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Impact loading of thin-walled beams

Edward A. Zanoni
Charles G. Culver

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INTRODUCTION

The behavior of individual thin compression elements subjected to a time varying load applied in the middle plane was discussed in a previous paper (2). The purpose of this paper is to present an analytical method for determining the dynamic response of thin-walled flexural members subjected to short duration impact loading. Results obtained from this mathematical model are compared with experimental values obtained from beams subjected to impact loading.

BEHAVIOR OF THIN-WALLED BEAMS

For illustrative purposes in this paper, the thin-walled beam with a "hat" shaped cross section shown in Fig. 1 will be considered. Note, however, that the general mathematical method described herein is also applicable to any thin-walled beam with singly symmetric cross section loaded in the plane of symmetry.

The sign convention used for positive bending moment is shown at the top of Fig. 1. In the following, the term "as built" will be used when referring to the cross sectional properties (moment of inertia, section modulus, etc.) of this initial unstressed cross section. When the compressive bending stresses reach a certain value, the plate element comprising the top flange of the cross section buckles and the concept of "effective width" is used to evaluate the properties of the buckled cross section. Referring to Fig. 1, the cross sectional dimensions such as width, b, referred to as "flange widths" are used to establish the stress level at which this buckling occurs.

For the specific cross section in Fig. 1, it will be assumed that only a positive bending moment will cause buckling, i.e., the f/h ratio is so small that any compressive bending stress in the bottom flange due to a negative bending moment will not exceed the buckling stress σ_{cr}.

An empirical formulation of σ_{cr} is available:

\[ \sigma_{cr} = \frac{11.6566E}{(w/h)^2} \]  

Eq. 1 is usually expressed in the form of a limiting width-to-thickness ratio, \( (w/h)_{lim} \), below which the plate will not buckle. Above this stress level, an effective width, w, must be calculated according to the following formula:

\[ t = 1.9h \sqrt{\frac{E}{\sigma_{max}}} \left( 1 - 0.415(h/w) \right) \sqrt{\frac{E}{\sigma_{max}}} \]  

In Eqs. 1, 2, all dimensions are in inches and the stress and modulus of elasticity are in ksi. Substituting \( E = 29.5 \times 10^3 \) ksi, and \( \sigma_{max} = 1.67\sigma_c \), 1.67 being the factor of safety, in Eq. 2 gives the formula for the effective design width of stiffened compression elements in the current specification (9). Thus two distinct stress conditions are possible, depending on the magnitude of the moment as noted at the bottom of Fig. 1. Computation of the section properties using Eqs. 1, 2 requires some iteration since the actual stress must reflect the reduced cross sectional properties (12).

If the same type of cross section is used often enough, it may be desirable to develop nondimensional curves for the section properties as shown in Fig. 2. In plotting these curves, the as built cross sectional properties, \( I_o, S_{10}, S_{20} \), were used as reference values. The reduction in the cross sec-

1Senior Engr., Bettis Atomic Div., Westinghouse Electric Corp., Pittsburgh, Pa.,
3Numerals in parentheses refer to corresponding items in Appendix I - References.
tional properties as the stress level increases above \( \sigma_{cr} \), \( M > M_{cr} \), is apparent. Since the flat width ratio for the bottom flange elements was assumed to be small such that these elements do not buckle, the cross sectional properties of the beam subjected to negative bending are the same as the as built properties, \( M/M_{lb} < 0, I/I_{lb} = 1 \), etc. Note that despite that fact that the cross section buckles, the stress levels in the range where \( \sigma_{cr} > \sigma \) are within the elastic range of the material and yielding does not commence until the internal moments exceed the values indicated by the termination of the curves in Fig. 2. Since only a limited amount of rotation capacity is available after initial yield, the present specification for cold-formed beams does not take into account any redistribution of moments or plastic design. The analysis presented herein is also limited to the elastic range of the material, \( \sigma_{max} \leq \sigma_{cr} \).

As noted previously, the relationship for \( \sigma_{cr} \) presented in Ref. 9 and Eq. 1 is based on extensive static tests (11). However, a recent analytical study (2) indicated that the time duration of the stress level may be an important factor when considering the dynamic response of thin plates. Direct application of these results to beam response, however, would be difficult and was not considered in the development of the mathematical model herein. The concept of a dynamic effective width will be considered, however, in evaluating the test results.

**MATHEMATICAL MODEL**

Consider the simply supported beam in Fig. 3a subjected to a system of external forces \( P_1 \). The continuous system is idealized by a series of lumped masses connected by flexible elements. This idealization is advantageous since the resulting mathematical model consists of a system of simultaneous ordinary differential equations rather than a partial differential equation. Neglecting damping, the equations of motion of the system of mass points can be written in matrix form as:

\[
\begin{bmatrix}
\ddot{y}_1 \\
\vdots \\
\ddot{y}_n
\end{bmatrix} + \begin{bmatrix} A \end{bmatrix} \begin{bmatrix}
\ddot{P}_1 \\
\vdots \\
\ddot{P}_n
\end{bmatrix} = \begin{bmatrix} F_1 \\
\vdots \\
F_n
\end{bmatrix} 
\]

For a linear elastic system, the matrix \( \begin{bmatrix} A \end{bmatrix} \) referred to as the beam flexibility matrix, remain constant with time.

The basic difficulty with using Eq. 3 for thin-walled members subject to dynamic loading is that the flexibility matrix becomes time dependent and is continuously changing since the moment of inertia changes with the stress level. This difficulty was overcome by using an incremental form of solution. For each small time increment, it was assumed that the stiffness characteristics of the beam could be represented by stationary nonlinear relationships of the form shown in Fig. 2. For a particular time increment the beam flexibility matrix was assumed to remain constant and was evaluated using the distribution of moments from the preceding time step.

Since the moment of inertia varies along the beam according to the stress level, the beam is a nonprismatic structure. The beam flexibility matrix \( \begin{bmatrix} A \end{bmatrix} \) was therefore computed by subdividing the portions of the beam between the lumped masses in Fig. 3a into a series of flexible elements. The moment of inertia was assumed to vary linearly over the length of each element. Treating an individual element as a cantilever beam, assuming the moment of inertia at the free end \( x = 0 \), \( I(0) \), is larger than that at the fixed end, \( I(1) \), and using the principle of virtual work (1), the flexibility coefficients for deflection and rotation due to a shear force \( v_x \) and moment \( v_m \) applied at the free end become:

\[
\begin{bmatrix}
v_x \\
v_m
\end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\
\vdots & \vdots \\
a_{n1} & a_{n2}
\end{bmatrix} \begin{bmatrix} \ddot{x} \\
\ddot{\theta}
\end{bmatrix} 
\]

where:

\[
a_{11} = (1/E) \int_0^1 \left[ \frac{I}{I(0)} - m \right] dx 
\]

\[
a_{12} = a_{21} = (1/E) \int_0^1 \left[ \frac{I}{I(0)} - m \right] dx 
\]

\[
a_{22} = \left( \frac{1}{E} \right) I(0) \int_0^1 \left[ 1/(I(0) - m) \right] dx 
\]

Note that shearing deflections were neglected in establishing Eqs. 5. Similar expressions may be obtained for the case in which \( 2(I) > I(0) \) by redefining \( m \).

In general Eq. 4 may be written as:

\[
\begin{bmatrix} v_x \\
v_m
\end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} \ddot{x} \\
\ddot{\theta}
\end{bmatrix} 
\]

where the subscript \( e \) is used to denote element and the element flexibility matrix is denoted as \( \begin{bmatrix} A \end{bmatrix}_e \). The flexibility matrix for the entire beam is then computed in the following manner. By definition:

\[
\begin{bmatrix} v_x \\
v_m
\end{bmatrix} = \begin{bmatrix} A \end{bmatrix}_B (\begin{bmatrix} \ddot{x} \\
\ddot{\theta}
\end{bmatrix} + \begin{bmatrix} \ddot{x}_0 \\
\ddot{\theta}_0
\end{bmatrix}) 
\]

The matrix \( \begin{bmatrix} c \end{bmatrix} \) is determined from static equilibrium principles. The beam flexibility matrix is finally computed as:

\[
\begin{bmatrix} A \end{bmatrix}_B = \sum_e \begin{bmatrix} c \end{bmatrix}_e \begin{bmatrix} A \end{bmatrix}_e 
\]

where the beam is made up of \( n \) elements. The above procedure may be applied to the idealized lumped mass beam shown in Fig. 3a. As indicated in Fig. 3b, variable element lengths \( l_1, l_2, \ldots \), etc. may be used between the discrete mass points. This variation is permitted in the event that it is necessary to have more elements in one location of the model than another. For example, in regions where steep stress gradients are expected, more elements may be required than in regions where the stress is expected to be more uniform. If the moment of inertia at the ends of the elements, as well as the element length, are expressed in nondimensional form as \( \eta_1 \) and \( \eta_2 \), respectively, and the element flexibility coefficients, and the matrix \( \begin{bmatrix} c \end{bmatrix} \), evaluated accordingly. Eq. 9 will not yield \( \begin{bmatrix} A \end{bmatrix}_B \) but a nondimensional matrix \( \begin{bmatrix} A \end{bmatrix}_B \). However, the properties of this matrix are such that the following relationship is true:

\[
(1/L^2 \eta_2)^2 \begin{bmatrix} A \end{bmatrix}_B = \begin{bmatrix} A \end{bmatrix}_B 
\]

**Nondimensional Equations for Lumped Mass Model**

In order to illustrate the influence of the significant parameters governing the dynamic response, it is advantageous to nondimensionalize Eq. 3. For example, any load pulse acting at the mass points can be defined as a function of a characteristic magnitude. That is, the dimension of force may be factored out so that each load pulse is represented by a common load factor multiplied by the appropriate time varying coefficient. An example of a characteristic load pulse is shown in Fig. 3c. The characteristics of the pulse are the magnitude \( P_{ch} \) and the duration \( t_d \). In the following development the forcing functions acting on the lumped mass model will always be expressed in this form.

Also, since the lumped masses \( m_i \) represent some fraction of the total beam mass \( M \), the diagonal mass matrix has a common factor. Considering Eq. 10, the following relationships are obtained:

\[
\begin{bmatrix} \tilde{m}_i \end{bmatrix} = (M) \begin{bmatrix} m_i \end{bmatrix} 
\]

\[
\begin{bmatrix} A \end{bmatrix}_B = (2L^2 I_2 / \eta_2) \begin{bmatrix} A \end{bmatrix}_B 
\]
Since the nonlinearity of the beam was expressed in non-dimensional form in terms of $\frac{I/I_0}{N_{MB}}$. Fig. 2, the moment $M_e$ will be non-dimensionalized with respect to $N_{MB}$. Introducing the parameter $\alpha = \frac{N_e}{N_{MB}}$ where $N_e$ is some characteristic moment of the beam given by:

$$M_e = \frac{P_1}{E} \cdot \frac{1}{C_2}$$

Eq. 19 becomes:

$$\left(M_e/N_{MB}\right) = \frac{(2a) [C_{W}] \left( t_1 - (t_2/t_1) \right)}{C_2}$$

Note that the coefficient $\xi$ is introduced only as a matter of convenience so that the characteristic moment $M_e$ can be specified as a fraction of a maximum static moment. The calculation of $M_e/N_{MB}$ is not affected by the choice of this parameter.

The equations of motion, Eq. 16, were solved by a numerical integration technique developed by Newmark (5) using the assumption that the accelerations vary linearly within each time increment. Instead of an iterative type solution, appropriate recurrence relationships were employed (10). The number of lumped masses used to idealize the beam and the size of the time increment required to obtain the desired degree of accuracy were determined (5) from comparisons with existing solutions.

A seven mass model and a time increment equal to 10% of the lowest natural period of the system (13) were found to yield excellent results. Note that the time increment used remained constant throughout a particular dynamic response calculation. These equations were solved using a computer program written in FORTRAN IV for a UNIVAC 1108 digital computer. In addition to the digital output from this program, a plotting routine available with the computer was employed to obtain a graphical presentation of the time variation of the stresses and deflections for each problem. A listing of this program and the complete mathematical development of the equations presented herein is available elsewhere (13).

**EXPERIMENTAL PROGRAM**

In order to check the mathematical model, a series of static and dynamic beam tests were conducted. A total of eight beams cold-formed from sheet material (AISI H-1100) using the press-brake method were tested. The virgin sheet material had a sharp yielding stress strain behavior.

Two different values were selected for the flat width ratios of the compression flange of the specimens (Group A, Group B). These values were well above the limiting ratio, $\frac{1}{2} \left( \frac{h_b}{h_f} \right)$, in order to insure that the dynamic response of the beams would be in the postbuckling or nonlinear elastic range.

The nominal or ordered dimensions of the specimens are shown in Fig. 14. Average values of the cross sectional dimensions obtained from measurements at five locations along the span length of each beam were used for calculation purposes. The mechanical properties of the specimens were determined from tension and compression coupons cut from extra lengths of the fabricated beams as shown in Fig. 14.

**Static Tests**

Four of the eight specimens, two from each group, were tested statically to determine the relationship between the applied moment and the section properties, Fig. 2. The setup and instrumentation for these tests was similar to that employed in previous studies (3). Solid plate web stiffeners of the same thickness as the beams were used at the load points and end supports. In order to insure that the buckling pattern of the compression flange was not inhibited by any local restraint, the stiffeners were cut short of the compression flange and tack welded to the webs. Both positive and negative moment tests were conducted for each group of specimens.
AI Built Value

The data were determined for each value of the external load. Since there was a slight amount of twisting of the specimens, as indicated by the dial gages and strain gages, the calculated internal moment differed slightly (less than 5%) from the external moment computed from the testing machine load. This unavoidable twisting did not significantly affect the effective width determination, however. The moment indicated for the test points corresponds to the internal moment based on the calculations using the strain gage data.

For comparison purposes the calculated moment of inertia based on Eq. 2 and the average mechanical properties is also shown in Fig. 5. As shown, the section was slightly stiffer than indicated by Eq. 2. As the moment increased, however, the test values approached those determined from Eq. 2.

Since negative moment did not produce local buckling, no reduction in the moment of inertia occurred as indicated. The same general trend shown in Fig. 5 was observed for the specimens in group B.

The load deflection curve for the beams subjected to positive moment was nonlinear (3) due to local buckling effects. Attempts were made to determine whether this curve had a horizontal plateau or if any rotation capacity was available similar to that of the heavier hot rolled wide flange shapes. No rotation capacity was present and the load deflection curve began to decrease and the section unloaded after reaching a load approximately 6% higher than the load at which initial yielding of the beam was indicated by the strain gages.

Dynamic Tests

Two loading conditions were used for the dynamic tests, centerline loading and quarter point loading. For quarter point loading, a considerably longer length of the beam was subjected to moment greater than the local buckling moment than in the centerline loading case. A sketch of the dynamic test setup for centerline loading is shown in Fig. 6.

The dynamic loads for these tests were developed by dropping a cali-
The magnitude and time variation of the applied loads were obtained from photographic records of the oscilloscope traces produced by the strain gages attached to the loading rings. Dynamic strains in the beam were recorded in the same manner. All the strain gages used in the dynamic tests were SR-4 Type C6 dynamic gages. Dynamic deflections were measured by means of a linear variable differential transformer. The photocells shown in Fig. 6 were used to actuate the electric recording circuit as the weight struck the loading ring or spreader beam. Complete details of the test setup and test procedure are available elsewhere (13).

Typical dynamic test results are shown for centerline loading in Fig. 7 and quarterpoint loading in Fig. 9. The midplane stresses at the middle and edge of the compression flange, the midplane stress in the tension flange, the centerline deflection and the variation of the load as recorded by the loading rings are shown. These figures were obtained by enlarging and tracing photographs of the oscilloscope records. Stresses were obtained by multiplying the recorded strains by the average modulus of elasticity.

For the particular test shown in Fig. 7, the stress level was so low that very little if any local buckling occurred and the beam response was essentially linear elastic. For the test in Fig. 9, however, considerable local buckling occurred and the beam response was not linear as indicated. The variation of the top flange stress at the edge of the compression flange, $\sigma_{top}$, and the bottom flange stress, $\sigma_{bot}$, in Fig. 9 are smooth curves during the duration of the applied load and the response is similar to the variation of the external load pulses. The time variation for the stress at the center of the top flange, $\sigma_{top}$, however, is irregular and is not similar to the time variation of the external load pulse. The presence of the dip in this stress trace is an indication that the top flange has buckled. Also, the reduced magnitude of stress at this point compared to the magnitude of stress at the edge of the top flange is an indication that a reduction in effective width has taken place. Note that the top flange buckled while the edge stress increased. These oscillations are a result of the vibrations of the individual plate elements which comprise the cross section and are superimposed on the oscillations of the overall beam vibrating as a unit.

Calculated response curves for the tests in Figs. 7, 9 obtained using the computer program and plotting routine discussed earlier are shown in Figs. 8, 10. The load pulses obtained from the oscilloscope and the relationship between the moment and moment of inertia in the static tests were used as input to the program. Approximately fifty discrete values of the load were used at equal time intervals (0.50 milliseconds) over the duration of the positive phase of the load pulse. Since the maximum beam response, which is of primary interest, occurs in this time interval, only the positive portion of the load pulse was considered. Since the computer program replotted the input load pulse, it was possible to compare this with the oscilloscope trace in order to insure that a sufficient number of discrete points had been selected to adequately represent the experimental load pulse. Note that the added mass of the loading rings which vibrated with the beam was taken into account in these calculations.

Comparison of Figs. 7, 9 and 8, 10 indicate that the calculated maximum stresses and deflections are of the same order of magnitude as the experimental values. The calculated values were much larger than the experimental values after the load pulse was removed, however. This difference is due to the influence of structural damping and the negative portion of the load pulse which were neglected in the mathematical model. Note also that the oscillations of the stress at the middle of the compression flange in Fig. 9 are not present in Fig. 10 since the plate vibrations were not included in the mathematical model.

Additional comparisons between calculated and experimental values of the midplane maximum top flange stress, $\sigma_{top}$, bottom flange stress, $\sigma_{bot}$, and midplane deflection occurring during the positive phase of the load pulse are given in Table 1, 2. Calculated values based on the relationship between the moment and moment of inertia obtained in the static tests, the relationship using Eq. 2 and a linear elastic analysis neglecting local buckling, $I/I_0 =$ constant $= 1$, are given. Note that the calculated deflections obtained from the mathematical model account for the variation of the moment of inertia due to local buckling over the entire length of the beam by means of the discrete elements described previously.
Referring to the calculated stresses in Tables 1, 2, note that the resulting values differ for the three methods of analysis. The internal moment, not presented, obtained from the three methods, however, did not differ appreciably. Thus the method of computing the stress rather than the method used to calculate the internal moment is the more significant factor in these comparison studies. Also, the reduction in the effective width of the top flange has a greater influence on the top flange section modulus than on the bottom flange section modulus. This was indicated by the fact that in general the three methods of analysis predicted essentially the same bottom flange stress. The predicted top flange stresses, however, differed considerably.

The reduction in section modulus is influenced by the \( I/\beta \) curve. As indicated previously, the \( I/\beta \) curve based on test properties is stiffer than the \( I/\beta \) curve based on Eq. 2. Thus the analysis based on Eq. 2 generally predicted higher stresses than those predicted using the test properties. Since the internal moment predicted by all three analyses is approximately the same for a given case, the stresses in the top flange predicted by the linear elastic analysis were generally lower than those predicted using other test properties or Eq. 2. The differences between the predicted bottom flange stress and midspan deflection obtained from the three analyses were very small.

The tests in Tables 1, 2 are arranged in order of increasing stress level. The time durations of the load pulses varied between 0.0185 sec. and 0.045 sec. Note that since the stresses were within the elastic range of the material, several tests were conducted on each beam. The calculated local buckling stress based on the static tests was \( \sigma_{BL} = 6200 \) psi and \( \sigma_{BL} = 14000 \) psi.

### Table 1: Summary of Test Results - Midspan Loading

<table>
<thead>
<tr>
<th>Beam</th>
<th>Test Value</th>
<th>Linear Prop.</th>
<th>Analysis</th>
<th>Difference As ( % )</th>
<th>Linear Prop.</th>
<th>Analysis</th>
<th>Eq. 2</th>
<th>Test Prop.</th>
<th>Analysis</th>
<th>Eq. 2</th>
</tr>
</thead>
</table>

### Table 2: Summary of Test Results - Quarter Point Loading - Beam

<table>
<thead>
<tr>
<th>Beam</th>
<th>Test Prop.</th>
<th>Linear Prop.</th>
<th>Analysis</th>
<th>Difference As ( % )</th>
<th>Linear Prop.</th>
<th>Analysis</th>
<th>Eq. 2</th>
<th>Test Prop.</th>
<th>Analysis</th>
<th>Eq. 2</th>
</tr>
</thead>
</table>

### Conclusions

For beam A3, the percentage difference between the calculated values using the static test properties and the test values for both the top and bottom flange stresses was less than 10% up to a stress level of approximately 10,000 psi in the top flange. Above this stress level, the percent difference increased and was generally larger than that for beam A3. Also the percent difference was different for the top and bottom flange stresses. The values in Table 2 indicate that in some cases the test values agreed more closely with those based on a linear analysis even though the beam was obviously in the postbuckling range. The poor agreement between calculated and measured values for beam Bk is due to the form of the \( I/\beta \) curve in the flange.
the nonlinear range. Beam B4 was considerably stiffer than beam A3 and the slope
of the $1/T^2$ curve in the postbuckling range for B4 was much steeper than that for
A3 (Fig. 3). The stiffness of beam B4, therefore, was considerably more sensi-
tive than beam A3 to slight changes in the internal moment. At the higher stress
levels, agreement between calculated and measured values were similar for both
beams.

The percent difference between the calculated values using Eq. 2 and the
test values was larger than that obtained using the static test values due to
the differences between the moment versus moment of inertia relationship mentioned
previously (Fig. 5).

A presentation of all the static and dynamic test results obtained in
this investigation is given in Fig. 11. The form of this graph is the same as
that used by Winter (11) in establishing existing requirements for cal-
culating effective width. Reduction of the static test data for this graph was
the same as that used by Winter. For the dynamic tests, calculation of the
effective width $b$, and $c_y$ in Fig. 11 were based on the maximum midplane top flan-
ge stress which occurred during the positive phase of the load pulse. The dynamic
test points therefore represent the minimum effective width which occurred during
each test. The values of the edema above which the full width of the compression
flange is effective and also the values for which the maximum top flange stress
is less than the yield stress are also shown.

As noted previously, earlier studies (27) indicated that the effective
width in a dynamic test would be influenced by the vibration of the plate elements
comprising the cross section. In order to determine whether any trend existed,
the dynamic data in Fig. 11 was grouped according to the value of $2 \frac{S_y}{P}$
or the ratio of the time duration of the stress pulse (Fig. 9) to the fundamen-
tal period of the compression flange treated as a simply supported plate. The
dynamic test results in Fig. 11 do not indicate any such trend. The scatter of
the dynamic data is similar to that in the static tests. Also, with the excep-
tion of few of the dynamic tests for beam A3 at high stress levels, the rela-
tionship in Eq. 2 fits both the static and dynamic results to the same degree of
accuracy obtained in earlier studies (11). Based on these results, the use of
Eq. 2 for calculating the moment versus moment of inertia relationship to be
used for dynamic response calculations of thin-walled beams subjected to impact
loading appears justified.

In discussing the calculated values presented in Tables 1, 2, it was noted
that although the stresses and deflections obtained by the three methods differ
substantially when the beam is in the postbuckling range, the internal moments,
however, did not differ appreciably. This fact suggests that for design pur-
poses, it may be possible to forgo the complicated incremental type solution of
the equations of motion presented herein and simply use the as-built section
properties of the beam and well established linear elastic response techniques
such as normal mode superposition, etc. to determine the internal moments pro-
duced by the impact loads. Using these moments and the appropriate reduced sec-
tion properties based on Eq. 2, the internal stresses and deflections could then
be calculated in the same manner as is presently used for cold-formed beams sub-
jected to static loads. The adequacy of this technique for design purposes
as well as the influence of the various nondimensional parameters on the dynamic
response will be considered in a forthcoming paper based on completed studies (8).

**Summary and Conclusions**

The behavior of thin-walled, cold-formed beams subjected
to impact loading was studied both analytically and experimentally. The experi-
mental study consisted of static and dynamic tests. The results of this investi-
gation indicated that in order to determine the stresses and deflections of thin-
wall beams subjected to impact, it is necessary to take into account the post-
buckling behavior and include the concept of effective width. The use of exist-
ing expressions for this effective width which are based on static test results
appear adequate for calculating the dynamic response. Preliminary calculations
indicate that it may be possible to use linear elastic response techniques to
determine the internal forces in structures of this type.

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man, J. R. Scalzi and C. H. Clauer is gratefully acknowledged.
APPENDIX I - REFERENCES


12. Yu, W. W., "Design of Light Gauge Cold-Formed Steel Structures," Engineering Experiment Station, West Virginia University, 1965.


APPENDIX II - NOTATION

The following symbols are used in this paper:

- $t$ = time coefficient at load point;
- $t_0$ = nondimensional time coefficient at load point;
- $t_1$ = nondimensional load duration for beam response;
- $t_2$ = nondimensional load duration for plate response;
- $t_3$ = fundamental natural period of beam;
- $t_i$ = characteristic time at load point $i$;
- $w$ = flat width of top flange of beam;
- $x_1$ = nondimensional displacement of mass point $1$;
- $y_1$ = nondimensional acceleration of mass point $1$;
- $z_1$ = absolute displacement of mass point $1$;
- $a$ = nondimensional mass loading;
- $b$ = nondimensional load duration for plate response;
- $c$ = characteristic deflection;
- $d$ = characteristic moment;
- $e$ = characteristic parameter of mass point $1$;
- $f$ = characteristic parameter of mass point $i$;
- $g$ = characteristic parameter of beam;...

Matrix Notation

$[ ]$ = matrix;
$[ ]^T$ = transposed matrix;
$[ ]^{-1}$ = inverted matrix;
$[ ]^T$ = column vector;

Specially Defined Matrices

$[A]_b$ = beam flexibility matrix;
$[A]_e$ = element flexibility matrix;
$[C]_b$ = element connection matrix for shear and moment;
$[C]_e$ = nondimensional element connection matrix for moment;
$[F]_b$ = nondimensional beam flexibility matrix;
$[I]_b$ = identity matrix.