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Cold-formed steel structural elements subjected to time-dependent loading

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SUMMARY REPORT

COLD-FORMED STEEL STRUCTURAL ELEMENTS
SUBJECTED TO TIME-DEPENDENTLOADING

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SUMMARY REPORT

on

COLD-FORMED STEEL STRUCTURAL ELEMENTS
SUBJECTED TO TIME-DEPENDENT LOADING

by

Charles G. Culver

Report on a research project sponsored by
The American Iron and Steel Institute

Department of Civil Engineering
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Pittsburgh, Pennsylvania 15213

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This report summarizes the results obtained from a comprehensive study of the behavior of thin-walled, cold-formed members subjected to impact or shock loading. The influence of local buckling and the concept of effective width in the elastic post-buckling range on the dynamic response of these members is described. Analytical and experimental results for thin compression elements, beams and columns are discussed. Comparisons between the analytical and experimental results are presented. Application of the results obtained from this research to practical situations involving impact or shock loading of cold-formed members is also discussed.
ACKNOWLEDGMENTS

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The analytical and experimental studies described were conducted by R. Van Tassel, E. Zanoni, A. Osgood, J. Logue and N. Vaidya, formerly graduate students in the Department of Civil Engineering.

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<table>
<thead>
<tr>
<th>TABLE OF CONTENTS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>-i</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>-ii</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. SCOPE OF STUDY</td>
<td>3</td>
</tr>
<tr>
<td>3. THIN COMPRESSION ELEMENTS</td>
<td>5</td>
</tr>
<tr>
<td>4. BEAMS</td>
<td>11</td>
</tr>
<tr>
<td>4.1 General</td>
<td>11</td>
</tr>
<tr>
<td>4.2 Mathematical Model</td>
<td>11</td>
</tr>
<tr>
<td>4.3 Experimental Study</td>
<td>12</td>
</tr>
<tr>
<td>4.4 Analytical Study</td>
<td>15</td>
</tr>
<tr>
<td>5. COLUMNS</td>
<td>19</td>
</tr>
<tr>
<td>5.1 General</td>
<td>19</td>
</tr>
<tr>
<td>5.2 Mathematical Model</td>
<td>19</td>
</tr>
<tr>
<td>5.3 Experimental Study</td>
<td>19</td>
</tr>
<tr>
<td>5.4 Analytical Study</td>
<td>21</td>
</tr>
<tr>
<td>6. SUMMARY AND CONCLUSIONS</td>
<td>23</td>
</tr>
<tr>
<td>7. NOMENCLATURE</td>
<td>24</td>
</tr>
<tr>
<td>8. FIGURES</td>
<td>26</td>
</tr>
<tr>
<td>9. REFERENCES</td>
<td>45</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

Thin-walled, cold-formed members have found wide application in building structures (1,2)*. Both individual structural framing members and panels and decks are included in this category. The thickness of the elements comprising these members usually ranges from 28 gage (0.0149") to about 1/4 inch. The cold forming process permits the simple fabrication of a variety of cross sectional shapes.

Several differences exist between the behavior of cold-formed members and the heavier hot-rolled structural shapes. Cold working effects in the vicinity of corners and bends produced in the forming process result in localized increases in the yield strength of the member in these regions (3). This increase may have a substantial effect on the behavior and overall strength of these members (4). In addition, the width-thickness ratios of the elements comprising the cross sections of these members may be such that local buckling occurs at loads well below the yield load. Utilization of the post-buckling strength associated with these elements in designing cold-formed members is another difference from the procedure used for hot-rolled members.

Extensive analytical and experimental research studies have led to a comprehensive design specification for cold-formed members subjected to static loading (5). Cold-formed members, however, have also been used in structures subjected to loads which vary with time. Farm equipment (6), transportation equipment (railroad cars, shipping containers, ...

* Numbers in parentheses refer to references in Section 9.
etc.), highway products and also certain components in buildings fall in this category.

Although a considerable number of theoretical and experimental investigations dealing with structural response due to time-dependent loading have been conducted (7,8,9), it was not apparent that this information could be extended directly to thin-walled, cold-formed members. This was particularly true for members in which local buckling occurs under design loads. In order to establish information in this area, the American Iron and Steel Institute sponsored a research project on dynamic loading of thin-walled members at Carnegie-Mellon University from February 1967 to April 1971. The purpose of this report is to summarize the results of this research. More detailed descriptions of the individual phases of the entire program are available elsewhere (10,11,12,13,14,15,16,17).
2. SCOPE OF STUDY

Time-dependent loading conditions may be classified into two general categories. Periodic loading or loads which are essentially repetitive with time are included in the first category. Loads applied to bridge structures due to the passage of vehicles and structures supporting rotating or reciprocating machinery are examples of this type of loading. Structures subjected to rapidly applied short duration force, displacement, velocity or acceleration pulses are included in the second category. Loadings of this type are generally referred to as mechanical shock.

Design problems for the first type of loading usually involve considerations of fatigue failure. For the second category, overstressing, large permanent deformations or failure of individual structural components are important.

This study was limited to the second type of loading, i.e., short duration mechanical shock or impact loading. Of primary interest was the effect of local buckling and the associated elastic post-buckling strength on the dynamic response of thin-walled members subjected to this type of loading. Note that the energy absorption capabilities and collapse modes for these members (18) were not included in the study. Furthermore, only individual structural framing members, beams and columns, were studied. Panels, corrugated sheets, diaphragm type elements and structural systems consisting of several members were not studied. The information obtained, however, may be extended in certain cases to cover these problems or may be used as a basis for planning further studies of these more complicated problems.
In order to achieve the objective cited above, the study was broken down into the following three phases:

1. The behavior of individual thin compression elements
2. The behavior of cold-formed beams
3. The behavior of thin-walled columns.

Both analytical and experimental work was conducted. Significant results obtained from each phase are discussed in the following sections of this report.
3. THIN COMPRESSION ELEMENTS

The fundamental difference between the design of thin-walled members and the heavier hot-rolled shapes involves the post-buckling strength of the elements comprising the cross section of the member. For thin-walled members in which local buckling occurs at loads well below the yield load, the concept of "effective width" is employed for design purposes (2). The analytical and experimental studies in this area and the equations developed for determining effective width were reviewed by Jombock and Clark (19).

In the post-buckling range the load-deformation characteristics of thin-walled members is nonlinear. The first step in the study reported herein, therefore, was to determine the influence of this nonlinearity on the dynamic response of thin-walled members. The simply supported compression element shown in Fig. 1 was used for this purpose. The element represents only one portion of the cross section of a thin-walled member. The simply supported conditions along the boundaries are obviously an idealization of the practical case since the restraining effects which occur at the junctions of the segments in a formed section (web-flange juncture, etc.) are not present. In addition, the simple forcing functions shown in Fig. 1 are also idealizations of the forces produced in practical situations of shock loading. Despite these limitations the model shown in Fig. 1 may be used to study at least qualitatively the influence of post-buckling behavior on dynamic response.

Considering inertia effects normal to the plane of the plate, neglecting stress wave propagation effects and using the large deflection
plate equations, Van Tassel (10, 11) derived a nonlinear differential equation with variable coefficients for determining the dynamic response of the model in Fig. 1. Solutions of this equation were obtained using numerical techniques. Results for the time variation of the strains and transverse deflections were presented. By neglecting the inertia terms, the post-buckling response of a plate subjected to static loading was also determined. It should be noted that these expressions derived for the effective width for static loading were quite close to those determined experimentally by Winter (11).

Typical results from this investigation are shown in Figs. 2, 3. The ratio of the maximum transverse deflection of the plate which occurs during the dynamic response when subjected to a triangular pulse type load to the maximum transverse deflection produced by a static load equal to the maximum value of the dynamic load, $S_{\text{Dyn.}}/S_{\text{Static}}$, is plotted as a function of the ratio of the time duration of the load pulse to the natural period of the plate, $\beta'$ in Fig. 2. By nondimensionalizing the results for the deflection in this manner, the effect of the dynamic loading or the impact factor is readily apparent. Curves are presented for several values of the ratio of the maximum load to the buckling load of the plate, $\alpha' = P_{\text{max}}/P_{\text{cr}}$, and for two cases of the in-plane boundary conditions along the supported edges (Case 1, Case 2). The parameter $S_0$ is the ratio of the initial deflection of the element prior to loading divided by the thickness and represents the initial imperfection or "out of flatness" ratio.

Referring to Fig. 2, note that the influence of the load duration, $\beta'$, on the impact factor for deflection is similar to that in a linear
elastic system. For a very short load duration, the dynamic effect is less than the static effect. As the load duration increases, the dynamic effect becomes more pronounced. The time durations required for the impact curves to reach their maximum values, however, are longer than those for a linear system (8). This behavior is in agreement with previous studies of the dynamic response of nonlinear systems.

The curves in Fig. 3 illustrate the effect of the various parameters on the impact factor for edge strains.

A complete discussion of the results obtained from this phase of the investigation was presented by Culver and Van Tassel (11). These studies indicated that the following parameters affect the dynamic response of thin-walled structures subjected to shock loading: the time variation of the load (pulse shape), load duration, load magnitude, initial imperfections, and the boundary conditions. Based on the analytical results obtained, the following conclusions related to these parameters were determined:

1. The influence of the time variation of the applied loads on the impact factors for deflections is qualitatively similar to that obtained for linear single mass oscillators.

2. The magnitude of the impact factor for edge strains decreases for short duration loads and increases for longer duration loads, as the maximum applied load increases.

3. In general, the influence of the in-plane boundary conditions on the mid-plane edge strains is the same as for static loading.

In order to relate the dynamic studies discussed above directly to the concept of effective width used for design purposes the results
were replotted in the form shown in Fig. 4. The ratio of static and dynamic edge strains in Fig. 3 for $\alpha' = 2.0$ were idealized as shown. Using this ratio as a modification factor for the stress used in computing the effective width (2), a dynamic effective width was calculated. Since the edge strains are changing constantly during the vibrational response of the plate the dynamic effective width varies with time. The ratio of the minimum value of this dynamic effective width to the static value for a stiffened compression element (5) is also shown in Fig. 4. For short duration loads the ratio of the strains or the modification factor for the stress is less than one and theoretically the dynamic effective width is greater than the static effective width. As the time duration of the load increases the modification factor increases and the dynamic effective width is less than the static value and then starts increasing for longer duration loads. The general form (increasing and decreasing) of the impact factor curve is similar to that obtained for linear systems. As the load duration is continually increased, the impact factor curves would oscillate about a value of unity as in the case of linear and nonlinear single mass oscillators.

The actual time durations of the dynamic loads represented in Fig. 4 may be evaluated by considering a specific example. For an 8" square 16 gage compression element, for example, the value of $\beta' = 3.2$ in Fig. 4 at which the static and dynamic effective widths differ by less than 2% corresponds to a load duration of 0.018 sec. For some practical problems of shock loading (20), this load duration would be considerably less than that which actually occurs and the dynamic and static effective widths would be nearly equal.
The influence of a dynamic effective width for the thin compression elements in a cold-formed beam may be established by considering the relationship between overall beam response and that of the elements comprising the cross section of the member when subjected to shock loading. A thin-walled beam subjected to shock loading will vibrate as a unit. If during these vibrations the compressive stress exceeds the local buckling stress of any of the elements of the beam cross section local buckling will occur and additional vibrations in the plane of cross section will be set up. Obviously these local vibrations will affect the overall vibrations of the member as a whole since local buckling reduces the bending rigidity of the member. The necessity of using a dynamic effective width in this case may be qualitatively determined by evaluating the influence of a particular shock load on both the beam response and the response of the compression elements of the cross section.

The two scales for $\beta$ in Fig. 4 illustrate this influence. Using the appropriate equations, the ratio of the natural frequency of a simply supported plate to that of a simply supported beam is given by

$$\frac{\omega_{\text{plate}}}{\omega_{\text{beam}}} = 2 \left( \frac{L}{W} \right)^2 \sqrt{\frac{h}{r_x}} \left( \frac{1}{12(1-v^2)} \right)$$  \hspace{1cm} (1)

Specifying Poisson's ratio, $v$, the ratio of the length of the beam to the width of the compression element, $L/W$, and the thickness of the cross sectional elements to the radius of gyration of the entire cross section, $h/r_x$, in Eq. 1, the time duration of the load pulse expressed in terms of the natural period of the plate, $\beta'$, may be written in terms of the natural period of the beam, $\beta$. The scales for $\beta$ in Fig. 4 were estab-
lished in this way for an I beam with \( h/r_x = 0.039 \) fabricated from two channels with stiffened flanges. In one case, \( L/r_x = 44 \), the length of the beam was 30 times the flange width of one channel and in the other case, \( L/r_x = 107 \), it was 75 times. Referring to Fig. 4 note that for both beams the time durations of the shock loads for the range in which the dynamic effective width is less than the static value, \( 1.3 < \beta' < 3.2 \) are considerably shorter than the natural periods of the beam, i.e., \( \beta << 1.0 \). Since the maximum impact factor for overall beam response occurs for shock loads with time durations nearly equal to the natural period of the beam, Fig. 4 indicates that dynamic effects associated with vibration of the elements of the cross section may not be important in this range. For extremely short duration loads, \( \beta << 1 \), local vibrations would be important. In this range, however, the impact factors for the overall beam vibrations would be less than one.

In order to determine the relationship between these localized and overall vibrations for beam sections the experimental and analytical studies described in the next section were conducted. Conclusions regarding the necessity of considering a dynamic effective width were established based on the results of these beam studies.
4. BEAMS

4.1 General

The objective of the beam studies phase of this investigation was to determine the effect of local buckling of the compression elements of thin-walled beams on the overall response of the member. Only the dynamic response in the elastic range was considered. Post yield behavior and energy absorption capacities were not included.

Both analytical and experimental studies were conducted. Experimental studies were conducted in order to verify the mathematical model developed in the analytical phase and establish the adequacy of this model for predicting dynamic response.

4.2 Mathematical Model

For thin-walled beams in which the width-thickness ratio of the compression elements exceeds certain limits, local buckling occurs at loads less than the design load. This local buckling is accompanied by a corresponding decrease in the overall flexural rigidity of the member. As the load increases the flexural rigidity continuously decreases and the load-deflection curve for the member is nonlinear. This behavior is indicated in Fig. 5 for a beam with a hat shaped cross section.

The relationship between the moment of inertia and section modulii and the applied moment is illustrated in Fig. 5. In plotting these curves, the cross sectional properties for moments greater than the local buckling moment, \( M_{LB} \), were determined using the effective width concept (2). The as built cross sectional properties \( (I_0, S_T, S_B) \) were used to nondimensionalize these curves and it was assumed that the flat width ratio for
the bottom flange elements was small enough such that they do not buckle under negative moment. Note that the termination points of these curves for positive moment corresponds to the moment at which the maximum stress reaches the yield stress.

For dynamic loading, the flexural rigidity varies with time for each cross section along the length of the member. For this reason, exact or closed form mathematical solutions of the response are not possible. An approximate solution for this problem was developed by Zanoni (12). The numerical solution was based on a lumped mass idealization of the beam and an incremental procedure. The acceleration of the mass points was assumed to vary linearly over each time increment and the beam stiffness between two mass points remained constant over the time increment. Since the internal moments varied with time, the beam stiffness also changed from one time increment to another. This procedure is analogous to subdividing the dynamic response into an extremely large number of static response calculations.

In calculating the beam stiffness Zanoni used expressions for the effective width based on static loading. No direct attempt was made to include the concept of dynamic effective width developed in the study of thin compression elements. A series of experiments was carried out, therefore, in order to determine the approximation introduced by neglecting this effect. Results obtained from this mathematical model will be discussed after reviewing the experimental results.

4.3 Experimental Study

The dimensions of the beam specimens used in Zanoni's tests are shown in Fig. 6. The flat width ratios of the top flange were selected
in order to insure that local buckling occurred in the elastic range. In order to have a direct basis of comparison for the dynamic tests, static tests were also conducted. The static tests were similar to those used to develop the existing design specification for cold-formed beams (5) and the results compared very well with these previous tests.

The test setup used for the dynamic tests is shown in Fig. 7. Shock loading was produced by dropping a weight on the test beams. The variation with time of the load magnitude, centerline deflection and strains over the cross section was measured using oscilloscopes and the instrumentation shown in Fig. 7.

Typical results obtained from these impact tests are shown in Fig. 8. A complete description of all the test results is presented elsewhere (12,13). A comparison of the measured stresses at the edge and the middle of the top flange in Fig. 8 illustrates an important aspect of the dynamic behavior observed in these tests. The irregular behavior of the stress at the middle of the flange which occurred at about 0.03 seconds is an indication that the top flange of the member was vibrating as a plate element. These vibrations were superimposed on the overall beam vibrations as discussed in Chapter 3.

In order to determine the influence of this plate-beam interaction and the significance of a dynamic effective width, all the test results, both static and dynamic, conducted in this investigation were compared as shown in Fig. 9 in a form similar to that used by Winter (21). The effective width, b/w, is plotted in Fig. 9 as a function of the stress level $\sigma_T$ and the width-thickness ratio of the compression element for each test. A total of 30 dynamic tests are shown. In plotting these
results the maximum value of $\sigma_T$ or equivalently the minimum effective width which occurred in the dynamic tests was used.

The static test results in Fig. 9 indicate a fair amount of scatter. This is typical of test results obtained for members of this type (21). Values of the effective width greater than the original width, $b/w > 1$, occurred in some of these tests due to experimental error. Note that the curves shown in Fig. 9 which represent the expressions used for the effective width of stiffened elements in the two editions of the AISI Specification provide a conservative estimate of the effective width for most of the tests.

The dynamic test results indicate about the same degree of scatter as the static tests. In three of the dynamic tests, effective widths greater than the original width were recorded. Although this could have been experimental error, it could also be due to the short load duration for which the impact factor is less than one and the dynamic load is less severe than a static load of equal magnitude. These dynamic tests were grouped according to the ratio of the time duration of the load pulse to the natural period of the compression flange, $\beta'$, in order to determine if any trend or indication of a dynamic effective width occurred. The data in Fig. 9 does not appear to indicate any such trend. This is not surprising in view of the small difference between the dynamic and static effective width indicated in Fig. 4 (+8% to -6%). The scatter in the test data was apparently large enough to mask any dynamic effects which may have occurred in the effective width.

Based on the above comparison, it seems reasonable to conclude that the static value for the effective width may be used for practical problems involving the dynamic response of thin-walled beams.

The dynamic tests were also used to evaluate the mathematical model discussed in the previous section. A comparison of the measured stresses and the stresses calculated using the mathematical model indicated per cent differences between the two ranging from -7% to 39%. These differences are not unusually high for dynamic
tests of this type (22). In almost all cases the per cent difference was posi-
tive indicating that the mathematical model was conservative and predicted larger
stresses than actually occurred in the tests. Because of this conservatism and
the agreement with test results, the mathematical model developed by Zanoni was
used to study the dynamic response of thin-walled beams.

4.4 **Analytical Study**

Preliminary results obtained by Zanoni (12) using the mathematical model in-
dicated that due to local buckling the maximum stresses which occurred in thin-
walled beams subjected to shock loading were considerably different than those
produced by the same loading for beams not subjected to local buckling. This
behavior is analogous to the case of static loading of thin-walled beams. The
internal moments produced by the shock loading, however, did not appear to be
affected by local buckling. Additional studies in this area were therefore con-
ducted by Osgood.

Osgood (14, 15) used the model discussed above to study the influence of the
following variables on the response of simply supported thin-walled beams to shock
loading: type of loading - uniform, concentrated; load magnitude; time variation
of applied load. Since the dynamic response of thin-walled beams is nonlinear,
it was also necessary to include the influence of the beam stiffness characteris-
tics in the study.

The dynamic loading used is shown in Fig. 10. The degree of nonlinearity in
a thin-walled beam is related to the length of the beam subjected to moments
which exceed the local buckling moment. Since this length is related to moment
gradient, the four loading conditions shown were selected. The idealized time
variations of the applied loading have been used in previous studies.

The beam stiffness characteristics are influenced by the type of
cross section and the width-thickness ratios for the various elements.
The three types of cross sections studied and the corresponding
stiffness relationships are shown in Figs. 11 and 12 respectively. The stiffness relationships in Fig. 12 are presented in the same form as those in Fig. 5 and cover the entire range of possibilities which could occur for cold-formed beams regardless of the shape of the cross section. In order to cover the practical range of proportions for cold-formed beams, the following cross sectional dimensions expressed in terms of the width-thickness ratios of the elements for the three sections in Fig. 11 were used: Section A -- $50 \leq w/h \leq 250$, $30 \leq d/h \leq 150$; Section B -- $30 \leq w/h \leq 60$, $30 \leq d/h \leq 150$; Section C -- $50 \leq w/h \leq 200$, $90 \leq d/h \leq 150$, $f/h = 40$. After obtaining curves similar to those in Fig. 12 for the above range of parameters, the one curve for each cross section with the greatest nonlinearity or the largest reduction in stiffness for a particular moment was used for the dynamic response study. The results obtained, therefore, illustrated the maximum influence of the nonlinear beam stiffness on the dynamic response. The influence of the effective width provisions of both the 1962 Edition and the 1968 Edition of the AISI Specification on the beam stiffness was studied.

Typical results from this phase of the study are shown in Figs. 13, 14. The results in these figures are presented in nondimensional form in order to illustrate the influence of nonlinearity on the internal moment and the deflection. In Fig. 13, for example, the ratio of the maximum internal moment obtained using the nonlinear beam stiffness, $M_{NL}$, produced in a hat-shaped beam, Section C, by a load equal to six times the load required to produce local buckling, $\alpha = 6$, to the maximum moment produced in the same beam neglecting local buckling, i.e. $M/M_{LB}$ vs. $I/I_0$ Fig. 5-constant, is shown. Results for the four types of external loading in Fig. 10 are plotted as a function of the time duration of the half sine pulse shock load to the natural period of the beam, $\beta$. The
maximum deviation between the two methods occurs for quarter-point loading. The greatest increase in moment for the nonlinear analysis over the linear analysis is less than 5%. Over most of the range of \( B \) the linear analysis predicted higher moments than the nonlinear analysis, \( \frac{M_{NL}}{M_L} < 1. \)

Based on all the results of this phase of the investigation it was concluded that the internal moments in thin-walled beams subjected to shock loading could be calculated with sufficient accuracy for design purposes using a linear elastic analysis neglecting the nonlinearity introduced by local buckling (15). For most cases such an approach would be conservative. For the remaining cases, the internal moment obtained in this fashion would be within 10% of the true moment including nonlinear effects. Calculation of the stresses produced by these moments would involve the same procedure currently used for thin-walled, cold-formed beams subjected to static loading (2,5).

Similar results for the maximum deflections produced by shock loading are presented in Fig. 14. The ratio of the maximum deflection obtained from the nonlinear dynamic analysis, \( \delta_{NL} \), to the maximum deflection from a linear analysis multiplied by the ratio of the original moment of inertia to the minimum value which occurred under the dynamic loading, \( \delta_L \left( \frac{I_0}{I} \right)_{\text{max}} \), is plotted as a function of \( B \). Note that again the linear analysis is either conservative or within a few percent of the correct values. Thus in order to calculate the maximum deflection of a thin-walled beam produced by shock loading, one could first calculate the maximum internal moment and deflection using a linear dynamic analysis. Using this internal moment and the effective width concept the reduced stiffness, \( I \), and the ratio of this stiffness to the original moment of
inertia, $I_0$, could be obtained, $(I_0/I)_{\max}$. Multiplying the linear deflection by $(I_0/I)_{\max}$ would then give a close estimate of the actual maximum deflection produced by the shock loading.

Although these studies dealt only with individual beams, it is believed that the conclusions regarding use of a linear analysis could be extended to other structures such as two dimensional frames consisting of several interconnected beams. For relatively simple structures such as beams and frames it might also be possible to eliminate the need to conduct a linear dynamic analysis for design purposes by developing impact factor relationships similar to those used in bridge design.
5. COLUMNS

5.1 General

As in the case of beams, the behavior of thin-walled columns subjected to static loading is affected by local buckling. The interaction between local and overall buckling has lead to the development of form factors to be used for cold-formed columns subject to local buckling (4,5,23). In addition, the influence of the cold-forming process on the material properties and column strength has been studied (4). Experimental (16) and analytical studies (17) were included in this investigation to determine the influence of these effects on columns subjected to shock loading.

5.2 Mathematical Model

The mathematical model developed by Vaidya for columns (17) was similar to that used by Zanoni for beams. In this case, however, the added effect of an axial load was included. The member was idealized as a system of lumped masses and the solution of the dynamic response obtained by means of an analog computer. In order to characterize the column stiffness it was necessary to develop moment-thrust-curvature relationships (M-P-\(\phi\) curves) for cold-formed members subjected to local buckling. The accuracy of this mathematical model was established using the column tests performed by Logue (16).

5.3 Experimental Study

The tests conducted by Logue were carried out on thin-walled columns subjected to both overall and local buckling as well as columns for which the width-thickness ratios of the elements were such that
local buckling did not occur prior to overall instability. Test specimens were fabricated by bolting two cold-formed channel sections back to back to form an I section. The dimensions of the specimens are given in Fig. 15. A total of eight columns of each type (Group A - \( Q < 1 \), Group B - \( Q = 1 \) ) were tested. Four of the columns, denoted by S in Fig. 15, were tested statically to provide a basis of comparison for the dynamic tests. The static tests on the columns with \( Q < 1 \) also provided additional information on the interaction between local and overall column buckling. The remaining four, denoted by D, were subjected to short duration impact type load superimposed on an existing static load. Static stub column tests were also performed for each cross section. The slenderness ratios for the members tested ranged from \( L/r_y = 44 \) to \( L/r_y = 126 \).

Details of the test setup and the experimental procedure are described elsewhere (16). In the dynamic tests, deflections and internal strains occurring during the dynamic response were measured. Typical data obtained from the dynamic tests is shown in Fig. 16. The deflection produced by the static load is indicated by the centerline deflection baseline in Fig. 16a. The variation with time of the additional deflection produced by the impact load was superimposed on this baseline. The time variation of the dynamic load is also shown. The time variation of the midplane strains at the two flange tips on the concave side of the column (side toward straight line through ends of member) is shown in Fig. 16b. The deflections and strains produced by a static load equal to the maximum impact load are also shown in Fig. 16.

The results of the dynamic test in Fig. 16 clearly indicate the effect of impact loading and the associated impact factor for deflections and strains. Although very little information was obtained from these tests on the maximum carrying capacity of columns subjected to impact loading, the tests did indicate that maximum loads in excess of the static failure load
could be carried dynamically for short load durations. Since failure in one of the dynamic tests occurred in the bolted connection between the two channel sections, the tests also indicated that additional work is required to determine the strength of connections for cold-formed members subjected to impact loading.

A comparison of the dynamic test results with calculated values obtained from the mathematical model developed by Vaidya (17) showed reasonable agreement. In general, the calculated and measured values agreed more closely for the columns with $Q = 1$ i.e. for columns not subjected to local buckling, than for those with $Q < 1$.

5.4 Analytical Study

The analytical model developed by Vaidya was used to determine dynamic load factors or impact factors which could be used for design. Using these factors, it would be possible to replace the dynamic or impact load by an equivalent static load for design calculations. These impact factors were presented graphically as response spectra as shown in Fig. 17. For a column with an initial out of straightness carrying a static load equal to the design load, $\xi_s = 0.51$, the ratio of the maximum moment produced by an impact load equal to a certain percentage of the Euler buckling load, $\xi$, to the moment produced by the same load applied statically is plotted versus the time duration of the load pulse divided by the natural period of the column. The nondimensional parameter $V_0$ characterizes the initial out of straightness. For $V_0 = 0.15$ in Fig. 17, the initial deflection prior to applying the load is 12% of the deflection produced by applying the yield moment to both ends of the member. As indicated, the impact factor,
\( \lambda_{\text{Mom}} \), increases for longer duration loads and is higher for lower values of the impact load.

For columns with \( Q = 1 \), the response spectra developed by Vaidya are applicable to a broad class of column cross sections. For columns with \( Q < 1 \), the width-thickness ratio of the plate elements comprising the cross section had a substantial effect on these response spectra and results were only obtained for a few specific cross sections.

The response spectra obtained in this study may be used to determine the influence of various parameters on the dynamic response of columns. The influence of the initial static load on the column prior to impact, for example, on the resulting stresses and deflections produced by the additional impact load was determined. Similarly, the time durations of the impact load which produce the most significant increase in stress and deflection were evaluated.

The mathematical model was also used to determine yield and failure envelopes similar to those shown in Fig. 18 for impact loads. For a column carrying a static load equal to the design load, the impact loads required to produce yielding and failure are plotted versus the time duration of the impact load. Using these curves, it is possible to determine whether a given impact load of a specific magnitude and time duration will produce yielding or failure of a column.
6. SUMMARY AND CONCLUSIONS

The information developed in this investigation and summarized herein concerned the dynamic response of thin-walled members subjected to shock or impact loading. In particular, the influence of local buckling on this response was evaluated. Analytical and experimental results obtained for flexural members indicated that the internal forces produced by impact loading may be determined with sufficient accuracy for design purposes by neglecting the effects of local buckling. Using these forces and existing design procedures for cold-formed members, deflections and stresses due to impact may be determined. For columns subjected to impact, results which may be used to determine impact factors or failure loads were obtained.

In order to utilize the results of this study for design purposes, it is suggested that design recommendations or design aids be prepared based on this information.

Several areas of impact loading also require further study. Because of the increased attention being given to severe dynamic loading situations e.g. tornado and earthquake loads on nuclear power plants, problems related to the energy absorption capabilities and problems of indentation or penetration of thin elements should be considered. In addition, the behavior of connections for thin-walled members subjected to impact load has not been studied. Although the results obtained in this investigation are not directly applicable to these problems, they may be used as a basis for future work in these areas.
7. NOMENCLATURE

The following notation is used in this report:

- $b$ = effective width;
- $E$ = modulus of elasticity;
- $h$ = thickness of element;
- $I$ = moment of inertia;
- $I_0$ = $I$ computed from original cross section properties;
- $L$ = length of member;
- $M$ = moment;
- $M_{LB}$ = local buckling moment;
- $M_L$ = internal moment obtained from linear analysis;
- $M_{NL}$ = internal moment obtained from nonlinear analysis;
- $P$ = axial load;
- $P_{cr}$ = critical axial load;
- $P_{DYN}$ = maximum dynamically applied impact load;
- $P_{max}$ = maximum axial load;
- $P_u$ = static failure load of column;
- $r_x$, $r_y$ = radii of gyration about x and y axes, respectively;
- $S_{DYN}$ = maximum deflection of plate under dynamic load;
- $S_{static}$ = maximum deflection of plate under static load;
- $S_o$ = initial imperfection parameter;
- $S_{To}$ = section modulus for top flange of beam;
- $S_{Bo}$ = section modulus for bottom flange of beam;
- $t$ = time;
- $t_d$ = time duration of load pulse;
- $V_o$ = nondimensional initial imperfection parameter;
$W$ = flat width of element;  
$\alpha$ = maximum moment in beam divided by local buckling moment;  
$\alpha'$ = maximum dynamic load in plate divided by critical load;  
$\beta$ = time duration of load pulse divided by natural period of beam or column;  
$\beta'$ = time duration of load pulse divided by natural period of plate;  
$\varepsilon_{\text{DYN}}$ = maximum dynamic strain;  
$\varepsilon_{\text{static}}$ = maximum static strain;  
$\delta_L$ = maximum deflection obtained from linear analysis;  
$\delta_{\text{NL}}$ = maximum deflection obtained from nonlinear analysis;  
$\sigma_T$ = maximum top flange beam stress;  
$\sigma_y$ = yield stress;  
$\nu$ = Poisson's ratio;  
$\omega_{\text{plate}}$ = natural frequency of lowest mode of plate;  
$\omega_{\text{beam}}$ = natural frequency of lowest mode of beam.  
$\xi$ = ratio of maximum impact load to Euler load;  
$\xi_s$ = ratio of static design load to Euler load;  
$\lambda_{\text{Mom}}$ = impact factor for moment;
<table>
<thead>
<tr>
<th>No.</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 1</td>
<td>Compression Element Subjected to Time-Dependent Loading</td>
<td>27</td>
</tr>
<tr>
<td>Fig. 2</td>
<td>Triangular Pulse Impact Factors - Deflection</td>
<td>28</td>
</tr>
<tr>
<td>Fig. 3</td>
<td>Triangular Pulse Impact Factors - Edge Strain</td>
<td>29</td>
</tr>
<tr>
<td>Fig. 4</td>
<td>Dynamic Effective Width</td>
<td>30</td>
</tr>
<tr>
<td>Fig. 5</td>
<td>Influence of Local Buckling on Bending Rigidity</td>
<td>31</td>
</tr>
<tr>
<td>Fig. 6</td>
<td>Dimensions of Beam Specimens</td>
<td>32</td>
</tr>
<tr>
<td>Fig. 7</td>
<td>Dynamic Test Setup</td>
<td>33</td>
</tr>
<tr>
<td>Fig. 8</td>
<td>Typical Dynamic Beam Test Results</td>
<td>34</td>
</tr>
<tr>
<td>Fig. 9</td>
<td>Comparison of Static and Dynamic Test Results</td>
<td>35</td>
</tr>
<tr>
<td>Fig. 10</td>
<td>Dynamic Loading Conditions</td>
<td>36</td>
</tr>
<tr>
<td>Fig. 11</td>
<td>Cross Sections Investigated</td>
<td>37</td>
</tr>
<tr>
<td>Fig. 12</td>
<td>Typical Beam Stiffness Relationships</td>
<td>38</td>
</tr>
<tr>
<td>Fig. 13</td>
<td>Influence of Load Location on Response Spectra-Moment</td>
<td>39</td>
</tr>
<tr>
<td>Fig. 14</td>
<td>Influence of Load Location on Response Spectra-Deflection</td>
<td>40</td>
</tr>
<tr>
<td>Fig. 15</td>
<td>Dimensions of Column Specimens</td>
<td>41</td>
</tr>
<tr>
<td>Fig. 16</td>
<td>Typical Dynamic Test Data Pictures</td>
<td>42</td>
</tr>
<tr>
<td>Fig. 17</td>
<td>Response Spectra for Moments</td>
<td>43</td>
</tr>
<tr>
<td>Fig. 18</td>
<td>Effect of Slenderness Ratio on Failure and Yield Envelopes</td>
<td>44</td>
</tr>
</tbody>
</table>
CASE 1 - Longitudinal Edges Free to Move Laterally (Mean Edge Stress = 0)
CASE 2 - Longitudinal Edges Cannot Move Laterally (Mean Edge Strain = 0)

FIGURE 1 - Compression Element Subjected to Time-Dependent Loading
FIGURE 2 - Triangular Pulse Impact Factors - Deflection
FIGURE 3 - Triangular Pulse Impact Factors - Edge Strain
Fig 4 - Dynamic Effective Width

\[ \frac{\varepsilon_{\text{DYN}}}{\varepsilon_{\text{Static}}} \]

\[ \frac{(b/w)_{\text{DYN}}}{(b/w)_{\text{Static}}} \]

\[ \alpha' = 2.0 \]

\[ S_0 = 0.1 \]

(Initial Imperfection Parameter)
As built properties:

- $I_0$ = Moment of Inertia
- $S_{TO}$ = Section modulus - top flange
- $S_{BO}$ = Section modulus - bottom flange
- $M_{LB}$ = Local buckling moment = $\sigma_{cr} S_{TO}$

Fig 5 - Influence of Local Buckling on Bending Rigidity
All dimensions are given in inches.

Fig 6 - Dimensions of Beam Specimens
Fig 7 - Dynamic Test Setup
Fig. 8 - Typical Dynamic Beam Test Results
Fig 9 - Comparison of Static and Dynamic Test Results
(a) LOADING CONDITIONS

(b) LOAD-PULSE VARIATION

Fig. 10 - Dynamic Loading Conditions
If \( \sigma_T > \sigma_{CR} \) and \( b < w \), the neutral axis for the as-built cross section is located as shown.

**Note:** Local buckling only under compressive stress.

**SECTION A**

If \( \sigma_T > \sigma_{CR} \) and \( b < w \), the neutral axis for the as-built cross section is located as shown.

**SECTION B**

If \( \sigma_T > \sigma_{CR} \) and \( b < w \), the neutral axis for the as-built cross section is located as shown.

**SECTION C**

If \( \sigma_T > \sigma_{CR} \) and \( g < f \), the neutral axis for the as-built cross section is located as shown.

**Fig 11 - Cross Sections Investigated**
TYPICAL $I/I_0$ VS. $M/M_{LB}$ CURVES

Fig. 12 - Typical Beam Stiffness Relationships
Fig 14 - Influence of Load Location on Response Spectra-Deflection
**Fig. 15**

**DIMENSIONS OF COLUMN SPECIMENS**

![Diagram of column specimen dimensions](image)

<table>
<thead>
<tr>
<th>Section Designation*</th>
<th>Length in Inches</th>
<th>D in Inches</th>
<th>B in Inches</th>
<th>t_F in Inches</th>
<th>t_W in Inches</th>
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<tbody>
<tr>
<td>B - STUB</td>
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<td>1.501</td>
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<tr>
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<td>0.068</td>
<td>0.138</td>
</tr>
</tbody>
</table>

*S Denotes Static Test Column  
D Denotes Dynamic Test Column
Section B-80-D
Test #4

(a) Load, Deflections

(b) Strains

FIG. 16 Typical Dynamic Test Data Pictures
Elastic

$\xi_s = 0.51$

$V_0 = 0.15$

Fig. 17 - Response Spectra for Moments
Fig. 18 - Effect of Slenderness Ratio on Failure and Yield Envelopes
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