tr>
<td>0.060</td>
<td>78.21</td>
<td>5.253</td>
<td>28.555</td>
<td>27.127</td>
<td>1.5646</td>
</tr>
<tr>
<td>0.048</td>
<td>98.26</td>
<td>3.343</td>
<td>21.795</td>
<td>20.705</td>
<td>1.5695</td>
</tr>
</tbody>
</table>

* $F_y = 33$ ksi, L = 60 in.

Channels with unstiffened flanges were also studied. Figure 11 shows the allowable load ratio versus dead-to-live load ratio for 6 in. x 1.5 in. standard channel sections with unstiffened flanges which are listed in Table 2 of Part V of the AISI Design Manual. Different curves represent the relationships for different thicknesses by using the same span length and material. Table IV.5 shows the sectional properties and calculated values used to obtain the curves which indicate that thinner members result in slightly higher values for the allowable load ratio.
Figure 7. Allowable Load Ratio vs. D/L Ratio for Combined Bending and Shear in Beams - Case A
Table IV.2
5 in. x 2 in. x 0.105 in. Channels With Stiffened Flanges for Various Lengths and Yield Points

<table>
<thead>
<tr>
<th>$F_v$ (ksi)</th>
<th>$L$ (in.)</th>
<th>$V_D$ (kips)</th>
<th>$M_D$ (K-in.)</th>
<th>$\phi M_n$ (K-in.)</th>
<th>$K_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>0</td>
<td>16.088</td>
<td>49.625</td>
<td>47.144</td>
<td>1.5065</td>
</tr>
<tr>
<td>25</td>
<td>16.088</td>
<td>49.625</td>
<td>47.144</td>
<td>1.5546</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>16.088</td>
<td>49.625</td>
<td>47.144</td>
<td>1.5733</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>16.088</td>
<td>49.625</td>
<td>47.144</td>
<td>1.5785</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>16.088</td>
<td>49.625</td>
<td>47.144</td>
<td>1.5805</td>
<td></td>
</tr>
</tbody>
</table>

| 50         | 0         | 19.803       | 75.190        | 71.430            | 1.5065|
| 25         | 19.803    | 75.190       | 71.430        | 1.5469            |
| 50         | 19.803    | 75.190       | 71.430        | 1.5691            |
| 75         | 19.803    | 75.190       | 71.430        | 1.5763            |
| 100        | 19.803    | 75.190       | 71.430        | 1.5792            |
Figure 8. Allowable Load Ratio vs. Span Length - Case A

5" x 2" x 0.105" Channel
With Stiffened Flanges
D/L = 1/5
Eq. (IV.56)
### Table IV.3
5 in. x 2 in. Channels With Stiffened Flanges for $F_y = 50$ ksi

<table>
<thead>
<tr>
<th>Section</th>
<th>h/t</th>
<th>$V_n$ (Kips)</th>
<th>$M_n$ (K-in.)</th>
<th>$\Phi M_n$ (K-in.)</th>
<th>$K_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5x2x0.135</td>
<td>32.26</td>
<td>32.735</td>
<td>93.640</td>
<td>88.958</td>
<td>1.5770</td>
</tr>
<tr>
<td>0.105</td>
<td>42.05</td>
<td>19.803</td>
<td>75.190</td>
<td>71.430</td>
<td>1.5729</td>
</tr>
<tr>
<td>0.075</td>
<td>62.17</td>
<td>10.103</td>
<td>54.626</td>
<td>51.895</td>
<td>1.5648</td>
</tr>
<tr>
<td>0.060</td>
<td>78.21</td>
<td>6.466</td>
<td>39.016</td>
<td>37.066</td>
<td>1.5615</td>
</tr>
<tr>
<td>0.048</td>
<td>98.26</td>
<td>3.343</td>
<td>30.687</td>
<td>29.153</td>
<td>1.5625</td>
</tr>
</tbody>
</table>

*L = 60 in.*

### Table IV.4
Channels With Stiffened Flanges - Case B.

<table>
<thead>
<tr>
<th>Section</th>
<th>h/t</th>
<th>$V_n$ (Kips)</th>
<th>$M_n$ (K-in.)</th>
<th>$\Phi M_n$ (K-in.)</th>
<th>$K_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9x3.25x0.105</td>
<td>80.14</td>
<td>16.088</td>
<td>152.534</td>
<td>144.907</td>
<td>1.5453</td>
</tr>
<tr>
<td>7x2.75x0.105</td>
<td>61.10</td>
<td>16.088</td>
<td>99.487</td>
<td>94.512</td>
<td>1.5607</td>
</tr>
<tr>
<td>5x2x0.105</td>
<td>42.05</td>
<td>16.088</td>
<td>49.625</td>
<td>47.144</td>
<td>1.5761</td>
</tr>
<tr>
<td>3.5x2x0.105</td>
<td>27.76</td>
<td>16.088</td>
<td>30.531</td>
<td>29.005</td>
<td>1.5804</td>
</tr>
</tbody>
</table>

* $F_y = 33$ ksi, $L = 60$ in.*
Figure 9. Allowable Load Ratio vs. h/t Ratio for Combined Bending and Shear in Beams - Case A

5" x 2" Channels With
Stiffened Flanges
D/L = 1/5
L = 60 in.
Eq. (IV.56)
Figure 10. Allowable Load Ratio vs. D/L Ratio for Combined Bending and Shear in Beams - Case B

Channels With Stiffened Flanges

$F_y = 33$ ksi
$L = 60$ in.

Eq. (IV.56)
In Figure 12, the span length was varied for a 6 in. x 1.5 in. x 0.105 in. channel with unstiffened flanges for D/L = 1/5 and F_y = 33 to 50 ksi. It can be seen that the material strength has small effect on the allowable load ratio. This figure also shows that for the channel section used in this comparison, the allowable load permitted by LRFD is about 2% less than that determined by ASD for various span lengths.

Figure 13 shows the allowable load ratio versus h/t ratio for the 6 in.-deep channels used in Figure 11 and Table IV.5 for a dead-to-live load ratio of 1/5 and a span length of 5 ft for F_y = 33 and 50 ksi. This figure shows that higher h/t ratios give slightly larger values of allowable load ratio.

<table>
<thead>
<tr>
<th>Section</th>
<th>h/t</th>
<th>V_M (Kips)</th>
<th>M_n (K-in.)</th>
<th>ΦM_n (K-in.)</th>
<th>K_w</th>
</tr>
</thead>
<tbody>
<tr>
<td>6x1.5x0.135</td>
<td>39.67</td>
<td>26.594</td>
<td>58.667</td>
<td>52.800</td>
<td>1.5002</td>
</tr>
<tr>
<td>0.105</td>
<td>51.57</td>
<td>16.088</td>
<td>46.637</td>
<td>41.973</td>
<td>1.5004</td>
</tr>
<tr>
<td>0.075</td>
<td>75.50</td>
<td>8.208</td>
<td>31.788</td>
<td>28.609</td>
<td>1.5008</td>
</tr>
<tr>
<td>0.060</td>
<td>94.88</td>
<td>5.253</td>
<td>24.129</td>
<td>21.716</td>
<td>1.5011</td>
</tr>
<tr>
<td>0.048</td>
<td>119.09</td>
<td>2.758</td>
<td>18.314</td>
<td>16.483</td>
<td>1.5123</td>
</tr>
</tbody>
</table>

* F_y = 33 ksi, L = 60 in.
Figure 11. Allowable Load Ratio vs. D/L Ratio for Combined Bending and Shear in Beams - Case C

6" x 1.5" x t Channels
With Unstiffened Flanges

$F_y = 33$ ksi
$L = 60$ in.

Eq. (IV.56)
Table IV.6
6 in. x 1.5 in. x 0.105 in. Channels With Unstiffened Flanges
for Various Lengths and Yield Points

<table>
<thead>
<tr>
<th>$F_Y$ (ksi)</th>
<th>$L$ (in.)</th>
<th>$V_n$ (Kips)</th>
<th>$M_n$ (K-in.)</th>
<th>$\Phi M_n$ (K-in.)</th>
<th>$K_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>0</td>
<td>16.088</td>
<td>46.637</td>
<td>41.973</td>
<td>1.5065</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>16.088</td>
<td>46.637</td>
<td>41.973</td>
<td>1.5021</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>16.088</td>
<td>46.637</td>
<td>41.973</td>
<td>1.5006</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>16.088</td>
<td>46.637</td>
<td>41.973</td>
<td>1.5003</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>16.088</td>
<td>46.637</td>
<td>41.973</td>
<td>1.5001</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
<td>19.803</td>
<td>68.840</td>
<td>61.956</td>
<td>1.5065</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>19.803</td>
<td>68.840</td>
<td>61.956</td>
<td>1.5026</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>19.803</td>
<td>68.840</td>
<td>61.956</td>
<td>1.5009</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>19.803</td>
<td>68.840</td>
<td>61.956</td>
<td>1.5004</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>19.803</td>
<td>68.840</td>
<td>61.956</td>
<td>1.5002</td>
</tr>
</tbody>
</table>

Table IV.7
6 in. x 1.5 in. Channels With Unstiffened Flanges, for $F_Y = 50$ ksi

<table>
<thead>
<tr>
<th>Section</th>
<th>$h/t$</th>
<th>$V_n$ (Kips)</th>
<th>$M_n$ (K-in.)</th>
<th>$\Phi M_n$ (K-in.)</th>
<th>$K_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6x1.5x0.135</td>
<td>39.67</td>
<td>32.735</td>
<td>88.890</td>
<td>80.001</td>
<td>1.5004</td>
</tr>
<tr>
<td>0.105</td>
<td>51.57</td>
<td>19.803</td>
<td>68.840</td>
<td>61.956</td>
<td>1.5006</td>
</tr>
<tr>
<td>0.075</td>
<td>75.50</td>
<td>10.103</td>
<td>45.584</td>
<td>41.026</td>
<td>1.5011</td>
</tr>
<tr>
<td>0.060</td>
<td>94.88</td>
<td>5.410</td>
<td>34.647</td>
<td>31.183</td>
<td>1.5117</td>
</tr>
<tr>
<td>0.048</td>
<td>119.09</td>
<td>2.758</td>
<td>26.283</td>
<td>23.655</td>
<td>1.5191</td>
</tr>
</tbody>
</table>

* $L = 60$ in.
Figure 12. Allowable Load Ratio vs. Span Length - Case C

6" x 1.5" x 0.105" Channel
With Unstiffened Flanges
D/L = 1/5
Eq. (IV.56)
Figure 13. Allowable Load Ratio vs. h/t Ratio for Combined Bending and Shear in Beams - Case C
For hat sections, one web was assumed to carry one-half of the load and, therefore, only half-sectional properties were used. Dimensions and sectional properties of numerous hat sections are given in Table 9 of Part V of the AISI Design Manual and Table IV.8 lists sectional properties and calculated member strengths used in this comparison. Figure 14 shows the relationships between allowable uniform load ratio and dead-to-live load ratio for three hat sections with a yield point of 33 ksi and a span length of 5 ft. All 4 in. deep hat sections resulted in the same curve regardless of h/t ratio. Hat sections with larger depths or larger h/t ratios resulted in smaller values of allowable load ratio.

I-sections made of two channels back-to-back would result in the same comparison and conclusions as the single channel sections.

From Figures 7 through 14, it can be seen that for dead-to-live load ratios less than about 1/10, the LRFD criteria for combined bending and shear are usually conservative compared with the allowable stress design method. For D/L = 0.5, the differences range from 2.7% to 7.8%. For large D/L ratios, ASD method is always conservative than LRFD. Yield point of steel has little effect on the allowable load ratio. The lower the yield point, the larger the difference. Span length has little effect on the allowable uniform load ratio as shown in Figs. 8 and 12. For channels and I-sections, smaller h/t ratios result in a slightly larger difference between allowable uniform loads obtained from these two design methods. For hat sections, smaller depths result in a larger difference between the allowable loads.
Table IV.8
Hat Sections (Positive Bending)

<table>
<thead>
<tr>
<th>Section</th>
<th>h/t</th>
<th>$V_p$ (Kips)</th>
<th>$M_p$ (K-in.)</th>
<th>$\phi M_p$ (K-in.)</th>
<th>$K_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4x2x0.075</td>
<td>48.83</td>
<td>8.208</td>
<td>15.086</td>
<td>14.331</td>
<td>1.5805</td>
</tr>
<tr>
<td>4x4x0.105</td>
<td>32.52</td>
<td>16.088</td>
<td>29.374</td>
<td>27.906</td>
<td>1.5806</td>
</tr>
<tr>
<td>4x4x0.075</td>
<td>48.83</td>
<td>8.208</td>
<td>17.237</td>
<td>16.375</td>
<td>1.5798</td>
</tr>
<tr>
<td>4x6x0.135</td>
<td>24.85</td>
<td>26.594</td>
<td>43.978</td>
<td>41.779</td>
<td>1.5810</td>
</tr>
<tr>
<td>4x6x0.105</td>
<td>32.52</td>
<td>16.088</td>
<td>30.756</td>
<td>29.218</td>
<td>1.5803</td>
</tr>
<tr>
<td>6x9x0.105</td>
<td>51.57</td>
<td>16.088</td>
<td>55.454</td>
<td>52.681</td>
<td>1.5745</td>
</tr>
<tr>
<td>10x5x0.075</td>
<td>128.83</td>
<td>6.225</td>
<td>72.671</td>
<td>69.037</td>
<td>1.5573</td>
</tr>
</tbody>
</table>

* $F_y = 33$ ksi, $L = 60$ in.

E. WEB CRIPLING STRENGTH

Beam webs should be checked for web crippling at locations of high intensity loads. This would occur under concentrated loads or support reactions.

1. Allowable Stress Design. To avoid crippling of unreinforced flat webs of flexural members having a flat width ratio, h/t, equal to or less than 200, concentrated loads and reactions shall not exceed the value of $P_a$ given in Table IV.9 according to Section C3.4 of the AISI Specification. Webs of flexural members for which h/t is greater than 200 shall be provided with adequate means of transmitting concentrated loads and/or reactions directly into the webs.
Figure 14. Allowable Load Ratio vs. D/L Ratio for Combined Bending and Shear in Beams - Case D
The formulas in Table IV.9 apply to beams when \( R/t \leq 6 \) and to deck when \( R/t \leq 7 \), \( N/t \leq 210 \) and \( N/h \leq 3.5 \).

<table>
<thead>
<tr>
<th>Opposing Loads Spaced</th>
<th>Shapes Having Single Webs</th>
<th>Shapes Having Multiple Webs(^{(1)})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stiffened Flanges</td>
<td>Unstiffened Flanges</td>
</tr>
<tr>
<td></td>
<td>Eq. IV.57</td>
<td>Eq. IV.58</td>
</tr>
<tr>
<td></td>
<td>Eq. IV.60</td>
<td>Eq. IV.60</td>
</tr>
<tr>
<td></td>
<td>Eq. IV.62</td>
<td>Eq. IV.62</td>
</tr>
<tr>
<td></td>
<td>Eq. IV.64</td>
<td>Eq. IV.64</td>
</tr>
</tbody>
</table>

Footnotes and Equation References to Table IV.9:

(1) I-sections made of two channels connected back to back or similar sections which provide a high degree of restraint against rotation of the web (such as I-sections made by welding two angles to a channel).

(2) At locations of one concentrated load or reaction acting either on the top or bottom flange, when the clear distance between the bearing edges of this and adjacent opposite concentrated loads or
reactions is greater than 1.5h.

(3) For end reactions of beams or concentrated loads on the end of cantilevers when the distance from the edge of the bearing to the end of the beam is less than 1.5h.

(4) For reactions and concentrated loads when the distance from the edge of bearing to the end of the beam is equal to or greater than 1.5h.

(5) At locations of two opposite concentrated loads or of a concentrated load and an opposite reaction acting simultaneously on the top and bottom flanges, when the clear distance between their adjacent bearing edges is equal to or less than 1.5h.

Equations for Table IV.9:

\[ t^2 k C_3 C_4 C_9 (179-0.33(h/t))[1+0.01(N/t)] \]  \hspace{1cm} (IV.57)

\[ t^2 k C_3 C_4 C_9 (117-0.15(h/t))[1+0.01(N/t)] \]  \hspace{1cm} (IV.58)

When \( N/t > 60 \), the factor \([1+0.01(N/t)]\) may be increased to \([0.71+0.015(N/t)]\)

\[ t^2 F_y C_6 (5.0+0.63\sqrt{N/t}) \]  \hspace{1cm} (IV.59)

\[ t^2 k C_1 C_2 C_9 (291-0.40(h/t))[1+0.007(N/t)] \]  \hspace{1cm} (IV.60)

When \( N/t > 60 \), the factor \([1+0.007(N/t)]\) may be increased to \([0.75+0.011(N/t)]\)

\[ t^2 F_y C_5 (0.88+0.12m)(7.50+1.63\sqrt{N/t}) \]  \hspace{1cm} (IV.61)

\[ t^2 k C_3 C_4 C_9 (132-0.31(h/t))[1+0.01(N/t)] \]  \hspace{1cm} (IV.62)

\[ t^2 F_y C_8 (0.64+0.31m)(5.0+0.63\sqrt{N/t}) \]  \hspace{1cm} (IV.63)

\[ t^2 k C_1 C_2 C_9 (417-1.22(h/t))[1+0.0013(N/t)] \]  \hspace{1cm} (IV.64)

\[ t^2 F_y C_7 (0.82+0.15m)(7.50+1.63\sqrt{N/t}) \]  \hspace{1cm} (IV.65)

In the above referenced formulas,
\( P_a = \) Allowable concentrated load or reaction per web, kips

\( C_1 = (1.22 - 0.22k) \) \hspace{1cm} (IV.66)

\( C_2 = (1.06 - 0.06R/t) \leq 1.0 \) \hspace{1cm} (IV.67)

\( C_3 = (1.33 - 0.33k) \) \hspace{1cm} (IV.68)

\( C_4 = (1.15 - 0.15R/t) \leq 1.0 \) but not less than 0.50 \hspace{1cm} (IV.69)

\( C_5 = (1.49 - 0.53k) \geq 0.6 \) \hspace{1cm} (IV.70)

\( C_6 = 1 + (h/t) / 750, \) when \( h/t \leq 150 \)

\[ = 1.20, \] when \( h/t > 150 \) \hspace{1cm} (IV.71)

\( C_7 = 1/k, \) when \( h/t \leq 66.5 \)

\[ = [1.10 - (h/t) / 665]/k, \] when \( h/t > 66.5 \) \hspace{1cm} (IV.72)

\( C_8 = [0.98 - (h/t) / 865]/k \) \hspace{1cm} (IV.73)

\( C_9 = 0.7 + 0.3(\theta/90)^2 \) \hspace{1cm} (IV.74)

\( F_y = \) Design yield stress of the web, ksi

\( h = \) Depth of the flat portion of the web measured along the plane of the web

\( k = F_y / 33 \) \hspace{1cm} (IV.75)

\( m = t / 0.075 \) \hspace{1cm} (IV.76)

\( t = \) Web thickness, inches

\( N = \) Actual length of bearing, inches. For the case of two equal and opposite concentrated loads distributed over unequal bearing lengths, the smaller value of \( N \) shall be taken

\( R = \) Inside bend radius

\( \theta = \) Angle between the plane of the web and the plane of the bearing surface \( \geq 45^\circ \), but not more than \( 90^\circ \)

2. LRFD Criteria. Section C3.4 of the LRFD Specification specifies that to avoid crippling of unreinforced flat webs of flexural
members having a flat width ratio, h/t, equal to or less than 200, concentrated loads and reactions determined according to the factored design loads shall not exceed the values of $\Phi W_n P_n$, with $\Phi_w = 0.75$ and 0.80 for single unreinforced webs and I-sections, respectively, and $P_n$ given in Table C3.4-1 of the LRFD Specification which is obtained from the above listed equations by using a factor of safety of 1.85 for single unreinforced webs and 2.0 for multiple webs. For webs of flexural members for which h/t is greater than 200 shall be provided with adequate means of transmitting concentrated loads and/or reactions directly into the webs.

3. **Comparison.** The unfactored concentrated load or reaction can be calculated for both methods by using Eq. (IV.79):

$$P_T = P_{DL} + P_{LL}$$  \hspace{1cm} (**IV.79**)

where

- $P_T =$ total unfactored load
- $P_{DL} =$ nominal dead load
- $P_{LL} =$ nominal live load

The total unfactored load should be less than or equal to the allowable load based on web crippling. For allowable stress design, the allowable load is $P_a$. For LRFD, the allowable load is computed from Eq. (II.6) and is as follows:

$$\left(P_a\right)_{LRFD} = \Phi_{W_n} \frac{P}{(D/L+1)/(1.2D/L+1.6)}$$  \hspace{1cm} (**IV.80**)
Figure 15. Allowable Load Ratio vs. D/L Ratio for Web Crippling
For shapes with single webs, the allowable load is derived from the ultimate value with a factor of safety of 1.85. For I-sections or similar shapes, the allowable load is derived from the ultimate web crippling load using a factor of safety of 2.0. Therefore, the allowable load ratio are as follows:

For shapes with single webs and \( \phi_w = 0.75 \),

\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.85 \phi_w \frac{D/L+1}{1.2D/L+1.6} = 1.39 \frac{D/L+1}{1.2D/L+1.6}
\]  

(IV.81)

For I-sections or similar shapes and \( \phi_w = 0.80 \),

\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.00 \phi_w \frac{D/L+1}{1.2D/L+1.6} = 1.60 \frac{D/L+1}{1.2D/L+1.6}
\]  

(IV.82)

Figure 15 shows the allowable load ratio versus dead-to-live load ratio for both types of beams based on the comparison of web crippling loads.

For single web beams, LRFD is always conservative as compared with ASD approach for \( D/L < 1.11 \). For I-sections, the ASD approach is always conservative than LRFD. For \( D/L = 0.5 \), the allowable load permitted by the allowable stress design method for I-sections is about 9% lower than that permitted by the LRFD criteria.

F. COMBINED BENDING AND WEB CRIPPLING STRENGTH. The interaction between bending and web crippling is similar to that of combined bending and shear and exists when a large bending moment is applied close to concentrated loads or support reactions. The web crippling capacity
may be reduced according to the following interaction equations provided in the specifications.

1. **Allowable Stress Design.** According to Section C3.5 of the AISI Specification, unreinforced flat webs of shapes subjected to a combination of bending and concentrated load or reaction shall be designed to meet the following requirements:

   (a) For shapes having single unreinforced webs:
   
   $1.2 \left( \frac{P}{P_a} \right) + \left( \frac{M}{M_a} \right) \leq 1.5$  \hspace{1cm} (IV.83)

   Exception: At the interior supports of continuous spans, the above formula is not applicable to deck or beams with two or more single webs, provided the compression edges of adjacent webs are laterally supported in the negative moment region by continuous or intermittently connected flange elements, rigid cladding, or lateral bracing, and the spacing between adjacent webs does not exceed 10 inches.

   (b) For shapes having multiple unreinforced webs such as I-sections made of two channels connected back-to-back, or similar sections which provide a high degree of restraint against rotation of the web (such as I-sections made by welding two angles to a channel);
   
   $1.1 \left( \frac{P}{P_a} \right) + \left( \frac{M}{M_a} \right) \leq 1.5$ \hspace{1cm} (IV.84)

   Exception: When $h/t \leq 2.33/\sqrt{(F_y/E)}$ and $\lambda \leq 0.673$, the allowable concentrated load or reaction may be determined by Section C3.4 of the AISI Specification.

In the above formulas,

$P = \text{Concentrated load or reaction in the presence of bending moment}$

$P_a = \text{Allowable concentrated load or reaction in the absence of}$
bending moment determined in accordance with Section C3.4
of the AISI Specification

\[ M = \text{Applied bending moment at, or immediately adjacent to, the} \]
\[ \text{point of application of the concentrated load or reaction} \]

\[ M_a = \text{Allowable bending moment determined according to Section} \]
\[ C3.1.1 \text{ if bending alone exists excluding lateral buckling} \]

\[ w = \text{Flat width of the beam flange which contacts the bearing} \]
\[ \text{plate} \]

\[ t = \text{Thickness of the web or flange} \]

\[ \lambda = \text{Slenderness factor} \]

2. **LRFD Criteria.** Section C3.5 of the LRFD Specification specifies that unreinforced flat webs of shapes subjected to a combination of bending and concentrated load or reaction shall be designed to meet the following requirements:

(a) For shapes having single unreinforced webs:

\[ 1.07 \left( \frac{P_D}{\phi_P} \right) + \left( \frac{M_D}{\phi_M} \right) \leq 1.42 \quad \text{(IV.85)} \]

Exception: At the interior supports of continuous spans, the above formula is not applicable to deck or beams with two or more single webs, provided the compression edges of adjacent webs are laterally supported in the negative moment region by continuous or intermittently connected flange elements, rigid cladding, or lateral bracing, and the spacing between adjacent webs does not exceed 10 inches.

(b) For shapes having multiple unreinforced webs such as I-sections made of two channels connected back-to-back, or similar sections which provide a high degree of restraint against rotation of the web (such as I-sections made by welding two angles to a channel);
\[ 0.82(P_D/\phi_P) + (M_D/\phi_M) \leq 1.32 \]  

(IV.86)

Exception: When \( h/t \leq 2.33/(F_y/E) \) and \( \lambda \leq 0.673 \), the nominal ultimate concentrated load or reaction may be determined by Section C3.4 of the LRFD Specification.

In the above formulas,

- \( \phi \) = Resistance factor for bending
- \( \phi_w \) = Resistance factor for web crippling
- \( P_D \) = Concentrated load or reaction computed on the basis of factored loads in the presence of bending moment.
- \( P_n \) = Nominal ultimate concentrated load or reaction in the absence of bending moment determined in accordance with Section C3.4 of the LRFD Specification.
- \( M_D \) = Applied bending moment at, or immediately adjacent to, the point of application of the concentrated load or reaction \( P_D \), computed on the basis of factored loads
- \( M_n \) = Nominal ultimate bending moment determined according to Section C3.1.1 of the LRFD Specification if bending alone exists
- \( w \) = Flat width of the beam flange which contacts the bearing plate
- \( t \) = Thickness of the web or flange
- \( \lambda \) = Slenderness factor
3. **Comparison.** A simply supported beam with a concentrated load at midspan was selected as a typical design example. This example has a maximum moment of $PL/4$ at midspan, under the concentrated load. The allowable loads, $P_T$, were calculated for both design methods. Since each design procedure utilizes separate design variable, the allowable loads were determined using nominal resistances.

The allowable load based on allowable stress design was calculated as follows:

\[
\frac{M}{M_a} = \frac{M_{TL}}{0.6M_n} = \frac{P_L/4}{0.6M_n} = \frac{0.4167P_L}{M_n} \tag{IV.87}
\]

For beams with single webs,

\[
\frac{P}{P_a} = \frac{P_T}{P_n/1.85} = \frac{1.85P_T}{P_n} \tag{IV.88}
\]

By substituting Eqs. (IV.87) and (IV.88) into Eq. (IV.83),

\[
\frac{P}{P_a} = \frac{M}{M_a} = \frac{2.22P_T}{P_n} + \frac{0.4167P_L}{M_n} = 1.5
\]

Therefore,

\[
(P_T)_{ASD} = \frac{3.6P_n}{5.328 + (P_L/M_n)} \tag{IV.89}
\]
For I-sections,

\[
\frac{P}{P_a} = \frac{P_T}{P_n/2.00} = \frac{2.00P_T}{P_n} \quad \text{(IV.90)}
\]

By substituting Eqs. (IV.87) and (IV.90) into Eq. (IV.84),

\[
1.1 \frac{P}{P_a} + \frac{M}{M_a} = \frac{2.20P_T}{P_n} + \frac{0.4167P_TL}{M_n} = 1.5
\]

Therefore,

\[
(P_T)_{ASD} = \frac{3.6P_n}{5.280 + (P_nL/M_n)} \quad \text{(IV.91)}
\]

The allowable load based on LRFD criteria was calculated as follows:

\[
\frac{M_D}{\Phi M_n} = \frac{1.2D/L+1.6}{D/L+1} \left( \frac{M_{TL}}{\Phi M_n} \right) = \frac{1.2D/L+1.6}{D/L+1} \left[ \frac{P_TL/4}{\Phi M_n} \right] \quad \text{(IV.92)}
\]

\[
\frac{P_D}{\Phi w P_n} = \frac{1.2D/L+1.6}{D/L+1} \left( \frac{P_T}{\Phi w P_n} \right) \quad \text{(IV.93)}
\]

For beams with single webs, Eqs. (IV.92) and (IV.93) were substituted into Eq. (IV.85) to obtain the following expression:

\[
1.07 \frac{P_D}{\Phi w P_n} + \frac{M_D}{\Phi M_n} = \frac{1.2D/L+1.6}{D/L+1} \left( \frac{1.07P_T}{\Phi w P_n} + \frac{0.25L}{\Phi M_n} \right) = 1.42
\]
Therefore,

\[
(P_T)_{LRFD} = \frac{D/L+1}{1.2D/L+1.6} \left[ \frac{5.680 \phi P_{w_n}}{4.280 + (\phi P_{w_n}/\phi M_n)} \right] \quad (IV.94)
\]

For I-sections, Eqs. (IV.92) and (IV.93) were substituted into Eq. (IV.86) to obtain the following expression:

\[
0.82 \frac{P_D}{\phi P_{w_n}} + \frac{M_D}{\phi M_n} = \frac{1.2D/L+1.6}{D/L+1} (P_T) \left[ \frac{0.82}{\phi P_{w_n}} + \frac{0.25L}{\phi M_n} \right] = 1.32
\]

Therefore,

\[
(P_T)_{LRFD} = \frac{D/L+1}{1.2D/L+1.6} \left[ \frac{5.280 \phi P_{w_n}}{3.280 + (\phi P_{w_n}/\phi M_n)} \right] \quad (IV.95)
\]

The allowable load ratios based on the design examples for combined bending and web crippling are given in Eqs. (IV.96) and (IV.97) for \( \phi = 0.95 \) and 0.90 for nominal section strength of sections with stiffened compression flanges and unstiffened compression flanges, respectively.

For beams with single webs (\( \phi_w = 0.75 \)),

\[
\frac{(P_T)_{LRFD}}{(P_T)_{ASD}} = \frac{D/L+1}{1.2D/L+1.6} \left[ \frac{6.305 + 1.183 (P_n L/M_n)}{4.280 + (0.75/\phi) (P_n L/M_n)} \right] \quad (IV.96)
\]
For I-sections ($\phi_w = 0.80$),

$$\frac{(P_T)_{LRFD}}{(P_T)_{ASD}} = \frac{D/L+1}{1.2D/L+1.6} \left[ 6.195 + 1.173 \left( \frac{P_n L/M_n}{P_n} \right) \right]$$  \hspace{1cm} (IV.97)

Eqs. (IV.96) and (IV.97) can be expressed in the following form:

$$\frac{(P_T)_{LRFD}}{(P_T)_{ASD}} = \frac{D/L+1}{1.2D/L+1.6} (K_w)$$  \hspace{1cm} (IV.98)

where $K_w$ is a variable determined from section properties, material strength, and span length for a particular design example.

Because the interaction combines moment and web crippling, the allowable load ratio is rather complex. It is not only a function of dead-to-live load ratio but is also a function of span length, sectional geometry, and material strength. Several individual beam sections with different conditions were studied due to the complexity involved in the comparison.

Figures 16 and 18 show the relationships between allowable load ratio and dead-to-live load ratio for various channel sections with stiffened flanges using $L = 5$ ft and $F_y = 33$ ksi. Tables IV.10 and IV.12 present section properties and calculated member strengths for several channel sections with stiffened flanges selected from Table 1 of Part V of the AISI Design Manual. In these two figures for $D/L = 0.5$, the allowable web crippling loads determined by LRFD are from 1.1% to 1.5% larger than that permitted by allowable stress design. The channel sections with the smaller h/t ratios resulted in larger values of al-
allowable load ratio. Therefore, with increasing h/t ratio, the difference between the allowable loads obtained from these two design methods decreases.

Figure 17 shows how the span length and yield point of steel affect the allowable load ratio for channels with stiffened flanges. Table IV.11 presents calculated member strengths for different span lengths and yield points. As shown in this figure, larger span lengths will result in slightly higher values of the allowable load ratio. Also from Figure 17, it can be seen that yield point of steel has a negligible effect on the allowable load ratio.

Figure 19 shows the relationship between allowable load ratio and dead-to-live load ratio for 8 in. x 2 in. x 0.105 in. channel with unstiffened flanges with L = 5 ft and Fy = 33 ksi. Table IV.13 presents section properties and calculated member strengths for this channel section. For D/L = 0.5, the allowable web crippling load determined by LRFD is 1.4% lower than that permitted by allowable stress design.

Figure 20 shows how the span length and yield point of steel affect the allowable load ratio for channels with unstiffened flanges. Table IV.14 presents calculated member strengths for different span lengths and yield points. As shown in this figure, larger span lengths will result in slightly lower values of the allowable load ratio. Also from Figure 17, it can be seen that yield point of steel has a negligible effect on the allowable load ratio.

For I-sections made from two channels back-to-back, Figure 21 shows the relationship between allowable load ratio and dead-to-live load ratio. Table IV.15 presents sectional properties and calculated
values for the cold-formed I-section with $F_y = 33$ ksi and $L = 5$ ft. For the I-section with stiffened flanges shown in Figure 21, LRFD would result in an allowable load about 7.1% higher than the load computed from allowable stress design for $D/L = 0.5$.

Figure 22 shows how the span length and yield point of steel affect the allowable load ratio. Table IV.16 presents calculated member strengths for different span lengths and yield points. A higher yield point of steel results in a larger value of the allowable load ratio. As shown in Figure 22, span length has a greater effect on the allowable load ratio for I-sections than it does on channel sections which are shown in Figures 17 and 20. In general, large span lengths result in lower values of the allowable load ratio.

<table>
<thead>
<tr>
<th>Section</th>
<th>h/t</th>
<th>$P_n$ (Kips)</th>
<th>$M_n$ (K-in.)</th>
<th>$\Phi M_n$ (K-in.)</th>
<th>$K_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8x3x0.105</td>
<td>70.62</td>
<td>7.144</td>
<td>124.769</td>
<td>118.531</td>
<td>1.4830</td>
</tr>
<tr>
<td>5x2x0.105</td>
<td>42.05</td>
<td>7.455</td>
<td>49.625</td>
<td>47.144</td>
<td>1.4890</td>
</tr>
</tbody>
</table>

* $F_y = 33$ ksi, $L = 60$ in., $N = 6$ in.
Stiffened Flanges

Channels With

$L = 60$ in.
$N = 6$ in.
$F_y = 33$ ksi
Eq. (IV.96)

Figure 16. Allowable Load Ratio vs. D/L Ratio for Combined Bending and Web Crippling - Case 1
### Table IV.11
5 in. x 2 in. x 0.105 in. Channels With Stiffened Flanges for Various Lengths and Yield Points

<table>
<thead>
<tr>
<th>$F_Y$ (ksi)</th>
<th>$L$ (in.)</th>
<th>$P_n$ (Kips)</th>
<th>$M_n$ (K-in.)</th>
<th>$\Phi M_n$ (K-in.)</th>
<th>$K_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>0</td>
<td>7.455</td>
<td>49.625</td>
<td>47.144</td>
<td>1.4731</td>
</tr>
<tr>
<td>25</td>
<td>7.455</td>
<td>49.625</td>
<td>47.144</td>
<td>1.4835</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>7.455</td>
<td>49.625</td>
<td>47.144</td>
<td>1.4878</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>7.455</td>
<td>49.625</td>
<td>47.144</td>
<td>1.4902</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>7.455</td>
<td>49.625</td>
<td>47.144</td>
<td>1.4917</td>
<td></td>
</tr>
</tbody>
</table>

| 50         | 0         | 10.015        | 75.190        | 71.430            | 1.4731 |
| 25         | 10.015    | 75.190        | 71.430        | 1.4828            |
| 50         | 10.015    | 75.190        | 71.430        | 1.4871            |
| 75         | 10.015    | 75.190        | 71.430        | 1.4896            |
| 100        | 10.015    | 75.190        | 71.430        | 1.4911            |

### Table IV.12
Channels With Stiffened Flanges, 5 in. Depths

<table>
<thead>
<tr>
<th>Section</th>
<th>$h/t$</th>
<th>$P_n$ (Kips)</th>
<th>$M_n$ (K-in.)</th>
<th>$\Phi M_n$ (K-in.)</th>
<th>$K_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5x2x0.075</td>
<td>62.17</td>
<td>4.443</td>
<td>36.917</td>
<td>35.071</td>
<td>1.4876</td>
</tr>
<tr>
<td>0.048</td>
<td>98.26</td>
<td>2.148</td>
<td>21.795</td>
<td>20.705</td>
<td>1.4863</td>
</tr>
</tbody>
</table>

* $F_Y = 33$ ksi, $L = 60$ in., $N = 6$ in.
Figure 17. Allowable Load Ratio vs. Span Length for Combined Bending and Web Crippling - Case 2

5" x 2" x 0.105" Channel
With Stiffened Flanges
D/L = 1/5
N = 6 in.
Eq. (IV.96)
Figure 18. Allowable Load Ratio vs. D/L Ratio for Combined Bending and Web Crippling - Case 2

- 5" x 2" x t Channels With Stiffened Flanges
- L = 60 in.
- N = 6 in.
- $F_y = 33$ ksi
- Eq. (IV.96)
### Table IV.13
Channel With Unstiffened Flanges

<table>
<thead>
<tr>
<th>Section</th>
<th>h/t</th>
<th>( P_n ) (Kips)</th>
<th>( M_n ) (K-in.)</th>
<th>( \phi M_n ) (K-in.)</th>
<th>( K_w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8x2x0.105</td>
<td>70.62</td>
<td>7.144</td>
<td>80.189</td>
<td>72.170</td>
<td>1.4458</td>
</tr>
</tbody>
</table>

* \( F_y = 33 \text{ ksi}, L = 60 \text{ in.}, N = 6 \text{ in.} \)

### Table IV.14
8 in. x 2 in. x 0.105 in. Channels With Unstiffened Flanges for Various Lengths and Yield Points

<table>
<thead>
<tr>
<th>( F_y ) (ksi)</th>
<th>( L ) (in.)</th>
<th>( P_n ) (Kips)</th>
<th>( M_n ) (K-in.)</th>
<th>( \phi M_n ) (K-in.)</th>
<th>( K_w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>0</td>
<td>7.144</td>
<td>80.189</td>
<td>72.170</td>
<td>1.4731</td>
</tr>
<tr>
<td>25</td>
<td>7.144</td>
<td>80.189</td>
<td>72.170</td>
<td>1.4569</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>7.144</td>
<td>80.189</td>
<td>72.170</td>
<td>1.4483</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>7.144</td>
<td>80.189</td>
<td>72.170</td>
<td>1.4429</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>7.144</td>
<td>80.189</td>
<td>72.170</td>
<td>1.4392</td>
<td></td>
</tr>
</tbody>
</table>

| 50             | 0            | 9.597            | 115.222           | 103.700              | 1.4731  |
| 25             | 9.597        | 115.222          | 103.700           | 1.4577               |
| 50             | 9.597        | 115.222          | 103.700           | 1.4492               |
| 75             | 9.597        | 115.222          | 103.700           | 1.4438               |
| 100            | 9.597        | 115.222          | 103.700           | 1.4400               |
Figure 19. Allowable Load Ratio vs. D/L Ratio for Combined Bending and Web Crippling - Case 3

8" x 2" x 0.105" Channel
With Unstiffened Flanges
L = 60 in.
N = 6 in.
F_y = 33 ksi
Eq. (IV.96)
Figure 20. Allowable Load Ratio vs. Span Length for Combined Bending and Web Crippling - Case 3

8" x 2" x 0.105" Channel
With Unstiffened Flanges
D/L = 1/5
N = 6 in.
Eq. (IV.96)
### Table IV.15
I-Section With Stiffened Flanges

<table>
<thead>
<tr>
<th>Section</th>
<th>h/t</th>
<th>( P_n ) (Kips)</th>
<th>( M_n ) (K-in.)</th>
<th>( \Phi M_n ) (K-in.)</th>
<th>( K_w )</th>
</tr>
</thead>
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<tr>
<td>8x6x0.105</td>
<td>70.62</td>
<td>28.966</td>
<td>249.538</td>
<td>237.061</td>
<td>1.5708</td>
</tr>
</tbody>
</table>

* \( F_y = 33 \text{ ksi} \), \( L = 60 \text{ in.} \), \( N = 6 \text{ in.} \)

### Table IV.16
8 in. x 6 in. x 0.105 in. I-Sections With Stiffened Flanges
for Various Lengths and Yield Points

<table>
<thead>
<tr>
<th>( F_y ) (ksi)</th>
<th>( L ) (in.)</th>
<th>( P_n ) (Kips)</th>
<th>( M_n ) (K-in.)</th>
<th>( \Phi M_n ) (K-in.)</th>
<th>( K_w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>0</td>
<td>28.966</td>
<td>249.538</td>
<td>237.061</td>
<td>1.8887</td>
</tr>
<tr>
<td>25</td>
<td>28.966</td>
<td>249.538</td>
<td>237.061</td>
<td>1.6770</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>28.966</td>
<td>249.538</td>
<td>237.061</td>
<td>1.5920</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>28.966</td>
<td>249.538</td>
<td>237.061</td>
<td>1.5462</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>28.966</td>
<td>249.538</td>
<td>237.061</td>
<td>1.5175</td>
<td></td>
</tr>
</tbody>
</table>

| 50             | 31.406        | 361.223          | 343.162          | 1.8887           |
| 25             | 31.406        | 361.223          | 343.162          | 1.7111           |
| 50             | 31.406        | 361.223          | 343.162          | 1.6272           |
| 75             | 31.406        | 361.223          | 343.162          | 1.5873           |
| 100            | 31.406        | 361.223          | 343.162          | 1.5463           |
Figure 21. Allowable Load Ratio vs. D/L Ratio for Combined Bending and Web Crippling - Case 4
$8'' \times 6'' \times 0.105''$ I-Section

With Stiffened Flanges

$D/L = 1/5$

$N = 6$ in.

Eq. (IV.97)

---

Figure 22. Allowable Load Ratio vs. Span Length for Combined Bending and Web Crippling - Case 4
V. CONCENTRICALLY LOADED COMPRESSION MEMBERS

A. GENERAL

Cold-formed steel concentrically loaded compression members have three possible modes of failure. Short and compact columns will fail by yielding. Local buckling of an individual element could occur if the flat-width to thickness ratio is large. Overall column buckling of intermediate and long columns could occur in one of three buckling modes: flexural buckling, torsional buckling, and torsional-flexural buckling.

B. ALLOWABLE STRESS DESIGN

Section C4 of the 1986 AISI Specification contains the following requirements for compression members in which the resultant of all loads acting on the member is an axial load passing through the centroid of the effective section calculated at the stress, $F_n$, defined in that section.

(a) The axial load shall not exceed $P_a$ calculated as follows:

$$P_a = \frac{P_n}{\Omega_c} \quad (V.1)$$

where

$$P_n = \frac{A}{e_n} \quad (V.2)$$

$A_{e}$ = Effective area at the stress $F_n$.

$F_n$ is determined as follows:

For $F_e > F_y/2$  \quad $F_n = F_y(1-F_y/4F_e)$ \quad (V.3)

For $F_e \leq F_y/2$ \quad $F_n = F_e$ \quad (V.4)
$F_e$ is the least of the elastic flexural, torsional and torsional-flexural buckling stress.

$\Omega_c =$ Factor of safety for axial compression

$= 1.92$, except when $F_e$ is determined according to Section C4.1 of the AISI Specification for fully effective sections having wall thickness greater than 0.09 inches and $F_e > F_y/2$. In this case,

$\Omega_c = \frac{5}{3} + \frac{3}{8}R - \frac{1}{8}R^3$

where

$R = \sqrt{\frac{F_y}{2F_e}}$

(b) For C and Z-shapes, and single-angle sections with unstiffened flanges, $P_n$ shall be taken as the smaller of $P_n$ calculated above and $P_n$ calculated as follows:

$P_n = \frac{An^2E}{[25.7(w/t)^2]}$ \hspace{1cm} (V.5)

where

$A =$ Area of the full, unreduced cross section

$w =$ Flat width of the unstiffened element

$t =$ Thickness of the unstiffened element

(c) Angle sections shall be designed for the applied axial load, $P$, acting simultaneously with a moment equal to $PL/1000$ applied about the minor principal axis causing compression in the tips of the angle legs.

(d) The slenderness ratio, $KL/r$, of all compression members preferably should not exceed 200, except that during construction only, $KL/r$ preferably should not exceed 300.
For doubly-symmetric sections, closed cross sections and any other sections which can be shown not to be subject to torsional or torsional-flexural buckling, the elastic flexural buckling stress, $F_e$, shall be determined as follows:

$$F_e = \frac{n^2E}{(KL/r)^2}$$  \hspace{1cm} (V.6)

where

$E$ = Modulus of elasticity

$K$ = Effective length factor

$L$ = Unbraced length of member

$r$ = Radius of gyration of the full, unreduced section

For sections subject to torsional or torsional-flexural buckling, $F_e$ shall be taken as the smaller of $F_e$ calculated above and $F_e$ calculated as follows:

$$F_e = \frac{((\sigma_{ex} + \sigma_t) - \sqrt{(\sigma_{ex} + \sigma_t)^2 - 4\beta \sigma_{ex} \sigma_t})}{(2\beta)}$$  \hspace{1cm} (V.7)

where

$\sigma_t$ and $\sigma_{ex}$ are as defined in Article IV of this report

$\beta = 1-(x_0/r_0)^2$  \hspace{1cm} (V.8)

Alternatively, a conservative estimate of $F_e$ can be obtained using the following equation:

$$F_e = \frac{\sigma_t \sigma_{ex}}{\sigma_t + \sigma_{ex}}$$  \hspace{1cm} (V.9)

For singly-symmetric sections, the $x$-axis is assumed to be the axis of symmetry.

For shapes whose cross sections do not have any symmetry, either about an axis or about a point, $F_e$ shall be determined by rational analysis.
C. LRFD CRITERIA

Based on Section C4 of the LRFD Specification, for members in which the resultant of all loads acting on the member is an axial load passing through the centroid of the effective section calculated at the stress, $F_n$, the factored axial strength, $\phi_c P_n$, shall be determined with $\phi_c = 0.85$ and the nominal axial strength, $P_n$, calculated exactly the same as that specified in Section C4 of the AISI Specification.

D. COMPARISON

The unfactored load applied to the member can be computed for both design methods by using the following formula:

$$P_T = P_{DL} + P_{LL}$$  \hspace{1cm} (V.10)

where

- $P_T$ = unfactored compressive load
- $P_{DL}$ = compressive load due to the nominal axial dead load
- $P_{LL}$ = compressive load due to the nominal axial live load

The total unfactored load should be less than or equal to the allowable loads computed from allowable stress design and LRFD. For allowable stress design, the allowable load is

$$\left(P_a\right)_{ASD} = \frac{P_n}{\Omega_c}$$  \hspace{1cm} (V.11)

For LRFD, the allowable axial load can be computed by using the following equation developed from Eq. (II.6):

$$\left(P_a\right)_{LRFD} = \phi_c P_n \frac{(D/L+1)}{(1.2D/L+1.6)}$$  \hspace{1cm} (V.12)
Then, the allowable load ratio can be determined as follow:

\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = \frac{\phi_c P_n}{P_n/\Omega_c} \left[ \frac{D/L+1}{1.2D/L+1.6} \right] = 0.85 \Omega_c \frac{D/L+1}{1.2D/L+1.6}
\]  

(V.13)

For fully effective sections having wall thickness greater than 0.09 in. and \( F_e > F_y/2 \),

\[ \Omega_c = \frac{5}{3} + (3/8)R - (1/8)R^2 \]

Therefore, the allowable load ratio is

\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 0.85 \left( \frac{5}{3} + \frac{3}{8}R - \frac{1}{8}R^2 \right) \frac{D/L+1}{1.2D/L+1.6}
\]

(V.14)

For all other cases, \( \Omega_c = 1.92 = 23/12 \), therefore the allowable load ratio is

\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 0.85(23/12) \frac{D/L+1}{1.2D/L+1.6} = 1.629 \frac{D/L+1}{1.2D/L+1.6}
\]

(V.15)

Figure 23 shows the allowable load ratio versus dead-to-live load ratio for the columns used to develop Eq. (V.15). For this case, the LRFD criteria always permit larger allowable loads than the allowable stress design. For \( D/L = 0.5 \), the LRFD criteria gives an allowable load about 11% greater than the load obtained by using allowable stress design.
Figure 23. Allowable Load Ratio vs. D/L Ratio for Column Buckling
Figure 24. Allowable Load Ratio vs. Slenderness Ratio for Flexural Buckling of Columns

- Fully Effective Section
- $t > 0.09$ in.
- $F_e > F_y/2$
- Eq. (V.14)

$F_y = 50$ ksi
$F_y = 33$ ksi

$D/L = 1/5$
The allowable load ratio versus slenderness ratio, $KL/r$, for columns having fully effective sections, $t \geq 0.09$ in., and $F_e > F_y/2$ is shown in Figure 24. For this case, the LRFD criteria were found to be conservative for short columns as compared with allowable stress design. As shown in Figure 24, higher yield point materials give slightly higher values of the allowable load ratio.
VI. BEAM-COLUMNS

A. GENERAL

Beam-columns are structural members subjected to combined axial load and bending. The structural behavior of beam-columns depends on the shape and dimensions of the cross section, the location of the applied eccentric load, column length, and condition of bracing. Interaction formulas are used to analyze beam-columns for flexural and torsional-flexural buckling.

B. ALLOWABLE STRESS DESIGN

In the 1986 edition of AISI Specification, the design criteria for combined axial load and bending are stated in Section C5 as follows:

The axial force and bending moments shall satisfy the following interaction equations:

\[
\frac{P}{P_a} + \frac{C_{M_x}}{M_{ax}} + \frac{C_{M_y}}{M_{ay}} \leq 1.0 \quad (VI.1)
\]

\[
\frac{P}{P_{ao}} + \frac{M_{ax}}{M_{x}} + \frac{M_{ay}}{M_{y}} \leq 1.0 \quad (VI.2)
\]

When \( \frac{P}{P_a} \leq 0.15 \), the following formula may be used in lieu of the above two formulas:

\[
\frac{P}{P_a} + \frac{M_{ax}}{M_{x}} + \frac{M_{ay}}{M_{y}} \leq 1.0 \quad (VI.3)
\]
where

\[ P = \text{Applied axial load} \]

\[ M_x \text{ and } M_y = \text{Applied moments with respect to the centroidal axes of the effective section determined for the axial load alone. For angle sections, } M_y \text{ shall be taken either as the applied moment or the applied moment plus } PL/1000, \text{ whichever results in a lower value of } P_a \]

\[ P_a = \text{Allowable axial load} \]

\[ P_{ao} = \text{Allowable axial load determined with } F_n = F_y \]

\[ M_{ax} \text{ and } M_{ay} = \text{Allowable moments about the centroidal axes} \]

\[ M_{axo} \text{ and } M_{ayo} = \text{Allowable moments about the centroidal axes excluding lateral buckling} \]

\[ 1/\alpha_x, 1/\alpha_y = \text{Magnification factors} \]

\[ = 1/[1-(\Omega_c P/P_{cr})] \quad (VI.4) \]

\[ \Omega_c = \text{Factor of safety used in determining } P_a \]

\[ P_{cr} = n^2EI_b/(K_b L_b)^2 \quad (VI.5) \]

\[ I_b = \text{Moment of inertia of the full, unreduced cross section about the axis of bending} \]

\[ L_b = \text{Actual unbraced length in the plane of bending} \]

\[ K_b = \text{Effective length factor in the plane of bending} \]

\[ C_{mx}, C_{my} = \text{Coefficients whose value shall be taken as follows:} \]

1. For compression members in frames subject to joint translation (sidesway)

\[ C_m = 0.85 \]

2. For restrained compression members in frames
braced against joint translation and not subject to transverse loading between their supports in the plane of bending

\[ C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) \]  \hspace{1cm} (VI.6)

where

\( M_1/M_2 \) is the ratio of the smaller to the larger moment at the ends of that portion of the member under consideration which is unbraced in the plane of bending. \( M_1/M_2 \) is positive when the member is bent in reverse curvature and negative when it is bent in single curvature.

3. For compression members in frames braced against joint translation in the plane of loading and subject to transverse loading between their supports, the value of \( C_m \) may be determined by rational analysis. However, in lieu of such analysis, the following values may be used:

(a) for members whose ends are restrained, \( C_m = 0.85 \),

(b) for members whose ends are unrestrained, \( C_m = 1.0 \).

C. LRFD CRITERIA

According to Section C5 of the LRFD Specification, the design values \( P_D \), \( M_{DX} \) and \( M_{DY} \) computed on the basis of factored loads, shall satisfy the following interaction equations:
When \( P_D/\Phi P_n \leq 0.15 \), the following formula may be used in lieu of the above two formulas:

\[
P_D/\Phi P_n + M_{DL}/\Phi M_{N} \leq 1.0
\]

where

\[ P_D \] = Factored design axial load

\[ M_{DL} \text{ and } M_{DY} \] = Factored design moments with respect to the centroidal axes of the effective section determined for the factored design axial load alone. For angle sections, \( M_{DY} \) shall be taken either as the factored moment or the factored moment plus \( P_D L/1000 \), whichever result in a lower value of \( P_n \).

\[ P_n \] = Nominal axial strength

\[ P_{no} \] = Nominal axial strength determined with \( F_n = F_y \)

\[ M_{nx} \text{ and } M_{ny} \] = Nominal beam strengths about the centroidal axes

\[ M_{nxo} \text{ and } M_{nyo} \] = Nominal beam strengths about the centroidal axes, excluding lateral buckling

\[ 1/\alpha_{nx}, 1/\alpha_{ny} \] = Magnification factors

\[ = 1/(1-P_D/\Phi P_{cE}) \] \hspace{1cm} (VI.10)

\[ \Phi \] = 0.95 or 0.90 for bending strength (Section C3.1.1)

or 0.90 for laterally unbraced beam (Section C3.1.2)
\[ \phi_c = 0.85 \]

\[ P_E = \frac{n^2EI_b}{(K_bL_b)^2} \]  \hspace{1cm} (VI.11)

\[ I_b = \text{Moment of inertia of the full, unreduced cross section about the axis of bending} \]

\[ L_b = \text{Actual unbraced length in the plane of bending} \]

\[ K_b = \text{Effective length factor in the plane of bending} \]

\[ C_{mx}, C_{my} = \text{Coefficients whose value shall be taken as follows:} \]

1. For compression members in frames subject to joint translation (sidesway)
   \[ C_m = 0.85 \]

2. For restrained compression members in frames braced against joint translation and not subject to transverse loading between their supports in the plane of bending
   \[ C_m = 0.6-0.4(M_1/M_2) \]  \hspace{1cm} (VI.12)

   where

   \[ M_1/M_2 \] is the ratio of the smaller to the larger moment at the ends of that portion of the member under consideration which is unbraced in the plane of bending. \( M_1/M_2 \) is positive when the member is bent in reverse curvature and negative when it is bent in single curvature.

3. For compression members in frames braced against joint translation in the plane of loading and subject to transverse loading between their supports, the value of \( C_m \) may be determined by
rational analysis. However, in lieu of such analysis, the following values may be used:

(a) for members whose ends are restrained,
\[ C_m = 0.85, \]
(b) for members whose ends are unrestrained,
\[ C_m = 1.0. \]

D. COMPARISON

Because of the complexity of the interaction formulas, the comparison was studied by using two different kinds of sections, namely, doubly-symmetric sections and singly-symmetric sections.

1. Doubly-Symmetric Sections. I-sections bending about the \( x \)-axis were considered. A typical design example was selected and the allowable axial loads were calculated by using three interaction equations for each design method. The example used a beam-column with equal moments applied to each end so that the member is bent in single curvature. Since the end moments are independent of the axial load, the ratio of the unfactored applied moment to the nominal moment capacity based on section strength, \( M_a/M_{no} \), was considered to be a parameter in the equations for determining the allowable loads.

For allowable stress design the allowable axial loads were computed as follows:

\[
\frac{P}{P_a} = c \frac{P_T}{P_n} = \frac{\Omega P_T}{P_n} \quad (VI.13)
\]
\[ \frac{M}{M_a} = \frac{M_T}{0.6M_n} = \frac{(M_T/M_{no})(M_{no}/M_n)}{0.6} \]  

(VI.14)

where

\(P_T\) = applied unfactored axial load

\(M_T\) = applied unfactored bending moment at each end of the member

\(\Omega_c\) = factor of safety of axially loaded compression members which is defined in Article V

Substitution of Eqs. (VI.13) and (VI.14) into Eq. (VI.1) results in the following expression:

\[ \frac{\Omega_c P_T}{P_n} + \frac{C_m(M_T/M_{no})(M_{no}/M_n)}{0.6(1-\Omega_c P_T/P_{cr})} = 1.0 \]  

(VI.15)

By solving for \(P_T\) in the first term of Eq. (VI.15), the following equation for allowable load is obtained:

\[ (P_T)_{ASD1} = \left[1 - \frac{C_m(M_T/M_{no})(M_{no}/M_n)}{0.6(1-\Omega_c P_T/P_{cr})}\right] \frac{P_n}{\Omega_c} \]  

(VI.16)

Equation (VI.16) is based on Eq. (VI.1) for failure at the midlength of the beam-column and requires a solution by iterations.

The following expressions were used to solve for the allowable load based on Eq. (VI.2):
Substitution of Eqs. (VI.17) and (VI.18) into Eq. (VI.2) results in the following expression:

\[
\frac{\Omega_c P_T}{P_{no}} + \frac{(M_T/M_{no})}{0.6} = 1.0
\]  

(VI.19)

By solving for \( P_T \) in Eq. (VI.19), the following equation for allowable load is obtained:

\[
(P_T)_{ASD} = \left[1 - \frac{(M_T/M_{no})}{0.6}\right] \frac{P_{no}}{\Omega_c}
\]  

(VI.20)

Equation (VI.20) is based on Eq. (VI.2) for failure at the braced points.

When \( P/P_a \leq 0.15 \), Eq. (VI.3) can be used in lieu of Eqs. (VI.1) and (VI.2). Equation (VI.3) can be written in the following form by using Eqs. (VI.13) and (VI.14):

\[
\frac{\Omega_c P_T}{P_n} + \frac{(M_T/M_{no})(M_{no}/M_n)}{0.6} = 1.0
\]  

(VI.21)
By solving for $P_T$ in Eq. (VI.21), the following equation for allowable load is obtained:

$$ (P_T)_{ASD3} = \left[ 1 - \frac{(M_T/M_{no})(M_{no}/M_n)}{0.6} \right] \frac{P_n}{\Omega_c} \tag{VI.22} $$

Equation (VI.22) is based on Eq. (VI.3) for flexural failure when the effect of the secondary moment is neglected.

For LRFD, the allowable axial loads were computed in accordance with Eq. (11.6) as follows:

$$ \frac{P_D}{\phi P_n} = \frac{1.2D/L+1.6}{D/L+1} \left[ \frac{P_T}{\phi P_n} \right] \tag{VI.23} $$

$$ \frac{M_D}{\phi M_n} = \frac{1.2D/L+1.6}{D/L+1} \left[ \frac{(M_T/M_{no})(M_{no}/M_n)}{\phi} \right] \tag{VI.24} $$

$$ \frac{P_D}{\phi P_E} = \frac{1.2D/L+1.6}{D/L+1} \left[ \frac{P_T}{\phi P_E} \right] \tag{VI.25} $$

Substitution of Eqs. (VI.23), (VI.24), and (VI.25) into Eq. (VI.7) results in the following expression:

$$ \frac{1.2D/L+1.6}{D/L+1} \left\{ \frac{P_T}{\phi P_n} + \frac{C_m(M_T/M_{no})(M_{no}/M_n)}{\phi(1-(1.2D/L+1.6)P_T/(D/L+1)\phi P_E)} \right\} = 1.0 \tag{VI.26} $$

By solving for $P_T$ in the first term of Eq. (VI.26), the following equation for allowable load is obtained:
Equation (VI.27) is based on Eq. (VI.7) for flexural failure at the midlength of the beam-column and requires a solution by iterations.

The following expressions were used to solve for the allowable load based on Eq. (VI.8):

\[
\frac{P_D}{\phi_{c_p_{no}}} = \frac{1.2D/L+1.6}{D/L+1} \left[ \frac{P_T}{\phi_{c_p_{no}}} \right] \tag{VI.28}
\]

\[
\frac{M_D}{\phi_{M_{no}}} = \frac{1.2D/L+1.6}{D/L+1} \left[ \frac{(M_T/M_{no})}{\phi} \right] \tag{VI.29}
\]

Substitution of Eqs. (VI.28) and (VI.29) into Eq. (VI.8) results in the following expression:

\[
\frac{1.2D/L+1.6}{D/L+1} \left[ \frac{P_T}{\phi_{c_p_{no}}} + \frac{(M_T/M_{no})}{\phi} \right] = 1.0 \tag{VI.30}
\]

By solving for \(P_T\) in Eq. (VI.30), the following equation for allowable load is obtained:

\[
(P_T)_{LRFD2} = \left[ \frac{D/L+1}{1.2D/L+1.6} - \frac{(M_T/M_{no})}{\phi} \right] \phi_{c_p_{no}} \tag{VI.31}
\]
Equation (VI.31) is based on Eq. (VI.8) for failure at the braced points.

When \( \frac{P_D}{(\Phi_c P_n)} \leq 0.15 \), Eq. (VI.9) can be used in lieu of Eqs. (VI.7) and (VI.8). Equation (VI.9) can be written in the following form by using Eqs. (VI.23) and (VI.24):

\[
\frac{1.2D/L+1.6}{D/L+1} \left[ \frac{P_T}{\Phi_c P_n} + \frac{(M_T/M_{no})(M_{no}/M_n)}{\Phi} \right] = 1.0 \quad \text{(VI.32)}
\]

By solving for \( P_T \) in Eq. (VI.32), the following equation for allowable load is obtained:

\[
(P_T)_{LRFD3} = \left[ \frac{D/L+1}{1.2D/L+1.6} - \frac{(M_T/M_{no})(M_{no}/M_n)}{\Phi} \right] \Phi_c P_n \quad \text{(VI.33)}
\]

Equation (VI.33) is based on (VI.9) for flexural failure when the effect of the secondary moment is neglected.

Equations (VI.16), (VI.20), and (VI.22) for determining the allowable axial load based on allowable stress design and Eqs. (VI.27), (VI.31), and (VI.33) for determining the allowable axial load based on LRFD are very complex and utilize iterations with multiple variables. The allowable load ratios, \( \frac{(P_T)_{LRFD}}{(P_T)_{ASD}} \), for various lengths combined with different applied end moment ratios, \( M_T/M_{no} \), with respect to the beam strength of the member were studied. Typical I-sections and their section properties used in this study were obtained from Tables 5 and 6 of Part V of the AISI Cold-Formed Steel Design Manual.
An I-section (3.5 in. x 4 in. x 0.105 in.) with stiffened flanges was studied with a yield point of 33 ksi. Figure 25 shows the allowable load ratio versus dead-to-live load ratio for a 4 ft length with various end moment ratios, $M_T/M_{no}$. This figure is based on Eqs. (VI.16) and (VI.27) for flexural failure at the midlength of the beam-column. For a D/L ratio around 0.35, the LRFD criteria gives an allowable load about 9% more than the value computed from allowable stress design for all end moment ratios indicated in the figure. For other values of the D/L ratio, the difference between the allowable loads computed by using these two methods depends on the end moment ratio as shown in Figure 25. For D/L > 0.35, the larger the end moment ratio, the higher the allowable load ratio. For example, for D/L = 0.5, the $(P_T^{LRFD}/P_T^{ASD})$ ratios are 1.137 and 1.117 for $M_T/M_{no} = 0.3$ and 0.1, respectively.

Figure 26 shows the allowable load ratio based on Eqs. (VI.20) and (VI.31) versus dead-to-live load ratio for the same I-section used in Figure 25. Figure 26 is based on failure at the braced points which corresponds to Eqs. (VI.20) and (VI.31). For D/L = 0.05, the LRFD criteria give an allowable load about 3% more than the value computed from allowable stress design for all end moment ratios shown in the figure. It also can be seen from this figure that LRFD design will always give a larger allowable load than allowable stress design.
3.5" x 4" x 0.105" I-Section
With Stiffened Flanges

$F_y = 33$ ksi
$L = 48$ in.

Eq. (VI.27)/Eq. (VI.16)

Figure 25. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case A
Figure 26. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case B

3.5" x 4" x 0.105" I-Section
With Stiffened Flanges

$F_y = 33$ ksi
$L = 48$ in.

Eq. (VI.31)/Eq. (VI.20)
Figures 27 and 28 show the relationships between allowable load ratio and dead-to-live load ratio for end moment ratios of 0.2 and 0.3, respectively. The different curves in each figure represent different lengths of the 3.5 in. x 4 in. x 0.105 in. I-section. With end moment ratio of 0.2 and D/L = 0.5, ASD would provide conservative values up to 12.9% for column lengths equal to 4 ft, 7 ft, and 9 ft as compared with the LRFD method. For the same column lengths and an end moment ratio of 0.3, ASD would be conservative (13.7% to 14.8%) as compared with the LRFD method for D/L = 0.5.

The relationships between the allowable load ratio and column length are shown in Figures 27 and 28 for various D/L ratios. Figures 29 and 30 show the allowable load ratio versus slenderness ratio, KL/r_y, for end moment ratios of 0.2 and 0.3, respectively. Each curve in the figure represents a different D/L ratio for the same I-section used in Figures 25 through 28. As shown in these two figures, the allowable load ratio increases with increasing slenderness ratios for large D/L ratios. For small D/L ratios, the slenderness ratio has small effect on the allowable load ratio. These two figures also show that for all three D/L ratios, the LRFD method would permit a larger load than the ASD method.
Figure 27. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case C

3.5" x 4" x 0.105" I-Section
With Stiffened Flanges

\( F_y = 33 \) ksi

\( \frac{M}{M_{\text{no}}} \geq 0.2 \)

Eq. (VI.27)/Eq. (VI.16)
3.5" x 4" x 0.105" I-Section
With Stiffened Flanges

$F_y = 33$ ksi

$M_{T}/M_{no} = 0.3$

Eq. (VI.27)/Eq. (VI.16)

Figure 28. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case D
Figure 29. Allowable Load Ratio vs. Slenderness Ratio for Beam-Columns - Case C
Figure 30. Allowable Load Ratio vs. Slenderness Ratio for Beam-Columns - Case D
A deeper I-section (6 in. x 5 in. x 0.105 in.) with stiffened flanges was also studied for a length of 5 ft. Figure 31 shows the allowable load ratio based on Eqs. (VI.16) and (VI.27) versus dead-to-live load ratio for various end moment ratios. This figure is also based on flexural failure at the midlength of the beam-column which governs the design for this case. The curves without star symbols are for $C_m = 1.0$. They are the same as those shown in Figure 25 for the 4 in. deep I-section. For this case, the yield point of steel would not affect the allowable load ratio. For $D/L = 0.5$ and $M_T/M_{no} = 0.1$, the allowable load computed from LRFD is 11.6% greater than the value determined from allowable stress design. However, for $D/L = 0.5$ and $M_T/M_{no} = 0.3$, the allowable load computed from LRFD is 13.6% higher than the value computed from allowable stress design.

The curves with star symbols in Figure 31 are for the same I-section except that the coefficient, $C_m$, is 0.85. The value of 0.85 is used for unbraced beam-columns and beam-columns with restrained ends subject to transverse loading between its supports. For small end moment ratios, the $C_m$ value has a negligible effect on the allowable load ratio. The effect of $C_m$ on the allowable load ratio increases as the end moment ratio increases as shown in Figure 31. It can be seen that for $D/L < 1/3$, the allowable load ratios computed for $C_m = 0.85$ are larger than those for $C_m = 1.0$.

Figure 32 shows the relationship between allowable load ratio and dead-to-live load ratio for the 6 in. deep I-section used in Figure 31 with a consideration of flexural failure at the braced points. This figure is the same as Figure 26 for the 4 in. deep I-section. The curves
shown in Figure 32 are applicable for yield points ranging from 33 to 50 ksi and all values of $C_m$.

I-sections with unstiffened flanges were studied in a similar manner. Figure 33 shows the allowable load ratio versus dead-to-live load ratio for an I-section (4 in. x 2.25 in. x 0.105 in.) having unstiffened flanges with $F_y = 33$ ksi and an effective column length of 4 ft. This figure is based on flexural failure at the midlength of the beam-column which would govern the design in this case. The allowable load ratio was determined from Eqs. (VI.16) and (VI.27). Figure 33 is similar to Figure 25 prepared for an I-section with stiffened flanges. For $D/L = 0.5$ and $M_L/M_{no} = 0.1$, the allowable load obtained from LRFD is 11.7% larger than the value obtained from allowable stress design. For $D/L = 0.5$ and $M_L/M_{no} = 0.3$, LRFD would result in an allowable load 14.6% higher than the value determined from allowable stress design.

Figure 34 shows the relationship between allowable load ratio and dead-to-live load ratio for the same I-section used in Figure 33 by considering flexural failure at the braced points. Equations (VI.20) and (VI.31) are used for this type of failure. For $D/L = 0.5$, the allowable loads obtained from LRFD are from 11.6% to 13.6% greater than the allowable loads determined from allowable stress design for end moment ratios from 0.1 to 0.3.
1. 25

Eq. (VI. 27)/Eq. (VI. 16)

With Stiffened Flanges

\[ F = 33-50 \text{ ksi} \]

\[ y = 60 \text{ in.} \]

\[ 6" \times 5" \times 0.105" \text{ I-Section} \]

\[ C = 0.85 \]

\[ C_\text{m} = 1.00 \]

\[ C_\text{m} = 0.85 \]

\[ M_{T}/M_\text{no} = 0.3 \]

\[ M_{T}/M_\text{no} = 0.2 \]

\[ M_{T}/M_\text{no} = 0.1 \]

6" x 5" x 0.105" I-Section

With Stiffened Flanges

\[ F_y = 33-50 \text{ ksi} \]

\[ L = 60 \text{ in.} \]

Eq. (VI.27)/Eq. (VI.16)

Figure 31. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case E
Figure 32. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case F

6" x 5" x 0.105" I-Section

With Stiffened Flanges

F_y = 33-50 ksi
L = 60 in.

Eq. (VI.31)/Eq. (VI.20)
Figure 33. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case G

4" x 2.25" x 0.105" I-Section
With Unstiffened Flanges

$F_y = 33$ ksi
$L = 48$ in.

Eq. (VI.27)/Eq. (VI.16)
With Unstiffened Flanges

4" x 2.25" x 0.105" I-Section

$F_y = 33$ ksi

$L = 48$ in.

Eq. (VI.31)/Eq. (VI.20)

Figure 34. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case H
Figures 35 and 36 show the allowable load ratio versus dead-to-live load ratio for end moment ratio of 0.1 and 0.2, respectively. Different curves represent different lengths of the I-section (4 in. x 2.25 in. x 0.105 in.) with $F_y = 33$ ksi. These two figures are similar to Figures 27 and 28 which were prepared for I-section with stiffened flanges. For the values of $\frac{M_T}{M_{no}}$ between 0.1 and 0.2 and $D/L = 0.5$, the allowable load values obtained from LRFD vary from 11.7% to 15.3% larger than the values obtained from the allowable stress design method.

Figure 37 shows the relationship between allowable load ratio and slenderness ratio, $KL/r_y$, for the same I-section used in previous figures and for an end moment ratio of 0.1. Each curve in the figure represents a different $D/L$ ratio. The relationship in Figure 37 is similar to the relationship indicated in Figures 29 and 30 which are used in the study of I-sections with stiffened flanges. As shown in this figure, the allowable load ratio increases with increasing slenderness ratio for large $D/L$ ratios. For small $D/L$ ratios, the slenderness ratio has small effect on the allowable load ratio. This figure also shows that for all three $D/L$ ratios, the LRFD method would permit a larger load than the ASD method.
With Unstiffened Flanges

4" x 2.25" x 0.105" I-Section

1. 30 1
1.25
0.95

$F_y = 33$ ksi

$M_T/M_{no} = 0.1$

Eq. (VI.27)/Eq. (VI.16)

Figure 35. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case I
With Unstiffened Flanges

$F = 33 \text{ ksi}$

$y / M_{n_0} = 0.2$

Eq. (VI.27)/Eq. (VI.16)

4" x 2.25" x 0.105" I-Section

With Unstiffened Flanges

$F_y = 33 \text{ ksi}$

$M_T / M_{n_0} = 0.2$

Eq. (VI.27)/Eq. (VI.16)

Figure 36. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case J
4" x 2.25" x 0.105" I-Section
With Unstiffened Flanges

$F_y = 33$ ksi

$M_t/M_{no} = 0.1$

Eq. (VI.27)/Eq. (VI.16)

Figure 37. Allowable Load Ratio vs. Slenderness Ratio for Beam-columns - Case I
A deeper I-section (6 in. x 3 in. x 0.105 in.) with unstiffened flanges was also included in this study for a length of 5 ft. The relationship between allowable load ratio and dead-to-live load ratio for the I-section is shown in Figure 38 for various end moment ratios. This figure is based on flexural failure at the midlength of the member. The curves computed for $F_y = 33$ ksi are similar to the curves shown in Figure 31 obtained for an I-section with stiffened flanges. For $D/L = 0.5$, the allowable load ratios vary from 1.12 to 1.14 for $M_T/M_{no}$ ratios ranging from 0.1 to 0.3. The lines with star symbols in Figure 38 represent the allowable load ratios determined for the same I-section by using $F_y = 50$ ksi. It can be seen that the allowable load ratios computed for $F_y = 50$ ksi are lower than that computed for $F_y = 33$ ksi when $D/L < 1/3$. This effect would be negligible for beam-columns with small end moment ratios as shown in Figure 38. This comparison does not agree with the results of a study of I-sections with stiffened flanges, for which the yield point had no significant effect on the allowable load ratio for the I-section with stiffened flanges illustrated in Figure 31.

Figure 39 shows how the $C_m$ coefficient affects the allowable load ratio for the I-section having unstiffened flanges. The curves without star symbols are plotted for $C_m = 1.0$. The lines with star symbols represent the allowable load ratios calculated by using $C_m = 0.85$. It should be noted that the relationship shown in Figure 39 is very similar to the relationship illustrated in Figure 31 obtained for an I-section with stiffened flanges. For $D/L < 1/3$, the allowable load ratios are larger for $C_m = 0.85$ as compared to the allowable load ratios computed.
with $C_m = 1.0$ In general, the effect of the $C_m$ value on the allowable load ratio is more important for beam-columns with large end moment ratios.

Figure 40 shows the allowable load ratio versus dead-to-live load ratio for the same I-section used in Figures 38 and 39 but for flexural failure at the braced points. For $D/L = 0.5$, the LRFD criteria result in a larger allowable load than the value obtained from allowable stress design. For $M_{T}/M_{D0}$ ratios ranging from 0.1 to 0.3, the differences vary from 11.6% to 13.6%.

2. Singly-Symmetric Sections. The allowable eccentric axial loads were calculated for allowable stress design and LRFD. The applied end moments are a result of the eccentric axial loads and can be calculated using the following equation:

$$M_T = e_T p_T$$  \hspace{1cm} (VI.34)

where

$$e_T = e - e_x$$  

$e$ = eccentricity of the axial load with respect to the centroidal axis of the full section, negative when on the shear center side of the centroid

$e_x$ = distance between the centroid of the full section and the centroid of the effective section, negative when on the shear center side of the centroid of the full section
Figure 38. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case K

6" x 3" x 0.105" I-Section

With Unstiffened Flanges

$F_y = 33$ ksi

$L = 60$ in.

$F_y = 50$ ksi

Eq. (VI.27)/Eq. (VI.16)
1.30
= 0.2
6" x 3" x 0.105" I-Section
With Unstiffened Flanges

Figure 39. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case L

F = 33 ksi
L = 60 in.

Eq. (VI.27)/Eq. (VI.16)
F = 33-50 ksi

L = 60 in.

With Stiffened Flanges

Eq. (VI. 31)/Eq. (VI. 20)

6" x 3" x 0.105" I-Section

With Stiffened Flanges

F_y = 33-50 ksi

L = 60 in.

Eq. (VI.31)/Eq. (VI.20)

Figure 40. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case M
Substitutions similar to the ones made to solve for the allowable loads of beam-columns with doubly-symmetric shapes in Part 1 of this section were used to solve for the allowable loads for members with singly-symmetric shapes.

Equation (VI.1) for allowable stress design is based on flexural failure at the midlength of the beam-column. Equations (VI.13) and (VI.34) were substituted into Eq. (VI.1) to obtain the following expression:

\[
\frac{\Omega}{P_n} C_T^m + \frac{C_T^m P_T}{0.6M_n(1-\Omega P_T/P_c)} = 1.0 
\]  

(VI.35)

By solving for \( P_T \) in Eq. (VI.35), the following equation for allowable load is obtained:

\[
(P_T)_{ASD1} = \frac{1.0}{\frac{\Omega}{P_n} C_T^m + \frac{C_T^m P_T}{0.6M_n(1-\Omega P_T/P_c)}}
\]  

(VI.36)

Equation (VI.36) requires a solution using iterations, since the allowable axial load is a function of the actual axial load, \( P_T \).

Equation (VI.2) for allowable stress design is based on flexural failure at the braced points. Equations (VI.17), (VI.18), and (VI.34) were substituted into equation (VI.2) to obtain the following expression:
By solving for $P_T$ in Eq. (VI.37), the following equation for allowable load is obtained:

$$\frac{\Omega_c P_T}{P_{no}} + \frac{e_T P_T}{0.6M_{no}} = 1.0$$  \hspace{1cm} (VI.37)

For allowable stress design, Eq. (VI.3) is based on flexural failure when the effect of secondary moment is neglected. Equations (VI.13) and (VI.34) were substituted into Eq. (VI.3) to obtain the following expression:

$$\frac{\Omega_c P_T}{P_n} + \frac{e_T P_T}{0.6M_n} = 1.0$$  \hspace{1cm} (VI.39)

The following equation for allowable load is obtained by solving for $P_T$ in Eq. (VI.39):

$$\frac{\Omega_c P_T}{P_n} + \frac{e_T P_T}{0.6M_n} = 1.0$$  \hspace{1cm} (VI.40)
For LRFD, Eq. (VI.7) is based on flexural failure at the midlength of the beam-column. Equations (VI.23), (VI.25), and (VI.34) were substituted into Eq. (VI.7) to obtain the following expression:

\[
\frac{1.2D/L+1.6}{D/L+1} \left\{ \frac{P_T}{\phi_c P_n} + \frac{C_m e_T P_T}{\phi_m [1-(1.2D/L+1.6)P_T/(D/L+1)\phi_c P_E]} \right\} = 1.0 \quad (VI.41)
\]

By solving for \( P_T \) in Eq. (VI.41), the following equation for allowable load is obtained:

\[
(P_T)_{LRFD} = \frac{1}{\phi_c P_n} + \frac{C_m e_T}{\phi_m [1-(1.2D/L+1.6)P_T/(D/L+1)\phi_c P_E]} \quad (VI.42)
\]

Equation (VI.42) requires a solution by using iterations, since the allowable axial load is also a function of the actual axial load.

Equation (VI.8) for LRFD is based on flexural failure at the braced points. The following expression was obtained by substituting Eqs. (VI.28), (VI.29), and (VI.34) into Eq. (VI.8):

\[
\frac{1.2D/L+1.6}{D/L+1} \left\{ \frac{P_T}{\phi_c P_{no}} + \frac{e_T P_T}{\phi_m P_{no}} \right\} = 1.0 \quad (VI.43)
\]

By solving for \( P_T \) in Eq. (VI.43), the following equation for allowable load is obtained:
Equation (VI.9) for LRFD is based on flexural failure when the effect of secondary moment is neglected. Equations (VI.23) and (VI.34) were substituted into Eq. (VI.9) to obtain the following expression:

\[
\frac{1.2D/L + 1.6}{D/L + 1} \left( \frac{P_T}{\phi_{c,n}} + \frac{e_T P_T}{\phi_{M,n}} \right) = 1.0
\]  

(VI.45)

The following equation for allowable load was obtained by solving for \( P_T \) in Eq. (VI.45):

\[
(P_T)_{LRFD} = \frac{(D/L+1)/(1.2D/L+1.6)}{1 + \frac{e_T}{\phi_{c,n}}}
\]  

(VI.46)

The equations to be used for the allowable eccentric axial load for allowable stress design and LRFD are very complex and utilize iterations with multiple variables. The allowable load ratios, \( (P_T)_{LRFD}/(P_T)_{ASD} \), for various lengths and eccentricities were studied. Typical channel sections and their section properties used in this study, were obtained from Tables 1 and 2 of Part V of the AISI Cold-Formed Steel Design Manual.
A channel (4 in. x 2 in. x 0.105 in.) with stiffened flanges was studied as a beam-column subjected to an eccentric load applied at each end. Figure 41 shows the allowable load ratio versus eccentricity for the channel with an effective length of 5 ft, D/L = 0.5, and C_m = 1.0. From this figure, it can be seen that the smaller the eccentricity the larger the allowable load ratio and this relationship holds for both positive and negative eccentricities.

The top line in Figure 41 represents the same channel section with a yield point of 50 ksi. The allowable load ratios in this case are slightly greater than that computed with F_y = 33 ksi.

Figure 42 shows the relationship between allowable load ratio and dead-to-live load ratio for the 4 in. deep channel with e = +1.29 in. The two curves represent yield points of 33 and 50 ksi for the 5 ft long beam-column. The higher yield point steels result in slightly higher values of the allowable load ratio as seen in Figure 41 and 42. From the computer output, the value of F_y has a negligible effect on the allowable load ratio for the same channel with -0.25 in. < e < +0.25 in. and effective length equals to 5 ft.

Figure 43 shows the allowable load ratio versus slenderness ratio, KL/r_y, for the channel (4 in. x 2 in. x 0.105 in.) with stiffened flanges and D/L = 1/5. The curves represent yield points of 33 and 50 ksi for the channel with e = +1.29 in. For F_y = 33 ksi, the allowable load ratio increases slightly as the slenderness ratio increases up to KL/r_y = 160. For KL/r_y > 160, the allowable load ratio decreases as the slenderness ratio increases. The slenderness ratio has a larger effect on the allowable load ratio for the channel with F_y = 50 ksi.
as compared with $F_y = 33$ ksi. For $F_y = 50$ ksi, the allowable load ratio increases as the slenderness ratio increases up to $KL/r_y = 130$. For $KL/r_y > 130$, the allowable load ratio decreases as the slenderness ratio increases.

A channel (6 in. x 2.5 in. x 0.105 in.) with stiffened flanges was also studied. The relationship between allowable load ratio and eccentricity for the channel with a length of 5 ft and $D/L = 0.5$ is shown in Figure 44. The bottom line represents the curve for $C_m = 1.0$ which would be used for braced frames. For this case, the curve is similar to that shown in Figure 41 for the 4 in. deep channel.

The top line in Figure 44 represents the same channel with $C_m = 0.85$. This value of $C_m$ is used for unbraced frames and beam-columns with restrained ends subjected to transverse loading between its supports. The curve for $C_m = 0.85$ is similar to the curve for $C_m = 1.0$ except that $C_m = 0.85$ results in a higher allowable load ratio than $C_m = 1.0$. The effect of the value of $C_m$ on the allowable load ratio is negligible for $-0.25$ in. $< e < +0.25$ in. as shown in Figure 44.

Figure 45 shows the allowable load ratio versus dead-to-live load ratio for the channel used in Figure 44. The curves represent the allowable load ratios for various eccentricities by using $F_y = 33$ ksi and $C_m = 1.0$. It can be seen from this figure that the eccentricity does not affect the shape of the curve but does affect the value of the allowable load ratio.
Figure 41. Allowable Load Ratio vs. Eccentricity for Beam-Columns - Case 1
Figure 42. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case 1

4" x 2" x 0.105" Channel
With Stiffened Flanges

\[ F_y = 50 \text{ ksi} \]
\[ F_y = 33 \text{ ksi} \]

\[ e = +1.29 \text{ in.} \]
\[ L = 60 \text{ in.} \]
Figure 43. Allowable Load Ratio vs. Slenderness Ratio for Beam-Columns - Case 1

4" x 2" x 0.105" Channel
With Stiffened Flanges
e = + 1.29 in.
D/L = 1/5

F_y = 33 ksi
F_y = 50 ksi

F = 33 ksi
The relationship between allowable load ratio and dead-to-live load ratio for the 6 in. deep channel (6 in. x 2.5 in. x 0.105 in.) is shown in Figure 46 for various lengths and eccentricities. The curves without star symbols represent the values of allowable load ratios for \( e = +1.73 \text{ in.} \) and effective lengths of 3 and 11 ft. It should be noted that the effective length has a small effect on the allowable load ratio.

The curves with star symbols in Figure 46 represent the allowable load ratios determined for the same channel section with \( e = -1.73 \text{ in.} \) and effective lengths of 3 and 11 ft. It can be seen that for larger effective length, the allowable load ratios determined for \( e = -1.73 \text{ in.} \) are larger than those determined for \( e = +1.73 \text{ in.} \). For smaller effective length, the allowable load ratios determined for \( e = -1.73 \text{ in.} \) are slightly smaller than those determined for \( e = +1.73 \text{ in.} \).

Channels with unstiffened flanges were studied in a similar manner. Figure 47 shows the allowable load ratio versus eccentricity for a channel (4 in. x 1.125 in. x 0.105 in.) with unstiffened flanges and an effective length of 5 ft. The curves in the figure are allowable load ratios computed for yield points of 33 and 50 ksi, respectively. These curves indicate similar relationships as compared with the curves in Figure 41 obtained from a 4 in. deep channel with stiffened flanges. As shown in Figure 47, allowable load ratios computed for \( F_y = 33 \text{ ksi} \) are larger than those computed for \( F_y = 50 \text{ ksi} \).
Figure 44. Allowable Load Ratio vs. Eccentricity for Beam-Columns - Case 2

6" x 2.5" x 0.105" Channel
With Stiffened Flanges

$F_y = 33$ ksi
$D/L = 0.5$
$L = 60$ in.
Figure 45. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case 2

6" x 2.5" x 0.105" Channel
With Stiffened Flanges
$F_y = 33$ ksi
$L = 60$ in.
Figure 46. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case 3
The curves in Figure 48 are for the same channel used in Figure 47 with an effective length of 4 ft. The yield points of steel vary from 33 to 50 ksi. This figure shows that the yield point has no effect and the sign of eccentricity has negligible effect on the allowable load ratio for various D/L ratios with L = 4 ft.

Figure 49 shows the allowable load ratio versus slenderness ratio, KL/r, for the same channel used in Figures 47 and 48 for D/L = 1/5 and e = + 1.20 in. The curves without star symbol represent the values of allowable load ratio for e = + 1.20 in; the curves with star symbols represent the values of allowable load ratio for e = - 1.20 in. The sign of eccentricity has negligible effect. The curve computed for yield point of 33 ksi indicates that the allowable load ratio increases slightly with increasing slenderness ratio up to KL/r = 170. For KL/r > 170, the allowable load ratio decreases as the slenderness ratio increases. The same relationships hold for Fy = 50 ksi but with the dividing point of KL/r = 130. The curves also show that for KL/r = 150.47 (L = 4 ft), the allowable load ratios for Fy = 33 and 50 ksi are the same. When KL/r < 150.47, the allowable load ratios computed for Fy = 50 ksi are larger than those computed for Fy = 33 ksi. For KL/r > 150.47, the allowable load ratio computed for Fy = 33 ksi are larger than those computed for Fy = 50 ksi.
$4" \times 1.125" \times 0.105"$ Channel
With Unstiffened Flanges
$L = 60$ in.
$D/L = 0.5$

Figure 47. Allowable Load Ratio vs. Eccentricity for Beam-Columns - Case 4
Figure 48. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case 4

4" x 1.125" x 0.105" Channel
With Unstiffened Flanges
L = 48 in.
F_y = 33-50 ksi

Dead-To-Live Load Ratio, D/L

Allowable Load Ratio, (P_T)_{IRFD}/(P_T)_{ASD}
Figure 49. Allowable Load Ratio vs. Slenderness Ratio for Beam-Columns - Case 4

4" x 1.125" x 0.105" Channel
With Unstiffened Flanges

D/L = 1/5

F_y = 33 ksi
F_y = 50 ksi
Figure 50. Allowable Load Ratio vs. Eccentricity for Beam-Columns - Case 5

6" x 1.5" x 0.105" Channel
With Unstiffened Flanges
D/L = 0.5
L = 60 in.
Figure 51. Allowable Load Ratio vs. D/L Ratio for Beam-Columns - Case 5

6" x 1.5" x 0.105" Channel

--- e = + 1.0 in. With Unstiffened Flanges
* e = - 1.0 in. $F_y = 33$ ksi

$F_v = 33$ ksi

$L = 7'$

$L = 3'$

Dead-To-Live Load Ratio, D/L

Allowable Load Ratio, $(P_L LRFB)/(P_A SD)$
A deeper channel (6 in. x 1.5 in. x 0.105 in.) with unstiffened flanges was also studied. Figure 50 shows the allowable load ratio versus eccentricity for the 5 ft long channel with D/L = 0.5 and $F_y = 33$ ksi. The curves shown in the figure are obtained for $C_m$ values of 1.0 and 0.85. It is similar in shape and magnitude to the allowable load ratio curve shown in Figure 47 for a 4 in. deep channel with unstiffened flanges and $F_y = 33$ ksi. Figure 50 shows that $C_m$ value has negligible effect on the allowable load ratio. Also as shown in Figures 47 and 50, small eccentricities will result in relatively high allowable load ratios.

The relationship between allowable load ratio and dead-to-live load ratio for the channel used in Figure 50 is shown in Figure 51 for various lengths. The curves without star symbol represent the values of allowable load ratio for e = +1.00 in; the curves with star symbols represent the values of allowable load ratio for e = -1.00 in. The effective lengths used in this figure are 3 and 7 ft. This figure is similar to Figure 46 which was obtained from a channel of equal depth but with stiffened flanges. As shown in this figure, the effective length has a small effect on the allowable load ratio. It also shows that the sign of eccentricity has no effect for L = 3 ft and has negligible effect for L = 7 ft on the allowable load ratio.
VII. CONNECTIONS

A. GENERAL

Connections are required for joining individual structural members together and are used to fabricate structural members from sheet steel or structural components. The AISI Specification and the Specification for Load and Resistance Factor Design include requirements for welded and bolted connections which are frequently used in cold-formed steel construction. All connections should be designed to transmit the maximum load with proper regard for eccentricity.

B. WELDED CONNECTIONS

Welds are classified as fusion welds and resistance welds. Weld shearing and plate tearing are the common failure modes for welded connections.

1. Allowable Stress Design. Welded connections shall be designed to transmit the maximum load in the connected member. Proper regard shall be given to eccentricity.

   a. Arc Welds. Arc welds are fusion welds produced by burning the metal to a molten state at the surface to be joined without the application of mechanical pressure or blows. Pekoz and McGuire studied the welding of sheet steel and provided most of the statistical data used for the development of the AISI design provisions for allowable stress design and the LRFD criteria for arc welds. According
to Section E2 of the AISI Specification, the load on each arc weld shall not exceed $P_a$ calculated as follows:

$$P_a = \frac{P_n}{\Omega_w}$$

(VII.1)

where

$\Omega_w = \text{Factor of safety for arc welded connections} = 2.50$

$P_n = \text{Nominal strength of welds determined according to the following formulas.}$

i. **Arc Spot Welds.** Arc spot welds are produced by burning a hole in the top sheet and filling it with weld metal which fuses it to the bottom sheet or structural member. They are sometimes referred to as puddle welds. Arc spot welds permitted by the AISI Specification are for welding sheet steel to thicker supporting members in the flat position. Arc spot welds (puddle welds) shall not be made on steel where the thinnest connected part is over 0.15 inch thick, nor through a combination of steel sheets having a total thickness over 0.15 inch.

Weld washers shall be used when the thickness of the sheet is less than 0.028 inch. Weld washers shall have a thickness between 0.05 and 0.08 inch with a minimum prepunched hole of 3/8-inch diameter.

Arc spot welds shall be specified by minimum effective diameter of fused area, $d_e$. Minimum allowable effective diameter is 3/8 inch.

The nominal shear load, $P_n$, on each arc spot weld between sheet or sheets and supporting member shall not exceed the smaller of either

$$P_n = 0.625d_e ^2 F_{xx} ' \text{ or }$$

(VII.2)

For $(d_e/t) \leq 0.815 \sqrt{(E/F_u)}$: 

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\[ P_n = 2.20 t d F_u \]  
(VII.3)

For \(0.815/(E/F_u) < (d/t) < 1.397/(E/F_u)\):

\[ P_n = 0.280[1+(5.59 t/E)/(d F_u)] t d F_u \]  
(VII.4)

For \((d/t) > 1.397/(E/F_u)\):

\[ P_n = 1.40 t d F_u \]  
(VII.5)

where

- \(d\) = Visible diameter of outer surface of arc spot weld
- \(d_a\) = Average diameter of the arc spot weld at mid-thickness of \(t\)
- \(d\_e\) = Effective diameter of fused area
  \[ d_e = 0.7d - 1.5t \text{ but } d_e \leq 0.55d \]  
(VII.6)
- \(t\) = Total combined base steel thickness (exclusive of coatings) of sheets involved in shear transfer
- \(F_{xx}\) = Stress level designation in AWS electrode classification
- \(F_u\) = Specified minimum tensile strength of steel

**ii. Arc Seam Welds.** Arc seam welds are produced in the same manner as arc spot welds except that a seam is formed. Arc seam welds covered by the AISI Specification apply only to the following joints:

(a) Sheet to thicker supporting member in the flat position.

(b) Sheet to sheet in the horizontal or flat position.

According to Section E2.3 of the AISI Specification, the shear load, \(P_n\), on each arc seam weld shall not exceed either

\[ P_n = (d_e^2/4 + L d_e/3)(2.5F_{xx}); \text{ or} \]  
(VII.7)
where
\[ d = \text{Width of arc seam weld} \]
\[ L = \text{Length of seam weld not including the circular ends (For computation purposes, } L \text{ shall not exceed } 3d) \]
\[ d_a = \text{Average width of seam weld} \]

where
\[ d_a = (d-t) \text{ for a single sheet, and} \]
\[ (d-2t) \text{ for a double sheet} \]

\[ d_e = \text{Effective width of arc seam weld at fused surfaces} \]
\[ d_e = 0.7d - 1.5t \]

### iii. Fillet Welds

Fillet welds are used to connect lap joints and T-joints. Fillet welds covered by the AISI Specification apply to the welding of joints in any position, either

(a) Sheet to sheet, or

(b) Sheet to thicker steel member.

According to Section E2.4 of the AISI Specification, the shear load, \( P_n \), on a fillet weld in lap and T-joints shall not exceed the following:

For longitudinal loading:

For \( L/t < 25 \):
\[ P_n = (1-0.01L/t)LF_u \] (VII.12)

For \( L/t \geq 25 \):
\[ P_n = 0.75tLF_u \] (VII.13)

For transverse loading:
\[ P_n = tLF_u \] (VII.14)

where
t = Least value of $t_1$ or $t_2$

In addition, for $t > 0.150$ inch the allowable load for fillet weld in lap and T-joints shall not exceed:

$$P_n = 0.75 t_w L F_{xx}$$  \hspace{1cm} (VII.15)

where

$L = \text{Length of fillet weld}$

$t_w = \text{Effective throat} = 0.707 w_1 \text{ or } 0.707 w_2$, whichever is smaller.

A larger effective throat may be taken if it can be shown by measurement that a given welding procedure will consistently give a larger value providing the particular welding procedure used for making the welds that are measured is followed.

$w_1$ and $w_2 = \text{leg on weld}$

iv. Flare Groove Welds. Flare groove welds are used in cold-formed steel construction to join rolled corners to sheets and to join two rolled corners. Flare groove welds covered by the AISI Specification apply to welding of joints in any position, either:

(a) Sheet to sheet for flare-V groove welds, or

(b) Sheet to sheet for flare-bevel groove welds, or

(c) Sheet to thicker steel member for flare-bevel groove welds.

According to Section E2.5 of the AISI Specification, the shear load, $P_n$, on a weld shall be governed by the thickness, $t$, of the sheet steel adjacent to the weld. The load shall not exceed:

For flare-bevel groove welds, transverse loading:

$$P_n = 0.833 t L F_u$$  \hspace{1cm} (VII.16)

For flare groove welds, longitudinal loading:

For $t \leq t_w < 2t$ or if the lip height is less than weld length, $L$:
\[ P_n = 0.75tLF \quad (VII.17) \]

For \( t_w \geq 2t \) and the lip height is equal to or greater than \( L \):

\[ P_n = 1.50tLF \quad (VII.18) \]

In addition, if \( t > 0.15 \) inch, then:

\[ P_n = 0.75t_LF \quad (VII.19) \]

b. **Resistance Welds.** Resistance welding is a group of welding processes wherein coalescence is produced by the heat obtained from resistance to electric current through the work parts held together under pressure by electrodes. They are mostly used for shop welding in cold-formed steel fabrication. According to Section E2.6 of the AISI Specification, in sheets joined by spot welding the allowable shear per spot, \( P_a' \), shall be as follows (the safety factor is included in Table VII.1):

<table>
<thead>
<tr>
<th>Thickness of Thinnest Outside Sheet, in.</th>
<th>Allowable Shear Strength per Spot, kips</th>
<th>Thickness of Thinnest Outside Sheet, in.</th>
<th>Allowable Shear Strength per Spot, kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>0.050</td>
<td>0.080</td>
<td>1.330</td>
</tr>
<tr>
<td>0.020</td>
<td>0.175</td>
<td>0.094</td>
<td>1.725</td>
</tr>
<tr>
<td>0.030</td>
<td>0.400</td>
<td>0.109</td>
<td>2.395</td>
</tr>
<tr>
<td>0.040</td>
<td>0.570</td>
<td>0.125</td>
<td>2.880</td>
</tr>
<tr>
<td>0.050</td>
<td>0.660</td>
<td>0.188</td>
<td>4.000</td>
</tr>
<tr>
<td>0.060</td>
<td>0.910</td>
<td>0.250</td>
<td>6.000</td>
</tr>
</tbody>
</table>

2. **LRFD Criteria**
a. **Arc Welds.** According to Section E2 of the LRFD Specification, the force on each weld computed on the basis of factored loads shall not exceed the factored nominal strength, \( \Phi P_n \), where

\[
\Phi = \text{Resistance factor for arc welded connections}
\]
\[
P_n = \text{Nominal strength of welds}
\]

i. **Arc Spot Welds.** Arc spot welds permitted by the LRFD Specification are for welding sheet steel to thicker supporting members in the flat position. Arc spot welds (puddle welds) shall not be made on steel where the thinnest connected part is over 0.15 inch thick, nor through a combination of steel sheets having a total thickness over 0.15 inch.

Weld washers shall be used when the thickness of the sheet is less than 0.028 inch. Weld washers shall have a thickness between 0.05 and 0.08 inch with a minimum prepunched hole of 3/8-inch diameter.

Arc spot welds shall be specified by minimum effective diameter of fused area, \( d_e \). Minimum allowable effective diameter is 3/8 inch.

The factored nominal shear strength, \( \Phi P_n \), of each arc spot weld between sheet or sheets and supporting member shall be determined by using the smaller of either

(a) \( \Phi = 0.60 \)

\[
P_n = 0.589d_e^{2.88}F_u ; \text{or} \tag{VII.20}
\]

(b) For \( (d_a/t) \leq 0.815 \sqrt{(E/F_u)} \):

\[
\Phi = 0.60
\]

\[
P_n = 2.20td_a^{F_u} \tag{VII.21}
\]

For \( 0.815 \sqrt{(E/F_u)} < (d_a/t) < 1.397 \sqrt{(E/F_u)} \):
\[ \phi = 0.50 \]

\[ P_n = 0.280 \left(1+5.59\sqrt{E/F_u/(d_a/t)}\right)td_a F_u \]  
(VII.22)

For \((d_a/t) \geq 1.397\sqrt{E/F_u}\):
\[ \phi = 0.50 \]
\[ P_n = 1.40td_a F_u \]  
(VII.23)

where

\(\phi\) = Resistance factor for welded connections

\(P_n\) = Nominal ultimate shear strength of an arc spot weld

\(d\) = Visible diameter of outer surface of arc spot weld

\(d_a\) = Average diameter of the arc spot weld at mid-thickness of \(t\)

(where \(d_a = (d-t)\) for a single sheet, and \((d-2t)\) for multiple sheets (not more than four lapped sheets over a supporting member))

\(d_e\) = Effective diameter of fused area

\[ d_e = 0.7d - 1.5t \text{ but } \leq 0.55d \]  
(VII.24)

\(t\) = Total combined base steel thickness (exclusive of coatings) of sheets involved in shear transfer

\(F\) = Stress level designation in AWS electrode classification

ii. Arc Seam Welds. Arc seam welds covered by the LRFD Specification apply only to the following joints:

(a) Sheet to thicker supporting member in the flat position.

(b) Sheet to sheet in the horizontal or flat position.

The factored nominal shear strength, \(\phi P_n\), of arc seam welds shall be determined by using the smaller of either

(a) \(\phi = 0.60\)
\[ P_n = (\frac{nd_e^2}{4} + Ld_e)(0.75F_{xx}); \]  
\[ \text{or} \]
\[ P_n = 2.5F_u(0.25L + 0.96d_a) \]

where

\[ \phi = \text{Resistance factor for welded connections} \]
\[ P_n = \text{Nominal ultimate shear strength of an arc seam weld} \]
\[ d = \text{Width of arc seam weld} \]
\[ L = \text{Length of seam weld not including the circular ends (For computation purposes, L shall not exceed 3d)} \]
\[ d_a = \text{Average width of seam weld} \]

iii. Fillet Welds. Fillet welds covered by the LRFD Specification apply to the welding of joints in any position, either

(a) Sheet to sheet, or
(b) Sheet to thicker steel member.

The factored nominal shear strength, \( \phi P_n \), of a fillet weld shall be determined as follows:

(a) For longitudinal loading:

For \( L/t < 25 \):
\[ \phi = 0.60 \]
\[ P_n = (1 - 0.01L/t)tLF_u \]  
\[ \text{(VII.27)} \]

For \( L/t \geq 25 \):
\[ \phi = 0.55 \]
\[ P_n = 0.75tLF_u \]  
\[ \text{(VII.28)} \]

(b) For transverse loading:
\[ \phi = 0.60 \]
\[ P_n = t L F_u \]  
(VII.29)

where

\[ t = \text{Least value of } t_1 \text{ or } t_2 \]

In addition, for \( t > 0.150 \text{ inch} \) the factored nominal strength determined above shall not exceed the following value of \( \Phi P_n \):

\[ \Phi = 0.60 \]

\[ P_n = 0.75 t_w L F_{xx} \]  
(VII.30)

where

\[ \Phi = \text{Resistance factor for welded connections} \]

\[ P_n = \text{Nominal ultimate strength of a fillet weld} \]

\[ L = \text{Length of fillet weld} \]

\[ t_w = \text{Effective throat} = 0.707w_1 \text{ or } 0.707w_2, \text{ whichever is smaller.} \]

A larger effective throat may be taken if it can be shown by measurement that a given welding procedure will consistently give a larger value providing the particular welding procedure used for making the welds that are measured is followed.

\[ w_1 \text{ and } w_2 = \text{leg on weld} \]

iv. Flare Groove Welds. Flare groove welds covered by the LRFD Specification apply to welding of joints in any position, either:

(a) Sheet to sheet for flare-V groove welds, or

(b) Sheet to sheet for flare-bevel groove welds, or

(c) Sheet to thicker steel member for flare-bevel groove welds.

The factored nominal shear strength, \( \Phi P_n \), of a flare groove weld shall be determined as follows:

(a) For flare-bevel groove welds, transverse loading:

\[ \Phi = 0.55 \]
(b) For flare groove welds, longitudinal loading:

1) For \( t \leq t_w < 2t \) or if the lip height is less than weld length, \( L \):
\[
\phi = 0.55 \\
P_n = 0.75tLF_u
\] (VII.32)

2) For \( t_w \geq 2t \) and the lip height is equal to or greater than \( L \):
\[
\phi = 0.55 \\
P_n = 1.50tLF_u
\] (VII.33)

In addition, if \( t > 0.15 \) inch, the factored nominal strength determined above shall not exceed the following value of \( \Phi P_n \):

\[
\phi = 0.60 \\
\Phi P_n = 0.75t_wLF_{xx}
\] (VII.34)

b. Resistance Welds. The factored nominal shear strength, \( \Phi P_n \), of spot welding shall be determined as follows:

\[
\phi = 0.65 \\
P_n = \text{Tabulated value given in Table VII.2}
\]
### TABLE VII.2
Nominal Shear Strength of Spot Welding

<table>
<thead>
<tr>
<th>Thickness of Thinnest Outside Sheet, in.</th>
<th>Shear Strength per spot, kips</th>
<th>Thickness of Thinnest Outside Sheet, in.</th>
<th>Shear Strength per spot, kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>0.125</td>
<td>0.080</td>
<td>3.325</td>
</tr>
<tr>
<td>0.020</td>
<td>0.438</td>
<td>0.094</td>
<td>4.313</td>
</tr>
<tr>
<td>0.030</td>
<td>1.000</td>
<td>0.109</td>
<td>5.988</td>
</tr>
<tr>
<td>0.040</td>
<td>1.425</td>
<td>0.125</td>
<td>7.200</td>
</tr>
<tr>
<td>0.050</td>
<td>1.650</td>
<td>0.188</td>
<td>10.000</td>
</tr>
<tr>
<td>0.060</td>
<td>2.275</td>
<td>0.250</td>
<td>15.000</td>
</tr>
</tbody>
</table>

3. **Comparison.** The allowable load per weld for allowable stress design is \( P_a \) computed from Eq. (VII.1). For the LRFD criteria, the allowable load per weld can be calculated from the following equation developed from Eq. (II.6):

\[
(P_a)_{LRFD} = \phi P_n (D/L+1)/(1.2D/L+1.6)
\]  
(VII.35)

a. **Arc Spot Welds.** Equation (VII.2) from allowable stress design and Eq. (VII.20) for LRFD criteria are based on shearing of the weld. The allowable load ratio based on shearing of arc spot welds and \( \phi = 0.60 \) is as follows:
\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = \frac{0.589}{2.5\phi} \cdot \frac{D/L+1}{1.2D/L+1.6} = \frac{1.414}{1.2D/L+1.6} \quad (VII.36)
\]

Figure 52 shows the allowable load ratio versus dead-to-live load ratio determined from Eq. (VII.36) for weld shear failure of arc spot welds. For \(D/L = 0.5\), the allowable load per spot determined from the LRFD criteria is 3.6% less than the value obtained from allowable stress design. As shown in the figure, LRFD is conservative for shear failure in arc spot welds for \(D/L < 0.9\).

Equations (VII.3), (VII.4), and (VII.5) from allowable stress design and Eqs. (VII.21), (VII.22), and (VII.23) for LRFD are based on failure in the plate. The allowable load ratios for plate failure are as follows:

For \((d_a/t) \leq 0.815 \sqrt{(E/F_U)}\) and \(\phi = 0.60\),
\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi \cdot \frac{D/L+1}{1.2D/L+1.6} = 1.50 \cdot \frac{D/L+1}{1.2D/L+1.6} \quad (VII.37)
\]

For \(0.815 \sqrt{(E/F_U)} < (d_a/t) < 1.397 \sqrt{(E/F_U)}\) and \(\phi = 0.50\),
\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi \cdot \frac{D/L+1}{1.2D/L+1.6} = 1.25 \cdot \frac{D/L+1}{1.2D/L+1.6} \quad (VII.38)
\]

For \((d_a/t) \geq 1.397 \sqrt{(E/F_U)}\) and \(\phi = 0.50\)
\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi \cdot \frac{D/L+1}{1.2D/L+1.6} = 1.25 \cdot \frac{D/L+1}{1.2D/L+1.6} \quad (VII.39)
\]

Equations (VII.37), (VII.38), and (VII.39) are shown in Figure 53 and are based on plate failure of arc spot welds. As seen from the figure, for \(D/L = 0.5\), the allowable load ratios computed from LRFD
and ASD vary from about 0.85 to 1.02 depending upon the $d_a/t$ ratio used in the connection. For the range of D/L ratios used in cold-formed steel, LRFD is conservative for the design of arc spot welds compared with allowable stress design.

b. Arc Seam Welds. Equation (VII.7) from allowable stress design and Eq. (VII.25) for LRFD criteria are based on shearing of the weld. The allowable load ratio based on shear failure of arc seam welds and $\phi = 0.60$ is as follows:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 0.75\phi \frac{D/L+1}{1.2D/L+1.6} = 1.414 \frac{D/L+1}{1.2D/L+1.6}$$ (VII.40)

Equation (VII.40) is identical to Eq. (VII.36) which is the allowable load ratio for arc spot welds based on weld shearing. Figure 52 shows the relationship between allowable load ratio and dead-to-live load ratio for this type of failure. As shown in the figure, LRFD is conservative for shear failure of arc seam welds compared with allowable stress design for $D/L < 0.90$.

Equation (VII.8) from allowable stress design and Eq. (VII.26) for LRFD criteria are based on plate tearing. The allowable load ratio for plate failure and $\phi = 0.60$ is as follows:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi \frac{D/L+1}{1.2D/L+1.6} = 1.50 \frac{D/L+1}{1.2D/L+1.6}$$ (VII.41)

Figure 54 shows the allowable load ratio versus dead-to-live load ratio determined from Eq. (VII.41) for plate tearing failure. Both design methods result in the same value of allowable load for a D/L ratio of 1/3. The allowable load based on LRFD is 2.3% greater than
the value based on allowable stress design for $D/L = 0.5$. However, LRFD is conservative for $D/L < 1/3$ compared with allowable stress design.

c. Fillet Welds. Equations (VII.12), (VII.13), and (VII.14) from allowable stress design and Eqs. (VII.27), (VII.28), and (VII.29) for LRFD design are based on plate tearing. The allowable load ratio can be computed using the following formula:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi \frac{D/L+1}{1.2D/L+1.6}$$ (VII.42)

For longitudinal loading with $L/t < 25$, the resistance factor is 0.60. Therefore, the allowable load ratio can be computed using the following equation:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.50 \frac{D/L+1}{1.2D/L+1.6}$$ (VII.43)

For longitudinal loading with $L/t \geq 25$, the resistance factor is 0.55. Therefore, the following equation can be used to calculate the allowable load ratio:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.375 \frac{D/L+1}{1.2D/L+1.6}$$ (VII.44)

For transverse loading with $\phi = 0.6$, Eq. (VII.45) can be used to calculate the allowable load ratio.

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.50 \frac{D/L+1}{1.2D/L+1.6}$$ (VII.45)
Figure 52. Allowable Load Ratio vs. D/L Ratio for Shear Failure of Arc Spot and Arc Seam Welds
Figure 53. Allowable Load Ratio vs. D/L Ratio for Arc Spot Welds
Figure 54. Allowable Load Ratio vs. D/L Ratio for Plate Tearing of Arc Seam Welds
The relationship between allowable load ratio and dead-to-live load ratio is shown in Figure 55 for plate tearing failure based on Eqs. (VII.43), (VII.44), and (VII.45). For longitudinally loaded fillet welds with \( L/t < 25 \) and \( D/L = 0.5 \), the allowable load computed from LRFD is 2.3% higher than the value computed from allowable stress design. For longitudinally loaded fillet welds with \( L/t \geq 25 \) and \( D/L = 0.5 \), the allowable load computed from LRFD is 6.1% lower than the value computed from allowable stress design.

For transverse loading of fillet welds, the allowable load based on the LRFD criteria is also 2.3% higher than the value based on allowable stress design for \( D/L = 0.5 \).

When the thickness of the plate is greater than 0.15 in., weld shearing has to be checked. Equation (VII.15) from allowable stress design and Eq. (VII.30) for LRFD design are based on weld shearing of fillet welds. The allowable load ratio can be computed using the following formula with \( \phi = 0.60 \):

\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi \frac{D/L+1}{1.2D/L+1.6} = 1.50 \frac{D/L+1}{1.2D/L+1.6} \quad (VII.46)
\]

The relationship between allowable load ratio and dead-to-live load ratio for weld failure of fillet welds is shown in Figure 56. From the figure, LRFD criteria result in an allowable load 2.3% larger than the value computed from allowable stress design for \( D/L = 0.5 \).
Figure 55. Allowable Load Ratio vs. D/L Ratio for Plate Tearing of Fillet Welds
Figure 56. Allowable Load Ratio vs. D/L Ratio for Weld Failure of Fillet and Flare Groove Welds

Eq. (VII.46) & Eq. (VII.50)
d. Flare Groove Welds. Equations (VII.16), (VII.17), and (VII.18) from allowable stress design and Eqs. (VII.31), (VII.32), and (VII.33) for LRFD design are based on plate failure. The allowable load ratio can be computed using the following formula:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi \frac{D/L+1}{1.2D/L+1.6} \quad (VII.47)$$

For flare-bevel groove welds loaded in the transverse direction and $\phi = 0.55$, the following equation can be used for allowable load ratio:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.375 \frac{D/L+1}{1.2D/L+1.6} \quad (VII.48)$$

For flare groove welds loaded in the longitudinal direction and $\phi = 0.55$, the allowable load ratio can be computed as follows:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.375 \frac{D/L+1}{1.2D/L+1.6} \quad (VII.49)$$

Figure 57 shows the relationship between allowable load ratio and dead-to-live load ratio computed from Eqs. (VII.48) and (VII.49). For transverse loading of flare-bevel groove welds and $D/L = 0.5$, the allowable load computed from LRFD is 6.3% lower than the value computed from allowable stress design. The same is true for flare groove welds loaded in the longitudinal direction. As shown in the figure, the LRFD criteria for flare groove welds are slightly conservative for the values of $D/L$ ratios generally used in cold-formed steel construction.

For flare groove welds on sheets thicker than 0.15 in., weld shearing may govern the design. Equation (VII.19) from allowable stress
design and Eq. (VII.34) for LRFD design are based on shear failure of the weld. With $\phi = 0.60$, the allowable load ratio can be computed as follows:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi \frac{D/L+1}{1.2D/L+1.6} = 1.50 \frac{D/L+1}{1.2D/L+1.6} \quad (VII.50)$$

Equation (VII.50) is identical to Eq. (VII.46) which is the allowable load ratio for fillet welds based on the same type of failure.

Figure 56 shows the allowable load ratio versus dead-to-live load ratio for weld failure of fillet and flare groove welds. The allowable load ratio based on LRFD is 2.3% larger than the value based on allowable stress design for $D/L = 0.5$.

e. Resistance Welds. The allowable loads per spot weld for allowable stress design in Table VII.1 were derived from the values in Table VII.2 using a factor of safety of 2.5. Therefore, the following equation for allowable load ratio can be used for $\phi = 0.65$:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi \frac{D/L+1}{1.2D/L+1.6} = 1.625 \frac{D/L+1}{1.2D/L+1.6} \quad (VII.51)$$

The relationship between allowable load ratio and dead-to-live load ratio is shown in Figure 58 for resistance welds. As shown from the figure, LRFD criteria always result in higher values of allowable load than allowable stress design for all dead-to-live load ratios. For $D/L = 0.5$, the difference between the allowable loads is 10.8%.
Figure 57. Allowable Load Ratio vs. D/L Ratio for Plate Tearing of Flare Groove Welds

Eq. (VII.48) & Eq. (VII.49)
Figure 58. Allowable Load Ratio vs. D/L Ratio for Resistance Welds
C. BOLTED CONNECTIONS

The AISI Specification and the LRFD Specification for bolted connections of cold-formed steel structural members apply to members in which the thickness of the thinnest connected part is less than 3/16 in. The AISC Specification should be used for bolted connections when the thickness of the thinnest connected part is greater than or equal to 3/16 in.

1. Allowable Stress Design. According to Section E3 of the AISI Specification, the following requirements govern bolted connections of cold-formed steel structural members in which the thickness of the thinnest connected part is less than 3/16 inch and there are no gaps between connected parts. For bolted connections in which the thinnest connected part is equal to or greater than 3/16 inch, refer to AISC Specification.

a. Spacing and Edge Distance. The minimum spacing and edge distance in the line of the stress has to be checked to prevent tearing of the steel sheet due to shear. According to Section E3.1 of the AISI Specification, the distance, e, measured in the line of force from the center of a standard hole to the nearest edge of an adjacent hole or to the end of the connected part toward which the force is directed shall not be less than the value of \( e_{\text{min}} \) determined as follows:

\[
e_{\text{min}} = e \Omega_e
\]

(VII.52)

where

\[
e = \frac{P}{F_u t}
\]

(VII.53)

(a) When \( \frac{F_u}{F_{sy}} \geq 1.15 \):

\[\Omega_e = \text{Factor of safety for sheet tearing}\]
(b) When \( F_u/F_{sy} < 1.15 \): 
\[ \Omega_e = \text{Factor of safety for sheet tearing} \]
\[ = 2.22 \]

where

\( P \) = Force transmitted by bolt
\( t \) = Thickness of thinnest connected part
\( F_u \) = Tensile strength of the connected part
\( F_{sy} \) = Specified yield point of the connected part

b. Tension in Connected Part. Tearing of the net section in tension is caused by stress concentrations resulting from the presence of holes and the concentrated force transmitted by the bolt to the sheets. According to Section E3.2 of the AISI Specification, the tension force on the net section of a bolted connection shall not exceed \( T_a \) from Section C2 of the AISI Specification or \( P_a \) calculated as follows:

\[ P_a = P_n/\Omega_t \]  \hspace{1cm} (VII.54)

where

\[ P_n = A_n F_t \]
\( A_n \) = Net section area
\( F_t \) and \( \Omega_t \) are defined as follows:

(a) When \( t \geq 3/16 \) in.:
Use AISC Specification

(b) When \( t < 3/16 \) inch and washers are provided under both the bolt head and nut

\[ F_t = (1.0-0.9r+3rd/s)F_u \leq F_u \]  \hspace{1cm} (VII.55)
\[ \Omega_t = \text{Factor of safety for tension on the net section} \]
\[ = 2.0 \text{ for double shear} \]
\[ = 2.22 \text{ for single shear} \]

(c) When \( t < \frac{3}{16} \text{ inch} \) and either washers are not provided under the bolt head and nut, or only one washer is provided under either the bolt head or nut

\[ F_t = (1.0 - r + 2.5rd/s)F_u \leq F_u \]  \hspace{1cm} (VII.56)

\[ \Omega_t = \text{Factor of safety for tension on the net section} \]
\[ = 2.22 \]

where

- \( r = \text{Force transmitted by the bolt or bolts at the section considered, divided by the tension force in the member at that section. If } r \text{ is less than 0.2, it may be taken equal to zero} \)
- \( s = \text{Spacing of bolts perpendicular to line of stress. In the case of a single bolt, } s = \text{width of sheet} \)
- \( F_t = \text{Nominal tension stress limit on net section} \)
- \( d = \text{Diameter of bolt} \)

\textbf{c. Bearing.} Bearing failure occurs when the steel sheet piles up in front of the bolts. This occurs when the edge distance or longitudinal spacing of the bolts is relatively large. According to Section E3.3 of the AISI Specification, the bearing force shall not exceed \( P_a \) calculated as follows:

\[ P_a = \frac{P_n}{\Omega_b} \]  \hspace{1cm} (VII.57)

where

\[ P_n = F_p \]  \hspace{1cm} (VII.58)
\[ \Omega_b = \text{Safety factor for bearing} \]
\[ F_p = \text{Nominal bearing stress as given in Tables VII.3 and VII.4} \]

Table VII.3
Nominal Bearing Stress for Bolted Connections
With Washers Under Both Bolt Head and Nut

<table>
<thead>
<tr>
<th>Thickness of connected part in.</th>
<th>Type of joint</th>
<th>( F_u / F_{sv} ) ratio of connected part</th>
<th>Nominal bearing stress, ( F_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \geq 0.024 ) but &lt; 3/16</td>
<td>Inside sheet of double shear connection</td>
<td>( \geq 1.15 )</td>
<td>3.33( F_u )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( &lt; 1.15 )</td>
<td>3.00( F_u )</td>
</tr>
<tr>
<td>( \geq 3/16 )</td>
<td>Single shear and outside sheets of double shear connection</td>
<td>No limit</td>
<td>3.00( F_u )</td>
</tr>
</tbody>
</table>

See AISC Specification
Table VII.4
Nominal Bearing Stress for Bolted Connections
Without Washers Under Both Bolt Head and Nut,
or With Only One Washer

<table>
<thead>
<tr>
<th>Thickness of connected part in.</th>
<th>Type of joint</th>
<th>$F_u/F_{sv}$ ratio of connected part</th>
<th>Nominal bearing stress, $F_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>≥ 0.036 but &lt; 3/16</td>
<td>Inside sheet of double shear connection</td>
<td>≥ 1.15</td>
<td>3.00$F_u$</td>
</tr>
<tr>
<td></td>
<td>Single shear and outside sheets of double shear connection</td>
<td>≥ 1.15</td>
<td>2.22$F_u$</td>
</tr>
<tr>
<td>≥ 3/16</td>
<td>See AISC Specification</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. **Shear and Tension in Bolts.** The strengths of the bolts in shear and tension have to be checked for bolted connections. According to Section E3.4 of the AISC Specification, the bolt force resulting from shear, tension or combination of shear and tension shall not exceed allowable bolt force, $P_a$, calculated as follows:

$$P_a = A_b F$$  \hspace{1cm} (VII.59)

where

$A_b =$ Gross cross-sectional area of bolt

$F$ is given by $F_v$, $F_t$ or $F_t'$ in Tables VII.5 and VII.6
### Table VII.5
Allowable Shear and Tension Stresses for Bolts

<table>
<thead>
<tr>
<th>Description of Bolts</th>
<th>Threads not Excluded from Shear Plane</th>
<th>Threads Excluded from Shear Plane</th>
<th>Allowable Tension Stress, $F_t$, ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>A325 Bolts</td>
<td>21</td>
<td>30</td>
<td>44</td>
</tr>
<tr>
<td>A354 Grade B Bolts</td>
<td>24</td>
<td>40</td>
<td>49</td>
</tr>
<tr>
<td>(1/4 in. ≤ d &lt; 1/2 in.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A449 Bolts</td>
<td>18</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>(1/4 in. ≤ d &lt; 1/2 in.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A490 Bolts</td>
<td>28</td>
<td>40</td>
<td>54</td>
</tr>
<tr>
<td>A307 Bolts, Grade A</td>
<td>9</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>(1/4 in. ≤ d &lt; 1/2 in.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A307 Bolts, Grade A</td>
<td>10</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>(d ≥ 1/2 in.)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When bolts are subject to a combination of shear and tension, the tension force shall not exceed to a combination of shear and tension, the tension force shall not exceed the allowable force, $P_a$, based on $F_t^*$, given in Table VII.6, where $f_v$, the shear stress produced by the same forces, shall not exceed the allowable value $F_v$ given in Table VII.5.
Table VII.6
Allowable Tension Stress, $F'_u$, for Bolts Subject to the Combination of Shear and Tension

<table>
<thead>
<tr>
<th>Description of Bolts</th>
<th>Threads Not Excluded from Shear Planes</th>
<th>Threads Excluded from Shear Planes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A325 Bolts</td>
<td>55 - $1.8f_v \leq 44$</td>
<td>55 - $1.4f_v \leq 44$</td>
</tr>
<tr>
<td>A354 Grade BD Bolts</td>
<td>61 - $1.8f_v \leq 49$</td>
<td>61 - $1.4f_v \leq 49$</td>
</tr>
<tr>
<td>A449 Bolts</td>
<td>50 - $1.8f_v \leq 40$</td>
<td>50 - $1.4f_v \leq 40$</td>
</tr>
<tr>
<td>A490 Bolts</td>
<td>68 - $1.8f_v \leq 54$</td>
<td>68 - $1.4f_v \leq 54$</td>
</tr>
<tr>
<td>A307 Bolts, Grade A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>When $1/4\text{in.} \leq d &lt; 1/2 \text{in.}$</td>
<td>23 - $1.8f_v \leq 18$</td>
<td></td>
</tr>
<tr>
<td>When $d \geq 1/2\text{in.}$</td>
<td>26 - $1.8f_v \leq 20$</td>
<td></td>
</tr>
</tbody>
</table>

2. LRFD Criteria. According to Section E3 of the LRFD Specification, the following LRFD design criteria govern bolted connections used for cold-formed steel structural members in which the thickness of the thinnest connected part is less than 3/16 inch. For bolted connections in which the thickness of the thinnest connected part is equal to or greater than 3/16 inch, refer to AISC Specification.

a. Spacing and Edge Distance. According to Section E3.1 of the LRFD Specification, the factored nominal shear strength, $\Phi P_n$, of the connected part along two parallel lines in the direction of applied force shall be determined as follows:

$$P_n = \Phi F_u$$  
(VII.60)

(a) When $F_u/F_{sy} \geq 1.15$:

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\[
\phi = 0.70
\]

(b) When \( \frac{F_u}{F_{sy}} < 1.15 \):
\[
\phi = 0.60
\]

where

\[
\phi = \text{Resistance factor}
\]

\[P_n = \text{Nominal resistance per bolt}\]

\[e = \text{The distance measured in the line of force from the center of a standard hole to the nearest edge of an adjacent hole or to the end of the connected part}\]

\[t = \text{Thickness of thinnest connected part}\]

\[F_u = \text{Tensile strength of the connected part}\]

\[F_{sy} = \text{Specified yield point of the connected part}\]

b. Tension in Connected Part. According to Section E3.2 of the LRFD Specification, the factored nominal tensile strength, \( \phi P_n \), on the net section of the connected part shall be determined as follows:

(a) Washers are provided under both the bolt head and the nut
\[
P_n = (1.0 - 0.9r + 3rd/s)F_u A_n \leq F_u A_n
\]
\[
\phi = 0.65 \text{ for double shear connection}
\]
\[
\phi = 0.55 \text{ for single shear connection}
\]

(b) Either washers are not provided under the bolt head and nut, or only one washer is provided under either the bolt head or nut
\[
P_n = (1.0 - r + 2.5d/s)F_u A_n \leq F_u A_n
\]
\[
\phi = 0.65
\]

In addition, the factored nominal tensile strength shall not exceed the following value:
\[
\phi = 0.95
\]
\[ P_n = F_{sy} A_n \]  

(VII.63)

where

\[ A_n = \text{Net area of the connected part} \]
\[ d = \text{Diameter of bolt} \]
\[ r = \text{Force transmitted by the bolt or bolts at the section} \]
\[ \text{considered, divided by the tension force in the member at that} \]
\[ \text{section. If } r \text{ is less than 0.2, it may be taken equal to zero} \]
\[ s = \text{Spacing of bolts perpendicular to line of stress. In the case} \]
\[ \text{of a single bolt, } s = \text{Width of sheet} \]

b. Bearing. According to Section E3.3 of the LRFD Specification, the factored nominal bearing strength, \( \phi P_n \), shall be determined by the values of \( \phi \) and \( P_n \) given in Tables VII.7 and VII.8 for the applicable thickness and \( F_u/F_{sy} \) ratio of the connected part and the type of joint used in the connection.

**TABLE VII.7**
Nominal Bearing Strength for Bolted Connections With Washers Under Both Bolt Head and Nut

<table>
<thead>
<tr>
<th>Thickness of Connected Part in.</th>
<th>Type of Joint</th>
<th>( F_u/F_{sy} ) ratio of Connected Part</th>
<th>Resistance Factor ( \phi )</th>
<th>Nominal Resistance ( P_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \geq 0.024 ) but ( &lt; 3/16 )</td>
<td>Inside sheet of double shear connection</td>
<td>( \geq 1.15 )</td>
<td>0.55</td>
<td>( 3.33F_u dt )</td>
</tr>
<tr>
<td></td>
<td>Single shear and outside sheets of double shear connection</td>
<td>( &lt; 1.15 )</td>
<td>0.65</td>
<td>( 3.00F_u dt )</td>
</tr>
<tr>
<td>( \geq 3/16 )</td>
<td>See Section E3</td>
<td>No limit</td>
<td>0.60</td>
<td>( 3.00F_u dt )</td>
</tr>
</tbody>
</table>
**TABLE VII.8**
Nominal Bearing Strength for Bolted Connections
Without Washers Under Both Bolt Head and Nut, or With Only One Washer

<table>
<thead>
<tr>
<th>Thickness of Connected Part in.</th>
<th>Type of Joint</th>
<th>$F_u/F_{sy}$ ratio of Connected Part</th>
<th>Resistance Factor $\phi$</th>
<th>Nominal Resistance $P_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq 0.036$ but $&lt; 3/16$</td>
<td>Inside sheet of double shear connection</td>
<td>$\geq 1.15$</td>
<td>0.70</td>
<td>$3.00 F_u dt$</td>
</tr>
<tr>
<td></td>
<td>Single shear and outside sheets of double shear connection</td>
<td>$\geq 1.15$</td>
<td>0.65</td>
<td>$2.22 F_u dt$</td>
</tr>
<tr>
<td>$\geq 3/16$</td>
<td>See Section E3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. Shear and Tension in Bolts. According to Section E3.4 of the LRFD Specification, the bolt force in shear or tension produced by factored loads shall not exceed the factored nominal strength, $\phi P_n$, determined as follows:

$$\phi = \text{Resistance factor given in Table VII.9}$$

$$P_n = A_b F_n$$  \hspace{1cm}  (VII.64)

where

- $A_b = \text{Gross cross-sectional area of bolt}$
- $F_n$ is given by $F_{nv}$ or $F_{nt}$ in Table VII.9.

When bolts are subject to a combination of shear and tension produced by factored loads, the tension force shall not exceed the factored nominal strength, $\phi P_n$, based on $\phi = 0.75$ and $P_n = A_b F'_{nt}$, where $F'_{nt}$ is given in Table VII.10, in which $f_v$ is the shear stress produced by the same factored loads. The shear force shall not exceed...
the factored shear strength, $\phi A_{bF_{nv}}$, determined in accordance with Table VII.9.

3. **Comparison.** The allowable load per bolt for allowable stress design can be determined as $P_{a} = P_{n}/\Omega$. For the LRFD criteria, the allowable load per bolt can be calculated from the following equation developed from Eq. (II.6):

$$ (P_{a})_{LRFD} = \phi P_{n} (D/L+1)/(1.2D/L+1.6) \tag{VII.65} $$

**a. Spacing and Edge Distance.** For allowable stress design, the allowable load can be computed for a given edge distance by solving for $P$ in Eqs. (VII.52) and (VII.53).

For $F_{u}/F_{sy} \geq 1.15$,

$$ (P_{a})_{ASD} = 0.5teF_{u} \tag{VII.66} $$

For $F_{u}/F_{sy} < 1.15$,

$$ (P_{a})_{ASD} = 0.45teF_{u} \tag{VII.67} $$

The allowable load for LRFD can be computed using Eq. (VII.65). The allowable loads from Eqs. (VII.66) and (VII.67) were derived from the ultimate load in Eq. (VII.60) using a factor of 2.00 and 2.22, respectively. Therefore, the allowable load ratios based on plate shearing around the bolt can be computed from the following:

For $F_{u}/F_{sy} \geq 1.15$, $\phi = 0.70$:

$$ \frac{(P_{a})_{LRFD}}{(P_{a})_{ASD}} = 1.4 \frac{D/L+1}{1.2D/L+1.6} \tag{VII.68} $$

For $F_{u}/F_{sy} < 1.15$, $\phi = 0.60$:

$$ \frac{(P_{a})_{LRFD}}{(P_{a})_{ASD}} = 1.332 \frac{D/L+1}{1.2D/L+1.6} \tag{VII.69} $$
<table>
<thead>
<tr>
<th>Description of Bolts</th>
<th>Tensile Strength</th>
<th>Shear Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Resistance</td>
<td>Resistance</td>
</tr>
<tr>
<td></td>
<td>Factor $\phi$</td>
<td>Stress $F_{nt}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Factor $\phi$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stress $F_{nv}$</td>
</tr>
<tr>
<td>A307 Bolts, Grade A (1/4 in. ≤ $d$ &lt; 1/2 in.)</td>
<td>0.75</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>40.5</td>
<td>24.0</td>
</tr>
<tr>
<td>A307 Bolts, Grade A (d ≥ 1/2 in.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>45.0</td>
<td>27.0</td>
</tr>
<tr>
<td>A325 bolts, when threads are not excluded from shear planes</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>90.0</td>
<td>54.0</td>
</tr>
<tr>
<td>A325 bolts, when threads are excluded from shear planes</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>90.0</td>
<td>72.0</td>
</tr>
<tr>
<td>A354 Grade B Bolts (1/4 in. ≤ $d$ &lt; 1/2 in.), when threads are not excluded from shear planes</td>
<td>101.0</td>
<td>59.0</td>
</tr>
<tr>
<td>A354 Grade B Bolts (1/4 in. ≤ $d$ &lt; 1/2 in.), when threads are excluded from shear planes</td>
<td>101.0</td>
<td>90.0</td>
</tr>
<tr>
<td>A449 Bolts (1/4 in. ≤ $d$ &lt; 1/2 in.), when threads are not excluded from shear planes</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>81.0</td>
<td>47.0</td>
</tr>
<tr>
<td>A449 Bolts (1/4 in. ≤ $d$ &lt; 1/2 in.), when threads are excluded from shear planes</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>81.0</td>
<td>72.0</td>
</tr>
<tr>
<td>A490 Bolts, when threads are not excluded from shear planes</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>112.5</td>
<td>67.5</td>
</tr>
<tr>
<td>A490 Bolts, when threads are excluded from shear planes</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>112.5</td>
<td>90.0</td>
</tr>
</tbody>
</table>
TABLE VII.10
Nominal Tension Stress, \( F'_{nt} \), for Bolts
Subject to the Combination of Shear and Tension

<table>
<thead>
<tr>
<th>Description of Bolts</th>
<th>Threads Not Excluded from Shear Planes</th>
<th>Threads Excluded from Shear Planes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A325 Bolts</td>
<td>113-2.4f_v ≤ 90</td>
<td>113-1.9f_v ≤ 90</td>
</tr>
<tr>
<td>A354 Grade BD Bolts</td>
<td>127-2.4f_v ≤ 101</td>
<td>127-1.9f_v ≤ 101</td>
</tr>
<tr>
<td>A449 Bolts</td>
<td>101-2.4f_v ≤ 81</td>
<td>101-1.9f_v ≤ 81</td>
</tr>
<tr>
<td>A490 Bolts</td>
<td>141-2.4f_v ≤ 112.5</td>
<td>141-1.9f_v ≤ 112.5</td>
</tr>
<tr>
<td>A307 Bolts, Grade A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>when 1/4 in. ≤ d &lt; 1/2 in.</td>
<td>47-2.4f_v ≤ 40.5</td>
<td></td>
</tr>
<tr>
<td>when d ≥ 1/2 in.</td>
<td></td>
<td>52-2.4f_v ≤ 45</td>
</tr>
</tbody>
</table>

Figure 59 shows the relationships between allowable load ratio and dead-to-live load ratio for Eqs. (VII.68) and (VII.69). For D/L = 0.5, the allowable loads based on the LRFD criteria are from 4.5% to 9.2% lower than the values based on allowable stress design.
Figure 59. Allowable Load Ratio vs. D/L Ratio for Minimum Edge Distance of Bolts
b. **Tension in Connected Parts.** For allowable stress design, the allowable tension on the net section can be computed by Eq. (VII.70).

\[
(P_a)_{ASD} = A_n F_t / \Omega_t
\]  
(VII.70)

For LRFD, the allowable tension on the net section can be computed using Eq. (VII.65).

The allowable load for double shear connections with washers based on allowable stress design was derived from the nominal tensile load and a factor of safety of 2.0. For single shear connections without washers, a factor of safety of 2.22 was used for allowable stress design. The yielding criteria for the net section was studied in Chapter III of this report. The allowable load ratios can be computed as follows:

For double shear connections with washers and \( \phi = 0.65 \),

\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.0\phi \frac{D/L+1}{1.2D/L+1.6} = 1.30 \frac{D/L+1}{1.2D/L+1.6}
\]  
(VII.71)

For single shear connections with washers and \( \phi = 0.55 \),

\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.22\phi \frac{D/L+1}{1.2D/L+1.6} = 1.221 \frac{D/L+1}{1.2D/L+1.6}
\]  
(VII.72)

For connections without washers and \( \phi = 0.65 \),

\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.22\phi \frac{D/L+1}{1.2D/L+1.6} = 1.443 \frac{D/L+1}{1.2D/L+1.6}
\]  
(VII.73)

Figure 60 shows the allowable load ratio versus dead-to-live load ratio for the three cases represented by Eqs. (VII.71), (VII.72), and (VII.73). As shown in the figure, the criteria for tension on the net section result in a wide range of allowable load ratios. For \( D/L = 0.5 \),
the allowable loads based on the LRFD criteria are from 1.8% to 16.7% lower than the values based on allowable stress design. The difference depends on the use of washers and the type of connections. Figure 60 also shows that LRFD is very conservative for connections with washers under the bolt head and nut compared with allowable stress design.

c. Bearing. The allowable load based on allowable stress design can be computed using the following equation:

\[
(P_a)_{ASD} = \frac{F_{ptd}}{Q_{b}}
\]

(VII.74)

For LRFD, Eq. (VII.65) can be used to calculate the allowable load.

The factor of safety used in the development of the allowable stress design formulas was 2.22. Therefore, the allowable load ratios can be computed as follows:

(i) Connections with washers:

For inside sheets of double shear connections with \( \frac{F_u}{F_{sy}} \geq 1.15 \) and \( \phi = 0.55 \),

\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.221 \frac{D/L+1}{1.2D/L+1.6} \quad \text{ (VII.75)}
\]

For inside sheets of double shear connections with \( \frac{F_u}{F_{sy}} < 1.15 \) and \( \phi = 0.65 \),

\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.443 \frac{D/L+1}{1.2D/L+1.6} \quad \text{ (VII.76)}
\]

For single shear and outside sheets of double shear connections with \( \phi = 0.60 \),

\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.332 \frac{D/L+1}{1.2D/L+1.6} \quad \text{ (VII.77)}
\]

(ii) Connections without washer or with only one washer:
For inside sheets of double shear connections with \( \frac{F_u}{F_{sy}} \geq 1.15 \) and \( \phi = 0.70 \),

\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.554 \frac{D/L+1}{1.2D/L+1.6} \quad (VII.78)
\]

For single shear and outside sheets of double shear connections with \( \frac{F_u}{F_{sy}} \geq 1.15 \) and \( \phi = 0.65 \),

\[
\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.443 \frac{D/L+1}{1.2D/L+1.6} \quad (VII.79)
\]

The relationships between allowable load ratio and dead-to-live load ratio for Eqs. (VII.75) through (VII.79) are shown in Figure 61. As shown in the figure, the criteria for bearing strength of bolted connections result in a wide range of values for allowable load ratio. For \( D/L = 0.5 \), the allowable loads based on LRFD are from 6\% higher to 16.7\% lower than the values obtained from allowable stress design. The difference between the allowable loads will depend upon the use of the washers, the shear conditions, and the \( \frac{F_u}{F_{sy}} \) ratio. Inside sheets of double shear bolted connection with washers designed using LRFD will be very conservative compared with allowable stress design.

d. Shear and Tension in Bolts. The allowable load based on allowable stress design can be computed as follows:

\[
(P_a)_{ASD} = A_b F \quad (VII.80)
\]

For LRFD, Eq. (VII.65) can be used to calculate the allowable load.

Therefore, the allowable load ratio for shear or tension of bolts is:
Figure 60. Allowable Load Ratio vs. D/L Ratio for Tension on Net Section
Figure 61. Allowable Load Ratio vs. D/L Ratio for Bearing Strength of Bolted Connections
Equation (VII.81) can be expressed in the following form:

\[
\frac{(P_a)_{\text{LRFD}}}{(P_a)_{\text{ASD}}} = \frac{\phi A_d F_n}{A_b F} \left( \frac{D/L+1}{1.2D/L+1.6} \right) = \frac{\phi F_n}{F} \left( \frac{D/L+1}{1.2D/L+1.6} \right) \tag{VII.81}
\]

where

\[
K_b = \frac{\phi F_n}{F} \tag{VII.83}
\]

Table VII.11 lists the values of \(K_b\) calculated from the values of \(F\), \(F_n\), and \(\phi\) provided in Tables VII.5 and VII.9. Figures 62 through 66 show the relationships between allowable load ratio and dead-to-live load ratio for the bolts in Table VII.11 using Eq. (VII.82).

Figure 62 shows the allowable load ratio versus dead-to-live load ratio for A325 bolts based on shear and tension strengths. As seen from this figure, for \(D/L = 0.5\), the allowable tensile load based on LRFD design is 4.6% larger than the value based on allowable stress design. Also for \(D/L = 0.5\), when threads are included in the shear plane, the allowable shear load based on LRFD design is 13.9% larger than the value based on allowable stress design; when threads are not included in the shear plane, the allowable shear load based on LRFD design is 6.4% larger than the value based on allowable stress design. It can also be seen from this figure that LRFD design will always result in a larger allowable shear load than allowable stress design when threads are included in the shear plane.
Table VII.11
K_b Values for Standard Bolts

<table>
<thead>
<tr>
<th>Description of Bolts</th>
<th>Shear Strength</th>
<th>Tension Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Threads not Excluded from Shear Plane</td>
<td>Threads Excluded from Shear Plane</td>
</tr>
<tr>
<td>A325 Bolts</td>
<td>1.671</td>
<td>1.560</td>
</tr>
<tr>
<td>A354 Grade B Bolts</td>
<td>1.598</td>
<td>1.463</td>
</tr>
<tr>
<td>(1/4 in. ≤ d &lt; 1/2 in.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A449 Bolts</td>
<td>1.697</td>
<td>1.560</td>
</tr>
<tr>
<td>(1/4 in. ≤ d &lt; 1/2 in.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A490 Bolts</td>
<td>1.567</td>
<td>1.463</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A307 Bolts, Grade A</td>
<td></td>
<td>1.733</td>
</tr>
<tr>
<td>(1/4 in. ≤ d &lt; 1/2 in.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A307 Bolts, Grade A</td>
<td>1.755</td>
<td>1.688</td>
</tr>
<tr>
<td>(d ≥ 1/2 in.)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 63 shows the allowable load ratio versus dead-to-live load ratio for A354 Grade B bolts (1/4 in. ≤ d < 1/2 in.) based on shear and tension strengths. As seen from this figure, for D/L = 0.5, the allowable tensile load based on LRFD design is 5.4% larger than the value based on allowable stress design. Also for D/L = 0.5, when threads are included in the shear plane, the allowable shear load based on LRFD design is 9% larger than the value based on allowable stress design; when threads are not included in the shear plane, the allowable shear load based on LRFD design is the same as the value based on allowable stress design.
stress design. It can also be seen from this figure that LRFD design will always result in a larger allowable shear load than allowable stress design when threads are included in the shear plane.

Figure 64 shows the allowable load ratio versus dead-to-live load ratio for A449 bolts (1/4 in. ≤ d < 1/2 in.) based on shear and tension strengths. As seen from this figure, for D/L = 0.5, the allowable tensile load based on LRFD design is 3.6% larger than the value based on allowable stress design. Also for D/L = 0.5, when threads are included in the shear plane, the allowable shear load based on LRFD design is 15.7% larger than the value based on allowable stress design; when threads are not included in the shear plane, the allowable shear load based on LRFD design is 6.4% larger than the value based on allowable stress design. It can also be seen from this figure that LRFD design will always result in a larger allowable shear load than allowable stress design when threads are included in the shear plane.

Figure 65 shows the allowable load ratio versus dead-to-live load ratio for A490 bolts based on shear and tension strengths. As seen from this figure, for D/L = 0.5, the allowable tensile load based on LRFD design is 6.6% larger than the value based on allowable stress design. Also for D/L = 0.5, when threads are included in the shear plane, the allowable shear load based on LRFD design is 6.8% larger than the value based on allowable stress design; when threads are not included in the shear plane, the allowable shear load based on LRFD design is the same as the value based on allowable stress design.
Figure 62. Allowable Load Ratio vs. D/L Ratio for Shear or Tension Strength on A325 Bolts
Shear Strength (Threads Included)

Tension Strength

Shear Strength (Threads Excluded)

Figure 63. Allowable Load Ratio vs. D/L Ratio for Shear or Tension Strength on A354 Grade B Bolts (1/4 in. ≤ d < 1/2 in.)
Figure 64. Allowable Load Ratio vs. D/L Ratio for Shear or Tension Strength on A449 Bolts (1/4 in. ≤ d < 1/2 in.)
Figure 65. Allowable Load Ratio vs. D/L Ratio for Shear or Tension Strength on A490 Bolts
Figure 66. Allowable Load Ratio vs. D/L Ratio for Shear or Tension Strength on A307 Bolts, Grade A
Figure 66 shows the allowable load ratio versus dead-to-live load ratio for A307 Grade A bolts based on shear and tension strengths. As seen from this figure, for D/L = 0.5, the allowable tensile load based on LRFD design is 15.1% larger than the value based on allowable stress design. Also for D/L = 0.5, when 1/4 in. ≤ d < 1/2 in., the allowable shear load based on LRFD design is 18.2% larger than the value based on allowable stress design; when d ≥ 1/2 in., the allowable shear load based on LRFD design is 19.7% larger than the value based on allowable stress design. It can also be seen from this figure that LRFD design will always result in a larger allowable load than allowable stress design for all three cases.

When bolts are subject to a combination of shear and tension, the unfactored shear force can be calculated for both ASD and LRFD methods using the following equation:

\[ V_T = V_{DL} + V_{LL} \]  \hspace{1cm} (VII.84)

where

- \( V_T \) = total unfactored shear force
- \( V_{DL} \) = shear force due to the nominal dead load
- \( V_{LL} \) = shear force due to the nominal live load

The factored shear force for LRFD design can be expressed as Eq. (VII.85) by using Eq. (II.6):

\[ V_D = \frac{1.2D/L+1.6}{D/L+1} \]  \hspace{1cm} (VII.85)
Therefore, the allowable load ratio for tensile strength when bolts are subject to a combination of shear and tension can be developed as follows by using Eq. (VII.65):

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = \frac{\phi(C_1 - D_1 f)}{C - D f} \frac{1.2D/L + 1.6}{D/L + 1} \frac{D/L + 1}{1.2D/L + 1.6}$$

(VII.86)

where

$\phi = 0.75$

$f = V_T / A_b$

$C$ and $D$ are tabulated in Table VII.6

$C_1$ and $D_1$ are tabulated in Table VII.10

Figures 67 through 76 show the relationships between allowable load ratio and dead-to-live load ratio for tensile strength of bolts which are subject to a combination of shear and tension by using Eq. (VII.86).

Figure 67 shows the allowable load ratio versus dead-to-live load ratio for A325 bolts when threads are included in shear plane. The different curves in this figure represent different unfactored shear stresses $f$. For $D/L$ ratio around 0.18, both design methods would result in the same allowable tensile load for the unfactored shear stresses $f$ shown in the figure. For $D/L > 0.18$, the larger the unfactored shear stress, the higher the allowable load ratio. For example, for $D/L = 0.5$, the $(P_a)_{LRFD}/(P_a)_{ASD}$ ratios are 1.162 and 1.066 for $f = 21$ ksi and 7 ksi, respectively.
Figure 67. Allowable Load Ratio vs. D/L Ratio for Tension Strength on A325 Bolts Subject to the Combination of Shear and Tension - Case A
Figure 68. Allowable Load Ratio vs. D/L Ratio for Tension Strength on A325 Bolts Subject to the Combination of Shear and Tension - Case B
Figure 69. Allowable Load Ratio vs. D/L Ratio for Tension Strength on A354 Grade BD Bolts Subject to the Combination of Shear and Tension - Case A
Figure 70. Allowable Load Ratio vs. D/L Ratio for Tension Strength on A354 Grade BD Bolts Subject to the Combination of Shear and Tension - Case B
Figure 71. Allowable Load Ratio vs. D/L Ratio for Tension Strength on A449 Bolts Subject to the Combination of Shear and Tension - Case A
Figure 68 shows the allowable load ratio versus dead-to-live load ratio for A325 bolts when threads are excluded in shear plane. The different curves in this figure represent different unfactored shear stresses \( f \). For D/L ratio around 0.28, the LRFD criteria gives an allowable load about 2% more than the value computed from allowable stress design for all unfactored shear stresses \( f \) shown in the figure. For D/L > 0.28, the larger the unfactored shear stress, the higher the allowable load ratio. For D/L = 0.5, the \( (P_a)_{LRFD}/(P_a)_{ASD} \) ratios are 1.143 and 1.062 for \( f = 29 \) ksi and 10 ksi, respectively.

Figure 69 shows the allowable load ratio versus dead-to-live load ratio for A354 Grade BD bolts when threads are included in shear plane. The different curves in this figure represent different unfactored shear stresses \( f \). For D/L ratio around 0.1, both design methods would result in the same allowable tensile load for the unfactored shear stresses \( f \) shown in the figure. For D/L > 0.1, the larger the unfactored shear stress, the higher the allowable load ratio. For D/L = 0.5, the \( (P_a)_{LRFD}/(P_a)_{ASD} \) ratios are 1.222 and 1.085 for \( f = 24 \) ksi and 8 ksi, respectively.

Figure 70 shows the allowable load ratio versus dead-to-live load ratio for A354 Grade BD bolts when threads are excluded in shear plane. The different curves in this figure represent different unfactored shear stresses \( f \). For D/L ratio around 0.2, the LRFD criteria give an allowable load about 2% more than the value computed from allowable stress design for all unfactored shear stresses \( f \) shown in the figure. For D/L > 0.2, the larger the unfactored shear stress, the higher the
allowable load ratio. For $D/L = 0.5$, the $(P_a)_{LRFD}/(P_a)_{ASD}$ ratios are 1.287 and 1.079 for $f = 36$ ksi and 10 ksi, respectively.

Figure 71 shows the allowable load ratio versus dead-to-live load ratio for A449 bolts when threads are included in shear plane. The different curves in this figure represent different unfactored shear stresses $f$. For $D/L$ ratio around 0.27, both design methods would result in the same allowable tensile load for the unfactored shear stresses $f$ shown in the figure. For $D/L > 0.27$, the larger the unfactored shear stress, the higher the allowable load ratio. For $D/L = 0.5$, the $(P_a)_{LRFD}/(P_a)_{ASD}$ ratios are 1.094 and 1.042 for $f = 18$ ksi and 6 ksi, respectively.

Figure 72 shows the allowable load ratio versus dead-to-live load ratio for A449 bolts when threads are excluded in shear plane. The different curves in this figure represent different unfactored shear stresses $f$. For $D/L$ ratio around 0.38, the LRFD criteria gives an allowable load about 2% more than the value computed from allowable stress design for all unfactored shear stresses $f$ shown in the figure. For $D/L > 0.38$, the larger the unfactored shear stress, the higher the allowable load ratio. For $D/L = 0.5$, the $(P_a)_{LRFD}/(P_a)_{ASD}$ ratios are 1.098 and 1.039 for $f = 29$ ksi and 10 ksi, respectively.

Figure 73 shows the allowable load ratio versus dead-to-live load ratio for A490 bolts when threads are included in shear plane. The different curves in this figure represent different unfactored shear stresses $f$. For $D/L$ ratio around 0.12, both design methods would result in the same allowable tensile load for the unfactored shear stresses $f$ shown in the figure. For $D/L > 0.12$, the larger the unfactored shear
stress, the higher the allowable load ratio. For $D/L = 0.5$, the $(P_a)_{LRFD}/(P_a)_{ASD}$ ratios are 1.211 and 1.079 for $f = 27$ ksi and 9 ksi, respectively.

Figure 74 shows the allowable load ratio versus dead-to-live load ratio for A490 bolts when threads are excluded in shear plane. The different curves in this figure represent different unfactored shear stresses $f$. For $D/L$ ratio around 0.22, the LRFD criteria give an allowable load about 2% more than the value computed from allowable stress design for all unfactored shear stresses $f$ shown in the figure. For $D/L > 0.22$, the larger the unfactored shear stress, the higher the allowable load ratio. For $D/L = 0.5$, the $(P_a)_{LRFD}/(P_a)_{ASD}$ ratios are 1.182 and 1.074 for $f = 36$ ksi and 12 ksi, respectively.

Figure 75 shows the allowable load ratio versus dead-to-live load ratio for A307 Grade A bolts when $1/4 \text{ in.} \leq d < 1/2 \text{ in.}$ The different curves in this figure represent different unfactored shear stresses $f$. For $D/L$ ratio around 0.2, both design methods would result in the same allowable tensile load for the unfactored shear stresses $f$ shown in the figure. For $D/L > 0.2$, the larger the unfactored shear stress, the higher the allowable load ratio. For $D/L = 0.5$, the $(P_a)_{LRFD}/(P_a)_{ASD}$ ratios are 1.152 and 1.059 for $f = 9$ ksi and 3 ksi, respectively.

Figure 76 shows the allowable load ratio versus dead-to-live load ratio for A307 Grade A bolts when $d \geq 1/2 \text{ in.}$ The different curves in this figure represent different unfactored shear stresses $f$. For $D/L$ ratio around 0.32, both design methods would result in the same allowable tensile load for the unfactored shear stresses $f$ shown in
the figure. For $D/L > 0.32$, the larger the unfactored shear stress, the higher the allowable load ratio. For $D/L = 0.5$, the $(P_a)^{LRFD}/(P_a)^{ASD}$ ratios are 1.074 and 1.030 for $f = 10$ ksi and 3.5 ksi, respectively.
Figure 72. Allowable Load Ratio vs. D/L Ratio for Tension Strength on A449 Bolts Subject to the Combination of Shear and Tension - Case B
Figure 73. Allowable Load Ratio vs. D/L Ratio for Tension Strength on A490 Bolts Subject to the Combination of Shear and Tension - Case A
Figure 74. Allowable Load Ratio vs. D/L Ratio for Tension Strength on A490 Bolts Subject to the Combination of Shear and Tension - Case B
Figure 75. Allowable Load Ratio vs. D/L Ratio for Tension Strength on A307 Bolts, Grade A, Subject to the Combination of Shear and Tension (1/4 in. ≤ d < 1/2 in.)
Figure 76. Allowable Load Ratio vs. D/L Ratio for Tension Strength on A307 Bolts, Grade A, Subject to the Combination of Shear and Tension (d ≥ 1/2 in.)
VIII. STIFFENERS

A. TRANSVERSE STIFFENERS

1. Allowable Stress Design. According to Section B6.1 of the AISI Specification, transverse stiffeners attached to beam webs at points of concentrated loads or reactions, shall be designed as compression members. Concentrated loads or reactions shall be applied directly into the stiffeners, or each stiffener shall be fitted accurately to the flat portion of the flange to provide direct load bearing into the end of the stiffener. Means for shear transfer between the stiffener and the web shall be provided according to Chapter E of the AISI Specification. The concentrated loads or reactions shall not exceed the smaller of the allowable loads, \( P_a \), given by (a) and (b) as follows:

(a) \( P_a = \frac{P_n}{\Omega_{st}} \)  

(b) \( P_a = \frac{P_n}{\Omega_c} \)

where

\( P_n = F_{wy}A_c \)  

\( \Omega_{st} = 2.00 \)  

\( A_c = 18t^2+A_s \), for transverse stiffeners at interior support and under concentrated load  

\( A_c = 10t^2+A_s \), for transverse stiffeners at end support

\( F_{wy} = \text{Lower value of beam web, } F_y \text{ or stiffener section, } F_{ys} \)
the AISI Specification with $A_e$ replaced by $A_b$

\[ \Omega_c = \text{Factor of safety for axial compression evaluated according to Section C4(a) of the AISI Specification} \]

\[ A_b = b_1 t + A_s, \text{ for transverse stiffeners at interior support and under concentrated load} \quad \text{(VIII.6)} \]

\[ A_b = b_2 t + A_s, \text{ for transverse stiffeners at end support} \quad \text{(VIII.7)} \]

\[ A_s = \text{Cross sectional area of transverse stiffeners} \]

\[ b_1 = 25t(0.0024(L_{st}/t)+0.72) \leq 25t \quad \text{(VIII.8)} \]

\[ b_2 = 12t(0.0044(L_{st}/t)+0.83) \leq 12t \quad \text{(VIII.9)} \]

\[ L_{st} = \text{Length of transverse stiffener} \]

\[ t = \text{Base thickness of beam web} \]

The $w/t_s$ ratio for the stiffened and unstiffened elements of cold-formed steel transverse stiffeners shall not exceed $1.28\sqrt{\frac{E}{F_{ys}}}$ and $0.37\sqrt{\frac{E}{F_{ys}}}$ respectively, where $F_{ys}$ is the yield stress, $F_y$, and $t_s$ the thickness of the stiffener steel.

2. LRFD Criteria. According to Section B6.1 of the LRFD Specification, transverse stiffeners attached to beam webs at points of concentrated loads or reactions, shall be designed as compression members. Concentrated loads or reactions shall be applied directly into the stiffeners, or each stiffener shall be fitted accurately to the flat portion of the flange to provide direct load bearing into the end of the stiffener. Means for shear transfer between the stiffener and the web shall be provided according to Chapter E of the LRFD Specification. The concentrated loads or reactions determined on the basis of factored loads shall not exceed the factored nominal strength, $\phi c P_n$, where $\phi_c = 0.85$ and $P_n$ is the smaller value given by provisions.
(a) and (b) in Section B6.1 of the LRFD Specification which are the same as those specified in Section B6.1 of the AISI Specification mentioned above.

3. **Comparison.** The unfactored load applied to the stiffener can be computed for both design methods by using the following formula:

\[ P_T = P_{DL} + P_{LL} \] (VIII.10)

where

- \( P_T \) = unfactored compressive load
- \( P_{DL} \) = compressive load due to the nominal axial dead load
- \( P_{LL} \) = compressive load due to the nominal axial live load

The total unfactored load should be less than or equal to the allowable loads computed from allowable stress design and LRFD. For allowable stress design, the allowable loads are

\[ (P_a)_{ASD1} = \frac{P_{n1}}{\Omega_{st}} \] (VIII.11)
\[ (P_a)_{ASD2} = \frac{P_{n2}}{\Omega_c} \] (VIII.12)

For LRFD, the allowable axial loads can be computed by using the following equations developed from Eq. (11.6):

\[ (P_a)_{LRFD1} = \phi_c \frac{P_{n1}}{(D/L+1)/(1.2D/L+1.6)} \] (VIII.13)
\[ (P_a)_{LRFD2} = \phi_c \frac{P_{n2}}{(D/L+1)/(1.2D/L+1.6)} \] (VIII.14)

where

- \( P_{n1} \) = Nominal compression strength specified in provision (a) of Section B6.1
- \( P_{n2} \) = Nominal compression strength specified in provision (b) of Section B6.1

In order to study the allowable load ratios, three different cases were considered:
(1) Case 1: $P_{n1} \leq P_{n2}$, then Eqs. (VIII.11) and (VIII.13) can be used to determine the allowable load ratio as follow:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = \frac{D/L+1}{1.2D/L+1.6} = 1.7 \frac{D/L+1}{1.2D/L+1.6}$$  \hspace{1cm} (VIII.15)

(2) Case 2: $P_{n1} > P_{n2}$ and $P_{n1}/\Omega_{st} > P_{n2}/\Phi_c$, then Eqs. (VIII.12) and (VIII.14) can be used to determine the allowable load ratio as follow:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = \frac{\Omega_c \Phi_c}{1.2D/L+1.6} = 0.85 \frac{\Omega_c \Phi_c}{1.2D/L+1.6}$$  \hspace{1cm} (VIII.16)

where

$$\Omega_c = \frac{5}{3} + (3/8)R - (1/8)R^3$$  \hspace{1cm} (VIII.17)

$$R = \sqrt{\frac{F_y}{2F_e}}$$  \hspace{1cm} (VIII.18)

(3) Case 3: $P_{n1} > P_{n2}$ and $P_{n1}/\Omega_{st} < P_{n2}/\Omega_c$, then Eqs. (VIII.11) and (VIII.14) can be used to determine the allowable load ratio as follow:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = \frac{P_{n2}}{P_{n1}} \left[ \frac{D/L+1}{1.2D/L+1.6} \right] = 1.7 \frac{P_{n2}}{P_{n1}} \left[ \frac{D/L+1}{1.2D/L+1.6} \right]$$  \hspace{1cm} (VIII.19)

Figure 77 shows the allowable load ratio versus dead-to-live load ratio for the compression strength of transverse stiffeners determined by Eq. (VIII.15). For this case, the LRFD criteria always permit larger allowable loads than the allowable stress design. For $D/L = 0.5$, the LRFD criteria give an allowable load about 16% greater than the load obtained by using allowable stress design.
Figure 77. Allowable Load Ratio vs. D/L Ratio for Compression Strength of Transverse Stiffeners-Case 1
Figure 78. Allowable Load Ratio vs. D/L Ratio for Compression Strength of Transverse Stiffeners—Case 2
Figure 79. Allowable Load Ratio vs. D/L Ratio for Compression Strength of Transverse Stiffeners—Case 3
Figure 78 shows the allowable load ratio versus dead-to-live load ratio for the compression strength of transverse stiffeners determined by Eq. (VIII.16). Different curves represent different values of \( \Omega_c \). For \( \Omega_c \) values from 1.67 to 1.92 and \( D/L = 0.5 \), the allowable loads determined by LRFD criteria are from 3.2% lower to 11.2% higher than the allowable loads determined by the allowable stress design.

Figure 79 shows the allowable load ratio versus dead-to-live load ratio for the compression strength of transverse stiffeners determined by Eq. (VIII.19). Different curves represent different values of \( P_{n2}/P_{n1} \). For \( P_{n2}/P_{n1} \) values from 0.835 to 1.0 and \( D/L = 0.5 \), the allowable loads determined by LRFD criteria are from 3.2% lower to 16% higher than the allowable loads determined by the allowable stress design.

B. SHEAR STIFFENERS

1. Allowable Stress Design. According to Section B6.2 of the AISI Specification, where shear stiffeners are required, the spacing shall be such that the web shear force shall not exceed the allowable shear force, \( V_a \), permitted by Section C3.2 of the AISI Specification, and the ratio \( a/h \) shall not exceed \( \sqrt{260/(h/t)} \) nor 3.0.

The actual moment of inertia, \( I_s \), of a pair of attached shear stiffeners, or of a single shear stiffener, with reference to an axis in the plane of the web, shall have a minimum value of

\[
I_{smin} = 5ht^2[h/a-0.7(a/h)] \geq (h/50)^4
\]  \hspace{1cm} (VIII.20)

The gross area of shear stiffeners shall be not less than

\[
A_{st} = \frac{((1-C_v)/2)[a/h-(a/h)^2/[(a/h)+\sqrt{1+(a/h)^2}]]YDht}{(a/h)^2 (a/h)^2} \]  \hspace{1cm} (VIII.21)
where

\[
C_v = 45,000k_y/[F_y(h/t)^2] \quad \text{when} \quad C_v \leq 0.8 \quad \text{(VIII.22)}
\]

\[
C_v = \left[190/(h/t)\right]\left(k_y/F_y\right) \quad \text{when} \quad C_v > 0.8 \quad \text{(VIII.23)}
\]

\[
k_v = 4.00 + 5.34/(a/h)^2 \quad \text{when} \quad a/h \leq 1.0 \quad \text{(VIII.24)}
\]

\[
k_v = 5.34 + 4.00/(a/h)^2 \quad \text{when} \quad a/h > 1.0 \quad \text{(VIII.25)}
\]

\(a = \text{Distance between transverse stiffeners}\)

\(Y = \text{Yield point of web steel/Yield point of stiffener steel}\)

\(D = 1.0\) for stiffeners furnished in pairs

\(D = 1.8\) for single-angle stiffeners

\(D = 2.4\) for single-plate stiffeners

2. **LRFD Criteria.** According to Section B6.2 of the LRFD Specification, where shear stiffeners are required, the spacing shall be such that the web shear force determined on the basis of factored loads shall not exceed the factored nominal shear strength, \(\phi V_n\), permitted by Section C3.2, and the ratio \(a/h\) shall not exceed \(\left[260/(h/t)\right]^2\) nor 3.0.

The requirements for the actual moment of inertia and gross area of shear stiffeners are the same as those specified in Section B6.2 of the AISI Specification.

3. **Comparison.** The unfactored shear force can be calculated for both ASD and LRFD methods by using the following equation.

\[
V_T = V_{DL} + V_{LL} \quad \text{(VIII.26)}
\]

where

\(V_T = \text{total unfactored shear force}\)

\(V_{DL} = \text{shear force due to the nominal dead load}\)

\(V_{LL} = \text{shear force due to the nominal live load}\)
This total unfactored shear force should be less than or equal to the allowable shear capacity. For allowable stress design, the allowable shear load is

\[(V_a)_{ASD} = \frac{V_n}{\Omega}\]  \hspace{1cm} (VIII.27)

For LRFD, the allowable shear load equation was developed from Eq. (II.6) and is

\[(V_a)_{LRFD} = \Phi_v V_n \frac{(D/L+1)}{(1.2D/L+1.6)}\]  \hspace{1cm} (VIII.28)

The allowable shear force, \(V_a\), for allowable stress design is determined from shear yielding with a factor of safety of 1.44, from the critical stress for elastic shear buckling with a factor of safety of 1.71, and from the critical stress for inelastic shear buckling with a factor of safety of 1.67. The limits of the \(h/t\) ratio were obtained by equating the formulas for the three shear failure modes for both allowable stress and LRFD criteria. Because each failure mode has a different factor of safety, the \(h/t\) limits are slightly different for both design criteria. For example, for \(h/t\) greater than \(1.38 \sqrt{\frac{E_k}{F_y}}\) and less than \(1.415 \sqrt{\frac{E_k}{F_y}}\), inelastic shear buckling will govern for LRFD.

The allowable shear ratios are:

For \(h/t \leq \sqrt[4]{\frac{E_k}{F_y}}\) and \(\Phi_v = 1.0,\)

\[
\frac{(V_a)_{LRFD}}{(V_a)_{ASD}} = 1.443\Phi_v \frac{D/L+1}{1.2D/L+1.6} = 1.443 \frac{D/L+1}{1.2D/L+1.6} \]  \hspace{1cm} (VIII.29)
For \( \sqrt{\frac{E_k}{F_y}} \) < h/t ≤ 1.38\( \sqrt{\frac{E_k}{F_y}} \) and \( \phi_v = 0.90 \)

\[
\frac{(V_a)_{LRFD}}{(V_a)_{ASD}} = 1.674\phi_v \frac{D/L+1}{1.2D/L+1.6} = 1.507 \frac{D/L+1}{1.2D/L+1.6}
\]

(VIII.30)

For h/t > 1.415\( \sqrt{\frac{E_k}{F_y}} \) and \( \phi_v = 0.90 \)

\[
\frac{(V_a)_{LRFD}}{(V_a)_{ASD}} = 1.712\phi_v \frac{D/L+1}{1.2D/L+1.6} = 1.541 \frac{D/L+1}{1.2D/L+1.6}
\]

(VIII.31)

Figure 80 shows the allowable shear ratio versus dead-to-live load ratio for the three failure modes. For D/L = 0.5, the allowable shears determined according to LRFD may be up to 5% higher than the values obtained from allowable stress design. For D/L < 0.17, LRFD is generally conservative. When D/L > 0.65, LRFD gives larger values of the allowable shear capacity. It can be seen that this figure is identical to Figure 5 which was obtained for the shear strength of the beam webs.
Figure 80. Allowable Shear Ratio vs. D/L Ratio for Shear Strength of Shear Stiffeners
A. WALL STUDS IN COMPRESSION

1. Allowable Stress Design. According to Section D4.1 of the AISI Specification, for studs having identical sheathing attached to both flanges, and neglecting any rotational restraint provided by the sheathing, the applied axial load, \( P \), shall not exceed \( P_a \) calculated as follows:

\[
P_a = \frac{A_e F_n}{\Omega_c}
\]  

(IX.1)

where

- \( A_e \) = Effective area determined at \( F_n \)
- \( \Omega_c \) = Factor of safety for axial compression
- \( F_n \) = The lowest value determined by the following three conditions:

(a) To prevent column buckling between fasteners in the plane of the wall, \( F_n \) shall be calculated according to Section C4 of the AISI Specification with \( K L \) equal to two times the distance between fasteners.

(b) To prevent flexural and/or torsional overall column buckling, \( F_n \) shall be calculated in accordance with Section C4 of the AISI Specification with \( F_e \) taken as the smaller of the two \( \sigma_{CR} \) values specified for the following section types, where \( \sigma_{CR} \) is the theoretical elastic buckling stress under concentric loading.

1. Singly-symmetric channels and C-Sections

\[
\sigma_{CR} = \sigma_{ey} + \sigma_a
\]  

(IX.2)

\[
\sigma_{CR} = \frac{1}{(2\beta)}\left(\sqrt{(\sigma_{ex} + \sigma_{tQ})^2 - (4\beta \sigma_{ex} \sigma_{tQ})} - (\sigma_{ex} + \sigma_{tQ})\right)
\]  

(IX.3)
(2) Z-Sections

\[ \sigma_{CR} = \sigma_r + \bar{Q}_t \tag{IX.4} \]

\[ \sigma_{CR} = 1/2 \left\{ \sigma_{ex} + \sigma_{ey} + \bar{Q}_a \right\} - \left[ \left( \sigma_{ex} + \sigma_{ey} + \bar{Q}_a \right)^2 - \Delta \left( \sigma_{ex} \sigma_{ey} + \sigma_{ex} \bar{Q}_a - \sigma_{ey} \bar{Q}_a \right) \right\}^{1/2} \tag{IX.5} \]

(3) I-Sections (doubly-symmetric)

\[ \sigma_{CR} = \sigma_{ey} + \bar{Q}_a \tag{IX.6} \]

\[ \sigma_{CR} = \sigma_{ex} \tag{IX.7} \]

In the above formulas

\[ \sigma_{ex} = \pi^2 E / (K_x L_x / r_x)^2 \tag{IX.8} \]

\[ \sigma_{exy} = (\pi^2 E I_{xy}) / (AL^3) \tag{IX.9} \]

\[ \sigma_{ey} = \pi^2 E / (K_y L_y / r_y)^2 \tag{IX.10} \]

\[ \sigma_t = 1 / (Ar_0^2) [GJ + n^2 Ec_w / (K_t L_t)^2] \tag{IX.11} \]

\[ \sigma_t Q = \sigma_t + \bar{Q}_t \tag{IX.12} \]

\[ \bar{Q} = \bar{Q}B = \text{Design shear rigidity for sheathing on both sides of the wall assembly} \tag{IX.13} \]

\[ \bar{q} = \text{Design shear rigidity for sheathing per inch of stud spacing} \]

(see Table IX.1)

\[ B = \text{Stud spacing} \]

\[ \bar{Q}_a = \bar{Q} / A \tag{IX.14} \]

\[ A = \text{Area of full unreduced cross section} \]

\[ L = \text{Length of stud} \]

\[ \bar{Q}_t = (\bar{Q}d^2) / (4Ar_0^2) \tag{IX.15} \]

\[ d = \text{Depth of section} \]

\[ I_{xy} = \text{Product of inertia} \]

(c) To prevent shear failure of the sheathing, a value of \( F_n \) shall be used in the following equations so that the shear strain of the
The shear strain, \( \gamma \), shall be determined as follows:

\[
\gamma = \frac{n}{L}[C_1 + \left( E_1 d/2 \right)] \tag{IX.16}
\]

where

\( C_1 \) and \( E_1 \) are the absolute values of \( C_1 \) and \( E_1 \) specified below for each section type:

1. **Singly-Symmetric Channels**

\[
C_1 = \frac{F_n C_o}{(\sigma_{ey} - F_n + \overline{q}_a)} \tag{IX.17}
\]

\[
E_1 = \frac{F_n \left( (\sigma_{ex} - F_n) (\overline{r}_n^2 \overline{E} - x_D - x_o D) - F_n x_o (D_o - x_o E_o) \right)}{(\sigma_{ex} - F_n) \overline{r}_n^2 (\sigma_{tQ} - F_n) - (F_n x_o)^2} \tag{IX.18}
\]

2. **Z-Sections**

\[
C_1 = \frac{F_n \left( C_o (\sigma_{ex} - F_n) - D_o \sigma_{exy} \right)}{(\sigma_{ey} - F_n + \overline{q}_a)(\sigma_{ex} - F_n) - \sigma_{exy}^2} \tag{IX.19}
\]

\[
E_1 = \frac{(F_n F_o)}{(\sigma_{tQ} - F_n)} \tag{IX.20}
\]

3. **I-Sections**

\[
C_1 = \frac{F_n C_o}{(\sigma_{ey} - F_n + \overline{q}_a)} \tag{IX.21}
\]

\[
E_1 = 0
\]

where

\( x_o \) = distance from shear center to centroid along principal x-axis, in. (absolute value)

\( C_o \), \( E_o \), and \( D_o \) are initial column imperfections which shall be assumed to be at least

\( C_o = L/350 \) in a direction parallel to the wall \( \tag{IX.22} \)

\( D_o = L/700 \) in a direction perpendicular to the wall \( \tag{IX.23} \)

\( E_o = L/(dx10,000) \), rad., a measure of the initial twist of the stud from the initial, ideal, unbuckled shape. \( \tag{IX.24} \)
If $F_n > 0.5F_y$, then in the definitions for $\sigma_{ey}$, $\sigma_{ex}$, $\sigma_{exy}$ and $\sigma_{tQ}$, the parameters $E$ and $G$ shall be replaced by $E'$ and $G'$, respectively, as defined below

$$E' = 4EF_n(F_y - F_n)/F_y^2$$  \hspace{1cm} (IX.25)

$$G' = G(E'/E)$$  \hspace{1cm} (IX.26)

Sheathing parameters $\bar{q}_o$ and $\bar{\gamma}$ may be determined from representative full-scale tests, conducted and evaluated as described by published documented methods, or from the small-scale-test values given in Table IX.1

<table>
<thead>
<tr>
<th>Sheathing</th>
<th>$\bar{q}$ (3)</th>
<th>$\bar{\gamma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/8 to 5/8 in. thick gypsum</td>
<td>2.0</td>
<td>0.008</td>
</tr>
<tr>
<td>Lignocellulosic board</td>
<td>1.0</td>
<td>0.009</td>
</tr>
<tr>
<td>Fiberboard (regular or impregnated)</td>
<td>0.6</td>
<td>0.007</td>
</tr>
<tr>
<td>Fiberboard (heavy impregnated)</td>
<td>1.2</td>
<td>0.010</td>
</tr>
</tbody>
</table>

(1) The values given are subject to the following limitations:

All values are for sheathing on both sides of the wall assembly.

All fasteners are No. 6, type S-12, self-drilling drywall screws with pan or bugle head, or equivalent, at 6-to 12-inch spacing.

(2) All sheathing is 1/2-inch thick except as noted.

(3) $\bar{q} = \bar{q}_o(2-s/12)$  \hspace{1cm} (IX.27)

where $s = $ fastener spacing, in.

For other types of sheathing, $\bar{q}_o$ and $\bar{\gamma}$ may be determined conservatively
from representative small-specimen tests as described by published
documented methods.

2. **LRFD Criteria.** According to Section D4.1 of the LRFD Spec-
ification, for studs having identical sheathing attached to both
flanges, and neglecting any rotational restraint provided by the
sheathing, the factored nominal axial strength, \( \Phi P_n \), shall be calcu-
lated as follows:

\[
\Phi = 0.85 \\
P_n = A F_n 
\]

\( P_n \) is the same as that specified in Section D4.1 of the AISI Specifi-
cation.

3. **Comparison.** The unfactored load applied to the member can
be computed for both design methods by using the following formula:

\[
P_T = P_{DL} + P_{LL} 
\]

where

- \( P_T \) = unfactored compressive load
- \( P_{DL} \) = compressive load due to the nominal axial dead load
- \( P_{LL} \) = compressive load due to the nominal axial live load

The total unfactored load should be less than or equal to the allowable
loads computed from allowable stress design and LRFD. For allowable
stress design, the allowable load is

\[
(P_a)_{ASD} = P_n / \Omega_c 
\]

For LRFD, the allowable axial load can be computed by using the fol-
lowing equation developed from Eq. (II.6):

\[
(P_a)_{LRFD} = \Phi_c P_n (D/L+1)/(1.2D/L+1.6) 
\]

Then, the allowable load ratio can be determined as follows:
For fully effective sections having wall thickness greater than 0.09 in. and $F_e > F_y/2$, 

$$\Omega_c = 5/3 + (3/8)R - (1/8)R^3$$

Therefore, the allowable load ratio is

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 0.85 \left( \frac{5}{3} + \frac{3}{8} - \frac{1}{8} R^3 \right) \frac{D/L+1}{1.2D/L+1.6}$$

For all other cases, $\Omega_c = 1.92 = 23/12$, therefore the allowable load ratio is

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 0.85(23/12) \frac{D/L+1}{1.2D/L+1.6} = 1.629 \frac{D/L+1}{1.2D/L+1.6}$$

Figure 81 shows the allowable load ratio versus dead-to-live load ratio for the wall studs used to develop Eq. (IX.34). For this case, the LRFD criteria always permit larger allowable loads than the allowable stress design. For $D/L = 0.5$, the LRFD criteria gives an allowable load about 11% greater than the load obtained by using allowable stress design.
Figure 81. Allowable Load Ratio vs. D/L Ratio for Wall Studs in Compression-Case 1

Eq. (IX.34)
Figure 82. Allowable Load Ratio vs. D/L Ratio for Wall Studs in Compression-Case 2
Figure 82 shows the allowable load ratio versus dead-to-live load ratio for the wall studs used to develop Eq. (IX.33). Different curves represent different values of $R$. For $R$ varies from 0 to 1 and $O/L = 0.5$, the allowable loads determined by LRFD criteria are from 3.2% lower to 16% higher than the allowable loads determined by the allowable stress design.

B. WALL STUDS IN BENDING

1. Allowable Stress Design. According to Section D4.2 of the AISI Specification, for studs having identical sheathing attached to both flanges, and neglecting any rotational restraint provided by the sheathing, the allowable moments are $M_{axo}$ and $M_{ayo}$, where

   $M_{axo}$ and $M_{ayo}$ = Allowable moments about the centroidal axes determined in accordance with Section C3.1 of the AISI Specification, excluding the provisions of Section C3.1.2 (lateral buckling)

2. LRFD Criteria. According to Section D4.2 of the LRFD Specification, for studs having identical sheathing attached to both flanges, and neglecting any rotational restraint provided by the sheathing, the factored nominal moments are $\phi M_{nxo}$ and $\phi M_{nyo}$ as follows:

   where

   $\phi = 0.95$ for sections with stiffened compression flanges
   $\phi = 0.90$ for sections with unstiffened compression flanges

   $M_{nxo}$ and $M_{nyo}$ = Nominal moments about the centroidal axes determined in accordance with Section C3.1, excluding the provisions of Section C3.1.2 (lateral buckling)
3. **Comparison.** The unfactored moment can be calculated by using Eq. (IX.35) for both methods (ASD and LRFD) for comparison.

\[
M_{TL} = M_{DL} + M_{LL} \quad \text{(IX.35)}
\]

where

- \(M_{TL}\) = total unfactored moment
- \(M_{DL}\) = moment due to the nominal dead load
- \(M_{LL}\) = moment due to the nominal live load

For allowable stress design, the allowable moment is determined from nominal section strength with a factor of safety of 1.67. Therefore, the allowable moment for beams is

\[
(M_a)_{ASD} = \frac{M_n}{\Omega_f} = \frac{M_n}{1.67} \quad \text{(IX.36)}
\]

For LRFD, the allowable moment can be computed by using the following equation developed from Eq. (II.6).

\[
(M_a)_{LRFD} = \frac{\phi M_n(D/L+1)/(1.2D/L+1.6)}{\phi} \quad \text{(IX.37)}
\]

The ratio of the allowable moments is

\[
\frac{(M_a)_{LRFD}}{(M_a)_{ASD}} = \frac{D/L+1}{1.67\phi \cdot 1.2D/L+1.6} \quad \text{(IX.38)}
\]

For sections with stiffened compression flanges, \(\phi = 0.95\)

\[
\frac{(M_a)_{LRFD}}{(M_a)_{ASD}} = \frac{D/L+1}{1.58 \cdot 1.2D/L+1.6} \quad \text{(IX.39)}
\]
Figure 83. Allowable Moment Ratio vs. D/L Ratio for Wall Studs in Bending-Case 1

Eq. (IX.39)
Figure 84. Allowable Moment Ratio vs. D/L Ratio for Wall Studs in Bending-Case 2
Figure 83 shows the allowable moment ratio versus dead-to-live load ratio for wall studs with stiffened compression flanges. For \( D/L = 1/25 \) both design methods will give the same value of allowable moment. However, LRFD will be conservative for \( D/L < 1/25 \) and unconservative for \( D/L > 1/25 \) as compared with the allowable stress design method.

For sections with unstiffened compression flanges, \( \phi = 0.90 \)

\[
\frac{(M_a)_{LRFD}}{(M_a)_{ASD}} = 1.50 \frac{D/L+1}{1.2D/L+1.6} \quad \text{(IX.40)}
\]

Figure 84 shows the allowable moment ratio versus the dead-to-live load ratio for this case. The two design methods give the same value for \( D/L = 1/3 \). For \( D/L = 0.5 \), the allowable moment based on LRFD is about 2.3\% larger than the value obtained from allowable stress design. When the dead-to-live load ratio for cold-formed steel is less than 1/3, the LRFD criteria are found to be conservative for sections with unstiffened compression flanges as compared with the allowable stress design method.

C. WALL STUDS WITH COMBINED AXIAL LOAD AND BENDING

1. Allowable Stress Design. According to Section D4.3 of the AISI Specification, the axial load and bending moments shall satisfy the interaction equations of Section C5 of the AISI Specification with the following redefined terms:

\( P_a = \) Allowable axial load determined according to Section D4.1 of the AISI Specification

\( M_{ax} \) and \( M_{ay} \) in Equations C5-1 and C5-3 shall be replaced by allowable moments, \( M_{axo} \) and \( M_{ayo} \), respectively.
2. **LRFD Criteria.** According to Section D4.3 of the LRFD Specification, the factored axial load and bending moments shall satisfy the interaction equations of Section C5 of the LRFD Specification with the following redefined terms:

\[ P_n = \text{Nominal axial strength determined according to Section D4.1 of the LRFD Specification} \]

\[ M_{nx} \quad \text{and} \quad M_{ny} \quad \text{in Equations C5-1 and C5-3 shall be replaced by nominal moments, } M_{nxo} \quad \text{and} \quad M_{nyo}, \text{ respectively.} \]

3. **Comparison.** Wall studs made by channel sections bending about the x-axis were considered. A typical design example was selected and the allowable axial loads were calculated by using three interaction equations for each design method. The example used a wall stud with equal moments applied to each end so that the member is bent in single curvature. Since the end moments are independent of the axial load, the ratio of the unfactored applied moment to the nominal moment capacity based on section strength, \[ \frac{M_T}{M_{n0}} \], was considered to be a parameter in the equations for determining the allowable loads.

For allowable stress design the allowable axial loads were computed as follows:

\[ \frac{P}{P_a} = \frac{P_T}{P_n} = \frac{\Omega_c P_T}{P_n} \quad (IX.41) \]

\[ \frac{M}{M_{ao}} = \frac{M_T}{0.6M_{n0}} = \frac{M_{T}/M_{n0}}{0.6} \quad (IX.42) \]
where

\[ P_T = \text{applied unfactored axial load} \]

\[ M_T = \text{applied unfactored bending moment at each end of the member} \]

\[ \Omega_c = \text{factor of safety of axially loaded compression members which is defined in Article V} \]

Substitution of Eqs. (IX.41) and (VI.42) into Eq. (VI.1) results in the following expression:

\[
\frac{\Omega_c P_T}{P_n} + \frac{C_m (M_T/M_{no})}{0.6(1-\Omega_c P_T/P_{cr})} = 1.0
\]  

(IX.43)

By solving for \( P_T \) in the first term of Eq. (VI.43), the following equation for allowable load is obtained:

\[
(P_T)_{ASD1} = \left[1 - \frac{C_m (M_T/M_{no})}{0.6(1-\Omega_c P_T/P_{cr})}\right] \frac{P_n}{\Omega_c}
\]  

(IX.44)

Equation (IX.44) is based on Eq. (VI.1) for failure at the midlength of the beam-column and requires a solution by iterations.

The following expression was used to solve for the allowable load based on Eq. (VI.2):

\[
\frac{P}{P_{ao}} = \frac{P_T}{P_{no}} = \frac{\Omega_c P_T}{P_{no}}
\]  

(IX.45)
Substitution of Eqs. (IX.45) and (IX.42) into Eq. (VI.2) results in the following expression:

\[
\frac{\Omega_c P_T}{P_{no}} + \frac{(M_T/M_{no})}{0.6} = 1.0
\]  

(IX.46)

By solving for \( P_T \) in Eq. (IX.46), the following equation for allowable load is obtained:

\[
(P_T)_{ASD2} = \left[1 - \frac{(M_T/M_{no})}{0.6}\right] \frac{P_{no}}{\Omega_c}
\]  

(IX.47)

Equation (IX.47) is based on Eq. (VI.2) for failure at the braced points.

When \( P/P_a \leq 0.15 \), Eq. (VI.3) can be used in lieu of Eqs. (VI.1) and (VI.2). Equation (VI.3) can be written in the following form by using Eqs. (IX.41) and (IX.42):

\[
\frac{\Omega_c P_T}{P_n} + \frac{M_T/M_{no}}{0.6} = 1.0
\]  

(IX.48)

By solving for \( P_T \) in Eq. (IX.48), the following equation for allowable load is obtained:

\[
(P_T)_{ASD3} = \left[1 - \frac{M_T/M_{no}}{0.6}\right] \frac{P_n}{\Omega_c}
\]  

(IX.49)
Equation (IX.49) is based on Eq. (VI.3) for flexural failure when the effect of the secondary moment is neglected.

For LRFD, the allowable axial loads were computed in accordance with Eq. (II.6) as follows:

\[
\frac{P_D}{\phi c_p_n} = \frac{1.2D/L + 1.6}{D/L + 1} \left( \frac{P_T}{\phi c_p_n} \right) \tag{IX.50}
\]

\[
\frac{M_D}{\phi M_{no}} = \frac{1.2D/L + 1.6}{D/L + 1} \left( \frac{M_T/M_{no}}{\phi} \right) \tag{IX.51}
\]

\[
\frac{P_D}{\phi c_p_E} = \frac{1.2D/L + 1.6}{D/L + 1} \left( \frac{P_T}{\phi c_p_E} \right) \tag{IX.52}
\]

Substitution of Eqs. (IX.50), (IX.51), and (IX.52) into Eq. (VI.7) results in the following expression:

\[
\frac{1.2D/L + 1.6}{D/L + 1} \left( \frac{P_T}{\phi c_p_n} + \frac{C_m(M_T/M_{no})}{\phi [1-(1.2D/L + 1.6)P_T/(D/L + 1)\phi c_p_E]} \right) = 1.0 \tag{IX.53}
\]

By solving for \( P_T \) in the first term of Eq. (IX.53), the following equation for allowable load is obtained:

\[
(P_T)_{LRFD} = \left\{ \frac{D/L + 1}{1.2D/L + 1.6} - \frac{C_m(M_T/M_{no})}{\phi [1-(1.2D/L + 1.6)P_T/(D/L + 1)\phi c_p_E]} \right\} \phi c_p_n \tag{IX.54}
\]
Equation (IX.54) is based on Eq. (VI.7) for flexural failure at the midlength of the beam-column and requires a solution by iterations.

The following expression was used to solve for the allowable load based on Eq. (VI.8):

\[
\frac{P_D}{\Phi_c P_{no}} = \frac{1.2D/L + 1.6}{D/L + 1} \left( \frac{P_T}{\Phi_c P_{no}} \right) \tag{IX.55}
\]

Substitution of Eqs. (IX.55) and (IX.51) into Eq. (VI.8) results in the following expression:

\[
\frac{1.2D/L + 1.6}{D/L + 1} \left[ \frac{P_T}{\Phi_c P_{no}} + \frac{(M_T/M_{no}^0)}{\Phi} \right] = 1.0 \tag{IX.56}
\]

By solving for \(P_T\) in Eq. (IX.56), the following equation for allowable load is obtained:

\[
(P_T)^{LRFD2} = \left[ \frac{D/L + 1}{1.2D/L + 1.6} - \frac{(M_T/M_{no}^0)}{\Phi} \right] \Phi_c P_{no} \tag{IX.57}
\]

Equation (IX.57) is based on Eq. (VI.8) for failure at the braced points.

When \(P_D/(\Phi_c P_{n}) \leq 0.15\), Eq. (VI.9) can be used in lieu of Eqs. (VI.7) and (VI.8). Equation (VI.9) can be written in the following form by using Eqs. (IX.50) and (IX.51):
By solving for $P_T$ in Eq. (IX.58), the following equation for allowable load is obtained:

$$\frac{1.2D/L+1.6}{D/L+1} \left[ \frac{P_T}{\phi P_n} + \frac{(M_T/M_{no})}{\phi} \right] = 1.0$$  \hspace{1cm} (IX.58)

Equation (IX.59) is based on (VI.9) for flexural failure when the effect of the secondary moment is neglected.

Equations (IX.44), (IX.47), and (IX.49) for determining the allowable axial load based on allowable stress design and Eqs. (IX.54), (IX.57), and (IX.59) for determining the allowable axial load based on LRFD are very complex and utilize iterations with multiple variables. The allowable load ratios, $(P_T)_LRFD/(P_T)_{ASD}$, for various lengths combined with different applied end moment ratios, $M_T/M_{no}$, with respect to the bending strength of the member were studied. The wall studs used in this study use 1/2 in. gypsum board with No. 6 type S-12 self-drilling screws at 12 in. spacing and the spacing of the channel is 24 in.. Typical channel sections and their section properties used in this study were obtained from Tables 1 and 2 of Part V of the AISI Cold-Formed Steel Design Manual.

A channel section (7 in. x 2.75 in. x 0.075 in.) with stiffened flanges was studied with a yield point of 50 ksi. Figure 85 shows the allowable load ratio versus dead-to-live load ratio for a 15 ft length.
with various end moment ratios, $M_T/M_{no}$. For a $D/L$ ratio around 0.05, the LRFD criteria give an allowable load about 3% more than the value computed from allowable stress design for all end moment ratios indicated in the figure. For other values of the $D/L$ ratio, the difference between the allowable loads computed by using these two methods depends on the end moment ratio as shown in Figure 85. For $D/L > 0.05$, the larger the end moment ratio, the higher the allowable load ratio. For example, for $D/L = 0.5$, the $(P_T)^{LRFD}/(P_T)^{ASD}$ ratios are 1.202 and 1.131 for $M_T/M_{no} = 0.3$ and 0.1, respectively.

Figure 86 shows the relationship between allowable load ratio and dead-to-live load ratio for end moment ratio of 0.2. The different curves in the figure represent different lengths of the 7 in. x 2.75 in. x 0.075 in. channel section. With end moment ratio of 0.2 and $D/L = 0.5$, ASD would provide conservative values up to 16.2% for effective lengths equal to 10 ft, 12 ft, 15 ft, and 20 ft as compared with the LRFD method. It can also be seen that effective length has a negligible effect on the allowable load ratio.

A shallower channel section (4 in. x 2 in. x 0.075 in.) with stiffened flanges was also studied for an effective length of 10 ft. Figure 87 shows the allowable load ratio versus dead-to-live load ratio for various end moment ratios. The curves without star symbols are for $F_y = 33$ ksi and the curves with star symbols are for $F_y = 50$ ksi. They are the same as those shown in Figure 85 for the 7 in. deep channel section. For this case, the yield point of steel would not affect the allowable load ratio. For $D/L = 0.5$ and $M_T/M_{no} = 0.1$, the allowable load computed from LRFD is 13.4% greater than the value determined from
allowable stress design. However, for D/L = 0.5 and \( \frac{M_e}{M_{no}} = 0.3 \), the allowable load computed from LRFD is 20.7% higher than the value computed from allowable stress design.

The curves without and with star symbols in Figure 88 are for \( C_m = 1.0 \) and 0.85, respectively, and for \( F_y = 33 \text{ ksi} \). The value of 0.85 is used for unbraced wall studs and wall studs with restrained ends subject to transverse loading between its supports. For small end moment ratios, the \( C_m \) value has a negligible effect on the allowable load ratio. The effect of \( C_m \) on the allowable load ratio increases as the end moment ratio increases as shown in Figure 88. It can be seen that for D/L < 0.05, the allowable load ratios computed for \( C_m = 0.85 \) are larger than those for \( C_m = 1.0 \).

The curves without and with star symbols in Figure 89 are for \( C_m = 1.0 \) and 0.85, respectively, and for \( F_y = 50 \text{ ksi} \). For small end moment ratios, the \( C_m \) value has a negligible effect on the allowable load ratio. The effect of \( C_m \) on the allowable load ratio increases as the end moment ratio increases as shown in Figure 88. It can be seen that for D/L < 0.05, the allowable load ratios computed for \( C_m = 0.85 \) are larger than those for \( C_m = 1.0 \).
Figure 85. Allowable Load Ratio vs. D/L Ratio for Wall Studs With Combined Axial Load and Bending-Case A

7" x 2.75" x 0.075" Channel
With Stiffened Flanges

$F_y = 50$ ksi
$C_m = 1.0$
$L = 180$ in.
Figure 86. Allowable Load Ratio vs. D/L Ratio for Wall studs With Combined Axial Load and Bending-Case B
Figure 87. Allowable Load Ratio vs. D/L Ratio for Wall Studs
With Combined Axial Load and Bending-Case C
1.4 F = 33 ksi

L = 120 in.

With Stiffened Flanges

4" x 2" x 0.075" Channel

C_m = 1.00

C_m = 0.85

F_y = 33 ksi

L = 120 in.

Figure 88. Allowable Load Ratio vs. D/L Ratio for Wall Studs With Combined Axial Load and Bending-Case D

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Figure 89. Allowable Load Ratio vs. D/L Ratio for Wall Studs With Combined Axial Load and Bending-Case E
Channel sections with unstiffened flanges were studied in a similar manner. Figure 90 shows the allowable load ratio versus dead-to-live load ratio for a channel section (7 in. x 1.5 in. x 0.075 in.) having unstiffened flanges with $F_y = 50$ ksi and an effective length of 15 ft. For $D/L = 0.5$ and $M_{T}/M_{no} = 0.1$, the allowable load obtained from LRFD is 11.7% larger than the value obtained from allowable stress design. For $D/L = 0.5$ and $M_{T}/M_{no} = 0.3$, LRFD would result in an allowable load 13.9% higher than the value determined from allowable stress design.

Figure 91 shows the allowable load ratio versus dead-to-live load ratio for end moment ratio of 0.2. Different curves represent different lengths of the channel section (7 in. x 1.5 in. x 0.075 in.) with $F_y = 50$ ksi. For $M_{T}/M_{no} = 0.2$ and $D/L = 0.5$, the allowable load values obtained from LRFD vary from 12.4% to 12.7% larger than the values obtained from the allowable stress design method. It can also be seen that effective length has a negligible effect on the allowable load ratio.

A shallower channel section (4 in. x 1.125 in. x 0.075 in.) with unstiffened flanges was also included in this study for an effective length of 10 ft. The relationship between allowable load ratio and dead-to-live load ratio for the channel section is shown in Figure 92 for various end moment ratios and $F_y$ values. The curves computed for $F_y = 33$ ksi are similar to the curves shown in Figure 90 which was obtained for 7 in. deep channel channel. For $D/L = 0.5$, the allowable load ratios vary from 1.12 to 1.14 for $M_{T}/M_{no}$ ratios ranging from 0.1 to 0.3.
The curves with star symbols in Figure 92 represent the allowable load ratios determined for the same channel section by using $F_y = 50$ ksi. It can be seen that the allowable load ratios computed for $F_y = 50$ ksi are the same as those computed for $F_y = 33$ ksi. From this figure it can also be seen that the yield point has no significant effect on the allowable load ratio for the channel section with unstiffened flanges.

Figures 93 and 94 show how the $C_m$ coefficient affects the allowable load ratio for the channel section having unstiffened flanges for $F_y = 33$ ksi and 50 ksi, respectively. The curves without star symbols are plotted for $C_m = 1.0$. The curves with star symbols represent the allowable load ratios calculated by using $C_m = 0.85$. For $D/L < 1/3$, the allowable load ratios are larger for $C_m = 0.85$ as compared to the allowable load ratios computed with $C_m = 1.0$. In general, the effect of the $C_m$ value on the allowable load ratio is more important for wall studs with large end moment ratios.
Figure 90. Allowable Load Ratio vs. D/L Ratio for Wall Studs With Combined Axial Load and Bending-Case F

7" x 1.5" x 0.075" Channel
With Unstiffened Flanges

\[ \frac{F_y}{F} = 50 \text{ ksi} \]
\[ C_m = 1.0 \]
\[ L = 180 \text{ in.} \]
Allowable Load Ratio, \( \frac{(P_T)_{LRFD}}{(P_T)_{ASD}} \)

![Graph showing allowable load ratio vs. dead-to-live load ratio for wall studs with combined axial load and bending.](image)

**Figure 91.** Allowable Load Ratio vs. D/L Ratio for Wall Studs With Combined Axial Load and Bending—Case G

- 7" x 1.5" x 0.075" Channel
- With Unstiffened Flanges
- \( F_y = 50 \text{ ksi} \)
- \( C_m = 1.0 \)
- \( \frac{M_T}{M_{no}} = 0.2 \)

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with unstiffened flanges

4" × 1.125" × 0.075" Channel

With Unstiffened Flanges

- \( F_y = 33 \) ksi
- \( F_y = 50 \) ksi

\( C_m = 1.0 \)
\( L = 120 \) in.

Figure 92. Allowable Load Ratio vs. D/L Ratio for Wall Studs
With Combined Axial Load and Bending-Case H

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Figure 93. Allowable Load Ratio vs. D/L Ratio for Wall Studs With Combined Axial Load and Bending-Case I
1. With Unstiffened Flanges

- $F_y = 50$ ksi
- $L = 120$ in.

4" x 1.125" x 0.075" Channel

Figure 94. Allowable Load Ratio vs. D/L Ratio for Wall Studs With Combined Axial Load and Bending-Case J
X. CONCLUSIONS

Currently, the 1986 Edition of the Specification for the Design of Cold-Formed Steel Structural Members published by the American Iron and Steel Institute applies to the design of cold-formed steel members and connections for load-carrying purposes in buildings. This specification provides design formulas for determining allowable load carrying capacities for tension members, compression members, flexural members, and connections based on appropriate factors of safety recommended by AISI for different types of structural members.

The Load and Resistance Factor Design method for cold-formed steel members and connections has recently been studied by using probabilistic and statistical techniques to account for the uncertainties in design, fabrication, material properties, and applied loads. The Load and Resistance Factor Design Specification was developed from a joint research project conducted at the University of Missouri-Rolla, Washington University, and the University of Minnesota.

This report compares these two methods for the design of cold-formed steel structural members using the proposed load and resistance factor design criteria and the allowable stress design criteria being used in the AISI Specification. Following a review of literature and discussion of different design variables used in both criteria, allowable loads using each design method were calculated for tension members, flexural members, compression members, beam-columns, connections, stiffeners, and wall studs. These allowable loads were then
compared in Chapters III through IX for different types of structural members and connections. For some cases, specific examples were used in this study due to the complexity of the analysis.

For all types of structural members only the dead and live load combination was studied in this investigation. It was found that the D/L ratio has a significant effect on the allowable load ratio. In general, the allowable load ratio, \( \frac{(P_a)_{LRFD}}{(P_a)_{ASD}} \), increases as the dead-to-live load ratio increases. Because cold-formed steel members are usually thin, the dead-to-live load ratios of such light weight members are expected to be lower than the ratios used for other building materials. In general practice, the dead-to-live load ratios used in building design of cold-formed steel members are less than 1/3. In view of the fact that the load factor used for live load is 1.6 which is larger than the load factor of 1.2 used for dead load, the LRFD criteria were found to be conservative for unusually small D/L ratios.

In addition to the effect of the dead-to-live load ratio, the resistance factors used in the LRFD criteria and the factors of safety used in allowable stress design also contribute to the differences between the allowable loads computed from two different methods. As the safety factor or resistance factor increases, the ratio of \( \frac{(P_a)_{LRFD}}{(P_a)_{ASD}} \) also increases. For a given set of statistical data and a selected safety index, the resistance factor can be determined by Eq. (II.5). This equation is a function of the mean value and coefficient of variation of the professional factor which is the ratio of the tested load to the predicted load. A low value of the resistance factor is resulted from a low value of \( P_m \) and a large value of \( V_p \) which
represents a big scatter of test results. This was the case for welded connections and plate failure of bolted connections.

The load and resistance factor design method is a rational approach for structural design. The research findings obtained from this comparative study of the current method based on allowable stress design and the proposed LRFD criteria can provide a useful reference for future revision of the current AISI Specification and the proposed LRFD Specification.
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