Load and resistance factor design of cold formed steel-calibrations of the design provisions on laterally unbraced beams and beam-columns

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LOAD AND RESISTANCE FACTOR DESIGN OF COLD-FORMED STEEL

CALIBRATION OF THE DESIGN PROVISIONS ON LATERALLY UNBRACED BEAMS AND BEAM-COLUMNS

by

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I. INTRODUCTION

Since the initiation of the project on load and resistance factor design in 1976, three progress reports have been submitted to American Iron and Steel Institute. These publications summarized the research findings on the statistical analysis of mechanical properties and thicknesses of materials accompanied with the calibration of the AISI design criteria on utilization of cold work, effective design width formulas for stiffened compression elements, allowable stress formulas for unstiffened compression elements, bolted connections, welded connections and axially loaded compression members. Previous reports have also presented the calibration of the proposed changes on bolted connections, unstiffened compression elements, axially loaded compression members and welded connections.

This report presents the progress made on the development of the load and resistance factor design of cold-formed steel during the past five months.

Article II.1 contains the calibration of the AISI design provisions on laterally unbraced beams.

The calibration of the current AISI design provisions on combined axial and bending stresses is presented in Article II.2.

Based on the progress made to date, a preliminary summary is presented in Article II.3.

This investigation was sponsored by American Iron and Steel Institute. The technical guidance provided by the AISI Task Group on Load and Resistance Factor Design (K.H. Klippstein, Chairman, D.H. Hall, and D.S. Wolford, members), the advisors for the AISI Task
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Special thanks are extended to Mrs. Alice Crangle for typing this report.
II. RESEARCH ACTIVITIES DURING THE PERIOD OF AUGUST THROUGH DECEMBER 1977

During the period of August through December 1977, the following activities were carried out on the load and resistance design of cold-formed steel:

(1) Calibration of the AISI design provisions on laterally unbraced beams.

(2) Calibration of the AISI design provisions on combined axial and bending stresses

Details of the above listed subjects are discussed in this Article.
II.1 CALIBRATION OF THE AISI DESIGN PROVISIONS ON LATERALLY UNBRACED BEAMS

II.1.1 AISI Design Provisions on Laterally Unbraced Beams

Cold-formed steel flexural members, when loaded in the plane of the web, may twist and deflect laterally as well as vertically if adequate braces are not provided. To prevent lateral buckling, the maximum compression stress, $F_b$, in kips per square inch, on extreme fibers of laterally unsupported straight flexural members shall not exceed the allowable stress as specified in Section 3.1 or 3.2 nor the following maximum stresses specified in Section 3.3 of the AISI Specification.

(a) When bending is about the centroidal axis perpendicular to the web for either I-shaped sections symmetrical about an axis in the plane of the web or symmetrical channel-shaped sections:

When $L_{xc}^2 / d_I y_c$ is greater than $0.36 \pi^2 E_b / F_y$ but less than $1.8 \pi^2 E_b / F_y$,

$$F_b = \frac{2}{3} F_y - \frac{F_y^2}{5.4 \pi^2 E_b} \left( \frac{L_{xc}^2}{d_I y_c} \right)$$  \hspace{1cm} (1.1)

When $L_{xc}^2 / d_I y_c$ is equal to or greater than $1.8 \pi^2 E_b / F_y$,

$$F_b = 0.6 \pi^2 E_b \frac{d_I y_c}{L_{xc}^2}$$  \hspace{1cm} (1.2)

(b) For point-symmetrical Z-shaped sections bent about the centroidal axis perpendicular to the web:

When $L_{xc}^2 / d_I y_c$ is greater than $0.18 \pi^2 E_b / F_y$ but less than $0.9 \pi^2 E_b / F_y$,
\[ F_b = \frac{2}{3} \frac{F_y}{y} - \frac{\frac{F_y^2}{2.7 \pi^2 E_c b}}{L^2 S_{xc}} \]  

(1.3)

When \( \frac{L^2 S_{xc}}{dI_{yc}} \) is equal to or greater than \( 0.9 \frac{\pi^2 E_c b}{F_y} \),

\[ F_b = 0.3 \frac{\pi^2 E_c b}{L^2 S_{xc}} \frac{dI_{yc}}{L^2 S_{xc}} \]  

(1.4)

In the above,

- \( L \) = the unbraced length of the member, in.
- \( I_{yc} \) = the moment of inertia of the compression portion of a section about the gravity axis of the entire section parallel to the web, in.\(^4\)
- \( S_{xc} \) = compression section modulus of entire section about major axis, \( I_x \) divided by distance to extreme compression fiber, in.\(^3\)
- \( C_b \) = bending coefficient which can conservatively be taken as unity or calculated from

\[ C_b = 1.75 + 1.05 \left( \frac{M_1}{M_2} \right) + 0.3 \left( \frac{M_1}{M_2} \right)^2 , \text{ but not more than 2.3} \]

where \( M_1 \) is the smaller and \( M_2 \) is the larger bending moment at the ends of the unbraced length, taken about the strong axis of the member, and where \( \frac{M_1}{M_2} \), the ratio of end moments, is positive when \( M_1 \) and \( M_2 \) have the same sign (reverse curvature bending) and negative when they are of opposite sign (single curvature bending). When the bending moment at any point within an unbraced length is larger than that at both ends of this length the ratio \( \frac{M_1}{M_2} \) shall be taken as unity.
For members subjected to combined axial and bending stresses, 
\( C_b \) shall be 1.

\( E = \text{modulus of elasticity} = 29,500 \text{ ksi} \)

\( d = \text{depth of section, in.} \)

II.1.2. Comparison Between the Theoretical Lateral Buckling Stress and 
That Used by the AISI Specification and the CSA Standard

In order to study the background information for the AISI design provisions on laterally unbraced beams, a comparison was made between the theoretical value and those used by the AISI Specification\(^{(1)}\) and the CSA Standard\(^{(2)}\). The member used for the comparison is a simply supported symmetric I-shaped beam under a uniform moment. The critical moment, \( M_{cr} \), under these conditions is computed by using Eq. (1.5) or Eq. (1.6)\(^{(3)}\)

\[
M_{cr} = \frac{\pi}{L} \sqrt{\frac{E I G J}{G J L^2}} + \frac{\pi^2 E I C W}{L^2} \tag{1.5}
\]

or

\[
M_{cr} = \frac{\pi^2 E}{L^2} \sqrt{\frac{I C W}{G J L^2}} + \frac{\pi^2 E}{\pi^2 E C W} \tag{1.6a}
\]

\[
= \frac{\pi^2 E}{L^2} \sqrt{\frac{I C W}{G J L^2}} + \sqrt{1 + \frac{G J L^2}{\pi^2 E C W}} \tag{1.6b}
\]

where,

\( E = \text{modulus of elasticity} \)

\( G = \text{shear modulus} = E/2(1 + \mu) \)
\( I_y = \text{moment of inertia about } y\text{-axis} \)

\( J = \text{St. Venant torsion constant of the cross section approximately determined by } 1/3 \sum b_i t_i^3 \)

\( C_w = \text{warping constant of torsion of the cross section} \)

\( L = \text{unbraced length of the beam} \)

In Eq. (1.6), the first term under the square root represents the strength due to lateral bending rigidity of the beam and the second term represents the St. Venant torsional rigidity. For thin-walled cold-formed steel sections, the first term usually exceeds considerably the second term. The second term under the square root of Eq. (1.6b) can be written as follows for the I-sections with unstiffened flanges (i.e., without edge stiffeners):

\[
\frac{GJL^2}{\pi^2 EC_w} = \frac{G}{\pi^2 E} \frac{JL^2}{I_y} \frac{4}{d^2}
\]

\[
= \frac{4G}{\pi^2 E} \frac{J}{A d^2} \frac{L^2}{I_y / A}
\]

\[
= \frac{4G}{\pi^2 E A d^2} \left( \frac{L}{r_y} \right)^2
\]

\[
= 0.156 \frac{J}{A d^2} \left( \frac{L}{r_y} \right)^2 \tag{1.7}
\]

in which, \( d \) is the depth, \( A \) is the area and \( r_y \) is the radius of gyration about \( y\)-axis of the entire cross section.

For the I-sections listed in the AISI Manuals, it was found that the values of \( J/Ad^2 \) are between \( 2 \times 10^{-5} \) and \( 100 \times 10^{-5} \).
Based on Eq. (1.6b), the theoretical critical buckling stress is

$$\sigma_{cr \text{theo}} = \frac{\pi^2 E}{S_x L^2} \sqrt{I_y C_w} \sqrt{1 + \frac{GJ L^2}{N^2 E C_w}}$$

(1.8)

However, the AISI Specification and the CSA Standard are based on the simplified formulas given in Eqs. (1.9) and (1.10), respectively.

$$\sigma_{cr \text{AISI}} = \frac{\pi^2 E d I_y}{2 L S_x}$$

(1.9)

$$\sigma_{cr \text{CSA}} = \frac{\pi^2 E d I_y}{2 L S_x} \left[ 1 + \frac{2GJ L^2}{N^2 I_y d^2 E} \right]$$

(1.10)

The following is a numerical example for the comparison of the theoretical critical buckling stress [Eq. (1.8)] and that used for the AISI Specification [Eq. (1.9)] and the CSA Standard [Eq. (1.10)]. In this example, a 12 in. x 7 in. x 0.135 in. I-section (Fig. 2) with L = 200 in. was used. From Table 5 of the AISI Manual, (19) the dimensions and sectional properties of the selected I-section with stiffened flanges (i.e., with edge stiffeners) are as follows:

- $d = 12$ in.
- $B = 3.5$ in.
- $C = 1.0$ in.
- $t = 0.135$ in.
- $A = 5.41$ in.$^2$
- $I_y = 12.5$ in.$^4$
- $S_x = 18.7$ in.$^3$
\( r_y = 1.52 \text{ in.} \)
\( L/r_y = 132 \)

\[ J = (2) \times (\text{J value for the } 12 \times 3.5 \times 0.135 \text{ in. channel given in Table 1 of the AISI Manual}) = (2)(0.0164) = 0.0328 \text{ in.}^4 \]

\[ C_w = \frac{I_y d^2}{4} \left[ 1 + \frac{C^2(4C + 6d)}{d^2(b + 3C)} \right] \]
\[ = \frac{12.5(12)^2}{4} \left[ 1 + \frac{1(4 + 6 \times 12)}{(12)^2(3.5 + 3 \times 1)} \right] \]
\[ = 487 \]

By using Eqs. (1.8), (1.9) and (1.10), the elastic critical lateral buckling stresses are computed as

\[ (\sigma_{cr})_{\text{theo}} = \frac{\pi^2 E}{S_L^2 L^2} \sqrt{I_y C_w} \sqrt{1 + \frac{GJL^2}{\pi^2 E C_w}} \]
\[ = \frac{(9.87)(29,500)}{(18.7)(200)^2} \sqrt{(12.5)(487)} \sqrt{1 + \frac{(11,300)(0.0328)(200)^2}{(9.87)(29,500)(487)}} \]
\[ = 31.92 \text{ ksi} \]

\[ (\sigma_{cr})_{\text{AISI}} = \frac{\pi^2 E d I_y}{2L^2 S_x} \]
\[ = \frac{(9.87)(29,500)(12)(12.5)}{2(200)^2(18.7)} \]
\[ = 29.19 \text{ ksi} \]

* The equation for \( C_w \) was derived for the I-section shown in Fig. 2.
\[
(\sigma_{cr})_{\text{CSA}} = \frac{\pi^2 E I_y}{2 L S_x} \left[ 1 + \frac{2 G J L^2}{\pi^2 I_y d^2 E} \right]
\]

\[
= \frac{(9.87)(29,500)(12)(12.5)}{2(200)^2(18.7)} \left[ 1 + \frac{2(11,300)(0.0328)(200)^2}{(9.87)(12.5)(12)^2(29,500)} \right]
\]

\[= 30.85 \text{ ksi} \]

It can be seen that the critical buckling stresses used by the AISI Specification and the CSA Standard are conservative as compared with the theoretical critical buckling stress, as pointed out already in the Commentary on the AISI Specification (Ref. 1). For large \(L/r_y\) ratios, the AISI Specification has been found to be quite conservative.

II.1.3 Tests on Lateral Buckling of Cold-Formed Steel Beams

A total of 74 tests on lateral buckling of cold-formed steel beams were reported in seven Cornell Progress Reports (12-18). Among these tests, the dimensions and cross-sectional properties of the 47 relatively long I-beams which failed in elastic buckling are as follows:

Thickness (t): 0.0598 in.
Depth (d): 4 in.
Width (2B): 2 in.
Area: 0.705 in.²

Moment of inertia about x-axis (Ix): 1.515 in.⁴
Moment of inertia about y-axis (Iy): 0.0806 in.⁴
Torsional constant (J): 0.00260 in.⁴
Radius of gyration about y-axis (ry): 0.338 in.
From the above data, the buckling load, \((P_u)_p\), of a beam can be predicted by using the following equation for the beam subjected to a concentrated load as shown in Fig. 3.\(^{(4)}\)

\[
(P_u)_p = \frac{1}{L^3} \left\{ \frac{2\pi^2 E C_b I_y d}{1 + C_b^2 + \frac{4}{\pi^2} \frac{G J}{E I_y} L^2 - C_2^2} \right\}^{1.11}
\]

in which the values of \(C_b\) and \(C_2\) are taken as 1.35 and 0.55, respectively.\(^{(4)}\)

The tested failure loads, \((P_u)_t\), and the predicted buckling loads, \((P_u)_p\), are presented in Table 1. A comparison of these tested and predicted failure loads is also included in this table and is shown in Fig. 1.

If all tests including the repeated tests on an identical specimen are used, the mean value and the coefficient of variation of the 47 test-to-prediction ratios, \((P_u)_t/(P_u)_p\), are

\[
P_m = 1.18
V_p = 0.19
\]

If only the lowest test result for each beam specimen is used, then the statistics of 15 test-to-prediction ratios are

\[
P_m = 1.05
V_p = 0.18
\]

If the average result for each specimen is used, then for the 15 different beams, the values of \(P_m\) and \(V_p\) are

\[
P_m = 1.19
V_p = 0.14
\]
For the purpose of calibration, use the following judgmental composite values:

\[ P_m = 1.15 \]
\[ V_P = 0.15 \]

II.1.4 Calibration

The procedures used for calibration of the AISI design provisions were presented in Ref. 6. The purpose of this calibration is to determine the value of the safety index \( \beta \) inherent in the current design for lateral buckling of beams as characterized by Section 3.3 of the AISI Specification. Similar to the previous work, the calibration has been performed for a combination of dead and live loads for laterally unbraced beams.

The safety index, \( \beta \), is defined in Ref. 5 as

\[ \beta = \frac{\lambda_n \left( R / Q_m \right)}{\sqrt{V_R^2 + V_Q^2}} \]  

(1.12)

where \( R_m \) is the mean resistance and \( V_R \) the coefficient of variation of the resistance, while \( Q_m \) and \( V_Q \) are the corresponding quantities for the load effects.

The mean load effects, \( Q_m \), for a combination of dead and live loads is assumed to be of the form

\[ Q_m = A_m \left( c_D D_m + c_L L_m \right) \]  

(1.13)

in which \( A_m \) is a mean value of a random variable representing the uncertainties in structural analysis, and \( B_m \) and \( C_m \) are mean values of random variables reflecting the uncertainties in the transformation of load intensities into load effects.
By assuming $C_m = B_m = 1.00$ and $c_D = c_L$, the coefficient of variation of load effects, $V_Q$, is

$$V_Q = \sqrt{V_A^2 + \frac{D_m^2 (V_C^2 + V_D^2)}{(D_m + L_m)^2} + \frac{L_m (V_L^2 + V_B^2)}{(D_m + L_m)^2}}$$ \hspace{1cm} (1.14)

In this equation, $V_A$, $V_C$, $V_D$ and $V_L$ are coefficients of variation associated with the uncertainties in structural analysis ($A$), dead load ($C,D$) and live load ($L,B$) random variables, respectively. Because of the use of relatively small tributary areas for cold-formed steel members, the mean values of dead and live loads, $D_m$ and $L_m$, may be assumed to be the specified values of $D_c$ and $L_c$, respectively. Eq. (1.14) can be written in the following form:

$$V_Q = \sqrt{V_A^2 + \frac{(D_c/L_c)^2 (V_C^2 + V_D^2) + (V_B^2 + V_L^2)}{(D_c/L_c)^2 + 2(D_c/L_c) + 1}}$$ \hspace{1cm} (1.15)

In the application of Eq. (1.15), the following assumed values of coefficients of variation of load effects are the same as those used and justified in Ref. 7.

$$V_A = 0.05$$
$$V_C = 0.04$$
$$V_D = 0.04$$
$$V_B = 0.10$$
$$V_L = 0.13$$ \hspace{1cm} (1.16)
By using Eq. (1.15) the values of $V_Q$ can be computed for four different $D_c/L_c$ ratios ranging from 0.1 to 3.0.

For $D_c/L_c = 0.1$, $V_Q = 0.16$
For $D_c/L_c = 0.3$, $V_Q = 0.13$
For $D_c/L_c = 1.0$, $V_Q = 0.10$
For $D_c/L_c = 3$, $V_Q = 0.08$

The mean resistance of the beam is

$$R_m = M_m F_m P_m R_n$$  \hspace{1cm} (1.17)

In the above equation, $M_m$, $F_m$ and $P_m$ are the mean values of the material factor, fabrication factor and professional factor, respectively, and $R_n$ is the nominal resistance determined on the basis of the current design specification. Since all test specimens used in this calibration failed in the elastic range, only the modulus of elasticity is involved in the material properties. Therefore, $M_m = 1.00$ and $V_M = 0.06$. \hspace{1cm} (7)

The mean value of the fabrication factor can be assumed to be unity with a coefficient of variation of 0.05. The mean value of the professional factor, $P_m = [(P_u)_t/(P_u)_p]_m$, for laterally unbraced beams is taken as 1.15 with a coefficient of variation of 0.15. The nominal resistance, $R_n$, determined on the basis of the AISI design provisions on laterally unbraced beams is equal to the load effect with a factor of safety $F_S = 1.67$, i.e.,

$$R_n = S F_n x c r$$
$$= 1.67c (D_c + L_c)$$

in which $D_c$ and $L_c$ are specified dead and live loads and $c$ is the influence coefficient. The mean resistance can be written as follows:
\[ R = (1.00)(1.00)(1.15)(1.67)c(D_c + L_c) \]
\[ = 1.92c(D_c + L_c) \]

and the mean load effect is
\[ Q_m = c(D_m + L_m) \]
\[ = c(D_c + L_c) \]

Then
\[ \frac{R_m}{Q_m} = 1.92 \]

The coefficient of variation of the resistance is
\[ V_R = \sqrt{V_M^2 + V_F^2 + V_P^2} \]
\[ = \sqrt{0.06^2 + 0.05^2 + 0.15^2} \]
\[ = 0.17 \]

For a selected ratio of \( D_c / L_c = 1/3 \), the safety index \( \beta \) is
\[ \beta = \frac{\ln 1.92}{\sqrt{(0.17)^2 + (0.13)^2}} \]
\[ = 3.04 \]

Similarly, for \( D_c / L_c = 0.1, 1.0 \) and \( 3.0 \) the values of safety index are 2.79, 3.31 and 3.54, respectively. See Table 4.
II.2 CALIBRATION OF THE AISI DESIGN PROVISIONS
ON COMBINED AXIAL AND BENDING STRESSES

II.2.1 AISI Design Provisions on Combined Axial and Bending Stresses

Thin walled cold-formed members exhibit a more complex behavior
than the generally more compact hot-rolled shapes when subjected to
combined axial compression and bending because of the possibility of
torsional flexural buckling. The AISI recognizes this difference in
Section 3.7 of the AISI Specification. (1)

Two types of limit states are considered for the design of beam-
columns. They are

(1) Torsional-flexural buckling (TFB)
(2) In-plane instability (IPI)

The structural behavior of beam-columns depends on the shape and
dimensions of the cross-section, the location of the applied eccentric
load, column length, condition of bracing, etc. For this reason, AISI
Specification separates the design criteria into the following four
cases according to the configuration of the cross section and the
type of buckling modes:

- Doubly symmetric shapes and shapes not subject to torsional
  or torsional-flexural buckling
- Singly symmetric shapes or intermittently fastened components
  of built-up shapes having Q=1.0 which may be subject to torsional-flexural
  buckling
- Singly symmetric shapes or intermittently fastened components
  of built-up shapes having Q < 1.0 which may be subject to torsional-
  flexural buckling
- Singly symmetric shapes which are unsymmetrically loaded
Following is an excerpt of Section 3.7 of the AISI Specification:

(1) When subject to both axial compression and bending, doubly-symmetric shapes or shapes which are not subject to torsional or torsional-flexural buckling shall be proportioned to meet the following requirements:

\[
\frac{f_a}{F_{al}} + \frac{C_{mx} f_{bx}}{(1 - \frac{a}{F_{ex}})F_{bx}} + \frac{C_{my} f_{by}}{(1 - \frac{a}{F_{ey}})F_{by}} \leq 1.0 \quad (2.1)
\]

\[
\frac{f_a}{F_{ao}} + \frac{f_{bx}}{F_{blx}} + \frac{f_{by}}{F_{bly}} \leq 1.0 \quad (2.2)
\]

When \(\frac{a}{F_{al}} < 0.15\), the following formula may be used in lieu of the above two formulas:

\[
\frac{f_a}{F_{al}} + \frac{f_{bx}}{F_{blx}} + \frac{f_{by}}{F_{bly}} \leq 1.0 \quad (2.3)
\]

The subscripts \(x\) and \(y\) in the above formulas indicate the axis of bending about which a particular stress or design property applies.

(2) Singly-symmetric shapes subject to both axial compression and bending applied in the plane of symmetry shall be proportioned to meet the following four requirements as applicable:

\[
\frac{f_a}{F_{al}} + \frac{f_{Cm}}{F_{bl}(1 - \frac{a}{F_{e}})} \leq 1 \quad (2.4)
\]

\[
\frac{f_a}{F_{ao}} + \frac{f_{b}}{F_{bl}} \leq 1 \quad (2.5)
\]
When \( \frac{f_a}{F_{a1}} < 0.15 \), the following formula may be used in lieu of the above two formulas

\[
\frac{f_a}{F_{a1}} + \frac{f_b}{F_{b1}} < 1
\]  

(2.6)

(ii) If the point of application of the eccentric load is located on the side of the centroid opposite from that of the shear center, i.e., if \( e \) is positive, then the average compression stress \( (f_a) \) also shall not exceed \( F_a \) given below:

\[
\sigma_{TF} > 0.5 F_y: \quad F_a = 0.522 F_y - \frac{f_b^2}{7.67 \sigma_{TF}}
\]  

(2.7)

\[
\sigma_{TF} \leq 0.5 F_y: \quad F_a = 0.522 \sigma_{TF}
\]  

(2.8)

where \( \sigma_{TF} \) shall be determined according to the formula:

\[
\frac{\sigma_{TF}}{\sigma_{TP0}} + \frac{C_{TF}b_1}{\sigma_b(1 - \frac{\sigma_{TF}}{\sigma_e})} = 1
\]  

(2.9)

(iii) Except for T- or unsymmetric I-sections, if the point of application of the eccentric load is between the shear center and the centroid, i.e., if \( e \) is negative, and if \( F_{a1} \) is larger than \( F_{a2} \), then the average compression stress \( (f_a) \) also shall not exceed \( F_a \) given below:

\[
F_a = F_{a2} + \frac{e}{x_0} (F_{aE} - F_{a2})
\]  

(2.10)

(iv) For T- and unsymmetric I-sections with negative eccentricities

(a) If the point of application of the eccentric load is between the shear center and the centroid, and if \( F_{a1} \) is larger than \( F_{a2} \), then the average compression stress \( (f_a) \) also shall not exceed \( F_a \) given below:
\[ F_a = F_{a2} + \frac{e}{x_0} (F_a - F_{a2}) \]  \hspace{1cm} (2.11)

(b) If the point of application of the eccentric load is located on the side of the shear center opposite from that of the centroid, then the average compression stress \( f_a \) also shall not exceed \( F_a \) given below:

\[
\sigma_{TF} > 0.5 \frac{F}{y} : \ F_a = 0.522 \frac{F^2}{y} - \frac{1}{7.67 \sigma_{TF}} 
\]  \hspace{1cm} (2.12)

\[
\sigma_{TF} \leq 0.5 \frac{F}{y} : \ F_a = 0.522 \sigma_{TF} 
\]  \hspace{1cm} (2.13)

where \( \sigma_{TF} \) shall be determined according to the formula:

\[
\frac{\sigma_{TF}}{\sigma_{ex}} + \frac{C_{TF}}{\sigma_{bc}} \left[ \frac{\sigma_{bl}}{\sigma_{TF}} - \frac{\sigma_{b2}}{\sigma_e} \right] = 1.0 \]  \hspace{1cm} (2.14)

In the above,

\( C_m \) = a coefficient whose value shall be taken as follows:

1. For compression members in frame subject to joint translation (sidesway) \( C_m = 0.85 \)

2. For restrained compression members in frames braced against joint translation and not subject to transverse loading between their supports in the plane of bending,

\[ C_m = 0.6 - 0.4 \frac{M_1}{M_2}, \text{ but not less than 0.4.} \]

where \( M_1/M_2 \) is the ratio of the smaller to larger moments at the ends of that portion of the member, unbraced in the plane of bending under consideration. \( M_1/M_2 \) is positive when the member is bent in reverse curvature and negative when it is bent in single curvature.
3. For compression members in frames braced against joint translation in the plane of loading and subject to transverse loading between their supports, the value of $C_m$ may be determined by rational analysis. However, in lieu of such an analysis, the following values may be used. (a) for members whose ends are restrained, $C_m = 0.85$, (b) for members whose ends are unrestrained, $C_m = 1.0$.

$C_{TF}$ = a coefficient whose value shall be taken as follows:

1. For compression members in frames subject to joint translation (sidesway) $C_{TF} = 0.85$

2. For restrained compression members in frames braced against joint translation and not subject to transverse loading between their supports in the plane of bending

$$C_{TF} = 0.6 - 0.4 \frac{M_1}{M_2}$$

where $M_1/M_2$ is the ratio of the smaller to larger moments at the ends of that portion of the member, unbraced in the plane of bending under consideration. $M_1/M_2$ is positive when the member is bent in reverse curvature and negative when it is bent in a single curvature.

c = distance from the centroidal axis to the fiber with maximum compression stress, negative when the fiber is on the shear center side of the centroid, in.

d = depth of section, in.

e = eccentricity of the axial load with respect to the centroidal axis, negative when on the shear center side of the centroid, in.
\[ F_a = \text{maximum average compression stress, ksi} \]
\[ F_{ac} = \text{average allowable compression stress determined by both requirements (i) and (ivb) if the point of application of the eccentric load is at the shear center, i.e. the calculated values of } f_a \text{ and } F_a, \text{ for } e = x_o, \text{ ksi} \]
\[ F_{aE} = \text{average allowable compression stress determined by requirement (i) if the point of application of the eccentric load is at the shear center, i.e. the calculated value of } f_a \text{ for } e = x_o, \text{ ksi} \]
\[ F_{a0} = \text{allowable compression stress under concentric loading determined by Section 3.6.1.1 for } L=0, \text{ ksi} \]
\[ F_{a1} = \text{allowable compression stress under concentric loading according to Section 3.6.1.1 for buckling in the plane of symmetry, ksi} \]
\[ F_{a2} = \text{allowable compression stress under concentric loading from Section 3.6.1.2, ksi} \]
\[ F_b = \text{maximum bending stress in compression that is permitted by this Specification where the bending stress only exists (Sections 3.1, 3.2, and 3.3), ksi} \]
\[ F_{b1} = \text{maximum bending stress in compression permitted by this Specification where bending stress only exists and the possibility of lateral buckling is excluded (Sections 3.1 and 3.2), ksi} \]
\[ F_e = \frac{12 \pi^2 E}{23(KL_b/r_b)^2} \text{ (may be increased one-third in accordance with Section 3.1.2), ksi} \]
\( f_a = \) axial stress = axial load divided by full cross-sectional area of member, P/A, ksi

\( f_b = \) maximum bending stress = bending moment divided by appropriate section modulus of member, M/S, noting that for members having stiffened compression elements the section modulus shall be based upon the effective design widths of such elements, ksi

\( I_{xc} = \) moment of inertia of the compression portion of a section about its axis of symmetry, in.\(^4\)

\( I_y = \) moment of inertia of the section about the y-axis, in.\(^4\)

\[ j = \frac{1}{2} \frac{1}{I_y} [\int_A x^3 dA + \int_A xy^2 dA] - x_o, \text{ in.}, \text{ where } x \text{ is the axis of symmetry and } y \text{ is orthogonal to } x \]

\( K = \) effective length factor in the plane of bending

\( L_b = \) actual unbraced length in the plane of bending, in.

\( M_c = -A \sigma_{ex} [\sqrt{j^2 + r_o^2 (\sigma_c / \sigma_{ex})}] = \) elastic critical moment causing compression on the shear center side of the centroid, kip-in.

\( M_T = -A \sigma_{ex} [\sqrt{j^2 + r_o^2 (\sigma_c / \sigma_{ex})}] = \) elastic critical moment causing tension on the shear center side of the centroid, kip-in.

\( r_b = \) radius of gyration about axis of bending, in.

\( r_{xc} = \) radius of gyration about the centroidal axis parallel to the web of that portion of the I-section which is in compression when there is no axial load, in.

\( S_{yc} = \) compression section modulus of entire section about axis normal to axis of symmetry, I \(_y\)/distance to extreme compression fiber, in.\(^3\)

\( x_o = x \) coordinate of the shear center, negative, in.
\[ \sigma_{bc} = \frac{M_c}{I_y} = \text{maximum compression bending stress caused by } M_c, \text{ ksi.} \]

For I-sections with unequal flanges \( \sigma_{bc} \) may be approximated by
\[ \frac{\pi^2 E dI}{L^2 S_{yc}} \]

\[ \sigma_{bT} = \frac{M_{Tc}}{I_y} = \text{maximum compression bending stress caused by } M_{Tc}, \text{ ksi.} \]

For I-sections with unequal flanges \( \sigma_{bT} \) may be approximated by
\[ \frac{\pi^2 E dI}{L^2 S_{yc}} \]

\[ \sigma_{b1} = \sigma_{TF} \frac{\pi c}{2 r_y} = \text{maximum compression bending stress in the section caused by } \sigma_{TF}, \text{ ksi.} \]

\[ \sigma_{b2} = \sigma_{TF} \frac{r_c}{2 r_y}, \text{ ksi} \]

\[ \sigma_e = \frac{\pi^2 E}{(KL_b/r_b)^2}, \text{ ksi} \]

\( \sigma_{TF} \) = average elastic torsional-flexural buckling stress, i.e., axial load at which torsional-flexural buckling occurs divided by the full cross-sectional area of member, ksi

\( A = \text{cross-sectional area, in}^2 \)

\( E = \text{modulus of elasticity} = 29,500 \text{ ksi} \)

\[ r_o = \sqrt{\frac{2}{r_x^2 + r_y^2 + x_0^2}} = \text{polar radius of gyration of cross-section about the shear center, in.} \]

\( r_x, r_y = \text{radii of gyration of cross-section about centroidal principal axes, in.} \)
\[ \beta = 1 - \left(\frac{x_0}{r_0}\right)^2 \]

\[ \sigma_{ex} = \frac{n^2E}{(KL/r_x)^2}, \text{ksi} \]

\[ \sigma_t = \frac{1}{Ar_o^2} \left[ Gd + \frac{nEw}{(KL)^2} \right], \text{ksi} \]

\[ \sigma_{TFO} = \frac{1}{2\beta} \left[ (\sigma_{ex} + \sigma_t) - \sqrt{(\sigma_{ex} + \sigma_t)^2 - 4\beta\sigma_{ex}\sigma_t} \right], \text{ksi} \]

(3) If \( Q < 1.0 \), singly-symmetric shapes or intermittently fastened components of built-up shapes subject to both axial compression and bending applied in the plane of symmetry can be conservatively proportioned by replacing \( F_y \) by \( QF_y \) in (2) or their strength may be determined by tests in accordance with Section 6. \( Q \) is defined in Section 3.6.1.1.

(4) Singly-symmetric shapes subject to both axial compression and bending applied out of the plane of symmetry must be designed according to Section 6.2, "Tests for Determining Structural Performance."

II.2.2 Background for AISI Provisions on Combined Axial and Bending Stresses

For in-plane instability (IPI), Ref. 4 gives the general background of research on beam-columns. Because there are no tests of cold-formed steel beam-columns failing by in-plane instability, one has to rely on the statistics and tests performed on the hot-rolled shapes. The statistical analysis on the test-to-prediction ratios of this type of hot-rolled beam-columns has been studied by Galambos and Ravindra. Stocky cold-formed beam-columns are here assumed to behave much the same as hot-rolled beam-columns. Future research may possibly modify this assumption.
For torsional-flexural buckling (TFB), the AISI design provisions for singly symmetric sections are based on an extensive investigation of torsional-flexural buckling of thin-walled sections under an eccentric load conducted by Pekoz, Winter and Celebi at Cornell University.\(^{(9,10)}\) This work was an expansion of the well established theory of elastic stability of thin-walled sections for the special case of cold-formed sections. The theoretical analysis for singly symmetric open shapes subjected to combined axial and bending stresses has been verified by a test program conducted by Pekoz and Celebi.\(^{(10)}\) These tests used the cold-formed steel beam-columns.

II.2.3 Testing Program and Statistics

II.2.3.1 Members Failing by In-Plane Instability

The best method of predicting in-plane instability failure is by performing numerical integration. In design, there are two interaction equations, each of which must be checked. For the case of laterally braced stocky I-shaped beam-columns subjected to bending about the major axis, these equations are\(^{(8)}\)

\[
\frac{P_u}{P_{cr}} + \frac{CM_u}{M_p (1 - \frac{u}{P_E})} \leq 1.0 \tag{2.15}
\]

\[
\frac{P_u}{P_y} + \frac{M_u}{1.18 M_p} \leq 1.0, \text{ but } \frac{u}{M_p} < 1.0 \tag{2.16}
\]

The terms used in Eqs. (2.15) and (2.16) are defined as follows:

- \(P_u, M_u\) = a point on the limit state interaction curve
- \(P_{cr}\) = the limit state axial load which can be supported by the member in the absence of bending moment
\[ P_E = \text{Euler buckling load} = \frac{P_y}{\lambda^2} \]

\[ P_y = AF_y \quad (2.17) \]

\[ P_{cr} = P_y (1-0.25 \lambda^2) \quad \text{for } \lambda \leq \sqrt{2} \quad (2.18) \]

\[ P_{cr} = P_y/\lambda^2 \quad \text{for } \lambda > \sqrt{2} \quad (2.19) \]

\[ M_p = Z F_x/y \quad (2.20) \]

- \( F_y = \text{yield point} \)
- \( A = \text{cross-sectional area} \)
- \( Z_x = \text{plastic section modulus} \)

\[ \lambda = \frac{L}{r_x} \left( \frac{1}{m} \right) \sqrt{\frac{F_y}{E}} \]

and \( C_m \) is defined in Article II.2.1.

For hot-rolled shapes, Eqs. (2.15) and (2.16) are used as the nominal resistance equations for the members failing by in-plane instability. The statistics of the test-to-prediction ratios were studied in Ref. 8. In this reference, it is shown that for the case of in-plane instability, the mean resistance of a beam-column is

\[ R_m = \left( \frac{P_u}{P_u} \right) \]

\[ = M_{mp} \frac{P}{m m u} \quad (2.21) \]

where, \( P_u = \text{predicted failure load by the interaction Eqs. (2.15) and (2.16)} \)

\[ M_m = \frac{\sigma_y}{F_y} \]

\[ F_m = \left( \frac{\text{mean cross-sectional properties}}{\text{nominal cross-sectional properties}} \right) \]
\[ P_m = \frac{(P_u)_t}{P_u} \] where \((P_u)_t\) and \(P_u\) are tested and predicted failure loads.

The following data are used in Ref. 8 for hot-rolled members:

\[ M_m = 1.05, V_M = 0.10 \]
\[ F_m = 1.00, V_F = 0.05 \]
\[ P_m = 1.02, V_P = 0.10 \]

It has also been shown in Ref. 8 that Eq. (2.15) can be used for members failing by lateral-torsional buckling if \(M_P\) is replaced by \(M_o\) which is the maximum moment capacity in the absence of compressive force. When the biaxial bending takes place, the linear interaction equation is

\[ \frac{P_u}{P_{cr}} + \frac{C_{mx} M_{ux}}{M_{ox} (1 - \frac{P}{P_{Ex}})} + \frac{C_{my} M_{uy}}{M_{oy} (1 - \frac{P}{P_{Ey}})} \leq 1 \] (2.22)

The subscripts \(x\) and \(y\) in the above formula indicate the axis of bending about which a particular load applies.

The statistics for one set of the data are:

\[ P_m = 1.11, V_P = 0.16 \]

For another set of the data, the mean and coefficient of variation are

\[ P_m = 1.16, V_P = 0.16 \]

It should be emphasized again that these data are for tests on hot-rolled wide-flange sections. For cold-formed steel compression members, local buckling and the effective width concept are essential; however, there are no tests of cold-formed steel beam-columns failing by
in-plane instability. It is, therefore, extremely difficult to generate the statistics required for the calibration.

For cold-formed beam-columns failing by in-plane instability, the following values are assumed in the investigation:

\[ M_m = 1.10, \ V_M = 0.10 \]
\[ F_m = 1.00, \ V_F = 0.05 \]
\[ P_m = 1.00, \ V_P = 0.13 \]

On the basis of these assumptions, the mean resistance and the coefficient of variation of a beam-column are:

\[ R_m = (P_u)_m = (1.10)(1.00)(1.00) P_u = 1.10 \ P_u \]
\[ V_R = \sqrt{0.10^2 + 0.05^2 + 0.13^2} \]

= 0.17

II.2.3.2 Members Failing by Torsional-Flexural Buckling

Eighteen hat-section tests of cold-formed steel beam-columns were carried out by Pekoz and Celebi at Cornell University. The tests were conducted with loads applied in the plane of symmetry with equal eccentricity at both ends. The sectional dimensions, eccentricities, lengths of column and the yield points of steels are presented in Table 2. The tested failure loads, \((P_u)^t\), were taken from Ref. 11. The predicted failure loads, \((P_u)^p\), were computed in accordance with Section 3.7 of the 1968 Edition of the AISI Specification.
See Appendix B for a sample calculation of $(P_u)$. These tested and predicted failure loads and their ratios, $(P_u)/(P_u)$, are also presented in Table 2. The mean value and its coefficient of variation of the professional factor are

$$P_m = 1.15$$
$$V_P = 0.07$$

For the purpose of calibration, the following conservative values of the mean and the coefficient of variation of the professional factor "$P$" will be assumed:

$$P_m = 1.10$$
$$V_P = 0.10$$

In view of the fact that the modulus of elasticity is the dominant material parameter for elastic buckling and the yield point of steel is a dominant material parameter for inelastic buckling, it is assumed that

$$M_m = 1.05$$
$$V_M = 0.10$$

The above values are based on $E_m = E, V_E = 0.06, (\sigma_y)_m = 1.10 F_y$ and $V_{\sigma_y/F_y} = 0.10$.

Therefore, the mean resistance and the coefficient of variation of a beam-column failing by torsional-flexural buckling is
II.2.4 Calibration

The calibration follows the procedures and steps presented in Article II.1.4 of this report by using Eqs. (1.12) through (1.17).

II.2.4.2 Members Failing by Torsional-Flexural Buckling (TFB)

The nominal resistance determined on the basis of the AISI design specification is equal to the load effect with a factor of safety \( = 1.92 \)

\[
R_n = (P_u)_p = 1.92c (D_c + L_c)
\]

in which \( D_c \) and \( L_c \) are specified dead and live loads and \( c \) is the influence factor. The mean resistance can be written as follows:

\[
R_m = (1.16)(1.92)c(D_c + L_c)
\]

Since \( Q_m = c(D_c + L_c) \), then

\[
R_m/Q_m = (1.16)(1.92)
\]

\[
= 2.21
\]

The coefficient of variation of the resistance is \( V_R = 0.15 \) and for \( D_c/L_c = 1/3 \) the coefficient of variation of the load effect is \( V_Q = 0.13 \), then the safety index, \( \beta \), is
Similarly, for $D_c/L_c = 0.1, 1.0$ and $3.0$, the values of safety index are $3.62, 4.41$ and $4.80$, respectively. These values of safety index are determined for $\lambda = 1.0$. Similar procedures can be used for other $\lambda$'s.

II.2.4.1 Members Failing by In-Plane Instability (IPI)

A calibration is performed for the beam-columns failing by in-plane instability. The following assumptions are made for this study.

1. No lateral-torsional buckling
2. Equal eccentricities
3. Compact section (i.e., $Q = 1.0$)
4. $t < 0.09$ in.
5. No plastic reserve capacities (i.e., $M_p = S F_x$)
6. Inelastic buckling (i.e., $L/r_x < C_c$)

On the basis of these assumptions, the predicted failure load of a beam-column failing by IPI can be computed from the following equation by substituting $M_u = P_u e$ and $M_p = S F_x y$ into Eq. (2.15), i.e.,

$$ \frac{P_u}{P_{cr}} + \frac{P_u e}{S_y (1 - \frac{P_u}{P})} = 1.0 \tag{2.23} $$

in which, $e$ is the eccentricity and $S_x$ is the elastic section modulus and other terms were defined previously (Article II.2.3.1).

Let,

$$ P_u = \frac{P_u}{P_y} \tag{2.24} $$
\[ p_{cr} = \frac{P_{cr}}{P_y} \]  
\[ P_E = \frac{P_E}{P_y} \]  
\[ e^* = \frac{ec}{r_x^2} = \frac{AF_{ex}}{I_y} = \frac{P_e}{S_F} \]  

Then, by using Eqs. (2.18) and (2.19),

\[ p_{cr} = 1 - \frac{\lambda^2}{4} \]  
\[ P_E = 1/\lambda^2 \]  

Eq. (2.23) can be rewritten as follows:

\[ \frac{p_u}{p_{cr}} + \frac{p_{e^*}}{1 - p_u/P_E} = 1.0 \]  

or

\[ p_u^2 - p_u(p_{cr} + P_E + e^*p_{cr}P_E) + p_{cr}P_E = 0 \]  
\[ p_u = \frac{1}{2} \left( b - \sqrt{b^2 - 4c} \right) \]  

in which

\[ b = p_{cr} + P_E + e^*p_{cr}P_E \]  
\[ c = p_{cr}P_E \]  

The AISI design capacity of a beam-column failing by IPI, \( P_D \), is determined from the following equation:

\[ \frac{P_D}{23} + \frac{P_{De}}{12 P_{cr} 0.6 S_F (1 - \frac{12 P_D}{23 P_E})} = 1.0 \]  

(2.35)
Let

$$P_D = \frac{P_D}{P_y}$$

Then, Eq. (2.35) can be written in the following form:

$$\frac{23}{12} \frac{P_D}{P_{cr}} + \frac{P_D e^*}{0.6 \left(1 - \frac{23}{12} \frac{P_D}{P_E}\right)} = 1$$

or

$$P_D - P_D \left[\frac{276}{529} \left(P_{cr} + P_E\right) + \frac{240}{529} e^* P_{cr} P_E\right] + \frac{144}{529} P_{cr} P_E = 0$$

$$P_D = \frac{1}{2} \left(b' - \sqrt{b'^2 - 4c'}\right)$$

in which

$$b' = \frac{276}{529} \left(P_{cr} + P_E\right) + \frac{240}{259} e^* P_{cr} P_E$$

$$c' = \frac{144}{529} P_{cr} P_E$$

Values of $P_u$ and $P_D$ can be computed by Eqs. (2.32) and (2.39) when numerical values of $\lambda$ and $e^*$ are known.

The nominal resistance determined on the basis of the AISI design specification is equal to the load effect with a factor of safety $SF = 1.92$.

$$R_n = P_u$$

The mean resistance of a beam-column failing by IPI is

$$R_m = 1.10 P_u = 1.10 \frac{P_u}{P_D} P_D$$

Since $Q_m = c(D_{cr} + L_c) = P_D$
where the values of $p_u$ and $p_D$ are computed by Eqs. (2.32) and (2.39), respectively.

The safety index is

$$
\beta = \frac{\ln \left( \frac{R_m}{Q_m} \right)}{\sqrt{\frac{V^2 + V_r^2}{R + Q}}}
$$

$$
= \frac{\ln (1.1 \frac{p_u}{p_D})}{\sqrt{(0.17)^2 + V_Q^2}}
$$

Various values of safety index, $\beta$, can be computed for different values of $D_c/L_c$, $\lambda$ and $e^*$. These values are presented in Table 3. From this table, it can be seen that for $D_c/L_c = 1/3$, the value of safety index is 3.49 for columns ($e^* = 0$) and is 2.85 for beams ($e^* = 1.00$).
II.3 SUMMARY AND FUTURE STUDY

During the period of August through December 1977, the research activities on load and resistance factor design of cold-formed steel includes: 1) calibration of the design provisions on laterally unbraced beams, and 2) calibration of the design provisions on combined axial and bending stresses.

The preliminary research findings are presented in this report. Values of safety index for beam-columns and beams subjected to lateral buckling have been evaluated on the basis of first order probabilistic theory by using the available test data on mechanical properties and the failure loads of these types of members. Representative values of $\beta$ are presented in Table 4. The selection of $\beta$ and the determination of the resistance factor, $\phi$, will be made at a later date after other design provisions are calibrated.

In the near future, it is planned to carry out the following studies:

1) Web design of beams
2) Web crippling of beams
3) Serviceability criteria
III. REFERENCES


IV. APPENDICES
APPENDIX A: Notation

A = Analysis factor = Cross-sectional area

B = Live load transformation factor = Width of section

C = Dead load transformation factor

$C_b, C_m, C_{TF}$ = Equivalent bending moment factors

$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$

$C_w$ = Warping constant

c = Influence factor = Distance from the centroidal axis to the fiber with maximum compression stress

e = subscript defining code - Specified loads

D = Dead load intensity

d = Depth of section

E = Modulus of elasticity

e = Eccentricity

e* = $ec/r_y^2$

F = Fabrication factor

$F_a$ = Maximum average compression stress

$F_{aC}, F_{aE}$ = Average allowable compression stresses under eccentric loading

$F_{a0}, F_{al}, F_{a2}$ = Allowable compression stresses under concentric loading

$F_b, F_{b1}$ = Maximum bending design stresses

FS = Factor of safety

$F_y$ = Yield stress

$F_e' = \frac{12 \pi^2 E}{23 (KL_b/r_b)^2}$
\[ f_a = \text{Axial stress} \]
\[ f_b = \text{Bending stress} \]
\[ G = \text{Shear modulus} \]
\[ I_{xc} = \text{Moment of inertia of the compression portion of a section about its axis of symmetry} \]
\[ I_y = \text{Moment of inertia of the section about the y-axis} \]
\[ I_{ye} = \text{Moment of inertia of the compression portion of a section about its gravity axis parallel to the web} \]
\[ J = \text{St. Venant torsional constant} \]
\[ j = \text{Section property, torsional flexural buckling} \]
\[ K = \text{Effective length factor} \]
\[ L = \text{Live load intensity} = \text{Unbraced length of beam} = \text{Length of column} \]
\[ L_b = \text{Actual unbraced length in the plane of bending} \]
\[ M = \text{Material factor} = \text{Bending moment} \]
\[ M_{cr}, M_c, M_t = \text{Critical moments} \]
\[ M_1, M_2 = \text{End moments} \]
\[ M_o = \text{Maximum moment capacity in the absence of compressive force} \]
\[ m = \text{Subscript defining mean value} = \text{Distance of shear center of channel from mid-plane of the web} \]
\[ n = \text{Subscript defining nominal resistance} \]
\[ P = \text{Professional factor} = \text{Axial load} \]
\[ P_D = \text{Design load} \]
\[ P_E = \text{Elastic buckling load} \]
\[ P_u = \text{Failure load capacity} \]
$P_y = \text{Yield load}$

$p = \text{Subscript defining predicted value}$

$Q = \text{Load effect = Stress and/or area reduction factor}$

$R = \text{Resistance of a structural member = Inside bend radius}$

$r_b, r_o, r_x, r_y = \text{Radii of gyration}$

$S_x = \text{Elastic section modulus}$

$S_{xc} = \text{Compression section modulus of entire section about x-axis}$

$S_{yc} = \text{Compression section modulus of entire section about y-axis}$

$s = \text{Spacing}$

$t = \text{Thickness of steel sheet = Subscript defining tested value}$

$V = \text{Coefficient of variation}$

$w = \text{Flat width of an element exclusive of fillets}$

$x_o = \text{x-coordinate of the shear center}$

$Z_x = \text{Plastic section modulus}$

$\alpha = \text{Linearization factor}$

$\beta = \text{Safety index = A constant equal to } 1 - (x_o/r_o)^2$

$\Theta = \text{Central safety factor}$

$\lambda = \text{Slenderness parameter}$

$\sigma_{bc}, \sigma_{bT} = \text{Maximum compression bending stresses caused by } M_c, M_T$

$\sigma_{b1}, \sigma_{b2} = \text{Maximum compression bending stresses caused by } a_{TF0}$

$\sigma_c = \frac{\bar{h}^2E/(KL_b/r_b)}{2}$

$\sigma_{ex} = \frac{\bar{h}^2E/(KL/r_x)}{2}$
\[ \sigma_t = \text{Torsional buckling stress} \]
\[ \sigma_{TF} = \text{Average elastic torsional-flexural buckling stress} \]
\[ \sigma_{TFO} = \text{Elastic torsional-flexural buckling stress} \]
\[ \sigma_y = \text{Tested yield stress} \]
APPENDIX B: Sample Calculations

The following is a detailed determination of the predicted failure load \((P_u)^p\) for specimen H2-1-2. This sample calculation follows the steps and formulas used in the AISI Design Manual.\(^{(19)}\)

**Given:**
- Steel \(\sigma_y = 45.50\) ksi \((F = 27.30\) ksi\)
- Hat section: \(A' = 1.200\) in., \(B' = 1.650\) in., \(C' = 0.450\) in., \(t = 0.058\) in., \(L = 52.5\) in., \(K_x = 0.5\), \(K_y = 1.0\), \(e_x = 0.90\) in.
- \((P_u)^t = 2.210\) kips

**Required:** \((P_u)^p\) and \((P_u)^t/(P_u)^p\)

**Sectional Properties**

1. **Basic parameters:**
   - Assume \(R=0\), \(r = \frac{t}{2} = 0.029\) in.
   - \(\bar{a} = A'-t = 1.142\) in.
   - \(a = A'-2(r + t) = 1.084\) in.
   - \(\bar{b} = B'-t = 1.592\) in.
   - \(b = B'-2(r + t) = 1.534\) in.
   - \(\bar{c} = C' - \frac{t}{2} = 0.421\) in.
   - \(c = C'-(r + \frac{t}{2}) = 0.392\) in.
   - \(u = 1.57 \times 0.0455\) in.

2. **Area:**
   \[
   A = t(a + 2b + 2c + 4u)
   \]
   \[
   = 0.058(1.084 + 2 \times 1.534 + 2 \times 0.392 + 4 \times 0.0455)
   \]
   \[
   = 0.2969\ in.\^2
   \]
3. Moment of inertia about x-axis:

\[ I_x = 2t \left( 0.0417 a^3 + b \left( \frac{a}{2} + r \right)^2 + u \left( \frac{a}{2} + 0.637 r \right)^2 + 0.149 r^3 + 0.0833 c^3 \right. \]
\[ + \frac{c}{4} \left( a + c + 4r \right)^2 + u \left( \frac{a}{2} + 1.363 r \right)^2 + 0.149 r^3 \}
\[ = 2x0.058(0.0417(1.084)^3 + 1.534(\frac{1.084}{2} + 0.029)^2 \]
\[ + 0.0455(\frac{1.084}{2} + 0.637x0.029)^2 + 0.149(0.029)^3 + 0.0833(0.392)^3 \]
\[ + \frac{0.392}{4} (1.084+0.392+4x0.029)^2 + 0.0455(\frac{1.084}{2} + 1.363x0.029)^2 \]
\[ + 0.149(0.029)^3 \}
\[ = 0.0970 \text{ in}^4 \]

4. Distance from centroid of section to centerline of web:

\[ \bar{x} = \frac{2t}{A} \left( b \left( \frac{b}{2} + r \right) + u(0.363 r) + u(b+1.637 r) + c(b+2r) \right) \]
\[ = \frac{2x0.058}{0.2969} \left( 1.534 \left( \frac{1.534}{2} + 0.029 \right) + 0.0455(0.363x0.029) \right. \]
\[ + 0.0455(1.534+1.637x0.029) + 0.392(1.534+2x0.029) \}
\[ = 0.7493 \text{ in.} \]

5. Moment of inertia about y-axis:

\[ I_y = 2t \left( b \left( \frac{b}{2} + r \right)^2 + 0.0833 b^3 + 0.356 r^3 + u(b+2r)^2 + u(b+1.637 r)^2 \right. \]
\[ + 0.149 r^3 \} - \bar{x}^2 \]
\[ = 2x0.058 \left( 1.534 \left( \frac{1.534}{2} + 0.029 \right)^2 + 0.0833(1.534)^3 + 0.356(0.029)^3 \right. \]
\[ + 0.392(1.534+2x0.029)^2 + 0.0455(1.534+1.637x0.029)^2 + 0.149(0.029)^3 \}
\[ - 0.2969(0.7493)^2 \]
6. Distance from shear center to center line of web:

\[ m = \frac{\delta_t}{12} \left\{ 6\bar{c}(\bar{a})^2 + 3\bar{b}(\bar{a})^2 - 8\bar{c}^3 \right\} \]

\[ = \frac{1.592 \times 0.058}{12 \times 0.0970} \left\{ 6(0.421)(1.142)^2 + 3(1.592)(1.142)^2 - 8(0.421)^3 \right\} \]

\[ = 0.7079 \text{ in.} \]

7. Distance from centroid to shear center

\[ x_o = -(\bar{x} + m) = -(0.7493 + 0.7079) \]

\[ = -1.4573 \text{ in.} \]

8. St. Venant torsion constant, J:

\[ J = \frac{t^3}{3} \left\{ a + 2b + 2c + 4u \right\} \]

\[ = \left( \frac{0.058}{3} \right)^3 \left\{ 1.084 + 2 \times 1.534 + 2 \times 0.392 + 4 \times 0.0455 \right\} \]

\[ = 0.000333 \text{ in.}^4 \]

9. Warping constant, C_w:

\[ C_w = \frac{\bar{a}^2}{4} \left\{ I_y + (\bar{x})^2 A(1 - \frac{(\bar{a})^2 A}{4 I_x}) + \frac{2(\bar{b})^2 t(\bar{c})^3}{3} - (\bar{a})(\bar{b})^2 (\bar{c})^2 t \right\} \]

\[ + \frac{(\bar{a})(\bar{b})(\bar{c})^3 A}{3 I_x} - \frac{4(\bar{b})^2 t^2 (\bar{c})^6}{9 I_x} \}

\[ = \frac{(1.142)^2}{4} \left\{ 0.1094 + (0.7493)^2 (0.2969) \left[ 1 - \frac{(1.142)^2 (0.2969)}{4(0.0970)} \right] \right\} \]

\[ + \frac{2(1.592)^2 (0.058)(0.421)}{3} - (1.142)(1.592)(0.421)^2 (0.058) \]

\[ + \frac{(1.142)^2 (1.592)(0.058)(0.421)^3 (0.7493)(0.2969)}{(3)(0.0970)} \]
\[
\frac{(4)(1.592)^2(0.058)^2(0.421)^6}{9(0.0970)}
\]

\[= 0.0200 \text{ in.}\]

10. Constant \(j\):

\[j = \frac{1}{2} I_y \left( \beta_w + \beta_f + \beta_\chi \right) - x_o\]

where

\[\beta_w = \{-0.0833 \, t \bar{x} (a)^3 + t(x)^3 \bar{a}\}\]

\[= \{-0.0833(0.058)(0.7493)(1.142)^3 + (0.058)(0.7493)^3(1.142)\}\]

\[= -0.0333 \text{ in.}^5\]

\[\beta_f = \frac{t}{2} \left\{ (\bar{b} - \bar{x})^4 - (\bar{x})^4 \right\} + \frac{t(a)^2}{4} \left\{ (\bar{b} - \bar{x})^2 - (\bar{x})^2 \right\}\]

\[= \frac{0.058}{2} \{1.592 - 0.7493\}^4 - (0.7493)^4\} + \frac{0.058}{4} (1.142)^2\]

\[= 0.0083 \text{ in.}^5\]

\[\beta_\chi = 2 \, c t (\bar{b} - \bar{x})^3 + \frac{2t}{3} (\bar{b} - \bar{x}) \left\{ \left( \frac{a}{2} + \bar{c} \right)^3 - \left( \frac{a}{2} \right)^3 \right\}\]

\[= 2(0.421)(0.058)(1.592 - 0.7493)^3 + \frac{2t(0.058)}{3} (1.592 - 0.7493)\]

\[\left\{ \left( \frac{1.142}{2} + 0.421 \right)^3 - \left( \frac{1.142}{2} \right)^3 \right\}\]

\[= 0.0550 \text{ in.}^5\]

\[j = \frac{1}{2(0.1094)} \{-0.0333 + 0.0083 + 0.0550\}\]

\[= 1.5943 \text{ in.}\]
11. Radii of gyration

\[ r_x = \sqrt{\frac{l_x}{A}} = \sqrt{\frac{0.0970}{0.2969}} = 0.5717 \text{ in.} \]

\[ r_y = \sqrt{\frac{l_y}{A}} = \sqrt{\frac{0.1094}{0.2969}} = 0.6071 \text{ in.} \]

\[ r_o = \sqrt{r_x^2 + r_y^2 + r_o^2} = \sqrt{(0.5717)^2 + (0.6071)^2 + (1.4573)^2} = 1.6790 \text{ in.} \]

12. Constant \( \beta \):

\[ \beta = 1 - \left( \frac{r_o}{r_o} \right)^2 = 1 - \left( \frac{1.4573}{1.6790} \right)^2 = 0.2467 \]

13. The member is composed of stiffened and unstiffened elements.

Therefore, paragraph (3) in Spec. Sec. 3.6.1.1(a) applies

For the web: \( \frac{w}{t} = \frac{a}{t} = \frac{1.084}{0.058} = 18.67 \)

For the flange: \( \frac{w}{t} = \frac{b}{t} = \frac{1.592}{0.058} = 27.45 \)

Since these are stiffened elements, Spec. Sec. 2.3.1.1 applies

\[ (\frac{w}{t})_{\text{lim}} = 171/\sqrt{\beta} = 171/\sqrt{27.3} = 32.73 \]

Both web and flanges are fully effective, \( Q_a = 1 \)

For the lip (Spec. Sec. 3.2) \( \frac{w}{t} = \frac{c}{t} = \frac{0.392}{0.058} = \)

\[ 6.76 < \frac{63.3}{\sqrt{45.5}} \]

Therefore, \( F_c = 0.6 \quad F_y = 27.3 \text{ ksi and } Q_s = 1 \)

The shape factor is \( Q = Q_a \times Q_s = 1 \)
Calculation of \((P_u)_p\) in accordance with AISI Spec.

In accordance with Spec. Sec. 3.7.3, singly-symmetric sections fail either by yielding as a result of continuous flexural beam-column behavior or by torsional-flexural buckling. Allowable stresses will be calculated for both modes of failure and the smaller of the two will govern.

1. Continuous flexural beam-column behavior

The allowable load for continuous beam-column behavior is calculated according to Spec. Sec. 3.7.2 (i).

Assume \(f_a/F_{al} > 0.15\), then the first relation to be satisfied is the following:

\[
\frac{f_a}{F_{al}} + \frac{f_b}{F_{bl}} \leq 1
\]

This expression can be rewritten in the form,

\[
\frac{P/A}{F_{al}} + \frac{(P_e/S_y)C_m}{F_{bl}(1-f_a/F_{ae})} \leq 1
\]

In order to solve for \(P\), a value must be assumed for \(f_a\).

\[
\frac{K_yL}{r_y} = \frac{(1)(52.5)}{0.6071} = 86.48
\]

\[
C_c = \sqrt{2\pi^2E/F_y} = \sqrt{2\times9.87\times29,500/45.5} = 113.13
\]

\[
C_c/Q = 113.13/1 = 113.13
\]

\[
F_{al} = 0.522 QF_y - \frac{QF_{K_yL/r_y}^2}{1494}
\]
Assume $f_a = 4$ ksi (first approximation)

Noting that the maximum compression stress due to bending and axial loading is at the lips, the full section modulus can be used to calculate $f_b$

$$S = \frac{I}{y c} = \frac{0.1094}{1.650-(\bar{x} + \frac{t}{2})} = 0.1255 \text{ in}^3$$

$$f_b = \frac{P(\bar{x})}{S_y} = \frac{P(0.90)}{0.1255} = \frac{P}{0.1394}$$

Since $w/t$ (at lips) = 0.392/0.058 = 6.76 < 63.3/√45.5, $f_{bl}$ is determined in accordance with Spec. Sec. 3.2, i.e.

$$F_{bl} = 0.6 F_y = 27.3 \text{ ksi}$$

$$C_m = 1.0$$

$$F'_e = \frac{12 \pi^2 E}{23 (K_y L/r_y)^2} = \frac{12 \times 9.87 \times 29,500}{23 (86.48)^2} = 20.31 \text{ ksi}$$

The relation to be satisfied can then be written as follows:

$$\frac{P/0.2969}{16.81} + \frac{P/0.1394}{27.3 (1-4/20.31)} = 1.0$$

Solving this expression for $P$ leads to

$$P = 1.90 \text{ kips}$$

Based on this value of $P$, $f_a = \frac{1.90}{0.2969} = 6.38 \text{ ksi}$ (compare with 4 ksi for the first approximation)
Assume $f_a = 5.9$ ksi (second approximation)

$$\frac{P}{0.2969} + \frac{P}{0.1394} = 1.0$$

$$P = 1.75 \text{ kips}$$

$$f_a = \frac{1.75}{0.2969} = 5.90 \text{ ksi}$$

Therefore, $P = 1.75$ kips [based on the first interaction equation in Spec. Sec. 3.7.2(1) for $f_a/F_{al} > 0.15$]

A second value of $P$ is obtained from the relation

$$\frac{f_a}{F_{a0}} + \frac{f_y}{F_{b1}} < 1.0$$

In order to solve for the allowable value of $P$, this relation can be rewritten as follows:

$$\frac{P/A}{F_{a0}} + \frac{Pe_x/S_y}{F_{b1}} < 1.0$$

$$F_{a0} = 0.522 \text{ QF} = 0.522x45.5 = 23.75 \text{ ksi when L=0}$$

Hence, the second relation to be satisfied can be written as follows:

$$\frac{P}{0.2969} + \frac{P}{0.1394} = 1.0$$

From which, $P = 2.47$ kips

The lower value of $P = 1.75$ kips obtained from the first relation governs for the continuous beam-column failure mode.

2. **Torsional-Flexural Buckling**

Since the axial load is located on the side of the centroid away from the shear center, the allowable load for torsional-flexural
Buckling is calculated in accordance with Spec. Sec. 3.7.2(ii)

\[
\frac{KL}{r_x} = \frac{(0.5)(52.5)}{0.5717} = 45.92
\]

\[
\sigma_{ex} = \frac{\pi^2 E}{(KL/r_x)^2} = \frac{9.87 \times 29,500}{(45.92)^2} = 138.09 \text{ ksi}
\]

\[
\sigma_t = \frac{1}{A_r} \left\{ 11,300 J + \pi^2 E \left( KL/r_x \right)^2 \right\}
\]

\[
= \frac{1}{0.2969(1.6790)^2} \left\{ 11,300(0.000333) + \frac{9.87 \times 29,500 \times 0.0200}{(26.25)^2} \right\}
\]

\[
= 14.60 \text{ ksi}
\]

\[
\sigma_{TFO} = \frac{1}{2\beta} \left[ (\sigma_{ex} + \sigma_t) - \sqrt{(\sigma_{ex} + \sigma_t)^2 - 4 \beta \sigma_t \sigma_{ex}} \right]
\]

\[
= \frac{1}{2(0.2469)} \left[ (138.09 + 14.60) - \sqrt{(138.09 + 14.60)^2 - 4(0.2469)(138.09)(14.60)} \right]
\]

\[
= 13.50 \text{ ksi}
\]

\[
\sigma_e = \frac{\pi^2 E}{(KL/r_y)^2} = \frac{9.87 \times 29,500}{(86.48)^2}
\]

\[
= 38.93 \text{ ksi}
\]

\[
M_T = -A \sigma_{ex} \left[ j - \sqrt{j^2 + \frac{r_o^2 \sigma_t}{\sigma_{ex}}} \right]
\]

\[
= -(0.2969)(138.09) \left\{ 1.5943 - \sqrt{(1.5943)^2 + (1.6790)^2} \frac{14.60}{138.09} \right\}
\]

\[
= 3.72 \text{ in.-kips}
\]
\[ \phi_1 = \sigma_{TFO} \sigma_e \]
\[ = (13.50)(38.93) \]
\[ = 525.40 \]

\[ \phi_2 = \sigma_{TFO} + \sigma_e + \frac{G_F \bar{e} \phi_1}{M_T} \]

\[ C_{TF} = 1.0 \text{ since there is no relative point translation} \]

\[ \phi_2 = 13.50 + 38.93 + \frac{(1)(0.90)(0.2969)(525.40)}{3.73} \]

\[ = 90.11 \]

\[ \sigma_{TF} = \frac{1}{2} \left[ \phi_2 - \sqrt{\phi_2^2 - 4\phi_1} \right] \]

\[ = \frac{1}{2} \left[ 90.11 - \sqrt{(90.11)^2 - 4(525.56)} \right] \]

\[ = 6.27 \text{ ksi} \]

\[ F_a = 0.522 \sigma_{TF} \]

\[ = 3.27 \text{ ksi} \]

\[ P = 0.971 \text{ kips} \]

\[ (P_u)_p = 1.92 \ P \]

\[ = 1.864 \text{ kip} \]

\[ \frac{(P_u)_t}{(P_u)_p} = \frac{2.21}{1.864} = 1.1854 \]
APPENDIX C: Evaluation of the Linearization Factor for Cold-Formed Steel Members

The first order probabilistic design criterion is

\[ R_m = \theta Q_m \]  \hspace{1cm} (C1)

where \( R_m \), \( Q_m \) are mean values of the resistance and the load effect and \( \theta \) is the central safety factor which is given by the following equation. (7)

\[ \theta = \exp \left( \beta \sqrt{V_R^2 + V_Q^2} \right) \]  \hspace{1cm} (C2)

in which \( \beta \) is the safety index and \( V_R \) and \( V_Q \) are coefficients of variation of the resistance and load effect, respectively.

The mean load effect, \( Q_m \), for a combination of dead and live loads is assumed to be of the following form

\[ Q_m = A_m (c_D D_m + c_L L_m) \]  \hspace{1cm} (C3)

in which \( A_m \) is a mean value of a random variable representing the uncertainties in structural analysis, and \( B_m \) and \( C_m \) are mean values of random variables reflecting the uncertainties in the transformation of load intensities into load effects and \( c_D \) and \( c_L \) are influence coefficients.

By assuming that \( A_m = B_m = C_m = 1 \) and \( c_D = c_L = c \), the mean value and the coefficient of variation of load effect are

\[ Q_m = cL_m \left( \frac{D_m}{L_m} + 1 \right) \]  \hspace{1cm} (C4)
in which, \( V_A \), \( V_C \), \( V_D \) and \( V_L \) are coefficients of variation associated with the uncertainties in structural analysis (A), dead load (C,D) and live load (L,B) random variables, respectively. Because of the use of relatively small tributary areas for cold-formed steel members, the mean values of dead and live loads, \( D_m \) and \( L_m \), may be assumed to be the specified values of \( D_c \) and \( L_c \), respectively. Eqs. (C4) and (C5) can be written in the following forms:

\[
Q_m = c L_c \left( \frac{D_c}{L_c} + 1 \right) \tag{C6}
\]

\[
V_Q = \frac{\sqrt{V_A^2 + \frac{(D_m/L_m)^2 (V_C^2 + V_D^2) + (V_B^2 + V_L^2)}{(D_m/L_m + 1)^2}}}{(D_c/L_c + 1)^2} \tag{C7}
\]

The design criterion can be written as follows:

\[
\frac{R_m}{\exp \alpha \beta V_R} = \exp \alpha \beta V_A \left\{ c D_c [1 + \alpha \beta \sqrt{V_C^2 + V_D^2}] \right. \\
+ c L_c [1 + \alpha \beta \sqrt{V_B^2 + V_L^2}] \} \tag{C8}
\]

in which \( \alpha \) is a linearization factor determined by minimizing a function of error in the approximation.

The approximate central safety factor, \( \theta_a \), is
The value of \( \alpha \) is so chosen as to minimize a function of the approximation, \( \varepsilon = 100(\theta_a - \theta_a)/\theta_a \). The function to be minimized could be the maximum error in the domain of all design situations. A design situation is characterized by the values of data variables \( D_c/L_c \), the coefficients of variation \( V^c_R \), \( V^c_A \), \( \sqrt{V^2_C + V^2_D} \), \( \sqrt{V^2_B + V^2_L} \), and the safety index \( \beta \). The minimization of maximum error is biased towards the combinations giving maximum errors. In order to obtain a more representative set of \( \alpha \)'s, the frequencies of occurrence of data variables have to be considered. The ranges of data variables were chosen as follows:

\[
D_c/L_c = 0.1, 0.4, 0.7, 1.0, 1.3
\]
\[
\sqrt{V^2_B + V^2_L} = 0.15, 0.20, 0.25
\]
\[
\sqrt{V^2_C + V^2_D} = 0.04, 0.06
\]
\[
V_A = 0.03, 0.05
\]
\[
V_R = 0.10, 0.15, 0.20
\]
\[
\beta = 2.5, 3.0, 4.0
\]

With \( \alpha = 0.55 \), the distribution of the error involved in this approximation is shown in Figs. C1 through C4.

From this study, it is concluded that \( \alpha = 0.55 \) can be also a reasonable linearization factor used for cold-formed steel members.
TABLE 1
Comparison of the Tested and Predicted Failure Loads of Cold-Formed Steel I-Beams Subjected to Elastic Lateral Buckling

<table>
<thead>
<tr>
<th>Specimen</th>
<th>L (in.)</th>
<th>(P_u) t (lbs)</th>
<th>(P_u) p (lbs)</th>
<th>(P_u) t / (P_u) p</th>
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<td>S-1/1</td>
<td>90</td>
<td>520</td>
<td>491</td>
<td>1.06</td>
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<td>720</td>
<td>&quot;</td>
<td>1.47</td>
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Mean value of \( \frac{(P_u)_t}{(P_u)_p} \): \( P_m = 1.18 \)

Coefficient of variation \( V_P = 0.19 \)
TABLE 2

Comparison of the Tested and Predicted Failure Loads of Cold-Formed Steel Columns Subjected to Torsional-Flexural Buckling Under Eccentric Load

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<th>( \sigma_y) (ksi)</th>
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| Mean of \(\frac{(P_u)^*}{P_u}\)/\(\frac{P_u}{P_t}\) | \(P_u = 1.15\) |
| Coefficient of variation | \(V_P^m = 0.07\) |

\* \(\frac{(P_u)^*}{P_u}\) was computed on the basis of Section 3.7 of the AISI Specification. See Appendix B for sample calculation.

For dimensions see Fig. 4.
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$^+$ These values of safety index are computed for $\lambda = 1.00$

$^{++}$ These values of safety index are computed for $\lambda = 1.60$ and $e^* = 1.00$
Fig. 1. Comparison of the Tested and Predicted Failure Loads of Cold-Formed Steel I-Beams Subjected to Elastic Lateral Buckling
Fig. 2. Lipped I-Section

Fig. 3. Loading of Beams
Fig. 4. Dimensions of Test Specimens
\[ \beta = 2.5 \]

\[ V_R = 0.1, 0.15, 0.2 (0.1, 0.5, 0.4) \]

\[ \alpha = 0.55 \]

\[ V_E = 0.03, 0.05 (0.9, 0.4) \]

\[ \sqrt{V_A^2 + V_D^2} = 0.04, 0.06 (0.1, 0.9) \]

\[ \sqrt{V_B^2 + V_L^2} = 0.15, 0.2, 0.25 (0.1, 0.5, 0.4) \]

\[ \frac{D}{L} = 0.1, 0.4, 0.7, 1.0, 1.3 (0.1, 0.2, 0.3, 0.2, 0.2) \]

\[ \epsilon = \frac{\theta - \theta_0}{\theta_0} \times 100 \]

Fig. C1
\( \beta = 2.5 \)

\( \alpha = 0.55 \)

\( V_R = 0.1, 0.15, 0.2 \ (0.2, 0.5, 0.3) \)

\( V_E = 0.03, 0.05 \ (0.5, 0.5) \)

\[ \sqrt{V_{A+D}^2} = 0.04, 0.06 \ (0.5, 0.5) \]

\[ \sqrt{V_{B+L}^2} = 0.15, 0.2, 0.25 \ (0.4, 0.4, 0.2) \]

\( D/L = 0.1, 0.4, 0.7, 1.0, 1.3 \ (0.1, 0.3, 0.3, 0.2, 0.1) \)

\[ \epsilon = \frac{\theta - \bar{\theta}}{\bar{\theta}} \times 100 \]

Fig. C2
\[ \beta = 3.0 \quad V_R = 0.10, 0.15, 0.20 \ (0.2, 0.5, 0.3) \]
\[ \alpha = 0.55 \quad V_E = 0.03, 0.05 \ (0.5, 0.5) \]

\[ \sqrt{V_{A,D}^2 + V_{D}^2} = 0.04, 0.06 \ (0.5, 0.5) \]

\[ \sqrt{V_{B,L}^2 + V_{L}^2} = 0.15, 0.2, 0.25 \ (0.4, 0.4, 0.2) \]

\[ D/L = 0.1, 0.4, 0.7, 1.0, 1.3 \ (0.1, 0.3, 0.3, 0.2, 0.1) \]

\[ \epsilon = \frac{\theta - \bar{\theta}}{\bar{\theta}} \times 100 \]

Fig. C3
$\beta = 4.0 \quad V_R = 0.1, 0.15, 0.2 (0.2, 0.5, 0.3)$

$\alpha = 0.55 \quad V_E = 0.03, 0.05 (0.5, 0.5)$

$\sqrt{V_A^2 + V_D^2} = 0.04, 0.06 (0.5, 0.5)$

$\sqrt{V_B^2 + V_L^2} = 0.15, 0.2, 0.25 (0.4, 0.4, 0.2)$

$D/L = 0.1, 0.4, 0.7, 1.0, 1.3 (0.1, 0.3, 0.3, 0.2, 0.1)$

$\epsilon = \frac{\theta - \theta_0}{\theta} \times 100$

Fig. C4