Aug 20th, 12:00 AM

Buckling Behavior and Post-Buckling Strength of Perforated Stiffened Compression Elements

W. W. Yu

C. S. Davis

Follow this and additional works at: http://scholarsmine.mst.edu/isccss

Part of the Structural Engineering Commons

Recommended Citation

http://scholarsmine.mst.edu/isccss/1iccfs/1iccfs-session2/5
INTRODUCTION

During recent years, cold-formed steel structural members have been widely used in building construction and other areas. The design of such members is based on the "Specification for the Design of Cold-Formed Steel Structural Members" issued by American Iron and Steel Institute (1).

In cold-formed steel structural members, holes are sometimes provided in webs and/or flanges of beams and columns for duct work, piping and other construction purposes as shown in Fig. 1. For steel storage racks, various types of holes are often used for the purpose of easy assembly. The presence of such holes may result in a reduction of strength of individual component elements and overall strength of the member depending on the shape and arrangement of holes, the geometric configuration of the cross section and mechanical properties of the material used.

The exact analysis and design of steel sections with perforated elements are complex particularly when the shape of holes and their arrangement are unusual. Even though limited design information is available for relatively thick steel sections (2, 3, 4, 5), these design criteria may not be applicable completely to perforated cold-formed steel sections due to the fact that local buckling is usually a major concern for thin-walled structural members. For this reason, no design provisions are presently included in the AISI Specification for perforated cold-formed steel members. The load-carrying capacity of such members with perforated elements must be determined on the basis of special tests.

In order to investigate the structural behavior of cold-formed steel beams and columns with perforated elements and to study the influence of holes on the load-carrying capacity of the perforated cold-formed steel members, a research project was initiated in 1970 at the University of Missouri-Rolla under the sponsorship of American Iron and Steel Institute. This study includes both analytical and experimental investigations. It is intended to develop necessary background information for the analysis and design of cold-formed steel structural members having perforated elements.

This paper deals only with the buckling load and post-buckling strength of stiffened compression elements having circular and square holes.

LITERATURE SURVEY

As the first step of this investigation, a literature survey was conducted to review the previous work relative to buckling and ultimate strength of thin compression plates with and without holes, and the structural behavior of cold-formed members without perforations in the component elements, mostly conducted at Cornell University. Results of the extensive studies of built-up columns with perforated plates conducted at Lehigh University and the National Bureau of Standards were also reviewed in detail. The Lehigh and NBS studies have concentrated on several subjects concerning (a) axial rigidity, (b) bending stiffness, (c) buckling and yield loads of concentrically loaded columns, (d) design of perforated plates for shear, (e) local buckling of plate elements and (f) stress concentrations due to perforations (7, 8, 9, 10).

ANALYTICAL INVESTIGATION

For perforated cold-formed steel structural members, the load-carrying capacity of the member is often governed by the buckling behavior and post-buckling strength of the component elements. These two subjects for perforated stiffened elements will be discussed separately as follows:

1. Local Buckling

The critical buckling loads for perforated plates have been studied by several investigators using the energy method and finite element method (11, 12, 13, 14, 15, 16). Since 1964, consideration has been given to two different approaches, i.e., uniform stress approach and uniform displacement approach.

For a simply-supported square plate having a central circular hole, the relationship between the buckling coefficient, $k_c$, and the $d/w$ ratio ($d$ being the diameter of holes and $w$ being the overall width of the plate) is shown in Fig. 2, in which both approaches indicate that when the diameter of a hole is small compared with the width of the plate (i.e., about 20 per cent of the total width of the plate or less), the buckling load of the plate is reduced.
only by a small amount due to the presence of holes. When the holes are considerably large in size, it may be assumed that most of the applied load is to be carried by two narrow unstiffened strips along the side boundaries. For this reason, different structural behavior should be expected for plates having different sizes of holes. Reference 15 has suggested a value of $d/w = 0.7$ as the limiting value, beyond which the assumption discussed above may be used.

For a simply-supported square plate having a central square hole, the corresponding relationship between the buckling coefficient, $k_5$, and the $h/w$ ratio ($h$ being the width of the square hole) is shown in Fig. 3. A comparison of Figs. 2 and 3 indicates that if the diameter of a circular hole is the same as the width of a square hole, the buckling load for the perforated plates having square holes is less than that for the perforated plates having circular holes. This is due to the difference in stress concentration and the shape of two different types of holes.

2. Post-Buckling Strength

It is well known that thin plates will not fail at the local buckling load, but will continue to carry additional load due to the post-buckling strength. The post-buckling strength can easily be determined by the well known “effective width” concept, which is to replace the non-uniform stress distribution which occurs after exceeding the stability limit of the plate by two rectangular blocks as shown in Fig. 4. In this figure, the solid line is the actual stress distribution over the entire width of the element, and the dashed line represents the equivalent uniform stress distribution.

For thin plates without holes, Equation 1 developed by Winter (18) has long been used by AISI as the basis for determination of the effective width for stiffened compression elements:

$$b = 1.97 \sqrt{\frac{t}{f_{\max}}} \left[ 1 - 0.475 \left( \frac{t}{h} \right)^2 \right]$$

(1)

where $b = \text{effective width of the stiffened compression element}$

$t = \text{thickness of steel}$

$E = \text{modulus of elasticity}$

$f_{\max} = \text{maximum edge stress}$

$w = \text{overall width of the element}$

In 1968, Equation 1 was slightly modified as shown in Equation 2: (19)

$$b = 1.97 \sqrt{\frac{E}{f_{\max}}} \left[ 1 - 0.415 \left( \frac{t}{h} \right)^2 \right]$$

(2)

For unstiffened compression elements, the effective width can be determined by using Equation 3 (18) even though this equation has not been directly used in the AISI Specification:

$$b = 0.8 \sqrt{\frac{t}{f_{\max}}} \left[ 1 - 0.202 \left( \frac{t}{h} \right)^2 \right]$$

(3)

With regard to perforated plates, previous discussion on local buckling indicated that the structural behavior of perforated plates is affected by the size and shape of holes. Consequently, different equations should be used for plates having circular and square holes in different sizes.

Considering the post-buckling behavior of perforated plates, Equations 4 to 8 may be used to determine the effective width of perforated stiffened compression elements having circular holes on the basis of $d/w$ ratios:

$$(A) \quad \frac{d}{w} \leq 0.11$$

$$1. \quad w/t \leq (w/t)_{\text{lim}}$$

$$b = w$$

(4)
2. \( \frac{w}{t} > \frac{(w/t)_{\text{lim}}}{0.788(1 - 0.226 \frac{d}{t})^2} \)

\[
b = 1.6t \sqrt{\frac{E}{f_{\text{max}}}} \left[ 1 - 0.202(\frac{w - d}{w})^2 \sqrt{\frac{E}{f_{\text{max}}}} \right]
\]

(6)

If \( \frac{w - d}{w} \leq 0.7 \)

(7)

2. \( \frac{w - d}{w} \geq 0.7 \)

Use Equation 6 provided that \( \frac{w - d}{w} > 63.3 \frac{d}{f_{\text{max}}} \).

\[
b = w - d
\]

(8)

\( \frac{d}{w} \leq 0.7 \)

In Item (A), the limiting width-thickness ratio, \( (w/t)_{\text{lim}} \), can be computed as follows:

\[
(w/t)_{\text{lim}} = \frac{b}{2} \left( 1 - \frac{4}{\pi} \right)
\]

(9)

FIGURE 3. BUCKLING COEFFICIENT VS h/w RATIO

FIGURE 4. EFFECTIVE WIDTH OF STIFFENED COMPRESSION ELEMENTS

\[
a_c = 1.9(1 - 0.226 \frac{d}{t})
\]

\[
a_c = 0.788(1 - 0.226 \frac{d}{t})(1 - 0.037 \frac{d}{t})/(1 - \frac{d}{t})
\]

Based on Equations 4 to 8, the influence of size of circular holes on the effective width of stiffened compression elements can be shown in Fig. 5.

For perforated plates having square holes, a study of local buckling indicated that a value of 0.5 may be used for the specific \( \frac{b}{w} \) ratio, beyond which the applied load can be assumed to be carried by two narrow unstiffened strips. Consequently, Equations 9 to 12 may be used for the determination of the effective width of perforated stiffened compression elements having square holes:

\( \frac{b}{w} \leq 0.11 \)

1. \( w/t \leq (w/t)_{\text{lim}} \)

\[
b = w
\]

(9)
2. \( \frac{w}{t} > \left( \frac{w}{t} \right)_{\text{lim}} \)

\[
b = 1.9t \sqrt{\frac{t}{f_{\text{max}}}} \left( 1 - 0.316 \left( 1 - \frac{1}{2} \sqrt{1 - 0.053 \frac{d}{w}} \right) \right)
\]

\[
\sqrt{\left( 1 - 0.053 \frac{d}{w} \right)}
\]

(10)

(b) \( \frac{b}{w} > 0.5 \)

Use Equation 10 provided that \( \frac{w - n}{2t} > \frac{63.3}{f_{\text{max}}} \)

\[
b = w - h
\]

(11)

(c) \( \frac{b}{d} > 0.5 \)

1. \( \frac{w - h}{2t} > \frac{63.3}{f_{\text{max}}} \)

\[
b = 1.6t \sqrt{\frac{t}{f_{\text{max}}}} \left( 1 - 0.202 \left( \frac{d}{w} - 1 \right) \right)
\]

(12)

2. \( \frac{w - h}{2t} = \frac{63.3}{f_{\text{max}}} \)

Use Equation 11

where \( h \) is the width of square holes.

\[
\left( \frac{w}{t} \right)_{\text{lim}} = \sqrt{\left( \frac{a}{h} \right) \left( \frac{b}{d} \right) \frac{1}{2}}
\]

(13)

\[
a = 1.9(1 - 0.316 \frac{d}{w})
\]

\[
b = 0.788(1 - 0.316 \frac{d}{w})(1 - 0.053 \frac{d}{w})(1 - \frac{d}{w})
\]

Other symbols were previously defined.

It should be noted that Equations 5 and 10 are the modified formulas for stiffened compression elements, while Equations 7 and 17 are based on the formula for the effective width of two narrow unstiffened elements along side boundaries.

When "d" or "h" is equal to zero (i.e., for solid plates), Equations 5 and 10 will be identical to Equation 2.

**EXPERIMENTAL INVESTIGATION**

In order to verify the effect of holes on the buckling load and post-buckling strength of the perforated stiffened elements as discussed in the analytical investigation, twenty-eight short column tests and eight beam tests have been conducted to cover the following parameters:

a) Shape of holes: circular and square holes
b) Overall width-to-thickness ratio: 36.6 to 73.8
c) Hole opening to overall width ratio (d/w or h/w): 0 to 0.722
d) Yield point of steel: 34.4 to 59.3 ksi

**1. Column Tests**

The short column specimens (approximately 20 in. long) were tested under flat end condition. Each test specimen composed of two C-shaped channels (6-1/2 x 2-1/2 in., nominal size), which were connected by 10-1/4" bolts through the simple lips. Central perforations, either circular or square, were cut in the 6-1/2 in. (nominal) stiffened elements. 1/4" foil strain gages were placed on the specimens as shown in Fig. 6. All specimens were tested in the 200,000 pound Tinius Olsen universal testing machine. Fig. 7 shows the set up of testing. During the test, strain gage readings were recorded and printed out on tape by using a 60-channel Data Acquisition System. Dial gages were used for measurements of lateral deflection and axial deformation of specimens.

Based on the results obtained from the short column tests, the critical buckling strain was determined by using the modified strain reversal method (20, 21). This was done by the application of a pair of strain gages placed on both sides of the plate along both edges of perforations. The critical strain was taken as the maximum compressive strain on the convex side of the buckled plate. The
buckling coefficient ratios were computed from the critical loads. It can be seen from Fig. 8 that the buckling coefficient ratios for specimens with circular holes are compared favorably with the uniform stress approach. For specimens with square holes, the buckling coefficient ratios are close to the theoretical values determined by Yang's method as shown in Fig. 9.

With regard to the post-buckling strength of the perforated stiffened compression elements, the effective width was computed on the basis of the tested ultimate load. In the calculation of effective widths, it was assumed that the stresses in lips, corners and the 2-1/2 in. stiffened elements reached to the yield point of steel because their width-thickness ratios are less than the governing limits for local buckling and are therefore fully effective. In Fig. 10, the values of the effective width determined from the column tests are compared with the results obtained from the tests made by Winter and Sechler (18).

2. Beam Tests

The beam specimens used in the test program were track sections. Circular holes ranging from 1 to 4 inches in diameter were cut in the stiffened compression flange. Dimensions of beam test specimens are shown in Fig. 11.

All beam specimens were tested in the 10,000 pound Tinius Olsen beam testing machine. Fig. 12 shows the set up of beam tests. Strain gage readings were also recorded and printed out by using the Data Acquisition System.

The buckling loads and buckling coefficient ratios were determined from the results of tests by using the same method applied to the column tests. These values are compared with the analytical results as shown in Fig. 8.
With regard to the verification of the tested moments, Equations 5 and 7 were used to compute the effective widths of the compression flanges. The average difference between the computed yield moments and the tested values was found to be 5.5 per cent.

SUMMARY

Analytical and experimental investigations have been conducted to study the structural behavior of cold-formed compression and flexural members having perforated stiffened elements. The following conclusions may be drawn from the preliminary results:

1. The presence of holes may reduce the buckling load of the stiffened compression elements.
2. The reduction of buckling load of the stiffened compression elements is more pronounced for square holes than for circular holes due to the difference in stress concentration and the shape of holes.
3. Test data indicated that for stiffened compression
elements with circular holes, the uniform stress approach may be used to predict the buckling load.

4. Winter’s effective width equation for solid plates can be modified for determination of the effective width of perforated stiffened elements.

5. Even though the buckling load for the perforated stiffened elements is affected more by the square holes than circular holes, the post-buckling strength of the elements with square and circular holes are found to be nearly the same if the diameter of a circular hole is the same as the width of a square hole.

ACKNOWLEDGMENTS

This investigation is sponsored by American Iron and Steel Institute through a special Engineering cooperation. Messrs. D. F. Thomure and H. Hollingsworth, Engineering, and Dr. A. L. Johnson, Senior Research Engineer, of American Iron and Steel Institute for their advice and cooperation.

Appreciation should also be expressed to Dr. J. H. Sensen, Chairman of the Department of Civil Engineering of the University of Missouri-Rolla for his guidance, and to Messrs. D. F. Thomure and H. Hollingsworth, Staff of the Department of Civil Engineering, and Mr. P. Honghiman, graduate student of the University of Missouri-Rolla, for their help in conducting the tests.

Some test specimens used in the experimental study were donated by Butler Manufacturing Company in Kansas City, Missouri.

APPENDIX I - REFERENCES


APPENDIX II - NOTATIONS

B = flange width of C-shaped channels
b = effective design width
D = total depth of C-shaped channels
d = diameter of circular holes
E = modulus of elasticity
f_max = maximum edge stress in stiffened compression elements
h = width of square holes
k_c = buckling coefficient for perforated plate having circular holes
k_b = buckling coefficient for perforated plate having square holes
L = length of column specimens
L_D = overall depth of simple lip
P = applied load
t = thickness of material
w = flat width

\( a_c, b_c \) = terms for determining the \((w/t)_{lim}\) ratio for perforated stiffened elements having circular holes

\( a_b, b_b \) = terms for determining the \((w/t)_{lim}\) ratio for perforated stiffened elements having square holes